Nuclear symmetry energy from QCD sum rules

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We calculate the nucleon self-energies in isospin-asymmetric nuclear matter using QCD sum rules. Taking the difference of these for the neutron and proton enables us to express the potential part of the nuclear symmetry energy in terms of local operators. We find that the scalar (vector) self-energy part gives a negative (positive) contribution to the nuclear symmetry energy which is consistent with the results from relativistic mean-field theories. Moreover, we find that an important contribution to the negative contribution of the scalar self-energy comes from the twist-4 matrix elements, whose leading density dependence can be extracted from deep inelastic scattering experiments. This suggests that the twist-4 contribution partly mimics the exchange of the δ meson and that it constitutes an essential part in the origin of the nuclear symmetry energy from QCD. Our result also extends an early success of QCD sum-rule method in understanding the symmetric nuclear matter in terms of QCD variables to the asymmetric nuclear matter case.

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I. INTRODUCTION

There is a renewed interest in the study of nuclear symmetry energy recently, as the next-generation low-energy rare isotope accelerators are being constructed and planned worldwide [1]. Understanding the details of the nuclear symmetry energy is intricately related to understanding a wide range of subjects ranging from rare isotopes to neutron-rich nuclear matter such as the neutron star [2,3]. One of the main puzzles to be solved currently is the behavior of the nuclear symmetry energy at high density [4,5].

From a phenomenological point of view, the nuclear symmetry energy can be obtained by looking at the nuclear binding energy within the semiempirical mass formula in the limit of a large nucleon number [6]. There, the symmetry energy can be understood as originating from the energy difference between the proton and the neutron in isospin-asymmetric nuclear matter. Hence, in this picture, the nuclear symmetry energy can be obtained from the nucleon optical potential or by calculating the energy of the nucleon quasiparticle near the Fermi surfaces in asymmetric nuclear matter.

In the Dirac phenomenology of nucleon-nucleus scattering [7,8], the optical potential of the nucleon is composed of a vector and scalar part, $U \simeq S + V\gamma^0$. It is well known that in order to fit the spin observable, one needs a strong scalar attraction (Re $S < 0$) and a strong vector repulsion (Re $V > 0$), both of several hundred MeV but such that the combined sum to the energy is only a few tens of MeV, a result consistent with traditional low-energy nuclear physics. The strong scalar and vector potentials appear naturally in the relativistic mean-field theories (RMFT), where meson exchange interactions between nucleons on the Fermi sea produces the strong scalar and vector potentials for the nucleons.

But it was only after the works in QCD sum rules that the strong optical potentials were found to have a basis in QCD. The application of QCD sum rules [9,10] to the nucleon in the vacuum was developed in Refs. [11,12]. The first pioneering work of applying the QCD sum-rule method to nucleons in medium was performed by Drukarev and Levin [13,14]. Here, the operator product expansion (OPE) was performed in the light cone direction where $-q^2, qu \to \infty$ with their ratio finite, where q^μ, u^μ are the external momenta and the medium four-vector, respectively. Later, the relation became clearer through the work by Cohen, Furnstahl, and Griegel [15], who showed that the strong scalar-vector self-energy appearing in the quasinucleon pole in the symmetric nuclear matter can be traced back to the scalar-vector quark condensate in the nuclear medium. The OPE in this work was based on the short distance expansion, where $-q^2 \to \infty$, while qu is held fixed. For the medium at rest, this expansion is equivalent to taking the energy to be large and imaginary at a fixed finite three momentum [16–19] and, hence, the comparison to the self-energy obtained in the RMFT approaches becomes more direct through the use of the energy dispersion relation.[1]

Motivated by these results, and to express and elucidate the origin of nuclear symmetry energy directly from QCD, we have applied the QCD sum rule to calculate the neutron and proton energy in asymmetric nuclear matter. Identifying the difference with appropriate factors to the nuclear symmetry energy, we show that this energy can be expressed in terms of quark and gluon degrees of freedom. Results based on the first formalism to calculate the nucleon mass in asymmetric matter using QCD sum rules were reported before [20–22]. But here, we will follow the second formalism adopted in Ref. [15]. We have performed the OPE up to dimension-six operators and have identified all the independent twist-4 operators. Independent twist-4 operators and their relation to moments of structure functions appearing in deep inelastic scattering (DIS) were identified before [23–29]. In a later work by one of us [30,31], the available experimental data on twist-4 effects were collected to constrain the independent matrix elements. Using this information, we have calculated the leading density dependence on the nucleon sum rules coming

[1]One should caution, however, that the strong medium dependence of the scalar-scalar four-quark condensate obtained from a naive factorized form leads to a result that does not agree well with the nuclear phenomenology [18].

from the twist-4 effects. From the QCD sum-rule analysis, we find that the scalar (vector) self-energy part gives a negative (positive) contribution to the nuclear symmetry energy, which is consistent with the results from relativistic mean-field theories. Moreover, we find that an important contribution to the negative contribution of the scalar self-energy comes from the twist-4 matrix elements, whose higher-density behavior will determine the still-controversial property of the symmetry energy at these densities. Our result also extends an early success of the QCD sum-rule method in understanding the symmetric nuclear matter in terms of QCD operators to the asymmetric nuclear matter case.

The paper is organized as follows. In Sec. II, we start with a brief review and a simple idea for the nuclear symmetry energy. In Sec. III, we develop the QCD sum-rule formalism and discuss the OPE and its matrix elements. The results for the QCD sum-rule analysis for the nucleons in asymmetric nuclear matter and the nuclear symmetry energy are presented in Sec. IV. Finally, the conclusion is given in Sec. V.

II. A SIMPLIFIED DESCRIPTION FOR THE NUCLEAR SYMMETRY ENERGY

We start from a finite nuclei with A nucleons. The Bethe-Weizsäker formula for the nuclear binding energy is given as

$$\begin{aligned} m_{\rm tot} &= Nm_n + Zm_p - E_B/c^2, \\ E_B &= a_V A - a_S A^{\frac{2}{3}} - a_C[Z(Z-1)]A^{-\frac{1}{3}} \\ &\quad - a_A I^2 A + \delta(A,Z), \end{aligned} \tag{1}$$

where $I = (N - Z)/A$. The fourth term accounts for the total shifted energy due to the neutron number excess. Taking the infinite nuclear matter limit of this formula, one notes that a_A reduces to the nuclear symmetry energy [6].

To derive the formula for a_A that can be generalized to the infinite nuclear matter, we start from a simple formula for the total energy,

$$\begin{aligned} E_{\rm tot} &= N\overline{E}_n + Z\overline{E}_p = \tfrac{1}{2}A(1+I)\overline{E}_n + \tfrac{1}{2}A(1-I)\overline{E}_p \\ &= \tfrac{1}{2}A(\overline{E}_n + \overline{E}_p) + \tfrac{1}{2}AI(\overline{E}_n - \overline{E}_p), \end{aligned} \tag{2}$$

where $\overline{E}_n$ ($\overline{E}_p$) is the average neutron (proton) quasiparticle energy in asymmetric nuclear matter. Now the core of the model is what approximation goes into calculating the average energy.

The symmetry energy in asymmetric nuclear matter is defined as

$$E_{\rm tot}(\rho, I) = E_0(\rho)A + E_{\rm sym}(\rho)I^2 A + O(I^4), \tag{3}$$

where ρ is the nuclear medium density and $I = (N - Z)/A \rightarrow (\rho_n - \rho_p)/(\rho_n + \rho_p)$ and the neutron and proton densities are $\rho_n = \frac{1}{2}\rho(1+I)$, $\rho_p = \frac{1}{2}\rho(1-I)$, respectively. Therefore, in Eq. (2), the symmetry energy will have contributions from the term proportional to I in $(\overline{E}_n - \overline{E}_p)$ and the term proportional to I^2 in $(\overline{E}_n + \overline{E}_p)$.

For a noninteracting Fermi gas of nucleons, each with mass m_N, calculating the average nucleon energy will give $\overline{E} = \frac{3}{5}E_F$, where E_F is the nucleon Fermi energy. Following the procedure described above and extracting the term proportional to I^2 gives a nuclear symmetry energy of $\frac{1}{3}E_F$.

Going back to finite nuclei, assuming a "Fermi well" with constant energy difference Δ between adjacent nucleon energy levels, the symmetry energy can be obtained from the second term of Eq. (2). That is, using $(\overline{E}_n - \overline{E}_p) = \frac{1}{4}IA\Delta$, we have

$$a_A = \frac{1}{8}A\Delta = \frac{1}{4I}[E_n(A,I) - E_p(A,I)]. \tag{4}$$

For the infinite nuclear matter case, we can calculate $\overline{E}_N$ from

$$\overline{E}_N = \frac{1}{\int d^3k_n d^3k_p}\int d^3k_n d^3k_p E_N(\rho_n, \rho_p), \tag{5}$$

and obtain the nuclear symmetry energy $E^{\rm sym}(\rho)$, as it appears in Eq. (3), by collecting coefficients of I^2 in Eq. (2). $E^{\rm sym}(\rho)$ can in general be decomposed into the kineticlike part and the potential-like part in the mean-field-type quasiparticle approximation. The kinetic part of $E^{\rm sym}$ can be obtained from the formula given in Ref. [32],

$$E^{\rm sym}_K = \frac{1}{6}\frac{k_F^2}{\sqrt{k_F^2 + E^2_{q,V(I=0)}}}, \tag{6}$$

where k_F is the Fermi momentum and $E_{q,V(I=0)}$ is the potential part of the quasinucleon self-energy in asymmetric nuclear matter.

A. Linear density approximation

In the present QCD sum-rule calculations, we will be using the linear density approximation, because the in-medium condensates in the QCD sum rule can be most reliably estimated to leading order in density. This means that for either the proton or the neutron, the mass will be given as follows:

$$\begin{aligned} E^n_V(\rho_n, \rho_p) &= m_0 + a\rho_p + b\rho_n \\ &= m_0 + \tfrac{1}{2}\rho(a+b) + \tfrac{1}{2}\rho I(b-a), \\ E^p_V(\rho_n, \rho_p) &= m_0 + \tfrac{1}{2}\rho(a+b) - \tfrac{1}{2}\rho I(b-a), \end{aligned} \tag{7}$$

where m_0 is the vacuum mass and a, b are the constants to be determined later. We then have

$$\begin{aligned} \overline{E}^N_V &= \frac{1}{\int d^3k_n d^3k_p}\int d^3k_n d^3k_p E^N_V(\rho_n, \rho_p) \\ &= m_0 + \frac{1}{2}a\rho_p + \frac{1}{2}b\rho_n \\ &= m_0 + \frac{1}{4}\rho(a+b) + \frac{1}{4}\rho I(b-a). \end{aligned} \tag{8}$$

Combining Eq. (8) with Eq. (2), we obtain the symmetry energy. That is, $(\overline{E}^n_V - \overline{E}^p_V) = \frac{1}{2}[E^n_V(\rho_n, \rho_p) - E^p_V(\rho_n, \rho_p)]$, hence,

$$E^{\rm sym}_V = \frac{1}{4I}\big[E^n_V(\rho_n, \rho_p) - E^p_V(\rho_n, \rho_p)\big], \tag{9}$$

which is similar to the relation given in Eq. (4). Therefore, to this order, the symmetry energy comes only from the energy difference in the proton and neutron at the Fermi surface in asymmetric nuclear matter as given in Eq. (9). However, when operators have higher density dependence, the factors appearing in Eq. (8) should be modified, and the symmetry energy will have contributions from both the sum and the difference of the nucleon energies.

The quantity of interest, namely $[E_V^n(\rho_n,\rho_p) - E_V^p(\rho_n,\rho_p)]$, can be obtained by looking at the pole of the nucleon propagator in nuclear medium,

$$G(q) = -i\int d^4x e^{iqx}\langle\Psi_0|T[\psi(x)\bar{\psi}(0)]|\Psi_0\rangle, \qquad (10)$$

where $|\Psi_0\rangle$ is the nuclear medium ground state and $\psi(x)$ is a nucleon field. A relativistic mean-field type of contribution will then appear in the self-energies. The nucleon propagator can be decomposed as

$$G(q) = G_s(q^2,qu) + G_q(q^2,qu)\not{q} + G_u(q^2,qu)\not{u}, \qquad (11)$$

where u^μ is the four-velocity of the nuclear medium ground state [16].

The nucleon self-energy can be decomposed similarly as [15–18]

$$\Sigma(q) = \tilde{\Sigma}_s(q^2,qu) + \tilde{\Sigma}_v^\mu(q)\gamma_\mu, \qquad (12)$$

where

$$\tilde{\Sigma}_v^\mu(q) = \Sigma_u(q^2,qu)u^\mu + \Sigma_q(q^2,qu)q^\mu. \qquad (13)$$

In the mean-field approximation Σ_s and Σ_v are real and momentum independent and Σ_q is negligible. Hence,

$$\Sigma_v \equiv \frac{\Sigma_u}{1-\Sigma_q} \sim \Sigma_u, \quad M_N^* \equiv \frac{M_N + \tilde{\Sigma}_s}{1-\Sigma_q} \sim M_N + \tilde{\Sigma}_s. \quad (14)$$

The phenomenological representation of the nucleon propagator can then be written as

$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \to \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}, \qquad (15)$$

where λ is unity in this discussion. But if one includes the effect of Σ_q, $\lambda^2 = (1-\Sigma_q)^{-1}$. E_q and $\bar{E}_q$ are the positive and negative energy poles, respectively,

$$E_q = \Sigma_v + \sqrt{\vec{q}^2 + M_N^{*2}}, \qquad (16)$$

$$\bar{E}_q = \Sigma_v - \sqrt{\vec{q}^2 + M_N^{*2}}. \qquad (17)$$

With fixed $|\vec{q}|$, $G(q)$ depends only on q_0. One can extract self-energy near $\sim E_q$ with analytic properties of the nucleon propagator.

III. QCD SUM RULE AND MATRIX ELEMENTS IN THE ASYMMETRIC NUCLEAR MEDIUM

A. Operator product expansion and Borel sum rule

To express the self-energies in terms of QCD variables, we start with analyzing the correlation function via the OPE. The correlator is defined as

$$\Pi(q) \equiv i\int d^4x e^{iqx}\langle\Psi_0|T[\eta(x)\bar{\eta}(0)]|\Psi_0\rangle, \qquad (18)$$

where $\eta(x)$ is an interpolating current of the nucleon and $|\Psi_0\rangle$ is the ground state of the asymmetric nuclear medium characterized by the rest frame medium density ρ, the medium four-velocity u^μ, and the asymmetry factor I. $|\Psi_0\rangle$ is assumed to be invariant under parity and time reversal. We will be using the Ioffe nucleon interpolating current given as in Refs. [11,15],

$$\eta(x) = \epsilon_{abc}\left[u_a^T(x)C\gamma_\mu u_b(x)\right]\gamma_5\gamma^\mu d_c(x). \qquad (19)$$

As in the case of the nucleon propagator, using Lorentz covariance, parity, and time reversal, one can decompose the correlator into three invariants [16],

$$\Pi(q) \equiv \Pi_s(q^2,qu) + \Pi_q(q^2,qu)\not{q} + \Pi_u(q^2,qu)\not{u}. \qquad (20)$$

The three invariants are functions of q^2 and qu, while the vacuum invariants depends only on q^2. For convenience, we set the nuclear medium at rest, which means $u^\mu \to (1,\vec{0})$, and keep $|\vec{q}|$ fixed. $\Pi_i(q^2,qu)$ then becomes a function of q_0 only, which means $\Pi_i(q^2,qu) \to \Pi_i(q_0,|\vec{q}| \to \text{fixed})$ $(i = \{s,q,u\})$.

