

Nuclear Drell-Yan effect in a covariant model

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We investigate effects of a nuclear medium on antiquark distribution in nuclei by applying the results of a recently developed relativistically covariant self-consistent model for the pion and the isobar. We take into account Fermi motion, including Pauli blocking and binding effects on the nucleons and medium effects on the isobar and pion, leading to modest enhancement of the pion light-cone-momentum distribution in large nuclei. As a consequence, the Drell-Yan cross-sectional ratio with respect to the deuteron exceeds one only for small values of the light-cone momentum.

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I. INTRODUCTION

Improving our understanding of the quark and gluon degrees of freedom of nucleons bound in nuclei necessitates further efforts in spite of numerous investigations and successes [1]. The planned nuclear Drell-Yan (DY) scattering experiment SeaQuest at Fermilab (E-906) [2–4] strongly motivates recent advancements in analysis of nuclear parton distributions [5] as well as attempts directed at more reliable and accurate calculation of experimentally observable quantities. In line with the second objective is our aim to update and extend to larger x values previous calculations of nuclear DY effect presented in Refs. [6,7]. Furthermore, our physics interest is twofold: (i) to investigate the role of nonperturbative physics (pion cloud) in the $\bar{u} - \bar{d}$ asymmetry in the nucleon and (ii) to study possible antiquark enhancement due to the presence of virtual mesons in the nuclear medium. From symmetry properties of QCD we may infer that the pion is one possible source of nonperturbative quark-antiquark physics, in particular $u - d$ asymmetry.

In conventional models based upon meson exchange, nuclear binding of nuclear matter comes from about 50% from (virtual) pions present in the nucleus. Can we observe these? Some indication for pions is present in the European Muon Collaboration (EMC) effect enhancement around $x = 0.1$, which can be ascribed to the fact that pions (and heavier mesons) carry a fraction of the momentum sum rule. Can one see these pions more explicitly, e.g., in the form of an enhancement of antiquarks in the nucleus? Antiquarks can be probed directly in Drell-Yan scattering but previous experiments [8] within the experimental uncertainty of about 10% did not show a nuclear enhancement; results of calculations varied strongly.

In practice one can distinguish two main types of theoretical interpretations of the classical EMC effect: (i) in terms of nucleon as constituents which are bound but not modified in the medium and (ii) in terms of off-shell nucleons with medium modified structure functions, e.g., through scalar and vector fields acting on the quarks. In the first category one has the nonrelativistic models which use a computed spectral function that accounts for large removal energy (50 MeV

in nuclear matter) and Fermi motion due to correlations. This approach can reproduce the observed slope of the reduction for $x < 0.5$ in the EMC ratio $2F_2^A(x)/AF_2^d(x)$ but not the behavior around the minimum in the ratio at $x = 0.8$. The latter seems to require *ad hoc* off-shell effects [9]. In the second category [10,11] one usually starts from the Walecka model in the mean-field approximation, which has a small net binding effect (8 MeV per nucleon) and hence yields a very small EMC effect, and then one adds the effects of external scalar and vector fields. Since in the present study we are interested in the antiquark distribution for $x < 0.4$ we rely on the conventional convolution approach using a parameterized nucleon distribution, which has the two above mentioned parameters related to the removal energy η and the Fermi momentum.

In Sec. II we study the antiquark distributions in the free nucleon and determine the off-shell πNN and $\pi N\Delta$ form factors which lead to good description of the isovector part of the proton antiquark distribution by the pion cloud. In Sec. III we turn to general discussion of the nuclear effects consisting of binding and Fermi motion of nucleons and modification of the pion cloud. Detailed consideration of the medium effects on the nucleon's pion cloud follows in Sec. IV where expressions are derived for the pion light-cone distributions originating from the πN and $\pi\Delta$ states. We emphasize the careful treatment of the in-medium Δ baryon based on a complete relativistically covariant basis for its dressed propagator. Numerical results for the in-medium pion distribution and DY cross-section ratios for nuclear targets relative to the deuteron are presented and discussed in Sec. V. Finally, Sec. VI contains a summary of our results.

II. ANTIQUARKS IN FREE NUCLEONS

Before turning to the nuclear case we investigate whether the pion cloud approach we will use in the medium can reproduce the observed flavor asymmetry in the free nucleon. The distribution of antiquarks in the nucleon can be decomposed into a flavor-symmetric isoscalar part (originating from gluon splitting and possibly meson cloud) and a nonperturbative isovector meson-cloud contribution. The latter can be

considered to be the source of the $\bar{u} - \bar{d}$ asymmetry. In addition to the pion cloud of the nucleon we include also the isobar with its pion cloud since it was shown to give substantial contributions [12]. In this way one can constrain the πNN and $\pi N\Delta$ form factors using the empirical antiquark $u - d$ flavor asymmetry. The physical free nucleon state is expressed approximately as

$$|N\rangle = \sqrt{Z}|N\rangle_{\text{bare}} + \alpha|N\pi\rangle + \beta|\Delta\pi\rangle. \quad (1)$$

By neglecting off-shell effects, the light-cone momentum distribution of a quark with flavor f in a proton can be written as ($B = N, \Delta$)

$$q_f(x) = Zq_{f,\text{bare}}(x) + \sum_{B,i} c_i \left[\int_x^1 \frac{dy}{y} f^{B_i/N}(y) q_{f,\text{bare}}^{B_i}(x/y) + \int_x^1 \frac{dy}{y} f^{\pi_i/N}(y) q^{\pi_i}(x/y) \right], \quad (2)$$

where c_i (i labels the charge states) are the appropriate isospin Clebsch-Gordan coefficients, $q_{f,\text{bare}}^{B_i}(x)$ is the parton distribution in the bare B_i baryon, and $q^{\pi_i}(x)$ is the pion parton distribution function.