As mentioned before, we will follow the formalism adopted in Ref. [15] and write the energy dispersion relation for the invariant functions at fixed three-momentum $|\vec{q}|$:

$$\Pi_i(q_0,|\vec{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega,|\vec{q}|)}{\omega - q_0} + \text{polynomials}, \qquad (21)$$

$$\begin{aligned}\Delta\Pi_i(\omega,|\vec{q}|) &\equiv \lim_{\epsilon\to 0^+}[\Pi_i(\omega+i\epsilon,|\vec{q}|) - \Pi_i(\omega-i\epsilon,|\vec{q}|)] \\ &= 2\text{Im}[\Pi_i(\omega,|\vec{q}|)].\end{aligned} \qquad (22)$$

The lowest-energy contribution to the discontinuity will be saturated by a quasinucleon and quasihole contribution in the positive and negative energy domains, respectively. Their contribution to the spectral density will be given as in Eq. (15), which will have the following contribution to the invariant functions:

$$\Pi_s(q_0,|\vec{q}|) = -\lambda_N^{*2}\frac{M_N^*}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots, \qquad (23)$$

$$\Pi_q(q_0,|\vec{q}|) = -\lambda_N^{*2}\frac{1}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots, \qquad (24)$$

$$\Pi_u(q_0,|\vec{q}|) = +\lambda_N^{*2}\frac{\Sigma_v}{(q_0-E_q)(q_0-\bar{E}_q)} + \cdots, \qquad (25)$$

where λ_N^{*2} is the residue at the quasinucleon pole, which accounts for the coupling of the interpolating current to the quasinucleon excitation state, and the omitted parts are the contributions from the higher excitation states, which will be accounted for through the continuum contribution after the Borel transformation.

The even and odd parts of the invariant functions are respectively related to the following parts of the discontinuity:

$$\begin{aligned}\Pi_i^E(q_0^2,|\vec{q}|) &= \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\omega\Delta\Pi_i(\omega,|\vec{q}|)}{\omega^2 - q_0^2} + \text{polynomials}, \\ \Pi_i^O(q_0^2,|\vec{q}|) &= \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega,|\vec{q}|)}{\omega^2 - q_0^2} + \text{polynomials},\end{aligned} \qquad (26)$$

where we have defined the invariants with different superscripts from the following decomposition according the the parity in q_0:

$$\Pi_i(q_0, |\vec{q}|) = \Pi_i^E\big(q_0^2, |\vec{q}|\big) + q_0\Pi_i^O\big(q_0^2, |\vec{q}|\big). \tag{27}$$

The OPE of the three invariants of both the even and odd parts can be expressed as

$$\Pi_i\big(q^2, q_0^2\big) = \sum_n C_n^i\big(q^2, q_0^2\big)\langle\hat{O}_n\rangle_{\rho,I}, \tag{28}$$

where $\langle\hat{O}_n\rangle_{\rho,I}$ is the ground-state expectation value of the physical operator in the asymmetric nuclear medium, $\langle\Psi_0|\hat{O}_n|\Psi_0\rangle_{\rho,I}$. We will be adopting the OPE at $q^2 \to -\infty$ at finite $|\vec{q}| \to$ fixed; this is equivalent to the limit of $q_0^2 \to -\infty$ at finite $|\vec{q}| \to$ fixed. The Wilson coefficients $C_n^i(q^2, q_0)$ thus can be calculated in QCD at short time [15].

The OPE of the invariants for the proton interpolating current are given as follows up to dimension-five operators:

$$\Pi_s^E\big(q_0^2, |\vec{q}|\big) = \frac{1}{4\pi^2}q^2\ln(-q^2)\langle\bar{d}d\rangle_{\rho,I} + \frac{4}{3\pi^2}\frac{q_0^2}{q^2}\langle\bar{d}\{iD_0iD_0\}d\rangle_{\rho,I}, \tag{29}$$

$$\Pi_s^O\big(q_0^2, |\vec{q}|\big) = -\frac{1}{2\pi^2}\ln(-q^2)\langle\bar{d}iD_0d\rangle_{\rho,I}, \tag{30}$$

$$\begin{aligned}\Pi_q^E\big(q_0^2, |\vec{q}|\big) = {}& -\frac{1}{64\pi^4}(q^2)^2\ln(-q^2) + \left[\frac{1}{9\pi^2}\ln(-q^2) - \frac{4}{9\pi^2}\frac{q_0^2}{q^2}\right]\langle\bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I} + \left[\frac{4}{9\pi^2}\ln(-q^2) - \frac{4}{9\pi^2}\frac{q_0^2}{q^2}\right]\langle\bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I}\\ & - \frac{1}{32\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} - \frac{1}{144\pi^2}\left[\ln(-q^2) - \frac{4q_0^2}{q^2}\right]\left\langle\frac{\alpha_s}{\pi}[(uG)^2 + (u\tilde{G})^2]\right\rangle_{\rho,I},\end{aligned} \tag{31}$$

$$\begin{aligned}\Pi_q^O\big(q_0^2, |\vec{q}|\big) = {}& \frac{1}{6\pi^2}\ln(-q^2)[\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho I}] - \frac{2}{3\pi^2}\frac{q_0^2}{(q^2)^2}\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} - \frac{2}{3\pi^2}\frac{q_0^2}{(q^2)^2}\langle\bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I}\\ & - \frac{2}{3\pi^2}\frac{1}{q^2}\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} + \frac{1}{18\pi^2}\frac{1}{q^2}\langle g_s u^\dagger\sigma\mathcal{G}u\rangle_{\rho,I},\end{aligned} \tag{32}$$

$$\begin{aligned}\Pi_u^E\big(q_0^2, |\vec{q}|\big) = {}& \frac{1}{12\pi^2}q^2\ln(-q^2)[7\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}] + \frac{3}{\pi^2}\frac{q_0^2}{q^2}\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I} + \frac{1}{\pi^2}\frac{q_0^2}{q^2}\langle\bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I}\\ & - \frac{1}{6\pi^2}\ln(-q^2)\langle g_s u^\dagger\sigma\cdot\mathcal{G}u\rangle_{\rho,I} + \frac{1}{12\pi^2}\ln(-q^2)\langle g_s d^\dagger\sigma\mathcal{G}d\rangle_{\rho,I},\end{aligned} \tag{33}$$

$$\Pi_u^O\big(q_0^2, |\vec{q}|\big) = -\frac{4}{9\pi^2}\ln(-q^2)\langle\bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I} - \frac{16}{9\pi^2}\ln(-q^2)\langle\bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I} + \frac{1}{36\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}[(uG)^2 + (u\tilde{G})^2]\right\rangle_{\rho,I}. \tag{34}$$

The quark part and their flavor structure of the above OPE can be obtained by suitable substitutions of the corresponding OPE for the Σ given in Ref. [33]; by changing $q \to u$, $s \to d$, and neglecting terms proportional to m_s. Moreover, when both u and d quarks are identified to the generic light flavor q, our OPE also reduces to that given in Ref. [18].

The next task is to identify the nucleon self-energies in the asymmetric nuclear medium. We therefore have to concentrate on the quasinucleon pole and not on the quasihole nor the continuum excitations. To this end, we apply the Borel transformation with appropriate weighting function to the dispersion relation [16] and the corresponding differential operator $\mathcal{B}$ to the OPE side; details of Borel transformations are given in Appendix E. The Borel transformed invariants which contain the continuum corrections are as follows:

$$\begin{aligned}\bar{\mathcal{B}}\big[\Pi_s\big(q_0^2, |\vec{q}|\big)\big] = \lambda_N^{*2}M_p^* e^{-(E_q^2-\vec{q}^2)/M^2} = {}& -\frac{1}{4\pi^2}(M^2)^2E_1\langle\bar{d}d\rangle_{\rho,I} - \frac{4}{3\pi^2}\vec{q}^2\langle\bar{d}\{iD_0iD_0\}d\rangle_{\rho,I}L^{-\frac{4}{9}}\\ & + \bar{E}_q\left[-\frac{1}{2\pi^2}M^2E_0\langle\bar{d}iD_0d\rangle_{\rho,I}L^{-\frac{4}{9}}\right],\end{aligned} \tag{35}$$

$$\begin{aligned}\bar{\mathcal{B}}\big[\Pi_q\big(q_0^2, |\vec{q}|\big)\big] = \lambda_N^{*2}e^{-(E_q^2-\vec{q}^2)/M^2} = {}& \frac{1}{32\pi^4}(M^2)^3E_2L^{-\frac{4}{9}} - \left(\frac{1}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right)\langle\bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I}L^{-\frac{4}{9}}\\ & - \left(\frac{4}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right)\langle\bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I}L^{-\frac{4}{9}} + \frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I}E_0L^{-\frac{4}{9}}\\ & + \frac{1}{144\pi^2}(M^2E_0 - 4\vec{q}^2)\left\langle\frac{\alpha_s}{\pi}[(u\cdot G)^2 + (u\cdot\tilde{G})^2]\right\rangle_{\rho,I}L^{-\frac{4}{9}} + \bar{E}_q\left[\frac{1}{6\pi^2}M^2E_0L^{-\frac{4}{9}}[\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}]\right.\\ & - \frac{2}{3\pi^2}\left(1 - \frac{\vec{q}^2}{M^2}\right)\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I}L^{-\frac{4}{9}} - \frac{2}{3\pi^2}\left(1 - \frac{\vec{q}^2}{M^2}\right)\langle\bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I}L^{-\frac{4}{9}}\\ & \left. - \frac{2}{3\pi^2}\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I}L^{-\frac{4}{9}} + \frac{1}{18\pi^2}\langle g_s u^\dagger\sigma\mathcal{G}u\rangle_{\rho,I}L^{-\frac{4}{9}}\right],\end{aligned} \tag{36}$$

$$\begin{aligned}\mathcal{B}\big[\Pi_u\big(q_0^2,|\vec{q}|\big)\big] = \lambda_N^{*2}\Sigma_v^p e^{-(E_q^2-\vec{q}^2)/M^2} = &\frac{1}{12\pi^2}(M^2)^2[7\langle u^\dagger u\rangle_{\rho,I}+\langle d^\dagger d\rangle_{\rho,I}]E_1L^{-\frac{4}{9}}+\frac{3}{\pi^2}\vec{q}^2\langle\bar{u}\{\gamma_0 iD_0 iD_0\}u\rangle_{\rho,I}L^{-\frac{4}{9}}\\&+\frac{1}{\pi^2}\vec{q}^2\langle\bar{d}\{\gamma_0 iD_0 iD_0\}d\rangle_{\rho,I}L^{-\frac{4}{9}}-\frac{1}{6\pi^2}M^2\langle g_s u^\dagger\sigma\mathcal{G}u\rangle_{\rho,I}E_0L^{-\frac{4}{9}}+\frac{1}{12\pi^2}M^2\langle g_s d^\dagger\sigma\mathcal{G}d\rangle_{\rho,I}E_0L^{-\frac{4}{9}}\\&+\bar{E}_q\bigg[\frac{4}{9\pi^2}M^2\langle\bar{d}\{\gamma_0 iD_0\}d\rangle_{\rho,I}E_0L^{-\frac{4}{9}}+\frac{16}{9\pi^2}M^2\langle\bar{u}\{\gamma_0 iD_0\}u\rangle_{\rho,I}E_0L^{-\frac{4}{9}}\\&-\frac{1}{36\pi^2}M^2\bigg\langle\frac{\alpha_s}{\pi}[(uG)^2+(u\tilde{G})^2]\bigg\rangle_{\rho,I}E_0L^{-\frac{4}{9}}\bigg].\end{aligned}\tag{37}$$

Here, we include the corrections from the anomalous dimensions as

$$L^{-2\Gamma_\eta+\Gamma_{O_n}} \equiv \left[\frac{\ln(M/\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right]^{-2\Gamma_\eta+\Gamma_{O_n}}, \tag{38}$$

where Γ_η (Γ_{O_n}) is the anomalous dimension of the interpolating current η ($\hat{O}_n$), μ is the normalization point of the OPE, and $\Lambda_{\rm QCD}$ is the QCD scale [16,18].

Also, the continuum corrections are taken into account through the factors

$$E_0 \equiv 1 - e^{s_0^*/M^2}, \tag{39}$$

$$E_1 \equiv 1 - e^{s_0^*/M^2}(s_0^*/M^2+1), \tag{40}$$

$$E_2 \equiv 1 - e^{s_0^*/M^2}\big(s_0^{*2}/2M^4+s_0^*/M^2+1\big), \tag{41}$$

where $s_0^* \equiv \omega_0^2 - \vec{q}^2$ and ω_0 is the energy at the continuum threshold. We choose the continuum to be the same as the vacuum value $\omega_0 = 1.5$ GeV. This assumption will be justified later as the results do not have strong ω_0 dependence.

B. Condensates in the asymmetric nuclear medium

To estimate the matrix elements, we will use the linear density approximation in the asymmetric nuclear matter,

$$\begin{aligned}\langle\hat{O}\rangle_{\rho,I} &= \langle\hat{O}\rangle_{\rm vac}+\langle n|\hat{O}|n\rangle\rho_n+\langle p|\hat{O}|p\rangle\rho_p\\&=\langle\hat{O}\rangle_{\rm vac}+\tfrac{1}{2}(\langle n|\hat{O}|n\rangle+\langle p|\hat{O}|p\rangle)\rho\\&\quad+\tfrac{1}{2}(\langle n|\hat{O}|n\rangle-\langle p|\hat{O}|p\rangle)I\rho.\end{aligned}\tag{42}$$

The quark flavor of condensate becomes important in the asymmetric nuclear medium. Consider an operator $\hat{O}_{u,d}$ composed of either u or d quarks, respectively. Making use of the isospin symmetry relation,

$$\langle n|\hat{O}_{u,d}|n\rangle = \langle p|\hat{O}_{d,u}|p\rangle, \tag{43}$$

we can convert the neutron expectation value to the proton expectation value, thereby rewriting Eq. (42) for the two-quark operators as follows:

$$\langle\hat{O}_{u,d}\rangle_{\rho,I} = \langle\hat{O}_{u,d}\rangle_{\rm vac}+(\langle p|\hat{O}_0|p\rangle\mp\langle p|\hat{O}_1|p\rangle I)\rho. \tag{44}$$

Here, "−" and "+" are for the u and d quark flavors, respectively, and the isospin operators are defined as

$$\hat{O}_0 \equiv \tfrac{1}{2}(\hat{O}_u+\hat{O}_d), \quad \hat{O}_1 \equiv \tfrac{1}{2}(\hat{O}_u-\hat{O}_d). \tag{45}$$

Hence, we will convert all the expectation values in terms of the proton counterparts and denote them as $\langle p|\hat{O}|p\rangle \to \langle\hat{O}\rangle_p$, throughout this paper. The next task is to find $\langle\hat{O}_0\rangle_p$ and $\langle\hat{O}_1\rangle_p$ for all operators appearing in our OPE.

1. $\langle\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle$ type of condensates

Let us start by estimating the lowest-dimensional operators $\langle[\bar{q}q]_0\rangle_p$ and $\langle[\bar{q}q]_1\rangle_p$. To find $\langle[\bar{q}q]_1\rangle_p$, we will use an estimate based on using the QCD energy momentum tensor in the baryon octet mass relation to leading order in the quark mass [34]; Eq. (A3) in Appendix A. Using Eq. (A4), one finds

$$\begin{aligned}\langle[\bar{q}q]_1\rangle_p &= \frac{1}{2}(\langle p|\bar{u}u|p\rangle-\langle p|\bar{d}d|p\rangle)\\&=\frac{1}{2}\left[\frac{(m_{\Xi^0}+m_{\Xi^-})-(m_{\Sigma^+}+m_{\Sigma^-})}{2m_s-2m_q}\right].\end{aligned}\tag{46}$$

We will use the baryon masses as given in the Particle Data Group [35]: $m_{\Xi^0} = 1315$ MeV, $m_{\Xi^-} = 1321$ MeV, $m_{\Sigma^+} = 1190$ MeV, $m_{\Sigma^-} = 1197$ MeV. Using $m_s = 150$ MeV and $m_q \equiv \frac{1}{2}(m_u+m_d) = 5$ MeV, Eq. (46) becomes

$$\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}\left(\frac{249\text{ MeV}}{300\text{ MeV}-2m_q}\right) \sim 0.43. \tag{47}$$

For $\langle[\bar{q}q]_0\rangle_p$, we make use of the nucleon $\sigma_N = 45$ MeV term,

$$\langle[\bar{q}q]_0\rangle_p = \frac{1}{2}(\langle p|\bar{u}u|p\rangle+\langle p|\bar{d}d|p\rangle) = \frac{\sigma_N}{2m_q} \sim 4.5. \tag{48}$$

For convenience, one can introduce the parameter $\mathcal{R}_\pm(m_q)$, defined as

$$\langle p|\bar{u}u|p\rangle \pm \langle p|\bar{d}d|p\rangle = \mathcal{R}_\pm(m_q)\langle p|\bar{u}u|p\rangle, \tag{49}$$

which leads to

$$\langle[\bar{q}q]_1\rangle_p = \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\langle[\bar{q}q]_0\rangle_p. \tag{50}$$

Using the previously selected values with the explicit quark mass dependence, we have

$$\begin{aligned}\mathcal{R}_\pm(m_q) \equiv \Bigg[1\pm\left(\frac{\sigma_N}{m_q}-\frac{249\text{ MeV}}{300\text{ MeV}-2m_q}\right)\Bigg/\\\left(\frac{\sigma_N}{m_q}+\frac{249\text{ MeV}}{300\text{ MeV}-2m_q}\right)\Bigg],\end{aligned}\tag{51}$$

so $\mathcal{R}_\pm(m_q = 5\text{MeV}) = 1 \pm 0.68$.

Using this parametrization, we can express the u quark or d quark condensates as follows,

$$\langle[\bar{q}q]_{u,d}\rangle_{\rho,I} = \langle[\bar{q}q]_{u,d}\rangle_{\rm vac} + \left[1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right] \langle[\bar{q}q]_0\rangle_p \rho, \tag{52}$$

where $[\bar{q}q]_u = \bar{u}u$ and $[\bar{q}q]_d = \bar{d}d$. For $\langle\bar{q}q\rangle_{\rm vac}$, we use the Gellmann-Oakes-Renner relation,

$$2m_q\langle\bar{q}q\rangle_{\rm vac} = -m_\pi^2 f_\pi^2, \tag{53}$$

where $m_\pi = 138$ MeV and $f_\pi = 98$ MeV [18]. For $m_q = 5$ MeV, we have $\langle\bar{q}q\rangle_{\rm vac} = -(263\ {\rm MeV})^3$.

Likewise, we will further assume that the ratios between the isospin singlet and triplet operators remain the same for all two-quark operator expectation values with any number of covariant derivatives inserted,

$$\left\langle\left[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\right]_1\right\rangle_p = \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\left\langle\left[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\right]_0\right\rangle_p. \tag{54}$$

With this assumption, $\langle\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho,I}$ can be written as

$$\begin{aligned}\left\langle\left[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\right]_{u,d}\right\rangle_{\rho,I} &= \left\langle\left[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\right]_{u,d}\right\rangle_{\rm vac} \\ &\quad + \left[1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right]\left\langle\left[\bar{q}D_{\mu_1}\cdots D_{\mu_n}q\right]_0\right\rangle_p \rho. \end{aligned} \tag{55}$$

The symmetric and traceless part of the above type of expectation values constitute the moments of the twist-3 $e_n(x,\mu^2)$ structure function defined as follows [36]:

$$\left\langle\left[\bar{q}\left\{D_{\mu_1}\cdots D_{\mu_n}\right\}q\right]_0\right\rangle_p \equiv (-i)^n e_n(\mu^2)\left\{p_{\mu_1}\cdots p_{\mu_n}\right\}, \tag{56}$$

$$e_n(\mu^2) \equiv \int dx x^n e_n(x,\mu^2), \tag{57}$$

where $\{\mu_1\cdots\mu_n\}$ means symmetric and traceless indices. The two-quark twist-3 condensates in our sum rule then can be written as follows:

$$\begin{aligned}\langle[\bar{q}iD_{\mu'}q]_{u,d}\rangle_{\rho,I} &= \langle[\bar{q}iD_0q]_{u,d}\rangle_{\rho,I}u'_\mu \\ &= m_q\langle[q^\dagger q]_{u,d}\rangle_{\rho,I} = 0, \end{aligned} \tag{58}$$

$$\begin{aligned}\langle[\bar{q}\{iD_{\mu'}iD_{\nu'}\}q]_{u,d}\rangle_{\rho,I} &= \tfrac{4}{3}\langle[\bar{q}\{iD_0iD_0\}q]_{u,d}\rangle_{\rho,I} \\ &\quad\times\left(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}\right), \end{aligned} \tag{59}$$

where the in-medium rest frame $u'_\mu \equiv (1,\vec{0})$ has been taken and the matrix element is estimated as

$$\langle[\bar{q}\{iD_{\mu'}iD_{\nu'}\}q]_0\rangle_p = M_N^2 e_2(\mu^2)\left(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}\right), \tag{60}$$

where one can identify that $M_N^2 e_2(\mu^2) = \frac{4}{3}\langle[\bar{q}\{iD_0iD_0\}q]_0\rangle_p$, and $\langle[\bar{q}\{iD_0iD_0\}q]_{u,d}\rangle_{\rho,I}$ can be written as

$$\langle[\bar{q}\{iD_0iD_0\}q]_{u,d}\rangle_{\rho,I} \simeq \left[1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right] M_N^2 e_2(\mu^2)\rho. \tag{61}$$

Since there are no measurements on the twist-3 structure function, we will take the estimate for $M_N^2 e_2(\mu^2) \sim 0.3\ {\rm GeV}^2$ given in Refs. [18,37].

When spin indices are contracted, the operator becomes

$$\begin{aligned}\langle[\bar{q}D^2q]_{u,d}\rangle_{\rho,I} &= \frac{1}{2}\langle[g_s\bar{q}\sigma\mathcal{G}q]_{u,d}\rangle_{\rho,I} \\ &= \frac{1}{2}\left[1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)} I\right]\langle[g_s\bar{q}\sigma\mathcal{G}q]_0\rangle_p\rho, \end{aligned} \tag{62}$$

where $\langle[g_s\bar{q}\sigma\mathcal{G}q]_0\rangle_p$ is chosen to be $3\ {\rm GeV}^2$ as in Refs. [18,37].

2. $\langle\bar{q}\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}q\rangle$ type of condensates

The simplest condensate of this type is

$$\langle\bar{q}\gamma_\lambda q\rangle_{\rho,I} = \langle\bar{q}\not{u}'q\rangle_{\rho,I}u'_\lambda \to \langle q^\dagger q\rangle_{\rho,I}u'_\lambda. \tag{63}$$

For this, the ratio $\langle u^\dagger u\rangle_p/\langle d^\dagger d\rangle_p = 2$, and the isospin relation for $\langle q^\dagger q\rangle_{\rho,I}$ can be written as

$$\langle[q^\dagger q]_1\rangle_p = \tfrac{1}{3}\langle[q^\dagger q]_0\rangle_p, \tag{64}$$

which leads to the following matrix elements appearing in the sum rule:

$$\langle[q^\dagger q]_{u,d}\rangle_{\rho,I} = \left(1 \mp \tfrac{1}{3}I\right)\langle[q^\dagger q]_0\rangle_p\rho = \left(\tfrac{3}{2} \mp \tfrac{1}{2}I\right)\rho. \tag{65}$$

When covariant derivatives are included, one can estimate the two-quark twist-2 condensates from the corresponding parton distribution function,

$$\left\langle\bar{q}\left\{\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_n}\right\}q\right\rangle_p \equiv \frac{(-i)^{n-1}}{2M_N}A_n^q(\mu^2)\left\{p_{\mu_1}\cdots p_{\mu_n}\right\}, \tag{66}$$

where $A_n^q(\mu^2) = [A_n^u(\mu^2) + A_n^d(\mu^2)]/2$ is the reduced matrix element [38,39],

$$A_n^q(\mu^2) = 2\int_0^1 dx x^{n-1}[q(x,\mu^2) + (-1)^n\bar{q}(x,\mu^2)], \tag{67}$$

where $q(x,\mu^2)$ and $\bar{q}(x,\mu^2)$ are the distribution functions for quarks and antiquarks in the proton, respectively, and μ^2 is the renormalization scale. For the distribution functions, we used the leading-order (LO) parametrization given in Ref. [40].

Specifically, the spin-2 part can be written as [41]

$$\begin{aligned}\langle[\bar{q}\{\gamma_\mu iD_\nu\}q]_{u,d}\rangle_{\rho,I} &\to \langle[\bar{q}\{\gamma_{\mu'}iD_{\nu'}\}q]_{u,d}\rangle_{\rho,I} \\ &= \tfrac{4}{3}\langle[\bar{q}\{\gamma_0 iD_0\}q]_{u,d}\rangle_{\rho,I}\left(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}\right), \end{aligned} \tag{68}$$

where the in-medium rest frame has been taken. The matrix elements for each flavor in $\langle[\bar{q}\{\gamma_0 iD_0\}q]_{u,d}\rangle_p$ can be identified as

$$\langle\bar{u}\{\gamma_{\mu'}iD_{\nu'}\}u\rangle_p = \tfrac{1}{2}M_N A_2^u(\mu^2)\left(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}\right), \tag{69}$$

$$\langle\bar{d}\{\gamma_{\mu'}iD_{\nu'}\}d\rangle_p = \tfrac{1}{2}M_N A_2^d(\mu^2)\left(u'_\mu u'_\nu - \tfrac{1}{4}g_{\mu\nu}\right), \tag{70}$$

where $A_2^u(\mu^2) \simeq 0.74$ and $A_2^d(\mu^2) \simeq 0.36$ at $\mu^2 = 0.25\ {\rm GeV}^2$ (LO) [40].

One can introduce a ratio factor for $\langle\hat{O}_1\rangle_p$ as

$$\langle[\bar{q}\{\gamma_{\mu'}iD_{\nu'}\}q]_1\rangle_p = \mathcal{R}_{A_2}(\mu^2)\langle[\bar{q}\{\gamma_{\mu'}iD_{\nu'}\}q]_0\rangle_p, \tag{71}$$

where $\mathcal{R}_{A_2}(\mu^2) = (A_2^u - A_2^d)/(A_2^u + A_2^d) \simeq 0.35$ so $\langle[\bar{q}\{\gamma_0 i D_0\}q]_{u,d}\rangle_{\rho,I}$ can be written as

$$\begin{aligned}\langle[\bar{q}\{\gamma_0 i D_0\}q]_{u,d}\rangle_{\rho,I} &= \left[1 \mp \mathcal{R}_{A_2}(\mu^2)I\right]\langle[\bar{q}\{\gamma_0 i D_0\}q]_0\rangle_p\rho \\ &= \left[1 \mp \mathcal{R}_{A_2}(\mu^2)I\right]\tfrac{1}{2}M_N A_2^q(\mu^2)\rho. \quad (72)\end{aligned}$$

The spin-3 part can be written as

$$\begin{aligned}&\langle[\bar{q}\{\gamma_{\lambda'} i D_{\mu'} i D_{\nu'}\}q]_{u,d}\rangle_{\rho,I} \\ &\quad = 2\langle[\bar{q}\{\gamma_0 i D_0 i D_0\}q]_{u,d}\rangle_{\rho,I}\left[u'_\lambda u'_\mu u'_\nu - \tfrac{1}{6}(u'_\lambda g_{\mu\nu} \right.\\ &\qquad \left. + u'_\mu g_{\lambda\nu} + u'_\nu g_{\lambda\mu})\right], \quad (73)\end{aligned}$$

where the matrix elements for each flavor in $\langle[\bar{q}\{\gamma_0 i D_0 i D_0\}q]_{u,d}\rangle_p$ can be identified with

$$\begin{aligned}\langle\bar{u}\{\gamma_{\lambda'} i D_{\mu'} i D_{\nu'}\}u\rangle_p &= \tfrac{1}{2}M_N^2 A_3^u(\mu^2)\left[u_{\lambda'}u_{\mu'}u_{\nu'} - \tfrac{1}{6}(u_{\lambda'}g_{\mu'\nu'}\right. \\ &\quad \left. + u_{\mu'}g_{\lambda'\nu'} + u_{\nu'}g_{\lambda'\mu'})\right], \quad (74)\end{aligned}$$

$$\begin{aligned}\langle\bar{d}\{\gamma_{\lambda'} i D_{\mu'} i D_{\nu'}\}d\rangle_p &= \tfrac{1}{2}M_N^2 A_3^d(\mu^2)\left[u_{\lambda'}u_{\mu'}u_{\nu'} - \tfrac{1}{6}(u_{\lambda'}g_{\mu'\nu'}\right. \\ &\quad \left. + u_{\mu'}g_{\lambda'\nu'} + u_{\nu'}g_{\lambda'\mu'})\right], \quad (75)\end{aligned}$$

where $A_3^u(\mu^2) \simeq 0.22$ and $A_3^d(\mu^2) \simeq 0.07$ at $\mu^2 = 0.25$ GeV^2 (LO) [40]. Similar to the spin-2 condensate case, one can write $\langle\hat{O}_1\rangle_p$ for spin-3 condensate as

$$\langle[\bar{q}\{\gamma_{\lambda'} i D_{\mu'} i D_{\nu'}\}q]_1\rangle_p = \mathcal{R}_{A_3}(\mu^2)\langle[\bar{q}\{\gamma_{\lambda'} i D_{\mu'} i D_{\nu'}\}q]_0\rangle_p, \quad (76)$$

where $\mathcal{R}_{A_3}(\mu^2) = (A_3^u - A_3^d)/(A_3^u + A_3^d) \simeq 0.51$ and $\langle[\bar{q}\{\gamma_0 i D_0 i D_0\}q]_{u,d}\rangle_{\rho,I}$ can be written as

$$\begin{aligned}&\langle[\bar{q}\{\gamma_0 i D_0 i D_0\}q]_{u,d}\rangle_{\rho,I} \\ &\quad = \left[1 \mp \mathcal{R}_{A_3}(\mu^2)I\right]\langle[\bar{q}\{\gamma_0 i D_0 i D_0\}q]_0\rangle_p\rho \\ &\quad = \left[1 \mp \mathcal{R}_{A_3}(\mu^2)I\right]\tfrac{1}{2}M_N^2 A_3^q(\mu^2)\rho. \quad (77)\end{aligned}$$

Operators with contracted spin indices are

$$\langle[\bar{q}\, \not{D} q]_{u,d}\rangle_{\rho,I} = 0, \quad (78)$$

$$\begin{aligned}\langle[q^\dagger D^2 q]_{u,d}\rangle_{\rho,I} &= \tfrac{1}{2}\langle[g_s q^\dagger\sigma\mathcal{G}q]_{u,d}\rangle_{\rho,I} \\ &\simeq \tfrac{1}{2}\left(1 \mp \mathcal{R}_{A_3}I\right)\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p\rho, \quad (79)\end{aligned}$$

where $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$ is chosen to be -0.33 GeV^2 [18,37].

3. Gluon condensates

As for the gluon operators, because they do not carry quark flavors, the expectation values do not depend on I. These operators can be written as [17,18]

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} = \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rm vac} - 2\left\langle\frac{\alpha_s}{\pi}(\vec{E}^2 - \vec{B}^2)\right\rangle_p\rho, \quad (80)$$

$$\left\langle\frac{\alpha_s}{\pi}[(uG)^2 + (u\tilde{G})^2]\right\rangle_{\rho,I} = -\left\langle\frac{\alpha_s}{\pi}(\vec{E}^2 + \vec{B}^2)\right\rangle_p\rho, \quad (81)$$

where $\vec{E}$ and $\vec{B}$ are the color electric and color magnetic fields. For the expectation values of the gluon operators we take $\langle(\alpha_s/\pi)G^2\rangle_{\rm vac} = (0.33\ \text{GeV})^4$ [9], $\langle(\alpha_s/\pi)(\vec{E}^2 - \vec{B}^2)\rangle_p = 0.325 \pm 0.075$ GeV, and $\langle(\alpha_s/\pi)(\vec{E}^2 + \vec{B}^2)\rangle_p = 0.10 \pm 0.01$ GeV [17].

C. Dimension-six four-quark operators

In many previous QCD sum-rule studies, dimension-six four-quark condensates are assumed to have the factorized form as

$$\left\langle u_\alpha^a\bar{u}_\beta^b u_\gamma^c\bar{u}_\delta^d\right\rangle_{\rho,I} \simeq \left\langle u_\alpha^a\bar{u}_\beta^b\right\rangle_{\rho,I}\left\langle u_\gamma^c\bar{u}_\delta^d\right\rangle_{\rho,I} - \left\langle u_\alpha^a\bar{u}_\delta^d\right\rangle_{\rho,I}\left\langle u_\gamma^c\bar{u}_\beta^b\right\rangle_{\rho,I}, \quad (82)$$

$$\left\langle u_\alpha^a\bar{u}_\beta^b d_\gamma^c\bar{d}_\delta^d\right\rangle_{\rho,I} \simeq \left\langle u_\alpha^a\bar{u}_\beta^b\right\rangle_{\rho,I}\left\langle d_\gamma^c\bar{d}_\delta^d\right\rangle_{\rho,I}. \quad (83)$$

While large N_c arguments can be made to justify factorization in the vacuum, no such argument exists in the medium. For the in-medium case, a renewed approach was developed in which the in-medium four-quark condensates are evaluated within the PCQM [20,42,43]. In this method, the vacuum condensates are factorized as in Eq. (83) but in-medium terms are evaluated by including intermediate states that include pion clouds. There are some previous results to calculate the four-quark operators appearing in the nucleon OPE. For example, in Ref. [43], the expectation values were calculated within the PCQM. Another approach uses a Fierz rearrangement suitable for factorization as in our case [44].

In this study, after using the Fierz transformation as above, for the scalar four-quark operators, we change the four-quark operators to vary from a mild factorized form to a density-independent limit that preserves the consistent nucleon sum rule as in Ref. [19]. For the spin-2 four-quark (twist-4) operators, we use a Fierz rearrangement to extract the independent four-quark operators that can be related to higher twist effects in DIS data. Using the following steps, we have classified the four-quark condensates in terms of the independent operators and of different twist.

1. Twist-4 operators with a single quark flavor

The first type of four-quark operator appearing in the OPE of the nucleon sum rule involves quark operators with the same flavor and is of the color antitriplet diquark times triplet antidiquark form. Using the following Fierz transformation, one can identify the independent four-quark operators in terms of products of quark-antiquark pairs,

$$\begin{aligned}&\epsilon_{abc}\epsilon_{a'b'c}\left(u_a^T C\gamma_\mu u_b\right)\left(\bar{u}_{b'}\gamma_\nu C\bar{u}_{a'}^T\right) \\ &\quad = \epsilon_{abc}\epsilon_{a'b'c}\tfrac{1}{16}(\bar{u}_{a'}\Gamma^o u_a)(\bar{u}_{b'}\Gamma^k u_b){\rm Tr}\left[\gamma_\mu\Gamma_k\gamma_\nu C\Gamma_o^T C\right] \\ &\quad = \epsilon_{abc}\epsilon_{a'b'c}\tfrac{1}{16}\left\{(\bar{u}_{a'}u_a)(\bar{u}_{b'}u_b)(-4g_{\mu\nu})\right. \\ &\qquad + (\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\gamma_5 u_b)(4g_{\mu\nu}) \\ &\qquad + (\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b)(4S_{\mu\beta\nu\alpha}) \\ &\qquad - (\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)(4S_{\mu\beta\nu\alpha}) \\ &\qquad + (\bar{u}_{a'}\sigma^{\alpha\bar{\alpha}}u_a)(\bar{u}_{b'}\sigma^{\beta\bar{\beta}}u_b)\tfrac{1}{4}{\rm Tr}[\gamma_\mu\sigma_{\beta\bar{\beta}}\gamma_\nu\sigma_{\alpha\bar{\alpha}}] \\ &\qquad + (\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)(8i\epsilon_{\mu\alpha\nu\alpha}) \\ &\qquad - (\bar{u}_{a'}u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)(8ig_{\alpha\mu}g_{\bar{\alpha}\nu}) \\ &\qquad \left. - (\bar{u}_{a'}\gamma_5 u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)(4\epsilon_{\mu\nu\alpha\bar{\alpha}})\right\}, \quad (84)\end{aligned}$$

where $\Gamma = \{I, \gamma_\alpha, i\gamma_\alpha\gamma_5, \sigma_{\alpha\beta}, \gamma_5\}$ and $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}$.

When quarks of the same flavor combine into a diquark, certain combinations are not allowed due to Fermi statistics. From these conditions, one can extract constraints among four-quark operators that can be used to identify independent operators. Among several conditions, the most suitable constraint for our OPE can be obtained from the zero identity used in Ref. [44]. With the constraint Eq. (B2) in Appendix B, Eq. (84) can be simplified as

$$\begin{aligned}
&\epsilon_{abc}\epsilon_{a'b'c}\big(u_a^T C\gamma_\mu u_b\big)\big(\bar{u}_{b'}\gamma_\nu C\bar{u}_{a'}^T\big)\\
&\quad= \epsilon_{abc}\epsilon_{a'b'c}\tfrac{1}{16}\{[(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b)\\
&\qquad - (\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)](8S_{\mu\beta\nu\alpha})\\
&\qquad + (\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5 u_b)(16i\epsilon_{\mu\beta\nu\alpha})\}. \qquad (85)
\end{aligned}$$

The last term in Eq. (85) will be dropped as one should take an expectation value with respect to a parity-even nuclear medium ground state. Then only two types of four-quark operators remain. Each type can be written as

$$\begin{aligned}
&\epsilon_{abc}\epsilon_{a'b'c}(\bar{q}_{a'}\Gamma^\alpha q_a)(\bar{q}_{b'}\Gamma^\beta q_b)\\
&\quad= \epsilon_{abc}\epsilon_{a'b'c}\Big\{\tfrac{1}{9}\delta_{a'a}\delta_{b'b}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q)\\
&\qquad + \tfrac{2}{3}t^A_{aa'}\delta_{b'b}(\bar{q}\Gamma^\alpha t^A q)(\bar{q}\Gamma^\beta q) + \tfrac{2}{3}\delta_{a'a}t^B_{bb'}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta t^B q)\\
&\qquad + 4t^A_{aa'}t^B_{bb'}(\bar{q}\Gamma^\alpha t^A q)(\bar{q}\Gamma^\beta t^B q)\Big\}, \qquad (86)
\end{aligned}$$

where $\Gamma^\alpha = \{\gamma^\alpha, i\gamma^\alpha\gamma_5\}$ and t^A is the generator of SU(3) normalized as $\mathrm{Tr}[t^A t^B] = \frac{1}{2}\delta^{AB}$. Combined with the product of epsilon tensors $\epsilon_{abc}\epsilon_{a'b'c} = \delta_{bb'}\delta_{aa'} - \delta_{ba'}\delta_{ab'}$, one finds that the second and third term in the right-hand side of Eq. (86) vanish. In the last term, the product of the generators of SU(3) can be simplified using the following identity:

$$t^A_{a'a}t^B_{b'b} = \tfrac{1}{8}\delta^{AB}t^C_{a'a}t^C_{b'b} + \big[t^A_{a'a}t^B_{b'b} - \tfrac{1}{8}\delta^{AB}t^C_{a'a}t^C_{b'b}\big], \qquad (87)$$

where only the first term in the right-hand side of Eq. (87) survives after multiplying it with the epsilon tensors $\epsilon_{abc}\epsilon_{a'b'c}$. Then Eq. (86) can be simplified as follows:

$$\begin{aligned}
&\epsilon_{abc}\epsilon_{a'b'c}(\bar{q}_{a'}\Gamma^\alpha q_a)(\bar{q}_{b'}\Gamma^\beta q_b)\\
&\quad= \epsilon_{abc}\epsilon_{a'b'c}\Big\{\tfrac{1}{9}\delta_{a'a}\delta_{b'b}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q)\\
&\qquad + \tfrac{1}{2}t^A_{aa'}t^A_{bb'}(\bar{q}\Gamma^\alpha t^B q)(\bar{q}\Gamma^\beta t^B q)\Big\}\\
&\quad= \tfrac{2}{3}(\bar{q}\Gamma^\alpha q)(\bar{q}\Gamma^\beta q) - 2(\bar{q}\Gamma^\alpha t^B q)(\bar{q}\Gamma^\beta t^B q). \qquad (88)
\end{aligned}$$

One can take another Fierz rearrangement to $(\bar{u}\Gamma^\alpha t^A u)(\bar{u}\Gamma^\beta t^A u)$ type of operators in Eq. (85). Then one can obtain the following relations when taking the symmetric and traceless parts of the operator relations,

$$\begin{aligned}
(\bar{u}\gamma^\alpha t^A u)(\bar{u}\gamma^\beta t^A u)|_{\mathrm{s,t}} &= -\tfrac{5}{12}(\bar{u}\gamma^\alpha u)(\bar{u}\gamma^\beta u)|_{\mathrm{s,t}}\\
&\quad - \tfrac{1}{4}(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{u}\gamma^\beta\gamma_5 u)|_{\mathrm{s,t}}\\
&\quad + \tfrac{1}{4}\big(\bar{u}\sigma^\alpha_o u\big)(\bar{u}\sigma^{o\beta}u)|_{\mathrm{s,t}}, \qquad (89)\\
(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{u}\gamma^\beta\gamma_5 t^A u)|_{\mathrm{s,t}} &= -\tfrac{5}{12}(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{u}\gamma^\beta\gamma_5 u)|_{\mathrm{s,t}}\\
&\quad - \tfrac{1}{4}(\bar{u}\gamma^\alpha u)(\bar{u}\gamma^\beta u)|_{\mathrm{s,t}}\\
&\quad - \tfrac{1}{4}\big(\bar{u}\sigma^\alpha_o u\big)(\bar{u}\sigma^{o\beta}u)|_{\mathrm{s,t}}, \qquad (90)
\end{aligned}$$

where $|_{\mathrm{s,t}}$ means symmetric and traceless. Therefore, only three independent twist-4 (dimension-six spin-2) matrices remain. Using the twist-4 effects in the deep inelastic scattering data on the proton and neutron target, one can, in principle, extract two independent constraints to the three independent matrix elements. To determine all the matrix elements, we will additionally use one constraint adopted by Jaffe [45]: $(\bar{u}\sigma^\alpha_o u)(\bar{u}\sigma^{o\beta}u)|_{\mathrm{s,t}} = 0$.

2. Twist-4 operators with mixed quark flavor

The second type of four-quark operators appearing in the nucleon sum rule are of the following mixed-quark-flavor operator form:

$$\begin{aligned}
&\epsilon_{abc}\epsilon_{a'bc'}\gamma^5\gamma^\mu d_c\bar{d}^T_{c'}\gamma^\nu\gamma^5\big(u_a^T C\gamma_\mu\not{q}\gamma_\nu C\bar{u}^T_{a'}\big)\\
&\quad= \epsilon_{abc}\epsilon_{a'bc'}\tfrac{1}{16}(\gamma^5\gamma^\mu\Gamma_k\gamma^\nu\gamma^5)(\bar{u}_{a'}\Gamma^o u_a)(\bar{d}_{c'}\Gamma^k d_c)\\
&\qquad\times \mathrm{Tr}[\gamma_\mu\not{q}\gamma_\nu C\Gamma_0^T C]\\
&\quad\Rightarrow \epsilon_{abc}\epsilon_{a'bc'}\tfrac{1}{16}\{-8q_\alpha(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}d_c)\\
&\qquad - 8(q_\beta\gamma_\alpha + g_{\alpha\beta}\not{q})(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{d}_{c'}\gamma^\beta d_c)\\
&\qquad + 8(q_\beta\gamma_\alpha - g_{\alpha\beta}\not{q})(\bar{u}_{a'}\gamma^\alpha\gamma_5 u_a)(\bar{d}_{c'}\gamma^\beta\gamma_5 d_c)\}, \qquad (91)
\end{aligned}$$

where we have again used Fierz rearrangement to express the operators in terms of the quark-antiquark type and have neglected operators that are odd in parity and time-reversal symmetry.

As in the case with a single quark flavor, the four-quark condensates in Eq. (91) can be decomposed into two different color structures according to Eqs. (88) and (87). We cannot reduce the number of independent operators as in the previous subsubsection because performing a similar Fierz rearrangement as in Eqs. (89) and (90), we find new mixed-flavor operators of $(\bar{u}\Gamma^\alpha d)(\bar{d}\Gamma^\beta u)$ type.

3. Contributions of dimension-six four-quarks to the OPE

In summary, the independent four-quark condensates appearing in our nucleon sum rule are given in Table I. Not all the matrix elements are known.

As for the dimension-six spin-0 (scalar) operators, we will assume the factorized form as $\langle\bar{u}u\rangle^2_{\rho,I}$, although this assumption has not been justified. Keeping only the linear density terms, they can be written as

$$\begin{aligned}
\langle[\bar{q}q]_{u,d}\rangle^2_{\rho,I} &\Rightarrow \langle\bar{q}q\rangle^2_{\mathrm{vac}}\\
&\quad + 2f\left[1 \mp \frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}I\right]\langle\bar{q}q\rangle_{\mathrm{vac}}\langle[\bar{q}q]_0\rangle_p\,\rho, \qquad (92)
\end{aligned}$$

where f is a parameter introduced in Ref. [18].

TABLE I. Independent four-quark operators appearing in the nucleon OPE with Ioffe's interpolating current. 'q_1' and 'q_2' represent light quark flavors.

Quark flavor	$q_1 = q_2 = q$	$q_1 \neq q_2$
Dimension 6 spin-2 (twist-4)	$(\bar{q}\gamma^\alpha\gamma_5 q)(\bar{q}\gamma^\beta\gamma_5 q)\|_{\rm s,t}$ $(\bar{q}\gamma^\alpha q)(\bar{q}\gamma^\beta q)\|_{\rm s,t}$ $(\bar{q}\sigma^\alpha_o q)(\bar{q}\sigma^{o\beta} q)\|_{\rm s,t}$	$(\bar{q}_1\gamma^\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\beta\gamma_5 q_2)\|_{\rm s,t}$ $(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2\gamma^\beta q_2)\|_{\rm s,t}$ $(\bar{q}_1\gamma^\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\beta\gamma_5 t^A q_2)\|_{\rm s,t}$ $(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2\gamma^\beta t^A q_2)\|_{\rm s,t}$
Dimension 6 spin-1 (vector)		$(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2 q_2)$ $(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2 t^A q_2)$
Dimension 6 spin-0 (scalar)	$(\bar{q}\gamma_\alpha\gamma_5 q)(\bar{q}\gamma^\alpha\gamma_5 q)$ $(\bar{q}\gamma_\alpha q)(\bar{q}\gamma^\alpha q)$ $(\bar{q}\sigma_{o\alpha} q)(\bar{q}\sigma^{o\alpha} q)$	$(\bar{q}_1\gamma_\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\alpha\gamma_5 q_2)$ $(\bar{q}_1\gamma_\alpha q_1)(\bar{q}_2\gamma^\alpha q_2)$ $(\bar{q}_1\gamma_\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\alpha\gamma_5 t^A q_2)$ $(\bar{q}_1\gamma_\alpha t^A q_1)(\bar{q}_2\gamma^\alpha t^A q_2)$

Also, dimension six spin 1 (vector) operators are factorized up to linear density terms as in Ref. [18].

Dimension-six spin-2 are the twist-4 operators. The twist-4 operators appearing in the nucleon sum rule have similar structures as those appearing in the higher twist effects in deep inelastic scattering [26,27]. If the higher twist effects are measured with precision in DIS for the proton and neutron target, the nucleon expectation value of $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{d}\gamma^\beta\gamma_5 t^A d)|_{\rm s,t}$ can be estimated with the same precision [30]. With further plausible arguments (Appendix C) on the ratio of u quark and d quark content of the proton such as those used in Eq. (50), one can estimate the proton expectation value of $(\bar{u}\gamma^\alpha t^A u)(\bar{u}\gamma^\beta t^A u)|_{\rm s,t}$, $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{u}\gamma^\beta\gamma_5 t^A u)|_{\rm s,t}$, and $(\bar{u}\gamma^\alpha t^A u)(\bar{d}\gamma^\beta t^A d)|_{\rm s,t}$.

From these condensates, one can estimate the nucleon expectation value of all the twist-4 operators for the single flavor case given in the first column in Table I with the extra constraint discussed above. For the mixed-flavor condensates given in the second column, one cannot deduce all the matrix elements $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)|_{\rm s,t}$ from $(\bar{u}\gamma^\alpha\gamma_5 t^A u)(\bar{d}\gamma^\beta\gamma_5 t^A d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha t^A u)(\bar{d}\gamma^\beta t^A d)|_{\rm s,t}$. We will, however, neglect $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)|_{\rm s,t}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)|_{\rm s,t}$ in our present analysis, as these mixed-quark-flavor condensates do not give important contributions to the nuclear symmetry energy in the linear density order.

The proton expectation value of the deducible twist-4 operators can be parameterized into the following forms:

$$\langle(\bar{q}_1\gamma^\alpha\gamma_5 t^A q_1)(\bar{q}_2\gamma^\beta\gamma_5 t^A q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^1_{q_1q_2}, \tag{93}$$

$$\langle(\bar{q}_1\gamma^\alpha t^A q_1)(\bar{q}_2\gamma^\beta t^A q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^2_{q_1q_2}, \tag{94}$$

$$\langle(\bar{q}_1\gamma^\alpha\gamma_5 q_1)(\bar{q}_2\gamma^\beta\gamma_5 q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^3_{q_1q_2}, \tag{95}$$

$$\langle(\bar{q}_1\gamma^\alpha q_1)(\bar{q}_2\gamma^\beta q_2)\rangle_p|_{\rm s,t} = \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u^\alpha u^\beta - \frac{1}{4}g^{\alpha\beta}\right)T^4_{q_1q_2}, \tag{96}$$

where q_1 and q_2 represent quark flavors. We have extracted the T^is from the matrix elements estimated in Ref. [30] and listed in Table II.

Using the parametrization of the nucleon expectation value of the twist-4 operators together with the linear density approximation given in Eq. (42), the contributions to the correlation function from the four-quark operators can be

TABLE II. Two sets for T^is. The three different classifications of T^i follow that given in Ref. [30]. Detailed treatment is given in Appendix C. Units are in GeV^2.

	T^1_{uu}	T^1_{dd}	T^2_{uu}	T^2_{dd}	T^3_{uu}	T^3_{dd}	T^4_{uu}	T^4_{dd}	T^1_{ud}	T^2_{ud}
				First set						
$K^1_u = K^1_{ud}/\beta$	−0.132	−0.041	0.154	0.048	0.842	0.262	−0.875	−0.272	−0.042	0.049
$K^1_u = K^1_{ud}(\beta+1)/\beta$	−0.071	−0.012	0.070	0.012	0.424	0.072	−0.422	−0.072	−0.042	0.041
$K^1_u = K^1_{ud}$	−0.042	0.002	0.033	−0.002	0.240	−0.012	−0.233	0.012	−0.042	0.031
				Second set						
$K^1_u = -K^1_{ud}$	0.215	0.124	−0.432	−0.265	−1.778	−1.091	2.104	1.290	−0.042	0.057
$K^1_u = -K^1_{ud}(\beta+1)/\beta$	0.154	0.100	−0.337	−0.219	−1.336	0.868	1.610	1.046	−0.042	0.056
$K^1_u = -K^1_{ud}/\beta$	0.125	0.085	−0.297	−0.202	−1.137	−0.773	1.395	0.949	−0.042	0.058

written as

$$\Pi^{O}_{D=6,s}(q_0^2,|\vec{q}|) = -\frac{4}{3}\frac{1}{q^2}\langle\bar{q}q\rangle_{\rm vac}\cdot\left(\frac{3}{2}-\frac{1}{2}I\right)\rho, \tag{97}$$

$$\Pi^{E}_{D=6,q}(q_0^2,|\vec{q}|) = -\frac{2}{3q^2}\langle\bar{u}u\rangle^2_{\rho,I}+\frac{1}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left[T^1_{ud}-T^2_{ud}\right]\rho+\frac{1}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(\left[T^1_0-T^2_0\right]-\left[T^1_1-T^2_1\right]I\right)\rho - \frac{1}{3q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(\left[T^3_0-T^4_0\right]-\left[T^3_1-T^4_1\right]I\right)\rho, \tag{98}$$

$$\Pi^{O}_{D=6,u}(q_0^2,|\vec{q}|) = -\frac{4}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left[T^1_{ud}-T^2_{ud}\right]\rho-\frac{4}{q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(\left[T^1_0-T^2_0\right]-\left[T^1_1-T^2_1\right]I\right)\rho + \frac{4}{3q^2}\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(\left[T^3_0-T^4_0\right]-\left[T^3_1-T^4_1\right]I\right)\rho, \tag{99}$$

where $T^i_0=\frac{1}{2}(T^i_{uu}+T^i_{dd})$ and $T^i_1=\frac{1}{2}(T^i_{uu}-T^i_{dd})$. All the vacuum and scalar condensates are factorized as Eqs. (83) and (92). The corresponding Borel transformations are given as follows:

$$\bar{\mathcal{B}}[\Pi_{D=6,s}(q_0^2,|\vec{q}|)] = (-\bar{E}_q)\frac{4}{3}\langle\bar{q}q\rangle_{\rm vac}\cdot\left(\frac{3}{2}-\frac{1}{2}I\right)\rho, \tag{100}$$

$$\bar{\mathcal{B}}[\Pi_{D=6,q}(q_0^2,|\vec{q}|)] = \frac{2}{3}\langle\bar{u}u\rangle^2_{\rho,I}L^{\frac{4}{9}}-\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left\{\left[T^1_{ud}-T^2_{ud}\right]+\left(\left[T^1_0-T^2_0\right]-\left[T^1_1-T^2_1\right]I\right) -\frac{1}{3}\left(\left[T^3_0-T^4_0\right]-\left[T^3_1-T^4_1\right]I\right)\right\}\rho L^{-\frac{4}{9}}, \tag{101}$$

$$\bar{\mathcal{B}}[\Pi_{D=6,u}(q_0^2,|\vec{q}|)] = \frac{(4\bar{E}_q)}{4\pi\alpha_s}\frac{M_N}{2}\left\{\left[T^1_{ud}-T^2_{ud}\right]+\left(\left[T^1_0-T^2_0\right]-\left[T^1_1-T^2_1\right]I\right)-\frac{1}{3}\left(\left[T^3_0-T^4_0\right]-\left[T^3_1-T^4_1\right]I\right)\right\}\rho L^{-\frac{4}{9}}. \tag{102}$$

Here, we have neglected the scaling of the matrix elements coming from the anomalous dimension of the dimension-six operators Γ_{O_n}. Although the twist-4 matrix elements are estimated at the separation scale of 5 GeV, and the matrix element we need is at lower energy scale close to the Borel mass, we will neglect the running of the matrix elements through the anomalous dimension for operator Γ_{O_n}, because the present estimate of the matrix elements already contains $\pm 50\%$ uncertainty. Throughout this paper, we used $\alpha_s \simeq 0.5$ for these twist-4 matrix elements as in Refs. [26,30]. In principle, the coupling appearing in the twist-4 matrix element should run with the Borel mass. However, we neglect such running because within the region of Borel mass $1.0\ \mathrm{GeV}^2 \leqslant M^2 \leqslant 1.2\ \mathrm{GeV}^2$, $\alpha_s(M^2)\sim 0.4$ and, hence, the difference with what was used is within the uncertainty of the twist-4 matrix element.

IV. RESULTS FOR THE NUCLEON SUM RULE AND THE NUCLEAR SYMMETRY ENERGY

We have expressed the self-energy contributions of the nucleons that contribute to the nucleon energy as Eq. (16) in terms of the Borel-transformed OPE as given in Eqs. (35)–(37). The next step is to substitute Eqs. (16) to (8) to extract the symmetry energy as defined in Eq. (3). There then will be the trivial kinematic correction coming from the three-momentum dependence in the kinetic energy part of Eq. (16). This term is universal and corresponds to the term in Eq. (6). Instead of following the full procedure, in this work, we will just concentrate on the contribution coming from the scalar and vector self-energy. This corresponds to calculating the contribution to the nuclear symmetry energy from potentials in effective models.

A. QCD sum-rule formula

The quasinucleon self-energies in the rest frame can be obtained in QCD sum rules by taking the ratios Eq. (35)/Eq. (36) and Eq. (37)/Eq. (36) for both the proton and neutron as follows:

$$E_{q,V(I)} \equiv \Sigma_v + M^*_N = \frac{\mathcal{N}^{n,p}(\rho)}{\mathcal{D}^{n,p}(\rho)} = \frac{\bar{\mathcal{B}}[\Pi^{n,p}_s(q_0^2,|\vec{q}|)]+\bar{\mathcal{B}}[\Pi^{n,p}_u(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi^{n,p}_q(q_0^2,|\vec{q}|)]}, \tag{103}$$

where subscripts q, $V(I)$ are meant to represent the potential part of Eq. (16) in the asymmetric nuclear matter. To discuss different approximations of self-energies in terms of the density and the asymmetric factor, we introduce the following symbols $\mathcal{N}^{n,p}_{(\rho^m,I^l)}(\rho)$ and $\mathcal{D}^{n,p}_{(\rho^m,I^l)}(\rho)$:

$$\mathcal{N}^{n,p}(\rho) = \mathcal{N}^{n,p}_{(\rho^0,I^0)}+\mathcal{N}^{n,p}_{(\rho,I^0)}\rho+\left[\mathcal{N}^{n,p}_{(\rho,I)}\rho\right]I + \sum_2^m\sum_2^l\left[\mathcal{N}^{n,p}_{(\rho^m,I^l)}\rho^m\right]I^l, \tag{104}$$

$$\mathcal{D}^{n,p}(\rho) = \mathcal{D}^{n,p}_{(\rho^0,I^0)} + \mathcal{D}^{n,p}_{(\rho,I^0)}\rho + \left[\mathcal{D}^{n,p}_{(\rho,I)}\rho\right]I + \sum_{2}^{m}\sum_{2}^{l}\left[\mathcal{D}^{n,p}_{(\rho^m,I^l)}\rho^m\right]I^l, \tag{105}$$

where the superscripts n and p represent either the neutron or the proton, respectively. For the pair of subscripts (ρ^m, I^l), the first index represents the order of the density, while the second index represents the isospin. Due to isospin symmetry, the isoscalar terms have the following relations:

$$\mathcal{N}^n_{(\rho^m,I^l)} = (-1)^l\mathcal{N}^p_{(\rho^m,I^l)}, \tag{106}$$

$$\mathcal{D}^n_{(\rho^m,I^l)} = (-1)^l\mathcal{D}^p_{(\rho^m,I^l)}, \tag{107}$$

where l is the integer for the order of the isospin. All these terms are summarized in Appendix D.

Because the dominant term of $\mathcal{D}^{n,p}(\rho)$ is $\mathcal{D}^{n,p}_{(\rho^0,I^0)}$, one can expand the denominator in terms of $(1/\mathcal{D}^{n,p}_{(\rho^0,I^0)})$ times condensate. After rewriting this with powers of ρ and I, one can express the potential part of a single nucleon energy as

$$E_V^{n,p}(\rho, I) = E^{n,p}_{V,(\rho^0,I^0)} + \sum_{k=1}^{\infty}\sum_{i=0}^{\infty}\left(\left[E^{n,p}_{V,(\rho^k,I^i)}\rho^k\right]I^i\right), \tag{108}$$

where $E^{n,p}_{V,(\rho^k,I^i)}$ are written in terms of $\mathcal{N}^{n,p}(\rho)$ and $\mathcal{D}^{n,p}(\rho)$. Averaging Eq. (108) as Eq. (8) and collecting terms of I^2 from Eq. (2), one can extract $E_V^{\mathrm{sym}}(\rho)$ as follows:

$$\begin{aligned} E_V^{\mathrm{sym}}(\rho) = \tfrac{1}{2}\Big[&\tfrac{1}{2}\rho\left(E^n_{V,(\rho,I)} - E^p_{V,(\rho,I)}\right) \\ &+ \tfrac{1}{3}\rho^2\left(E^n_{V,(\rho^2,I)} - E^p_{V,(\rho^2,I)}\right) \\ &+ \tfrac{1}{4}\rho^3\left(E^n_{V,(\rho^3,I)} - E^p_{V,(\rho^3,I)}\right) + \cdots\Big] \\ &+ \tfrac{1}{2}\Big[\tfrac{1}{3}\rho^2\left(E^n_{V,(\rho^2,I^2)} + E^p_{V,(\rho^2,I^2)}\right) \\ &+ \tfrac{1}{4}\rho^3\left(E^n_{V,(\rho^3,I^2)} + E^p_{V,(\rho^3,I^2)}\right) + \cdots\Big]. \end{aligned} \tag{109}$$

For terms linear in density, one can see that the first term in the upper bracket of Eq. (109) corresponds to the form given in Eq. (9). The explicit expression in terms of $\mathcal{N}^n_{(\rho^m,I^l)}$ and $\mathcal{D}^n_{(\rho^m,I^l)}$ is

$$E^{\mathrm{sym}}_{V,\rho} = \frac{1}{4}\rho\left[\frac{1}{\mathcal{D}^p_{(\rho^0,I^0)}}\left(-2\mathcal{N}^p_{(\rho,I)}\right) - \frac{\mathcal{N}^p_{(\rho^0,I^0)}}{(\mathcal{D}^p_{(\rho^0,I^0)})^2}\left(-2\mathcal{D}^p_{(\rho,I)}\right)\right], \tag{110}$$

valid to leading order in density.

When higher density dependence of the condensates is calculated, Eq. (109) provides a systematic expression of $E_V^{\mathrm{sym}}(\rho)$ that includes higher $\rho^{n\geqslant 2}$ terms.

B. Sum-rule analysis

In principle, a physical quantity extracted from the QCD sum rule should not depend on the Borel parameter M^2. However, since we truncate the OPE at finite mass dimension, such a physical quantity should be obtained within a reliable range of M^2 (Borel window) with a "plateau." While we do not find the most stable "plateau" with an extremum in the appropriate Borel window, one finds that the results have only a weak dependence on M^2.

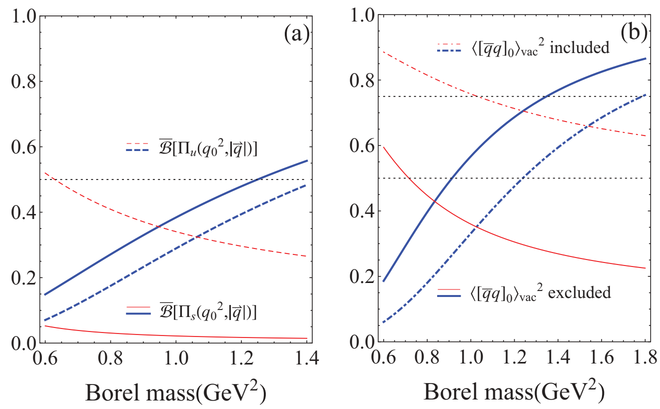

FIG. 1. (Color online) Borel window for (a) $\bar{\mathcal{B}}[\Pi_s(q_0^2, |\vec{q}|)]$ and $\bar{\mathcal{B}}[\Pi_u(q_0^2, |\vec{q}|)]$ and (b) $\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)]$. In both figures, the thick lines increasing with the Borel mass represent the ratio (the contribution of highest dimensional operators)/(the total OPE), and the thin lines decreasing with the Borel mass represent the ratio (the continuum contribution)/(total contribution). These graphs are obtained with T^is in the $K_u^1 = K_{ud}^1(\beta + 1)/\beta$ estimation from the first set of Table II.

The well-accepted Borel window for the nucleon sum rule is 0.8 GeV$^2 \leqslant M^2 \leqslant$ 1.4 GeV^2 [46]. But as our sum rule contains the newly added twist-4 four-quark operators, the Borel window needs to be re-examined. We determine the upper Borel window by requiring that the quasinucleon contribution is more than 50% of the total sum rule so the continuum contribution is less than 50%. As for the lower limit, for the same OPE, we restrict the contribution from the highest mass dimension operator to be less than 50% of the total contribution. For the quasinucleon energy in medium rest frame, we applied this prescription to the right-hand side of Eqs. (35)–(37).

The Borel curves for the three invariants [Eqs. (35)–(37)] are plotted in Fig. 1. Here all the graphs are obtained with the T^is using the $K_u^1 = K_{ud}^1(\beta + 1)/\beta$ estimates from the first set of Table II. From Fig. 1(a), one can get acceptable Borel windows for $\bar{\mathcal{B}}[\Pi_s(q_0^2, |\vec{q}|)]$ [Eq. (35)] and $\bar{\mathcal{B}}[\Pi_u(q_0^2, |\vec{q}|)]$ [Eq. (37)]. However, in Fig. 1(b), $\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)]$ do not provide an acceptable Borel window. While the usual Borel window is obtained by requiring that the power and continuum corrections are both less than 50% of the total OPE, we will loosen the condition to be less than 75% in this case.

This large power correction may be caused by an overestimated $\langle[\bar{q}q]_0\rangle^2_{\mathrm{vac}}$. As mentioned in the previous section, all the vacuum expectation values of four-quark operators are factorized as in Eq. (92). Only large N_c supports factorization in the vacuum. Hence, the generalization to the nuclear medium can be only an order of magnitude estimate with large uncertainty. For example, as one can see in Fig. 1(b), the lower and upper boundaries from $\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)]$ are already largely affected by whether the vacuum value $\langle[\bar{q}q]_0\rangle^2_{\mathrm{vac}}$ is included. Another reason for the larger uncertainty could be the neglected twist-4 matrix elements T^3_{ud} and T^4_{ud} for $(\bar{u}\gamma^\alpha\gamma_5 u)(\bar{d}\gamma^\beta\gamma_5 d)|_{\mathrm{s,t}}$ and $(\bar{u}\gamma^\alpha u)(\bar{d}\gamma^\beta d)|_{\mathrm{s,t}}$. If the vacuum expectation value of four-quark operators as well as the T^3_{ud} and T^4_{ud} can be determined well, we can discuss about the stability of our sum rule in a more reliable way. The second set of T^is from Table II do not produce any acceptable Borel window. In conclusion, we will use the results from the following Borel window: 1.0 GeV$^2 \leqslant M^2 \leqslant 1.2$ GeV^2.

In the analysis to follow, for the symmetric nuclear matter case, we will denote the twist-4 condensates contribution to the quasinucleon self-energy as Σ_T and the total quasinucleon self-energy in the rest frame as $E_{q,V(I=0)}$. For the asymmetric nuclear matter case, we will use two sum rules for $E_V^{\rm sym}$: one that includes contributions up to order ρ terms and another one up to ρ^2. The former sum rule will be called the linear ρ sum rule ($E^{\rm sym}_{V,\rho}$) and the latter the ρ^2 sum rule ($E^{\rm sym}_{V,\rho^2}$). As for the value for the antinucleon pole, an optimal "in-medium" value ranged -0.2 GeV $\leqslant \bar{E}_q \leqslant -0.4$ GeV will be used for each different estimation of twist-4 matrix elements in the sum rule for the quasinucleon self-energy, while the "bare" value $\bar{E}_q = -M_N$ will be used in the sum rule for the nuclear symmetry energy. This is so because the quasihole contribution in the nuclear symmetry energy comes with a term proportional to the density [Eq. (8)]. Nuclear matter density ρ is set at the saturation density $\rho_0 = 0.16$ fm^{-3} and the corresponding quasinucleon three-momentum $|\vec{q}|$ is taken to be 270 MeV, the Fermi momentum of a normal nucleus ($\rho_0 = 0.16$ fm^{-3}). The light quark (u, d quark) mass m_q is taken to be 5 MeV.

As for the density dependence of dimension-six spin-0 condensates, different f values are used for every estimation of twist-4 matrix elements. For the first set of Table II; $f = -0.2$ for $K^1_u = K^1_{ud}/\beta$ (corresponding $\bar{E}_q = -0.26$ GeV), $f = -0.12$ (corresponding $\bar{E}_q = -0.30$ GeV) for $K^1_u = K^1_{ud}(\beta + 1)/\beta$, and $f = -0.08$ for $K^1_u = K^1_{ud}$ (corresponding $\bar{E}_q = -0.34$ GeV). This parameter set of f's are chosen to satisfy the self-consistency constraint as given in Eq. (17) for the quasihole value. Again, the second set in Table II does not provide a set of f's which satisfies the constraint of Eq. (17). A detailed discussion for related parameters (f and $\bar{E}_q$) will be given in a later section.

1. Symmetric nuclear matter

First, we investigate the quasinucleon self-energies in the symmetric nuclear matter with twist-4 condensates. Throughout the analysis, we check the result against the $I = 0$ case. In Fig. 2, we plot the ratio to the nucleon mass in a vacuum of the in-medium scalar self-energy (M^*_N/M_N), the vector self-energy (Σ_v/M_N), the twist-4 condensate contribution (Σ_T/M_N), and the potential part of the total quasinucleon self-energy in the rest frame [$E_{q,V(I=0)}/M_N$]. For the twist-4 matrix elements, we take $K^1_u = K^1_{ud}(\beta + 1)/\beta$ from the first set in Table II and the corresponding $\bar{E}_q = -0.30$ GeV, which gives the average result. From our analysis, we find that the contribution of the the twist-4 condensates give enhancement of the quasinucleon self-energy by $\sim$50 MeV. When $f = 0$, we find the ratio $E_{q,V(I=0)}/M_N \simeq 0.96$, $M^*_N/M_N \simeq 0.58$, and $\Sigma_v/M_N \simeq 0.37$. By using the aforementioned parameter set for $f < 0$ and $\bar{E}_q$, the ratios become $E_{q,V(I=0)}/M_N \simeq 0.87$, $M^*_N/M_N \simeq 0.56$, and $\Sigma_v/M_N \simeq 0.30$, which are comparable with previous studies [15–17]. When the second set of Table II is used for the twist-4 matrix elements, we do not find a stable behavior in the same Borel window, in contrast to the case with the first set as shown previously. By setting $f > 0$ for the second set, $E_{q,V(I=0)}/M_N$ can be adjusted to $\sim$0.9, which is a typically acceptable value. But even so, there is no reasonable f and $\bar{E}_q$ for M^*_N/M_N and Σ_v/M_N which satisfies Eq. (17). The estimates for T^is given in the second set of Table II do not reproduce the aspect of the nucleon sum rule that is consistent with the Dirac phenomenology [15]. Hence, we will continue the present analysis with estimates for T^is given by the the first set in Table II.

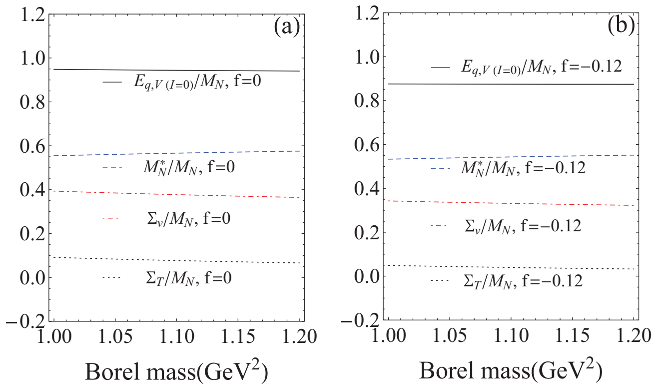

FIG. 2. (Color online) The ratios between quasinucleon self-energies and the vacuum mass. The different lines represent [M^*_N/M_N (dashed blue), Σ_v/M_N (dot-dashed red), Σ_T/M_N (dotted black), and $E_{q,V(I=0)}/M_N$ (solid black)], respectively.

The quasinucleon three-momentum dependence is plotted in Fig. 3(a) for the $f < 0$ case; one finds that the ratios Σ_v/M_N and Σ_T/M_N do not depend strongly on the quasinucleon three-momentum. On the other hand, M^*_N/M_N shows a significant change when $|\vec{q}| \geqslant 0.5$ GeV, as in Ref. [18]. So this sum-rule analysis works in the $0 \leqslant |\vec{q}| \leqslant 0.5$ GeV region, which is consistent with our phenomenological ansatz that assumes a momentum-independent self-energy.

As all the condensates in our nucleon sum rule are estimated to linear order in density, the results may be valid at least near the nuclear saturation density region. In Fig. 3(b), the density dependence of the quasinucleon self-energies is plotted for

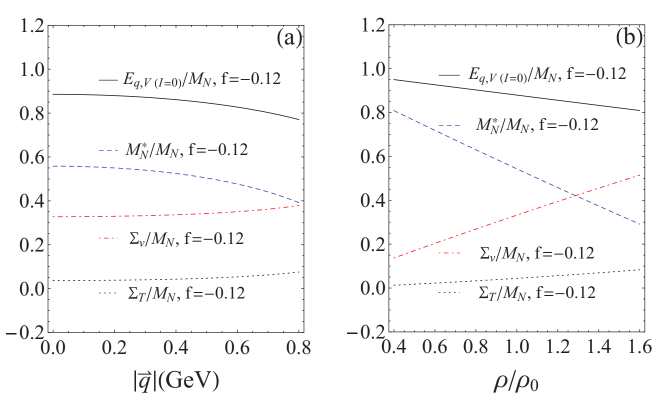

FIG. 3. (Color online) (a) $|\vec{q}|$ and (b) density dependence of the ratios between quasinucleon self-energies and the vacuum mass.

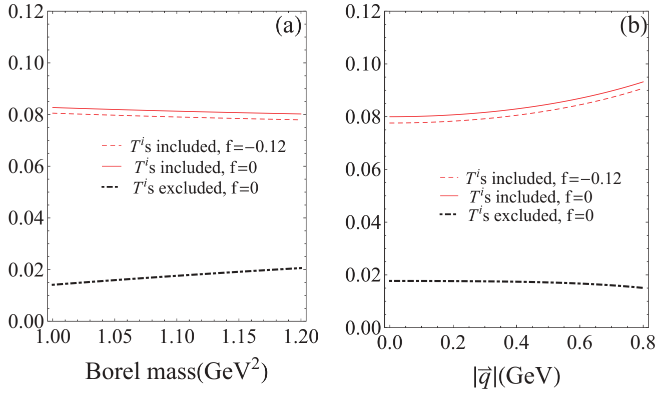

FIG. 4. (Color online) (a) Borel mass and (b) $|\vec{q}|$ dependence of $E^{\rm sym}_{V,\rho}$. The unit of the vertical axis is GeV.

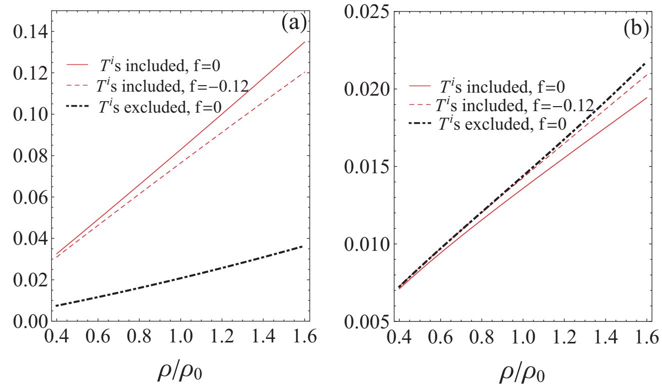

FIG. 5. (Color online) Density dependence of (a) $E^{\rm sym}_{V,\rho^2}$ and (b) $E^{\rm sym}_K$. The unit of the vertical axis is GeV.

$0.4 \leqslant \rho/\rho_0 \leqslant 1.6$. Here we used the parameter set $f = -0.12$ and $\bar{E}_q = -0.30$ GeV determined at the saturation density, as our nucleon sum rule does not depend strongly on $\bar{E}_q$ as long as it is varied within -0.6 GeV $\leqslant \bar{E}_q \leqslant -0.3$ GeV, which covers the naive estimates for $\bar{E}_q$ when $0.4 \leqslant \rho/\rho_0 \leqslant 1.6$. One also notes that the magnitude of both Σ_v/M_N and Σ_T/M_N increases with density while M_N^*/M_N reduces.

2. *Asymmetric nuclear matter*

In our sum rule, the nuclear bulk properties in the asymmetric nuclear matter are parameterized by the asymmetry factor I. If one plots the quasinucleon self-energy as a function of I to leading order in density, $E^{\rm sym}_{V,\rho}$ can be obtained from the difference between the slopes of the quasineutron and the quasiproton [Eq. (110)].

$E^{\rm sym}_{V,\rho}$ is plotted in Fig. 4. One finds that $E^{\rm sym}_{V,\rho}$ ranges from 15 to 80 MeV, which agrees with previous studies. The results in the figure also show that including the twist-4 contribution enhances the nuclear symmetry energy.

In Fig. 4(b), one finds that $E^{\rm sym}_{V,\rho}$ do not depend strongly on the quasinucleon three-momentum up to 0.5 GeV. This result agrees with the quasinucleon three-momentum dependence of the quasinucleon self-energy. When the second set of Table II is used for the T^is, we find that $E^{\rm sym}_{V,\rho}$ depends strongly on the quasinucleon three-momentum compared to the case when the first set is used.

One can also work out $E^{\rm sym}_{V,\rho^2}$, although with larger uncertainty than that for the $E^{\rm sym}_{V,\rho}$. The density dependence of $E^{\rm sym}_{V,\rho^2}$ and $E^{\rm sym}_K$ for $0.4 \leqslant \rho/\rho_0 \leqslant 1.6$ are plotted in Fig. 5. Here again, the four-quark condensates contribute nontrivially to the density behavior of $E^{\rm sym}_V$. For $f = 0$, the contribution of T^is gives enhancement to $E^{\rm sym}_{V,\rho^2}$ at higher nuclear density while for $f = -0.12$, it gives reduction to $E^{\rm sym}_{V,\rho^2}$ at higher density. This means that the scalar four-quark operators contribute importantly in providing attraction to the nuclear symmetry energy. However, $E^{\rm sym}_K$ is slightly reduced by T^is as the twist-4 matrix elements enhance M_N^*/M_N. The parameter set with $f < 0$ contributes differently to $E^{\rm sym}_V$ and $E^{\rm sym}_K$; reduction for $E^{\rm sym}_V$ and enhancement for $E^{\rm sym}_K$ for $0.4 \leqslant \rho/\rho_0 \leqslant 1.6$.

In Fig. 6, we plot the scalar $\left(\frac{\bar{\mathcal{B}}[\Pi_s(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]}\right)$ and vector $\left(\frac{\bar{\mathcal{B}}[\Pi_u(q_0^2,|\vec{q}|)]}{\bar{\mathcal{B}}[\Pi_q(q_0^2,|\vec{q}|)]}\right)$ self-energy part of $E^{\rm sym}_V$. In Fig. 6(a), we plot the result without the twist-4 contribution, while in Fig. 6(b), we include the contribution from twist-4 matrix elements. While both the scalar and vector self-energy give weak contribution to the self-energy in Fig. 6(a), one finds that in Fig. 6(b), the scalar and vector give enhanced negative and positive contributions, respectively. The result shown in Fig. 6(b) is consistent with the general trends in RMFT results [47], which show that the scalar self-energy part gives a negative contribution and the vector self-energy part gives a positive contribution from the exchange of δ and ρ meson exchanges, respectively. One can infer from this result that the twist-4 contribution mimics the exchange of the δ and ρ meson and that it constitutes an essential part in the origin of the nuclear symmetry energy from QCD.

3. *Uncertainties*

In general, there are two quark-gluon mixed operators with contracted spin indices, $\langle [g_s\bar{q}\sigma\mathcal{G}q]_0\rangle_p$ and $\langle [g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$, which are not accurately determined. $\langle [g_s\bar{q}\sigma\mathcal{G}q]_0\rangle_p$ does not appear in our sum rule with Ioffe's nucleon interpolating current [Eq. (19)]. As for the operator $\langle [g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$,

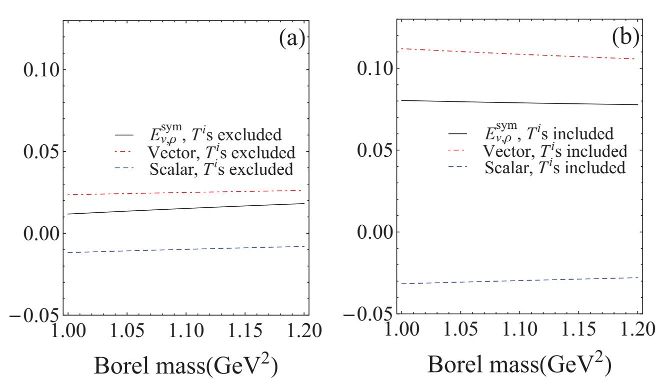

FIG. 6. (Color online) Scalar-vector self-energy decomposition of $E^{\rm sym}_{V,\rho}$ (a) without twist-4 contribution and (b) with twist-4 contribution. The unit of the vertical axis is GeV.

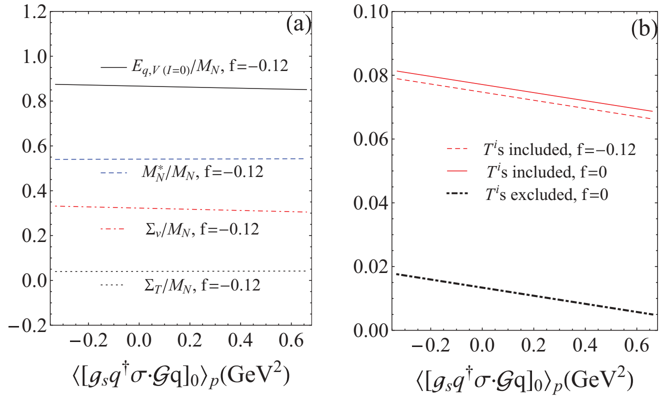

FIG. 7. (Color online) Sensitivity analysis under variation of the matrix element $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$ on (a) $E_{q,V(I=0)}$ and on (b) $E_{V,\rho}^{\rm sym}$. The unit of the vertical axis for the right figure is GeV.

the proton expectation value has been estimated in Refs. [17,18,24,48] to be in the range of $-0.33\ \mathrm{GeV}^2 \leqslant \langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p \leqslant 0.66\ \mathrm{GeV}^2$. Hence, we investigate the $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$ dependence in Fig. 7. The matrix element $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$ does not give an important contribution to the quasinucleon self-energies in the range $-0.33\ \mathrm{GeV}^2 \leqslant \langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p \leqslant 0.66\ \mathrm{GeV}^2$ as we are not interested in the accuracy of 10 MeV. However, such a magnitude in $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p$ gives nontrivial fractional change to $E_{V,\rho}^{\rm sym}$. We choose the value as $\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p = -0.33\ \mathrm{GeV}^2$ in this study as was done in Refs. [17,18].

In our analysis, we fixed the σ term to be $\sigma_N = 45$ MeV. In Fig. 8(a) we show that changing this number from $30\ \mathrm{MeV} \leqslant \sigma_N \leqslant 70\ \mathrm{MeV}$ changes $E_{V,\rho}^{\rm sym}$ by less than 5%. Also, in principle, the density dependence of the operators could also induce changes in the continuum. To investigate this possibility, we have allowed the continuum to vary $1.4\ \mathrm{GeV} \leqslant \omega_0 \leqslant 1.6\ \mathrm{GeV}$. As can be seen in the three lines in Fig. 8(b), the change in the symmetry energy is less than 10%. This suggest that reasonable density dependence will not appreciably modify the current result.

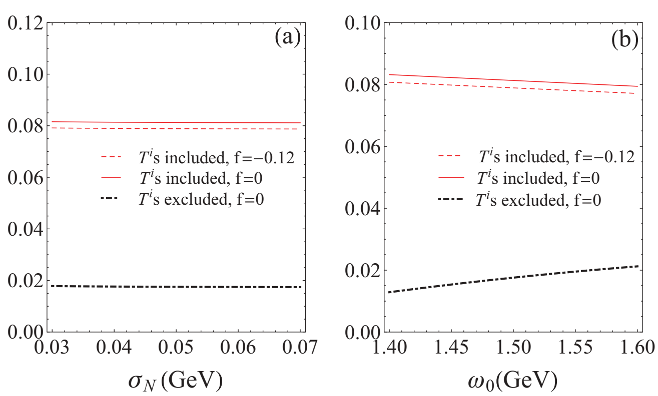

FIG. 8. (Color online) Variation of (a) $E_{q,V(I=0)}$ under change in σ_N and (b) $E_{V,\rho}^{\rm sym}$ under change in ω_0. The unit of the vertical axis is GeV.

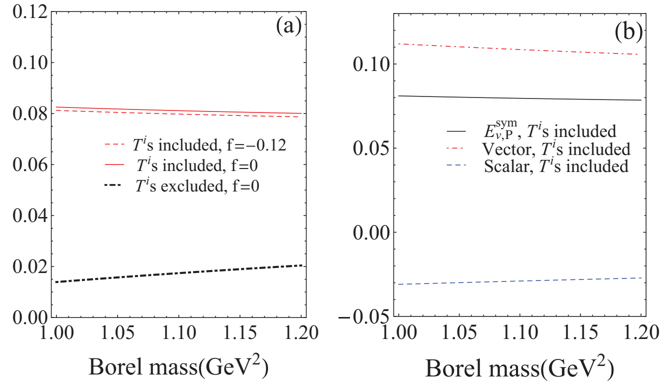

FIG. 9. (Color online) $E_{V,\rho}^{\rm sym}$ in which $\langle[\bar{q}q]_1\rangle_p$ is replaced with $\frac{1}{2}\zeta_N$ from PCQM [49]. (a) $E_{V,\rho}^{\rm sym}$ with $\zeta_N = 0.54$ and (b) scalar-vector decomposition of $E_{V,\rho}^{\rm sym}$ with $\zeta_N = 0.54$ (including T^is). The unit of the vertical axis is GeV.

4. Comparison with the result from PCQM

As mentioned in the Introduction, there were early studies about nucleon sum rule in the asymmetric nuclear matter using the first approach [20,21,43]. In comparison with this study, the two main differences are the followings. First, in Refs. [20,21,43], the OPE expansion was performed in the light cone direction and the q^2 dispersion relation was used. On the other hand, in this work, the OPE is a short distance expansion and the energy dispersion relation is used; consequently, the OPE totally differs. Second, in Refs. [20,21,43] $\langle[\bar{q}q]_1\rangle_p$ is obtained from PCQM [20,49] while we calculate $\langle[\bar{q}q]_1\rangle_p$ from the leading chiral expansion [34,50]. Because the OPEs totally differ, it is impossible to compare both results term by term in terms of the QCD condensates, but here we can compare the final results in $E_{V,\rho}^{\rm sym}$. From a phenomenological aspect, Refs. [20,21] give values for the nuclear symmetry energy, $E_V^{\rm sym} + E_K^{\rm sym} = 29$ MeV, which almost agrees with the phenomenological estimates. As one can check in Fig. 9, using the same values for $\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}\zeta_N$ as estimated from the PCQM [49], we find $E_{V,\rho}^{\rm sym} \sim 80$ MeV, similar to previous estimate. However, in our approach, we find interesting similarities with the main results from RMFT [47], namely strong vector repulsion and scalar attraction.

V. CONCLUSION

In this paper we studied the nuclear bulk properties in asymmetric nuclear matter by calculating the quasinucleon self-energies with QCD sum-rule approach. In particular, we identified all the twist-4 local condensates appearing in the nucleon sum rule. Using the existing estimates for the twist-4 matrix element from DIS, we were able to find the magnitudes of all the twist-4 matrix elements (T^i) in our sum rule except for two mixed-quark-flavor-type condensates. We have calculated the nuclear symmetry energy and found that twist-4 contributions are non-negligible and essential to give a phenomenologically consistent result with RMFT for the quasinucleon self-energy and the nuclear symmetry energy.

For the symmetric nuclear matter case, we found that $E_{q,V(I=0)}$ is enhanced by ~ 50 MeV with T^is in the first set of Table II. Because the T^is in the first set of Table II provides qualitatively reliable sum-rule results while T^is in the second set of Table II do not, we conclude that taking the sum-rule results with T^is in the first set is the reasonable choice. With parameter set $f < 0$, dimension-six spin-0 (scalar) operators reduces $E_{q,V(I=0)}/M_N$ to $\sim$0.87.

For the asymmetric nuclear matter case, we confirmed two meaningful facts. First, the QCD sum-rule technique can be used to successfully reproduce the acceptable result for the nuclear symmetry energy at the nuclear matter density. Second, dimension-six spin-2 (twist-4) condensates play important roles in making the scalar part contribute negatively to the self-energy and, thus, providing a consistent picture for the $E^{\rm sym}$ with the RMFT results [47],

$$E_V^{\rm sym} = \frac{1}{2}\left[f_\rho - f_\delta\left(\frac{m^*}{E_F^*}\right)\right]\rho_B, \tag{111}$$

where f_ρ is the isovector ρ meson coupling, f_δ the isoscalar δ (f_0) coupling, and ρ_B the nuclear matter density. Moreover, our approach provides a first attempt to understanding the origin of $E_{V,\rho}^{\rm sym}$ in terms of local operators directly from QCD. This extends the analogy between QCD sum rules to RMFT for the symmetric nuclear matter established in Refs. [15,17,18] to the asymmetric limit.

While the uncertainties in T^is and in the four-quark scalar operators with the f parametrization are still large, attempts to measure the twist-4 contribution in DIS at the future upgrade at Jefferson Lab is expected to lower the uncertainties and provide more insights to the value for the nuclear expectation value of the four-quark operators.

ACKNOWLEDGMENTS

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APPENDIX A: BARYON OCTET MASS RELATION

In this section, we summarize an essential argument for obtaining $\langle[\bar{q}q]_1\rangle_p = \frac{1}{2}(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle)$ from Refs. [34,50]. Phenomenologically, the nucleon mass can be expressed in terms of the matrix element of the trace of the energy-momentum tensor,

$$m_N\langle N|\bar{\psi}_N\psi_N|N\rangle = \langle N|\theta_\mu^\mu|N\rangle. \tag{A1}$$

Using the equations of motion, the trace of the energy-momentum tensor can be written as

$$\begin{aligned}\theta_\mu^\mu &= m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + \sum_{h=c,t,b} m_h\bar{h}h + \cdots \\ &= \left(\frac{\tilde{\beta}}{4\alpha_s}\right)G^2 + m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s + O(\mu^2/4m_h^2),\end{aligned} \tag{A2}$$

where the h's are the heavy quark fields and the gluonic term comes from the trace anomaly [51–53]. $\tilde{\beta} = -9\alpha_s^2/2\pi$ is the "reduced" Gellmann-Low function in which heavy quark contribution has been subtracted out using the heavy quark expansion [54].

Equation (A2) can be applied to the lowest-lying baryon octet. The baryon octet mass relations to first order in SU(3) flavor symmetry breaking are as follows:

$$\begin{aligned} m_p &= A + m_uB_u + m_dB_d + m_sB_s,\\ m_n &= A + m_uB_d + m_dB_u + m_sB_s,\\ m_{\Sigma^+} &= A + m_uB_u + m_dB_s + m_sB_d,\\ m_{\Sigma^-} &= A + m_uB_s + m_dB_u + m_sB_d,\\ m_{\Xi^0} &= A + m_uB_d + m_dB_s + m_sB_u,\\ m_{\Xi^-} &= A + m_uB_s + m_dB_d + m_sB_u,\end{aligned} \tag{A3}$$

where $A \equiv \langle(\tilde{\beta}/4\alpha_s)G^2\rangle_p$, $B_u \equiv \langle\bar{u}u\rangle_p$, $B_d \equiv \langle\bar{d}d\rangle_p$, and $B_s \equiv \langle\bar{s}s\rangle_p$. In this relation, correction terms for hyperon is neglected [55]. From Eq. (A3) one can obtain

$$\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle = \frac{(m_{\Xi^0}+m_{\Xi^-}) - (m_{\Sigma^+}+m_{\Sigma^-})}{2m_s - (m_u+m_d)}. \tag{A4}$$

APPENDIX B: A SIMPLE CONSTRAINT FOR TWIST-4 OPERATORS FROM ZERO IDENTITY

In this section, we show an explicit calculation for a simple constraint using the zero identity [44]. For the single quark flavor diquark structure,

$$\epsilon_{abc}\big(u_a^TC\Gamma u_b\big) = 0, \quad \text{if} \quad (C\Gamma)^T = -C\Gamma, \tag{B1}$$

where $(\Gamma = \{I, \gamma_5, i\gamma_\mu\gamma_5\})$ satisfies the above condition. Therefore, constraints for the four-quark operator can be obtained by requiring that the Fierz transformed form of the products of above diquarks are zero. An example is the following:

$$\begin{aligned}&\epsilon_{abc}\epsilon_{a'b'c}\big(u_a^TC\gamma_\mu\gamma_5u_b\big)\big(\bar{u}_{b'}\gamma_\nu\gamma_5C\bar{u}_{a'}^T\big)\\ &\quad= \epsilon_{abc}\epsilon_{a'b'c}\tfrac{1}{16}(\bar{u}_{a'}\Gamma^ou_a)(\bar{u}_{b'}\Gamma^ku_b)\mathrm{Tr}\big[\gamma_\mu\Gamma_k\gamma_\nu C\Gamma_o^TC\big]\\ &\quad= \epsilon_{abc}\epsilon_{a'b'c}\tfrac{1}{16}\big\{(\bar{u}_{a'}u_a)(\bar{u}_{b'}u_b)(4g_{\mu\nu})\\ &\qquad+(\bar{u}_{a'}\gamma_5u_a)(\bar{u}_{b'}\gamma_5u_b)(-4g_{\mu\nu})\\ &\qquad+(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta u_b)(4S_{\mu\beta\nu\alpha})\\ &\qquad-(\bar{u}_{a'}\gamma^\alpha\gamma_5u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5u_b)(4S_{\mu\beta\nu\alpha})\\ &\qquad-(\bar{u}_{a'}\sigma^{\alpha\bar{\alpha}}u_a)(\bar{u}_{b'}\sigma^{\beta\bar{\beta}}u_b)\tfrac{1}{4}\mathrm{Tr}[\gamma_\mu\sigma_{\beta\bar{\beta}}\gamma_\nu\sigma_{\alpha\bar{\alpha}}]\\ &\qquad+(\bar{u}_{a'}\gamma^\alpha u_a)(\bar{u}_{b'}\gamma^\beta\gamma_5u_b)(8i\epsilon_{\mu\alpha\nu\alpha})\\ &\qquad+(\bar{u}_{a'}u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)(8ig_{\alpha\mu}g_{\bar{\alpha}\nu})\\ &\qquad+(\bar{u}_{a'}\gamma_5u_a)(\bar{u}_{b'}\sigma^{\alpha\bar{\alpha}}u_b)(4\epsilon_{\mu\nu\alpha\bar{\alpha}})\big\} = 0,\end{aligned} \tag{B2}$$

where $S_{\mu\alpha\nu\beta} = g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\alpha\nu} - g_{\mu\nu}g_{\alpha\beta}$. By subtracting Eq. (B2) from Eq. (84), Eq. (84) can be simplified into Eq. (85).

TABLE III. Table for K_u^i from Ref. [30]. Units are in GeV^2.

K_u^1	K_u^2	K_u^1	K_u^2
$K_{ud}^1/\beta = -0.173$	0.203	$-K_{ud}^1 = 0.083$	−0.181
$K_{ud}^1(\beta+1)/2\beta = -0.112$	0.110	$-K_{ud}^1(\beta+1)/2\beta = 0.112$	−0.225
$K_{ud}^1 = -0.083$	0.066	$-K_{ud}^1/\beta = 0.173$	−0.318

APPENDIX C: ESTIMATION OF TWIST-4 MATRIX ELEMENTS

In this section, we provide a detailed treatment for extracting T^is from the values estimated in Ref. [30]. In Ref. [30], twist-4 operators which appear in our nucleon sum rule are given as

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_u^1 = \langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{u}\gamma_\beta\gamma_5 t^A u)\rangle_p|_{\rm s,t} + \langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{d}\gamma_\beta\gamma_5 t^A d)\rangle_p|_{\rm s,t}, \tag{C1}$$

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_u^2 = \langle(\bar{u}\gamma_\alpha t^A u)(\bar{u}\gamma_\beta t^A u)\rangle_p|_{\rm s,t} + \langle(\bar{u}\gamma_\alpha t^A u)(\bar{d}\gamma_\beta t^A d)\rangle_p|_{\rm s,t}, \tag{C2}$$

$$\frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right)K_{ud}^1 = 2\langle(\bar{u}\gamma_\alpha\gamma_5 t^A u)(\bar{d}\gamma_\beta\gamma_5 t^A d)\rangle_p|_{\rm s,t}, \tag{C3}$$

where we changed the normalization for the nucleon state appearing in Ref. [30] to the following:

$$\langle N(p)|N(p')\rangle = \frac{\omega_p}{M_N}(2\pi)^3\delta^3(\vec{p}-\vec{p}'), \tag{C4}$$

with $\omega_p = p_0 = \sqrt{\vec{p}^2 + M_N^2}$. Here only K_{ud}^1 is uniquely determined: $K_{ud}^1 = -0.083$ GeV^2. One can set a constraint $|K_d^1| = |K_u^1|\beta < |K_{ud}^1| < |K_u^1|$ with an ansatz that the ratio K_d^i/K_u^i is equal to the momentum fraction of the d and u quarks in the nucleon:

$$K_d^i/K_u^i \simeq \frac{\int x[d(x)+\bar{d}(x)]dx}{\int x[u(x)+\bar{u}(x)]dx} \equiv \beta = 0.476. \tag{C5}$$

Varying K_u^1 with the constraint above, one can estimate K_u^2 as a functions of K_u^1 from the constraints from DIS; the results are given in Table III.

T_{uu}^1 and T_{dd}^1 can be easily estimated by taking $T_{ud}^1 = \frac{1}{2}K_{ud}^1$ from K_u^1 and K_d^1:

$$T_{uu}^1 = K_u^1 - T_{ud}^1, \tag{C6}$$

$$T_{dd}^1 = K_d^1 - T_{ud}^1, \tag{C7}$$

$$T_{ud}^1 = \tfrac{1}{2}K_{ud}^1 = -0.042\ \text{GeV}^2. \tag{C8}$$

Similarly, one can try to obtain T_{uu}^2 and T_{dd}^2 from K_u^2 and K_d^2. As $T_{ud}^2 = \frac{1}{2}K_{ud}^2$ has not been determined uniquely as $T_{ud}^1 = \frac{1}{2}K_{ud}^1$, we assumed that the ratio T_{uu}^1/T_{dd}^1 is equal to T_{uu}^2/T_{dd}^2. Then, by the following relation, one can estimate T_{qq}^2s:

$$T_{uu}^2 = \left(K_u^2 - K_d^2\right)\left(1 - \frac{T_{dd}^1}{T_{uu}^1}\right)^{-1}, \tag{C9}$$

$$T_{dd}^2 = \left(\frac{T_{dd}^1}{T_{uu}^1}\right)T_{uu}^2, \tag{C10}$$

$$T_{ud}^2 = \frac{1}{2}\left(\left[K_u^2 + K_d^2\right] - \left[T_{uu}^2 + T_{dd}^2\right]\right), \tag{C11}$$

where $K_u^2 - K_d^2$ and $K_u^2 + K_d^2$ can be obtained from Table III.

For the single-quark-flavor case, T_{qq}^3 and T_{qq}^4 can be obtained from Eqs. (89) and (90). As discussed in Ref. [45], we neglect $(\bar{u}\sigma_o^{\alpha}u)(\bar{u}\sigma^{o\beta}u)|_{\rm s,t}$. Then T_{qq}^3 and T_{qq}^4 can be related as

$$T_{qq}^3 = -\tfrac{15}{4}T_{qq}^1 + \tfrac{9}{4}T_{qq}^2, \tag{C12}$$

$$T_{qq}^4 = -\tfrac{15}{4}T_{qq}^2 + \tfrac{9}{4}T_{qq}^1. \tag{C13}$$

T_{qq}^is can be classified as the three different classification of K^is given in Table III: $K_u^1 = \{K_{ud}^1/\beta, K_{ud}^1(\beta+1)/2\beta, K_{ud}^1\}$ and $K_u^1 = \{-K_{ud}^1, -K_{ud}^1(\beta+1)/2\beta, -K_{ud}^1/\beta\}$. T_{qq}^i's are classified in Table II according to these three classifications in the two sets.

APPENDIX D: QCD SUM-RULE FORMULAS FOR $E_{q,V(I)}$ AND $E_{V,\rho}^{\rm sym}$

In this section, we provide the detailed description for

$$E_{q,V(I)} = \frac{\mathcal{N}^{n,p}_{(\rho^0,I^0)} + \mathcal{N}^{n,p}_{(\rho,I^0)}\rho + \left[\mathcal{N}^{n,p}_{(\rho,I)}\rho\right]I}{\mathcal{D}^{n,p}_{(\rho^0,I^0)} + \mathcal{D}^{n,p}_{(\rho,I^0)}\rho + \left[\mathcal{D}^{n,p}_{(\rho,I)}\rho\right]I}, \tag{D1}$$

$$E_{V,\rho}^{\rm sym} = \frac{1}{4}\rho\left[\frac{1}{\mathcal{D}^p_{(\rho^0,I^0)}}\left(-2\mathcal{N}^p_{(\rho,I)}\right) - \frac{\mathcal{N}^p_{(\rho^0,I^0)}}{\left(\mathcal{D}^p_{(\rho^0,I^0)}\right)^2}\left(-2\mathcal{D}^p_{(\rho,I)}\right)\right], \tag{D2}$$

in QCD sum-rule formula. In this formula, $\mathcal{N}^p_{(\rho^m,I^l)}$ and $\mathcal{D}^p_{(\rho^m,I^l)}$ are as follows:

$$\mathcal{N}^p_{(\rho^0,I^0)} = -\frac{1}{4\pi^2}(M^2)^2 E_1\langle[\bar{q}q]_0\rangle_{\rm vac}, \tag{D3}$$

$$\begin{aligned}\mathcal{N}^p_{(\rho,I^0)} = &-\frac{1}{4\pi^2}(M^2)^2 E_1\langle[\bar{q}q]_0\rangle_p - \frac{4}{3\pi^2}\vec{q}^2\langle[\bar{q}\{iD_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}}\\ &+\frac{2}{3\pi^2}(M^2)^2\langle[q^\dagger q]_0\rangle_p E_1 L^{-\frac{4}{9}} + \frac{4}{\pi^2}\vec{q}^2\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}} - \frac{1}{12\pi^2}M^2\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_0\rangle_p E_0 L^{-\frac{4}{9}}\\ &+\bar{E}_q\left\{\frac{20}{9\pi^2}M^2\langle[\bar{q}\{\gamma_0 iD_0\}q]_0\rangle_p E_0 L^{-\frac{4}{9}} - \frac{1}{36\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}[(u\cdot G)^2+(u\cdot\tilde{G})^2]\right\rangle_p E_0 L^{-\frac{4}{9}}\right.\\ &\left.+\frac{1}{\pi\alpha_s}\frac{M_N}{2}\left([T^1_{ud}-T^2_{ud}]+[T^1_0-T^2_0]-\frac{1}{3}[T^3_0-T^4_0]\right)L^{-\frac{4}{9}} - \frac{4}{3}\langle\bar{q}q\rangle_{\rm vac}\langle[q^\dagger q]_0\rangle_p\right\},\end{aligned} \tag{D4}$$

$$\begin{aligned}\mathcal{N}^p_{(\rho,I)} = &-\frac{1}{4\pi^2}(M^2)^2 E_1\langle[\bar{q}q]_1\rangle_p - \frac{4}{3\pi^2}\vec{q}^2\langle[\bar{q}\{iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}}\\ &-\frac{1}{2\pi^2}(M^2)^2\langle[q^\dagger q]_1\rangle_p E_1 L^{-\frac{4}{9}} - \frac{2}{\pi^2}\vec{q}^2\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}} + \frac{1}{4\pi^2}M^2\langle[g_s q^\dagger\sigma\cdot\mathcal{G}q]_1\rangle_p E_0 L^{-\frac{4}{9}}\\ &+\bar{E}_q\left\{-\frac{4}{3\pi^2}M^2\langle[\bar{q}\{\gamma_0 iD_0\}q]_1\rangle_p E_0 L^{-\frac{4}{9}} + \frac{1}{\pi\alpha_s}\frac{M_N}{2}\left(-[T^1_1-T^2_1]+\frac{1}{3}[T^3_1-T^4_1]\right) + \frac{4}{3}\langle\bar{q}q\rangle_{\rm vac}\langle[q^\dagger q]_1\rangle_p\right\}L^{-\frac{4}{9}},\end{aligned} \tag{D5}$$

$$\mathcal{D}^p_{(\rho^0,I^0)} = \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rm vac}E_0 L^{-\frac{4}{9}} + \frac{2}{3}\langle[\bar{q}q]_0\rangle^2_{\rm vac}L^{\frac{4}{9}}, \tag{D6}$$

$$\begin{aligned}\mathcal{D}^p_{(\rho,I^0)} = &-\left(\frac{5}{9\pi^2}M^2 E_0 - \frac{8}{9\pi^2}\vec{q}^2\right)\langle[\bar{q}\{\gamma_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}}\\ &+\frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_p E_0 L^{-\frac{4}{9}} + \frac{1}{144\pi^2}(M^2E_0-4\vec{q}^2)\left\langle\frac{\alpha_s}{\pi}[(uG)^2+(u\tilde{G})^2]\right\rangle_p L^{-\frac{4}{9}}\\ &+\frac{4}{3}f\langle\bar{q}q\rangle_{\rm vac}\langle[\bar{q}q]_0\rangle_p L^{\frac{4}{9}} - \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left([T^1_{ud}-T^2_{ud}]+[T^1_0-T^2_0]-\frac{1}{3}[T^3_0-T^4_0]\right)L^{-\frac{4}{9}}\\ &+\bar{E}_q\left\{\frac{1}{3\pi^2}M^2E_0L^{-\frac{4}{9}}\langle[q^\dagger q]_0\rangle_p - \frac{4}{3\pi^2}\left(1-\frac{\vec{q}^2}{M^2}\right)\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0\rangle_{\rho,I}L^{-\frac{4}{9}}\right.\\ &\left.-\frac{2}{3\pi^2}\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_0\rangle_p L^{-\frac{4}{9}} + \frac{1}{18\pi^2}\langle[g_s q^\dagger\sigma\mathcal{G}q]_0\rangle_p L^{-\frac{4}{9}}\right\},\end{aligned} \tag{D7}$$

$$\begin{aligned}\mathcal{D}^p_{(\rho,I)} = &\frac{1}{3\pi^2}M^2E_0\langle[\bar{q}\{\gamma_0 iD_0\}q]_1\rangle_{\rho,I}L^{-\frac{4}{9}}\\ &-\frac{4}{3}f\frac{\mathcal{R}_-(m_q)}{\mathcal{R}_+(m_q)}\langle\bar{q}q\rangle_{\rm vac}\langle[\bar{q}q]_0\rangle_p L^{\frac{4}{9}} - \frac{1}{4\pi\alpha_s}\frac{M_N}{2}\left(-[T^1_1-T^2_1]+\frac{1}{3}[T^3_1-T^4_1]\right)L^{-\frac{4}{9}}\\ &+\bar{E}_q\left\{\frac{2}{3\pi^2}\langle[\bar{q}\{\gamma_0 iD_0 iD_0\}q]_1\rangle_p L^{-\frac{4}{9}} - \frac{1}{18\pi^2}\langle[g_s q^\dagger\sigma\mathcal{G}q]_1\rangle_p L^{-\frac{4}{9}}\right\}.\end{aligned} \tag{D8}$$

APPENDIX E: BOREL TRANSFORMATION

To emphasize the quasinucleon pole, the phenomenological side and the OPE side have to be Borel transformed. The transformation changes the phenomenological side to have the following weighed dispersion relation:

$$\mathcal{B}[\Pi_i(q_0,|\vec{q}|)] = \frac{1}{2\pi i}\int_{-\omega_0}^{\omega_0} d\omega\; W(\omega)\Delta\Pi_i(\omega,|\vec{q}|), \tag{E1}$$

$$W(\omega) = (\omega-\bar{E}_q)e^{-\omega^2/M^2}, \tag{E2}$$

where $\bar{E}_q$ is the quasihole pole which will be assigned to satisfy Eq. (17). The weighting function will de-emphasize the contribution from the quasihole, and the Borel transformation suppress the continuum contribution. Using Eq. (26), the OPE side of the sum rule can be obtained by taking the Borel transformation of $\Pi_i(q_0,|\vec{q}|) = \Pi^E_i(q_0^2,|\vec{q}|) - \bar{E}_q\Pi^O_i(q_0^2,|\vec{q}|)$. Here, we define the differential operator $\mathcal{B}$ for the Borel transformation of the OPE side as

$$\begin{aligned}\mathcal{B}[f(q_0^2,|\vec{q}|)] &\equiv \lim_{\substack{-q_0^2,\,n\to\infty\\ -q_0^2/n=M^2}}\frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n f(q_0^2,|\vec{q}|)\\ &\equiv \hat{f}(M^2,|\vec{q}|),\end{aligned} \tag{E3}$$

where M is the Borel mass [10]. Polynomial terms in the OPE side vanish after the Borel transformation.

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