Attributing the asymmetry in the \bar{u} and \bar{d} antiquark distributions to the nucleon meson cloud we are concerned with the pion light-cone distribution in the nucleon, which gets contributions from final states with either nucleon or isobar:

$$f^{\pi/N}(y) = f^{\pi NN}(y) + f^{\pi\Delta/N}(y). \quad (3)$$

The nucleon term was calculated by Sullivan [13]:

$$f^{\pi^0 NN}(y) = \frac{g_{\pi NN}^2}{16\pi^2} y \int_{M^2 y^2(1-y)}^{\infty} dt \frac{|F_{\pi NN}^{(\pi)}(t)|^2 t}{(t + m_\pi^2)^2}, \quad (4)$$

where $y = (k_0 + k_3)/M$ is the pion light-cone momentum fraction, M is the physical mass of the nucleon (as a convenient scale), $F_{\pi NN}^{(\pi)}(t)$ is the off-shell form factor of the πNN vertex, and $g_{\pi NN}$ is the $\pi^0 NN$ coupling. The free-pion propagator, D_π^0 , appears in the above expression in the form $(t + m_\pi^2)^{-1}$, where $t \equiv -q^2$ and where q denotes the pion four-momentum. The isobar contribution also plays an important role [12] despite the kinematical suppression coming from the isobar-nucleon mass difference. In Ref. [12] it was calculated using the free isobar propagator, i.e., neglecting its width. A complete relativistically covariant treatment of the isobar in vacuum and nuclear medium was introduced in Ref. [14] and we use that formalism to take into account the vacuum width, which is consistent with the measured pion-nucleon scattering phase shift in the spin-3/2 isospin-3/2 channel. The full Lorentz structure of the vacuum propagator of the Rarita-Schwinger field can be expressed in terms of 10 Lorentz scalar functions [14,15] which contain both spin-3/2 and spin-1/2 sectors [16]. However, using the convenient basis from Ref. [14] it turns out that a single term, namely the (on-mass-shell) positive-energy spin-3/2 contribution gives the dominant contribution and all others (some terms in the propagator are identically zero) are completely negligible. In the notation of Ref. [14] this is the coefficient of the projector sum $Q_{[11]}^{\mu\nu} \equiv Q_{[11]}^{\mu\nu} + P_{[55]}^{\mu\nu}$, which we denote by $G_{[11]}^{(Q)}(p)$. The pion light-cone distribution

originating from the $\Delta\pi$ state then can be expressed as

$$f^{\pi^- \Delta/N}(y) = \frac{yMg_{\pi N\Delta}^2}{6\pi^3} \int_{-\infty}^{-(My+m_\pi)} dp'_3 \times \int_0^\infty p'_\perp dp'_\perp F_{\pi N\Delta}^{(\pi)}(t)^2 F_{\pi N\Delta}^{(\Delta)}(p')^2 \times \frac{(M + p\hat{p}') [t + (k\hat{p}')^2]}{(t + m_\pi^2)^2} \text{Im} G_{[11]}^{(Q)}(p'), \quad (5)$$

where p and p' are the four-momenta of the nucleon and isobar, and $t \equiv -(p - p')^2$ and $F_{\pi N\Delta}^{(\pi, \Delta)}$ are the form factors of the $\pi N\Delta$ vertex. The form factor

$$F_{\pi N\Delta}^{(\Delta)}(p) = \exp \left[-\frac{p^2 - (M + m_\pi)^2}{\Lambda^2} \right] \quad (6)$$

with $\Lambda = 0.97 \text{ GeV}$ (and $g_{\pi N\Delta} = 20.2 \text{ GeV}^{-1}$) was used in Ref. [15] and was shown to give a good fit to the relevant pion-nucleon phase shift. For the πNN and $\pi N\Delta$ off-shell form factors which take into account the off-shell pion, we take a dipole form:

$$F_{\pi NX}^{(\pi)}(t) = \left(\frac{\Lambda_{\pi X}^2 - m_\pi^2}{\Lambda_{\pi X}^2 + t} \right)^2, \quad (7)$$

with X standing for N or Δ .

In order to calculate the $\bar{d} - \bar{u}$ distribution for the free proton we assume that the pion sea is isospin symmetric, leaving only the contribution of valence distributions. The final state with nucleon contributes through the presence of π^+ with distribution $2f^{\pi^0 NN}$, while the isobar final state can have also a π^+ with isospin weight 1/3 or a π^- with minus sign (because of the pion valence \bar{u} distribution) with respect to $f^{\pi^- \Delta/N}$, giving in total:

$$(\bar{d} - \bar{u})_p(x) = \int_x^1 \frac{dy}{y} \left(2f^{\pi^0 NN}(y) - \frac{2}{3}f^{\pi^- \Delta/N}(y) \right) q_v^\pi(x/y), \quad (8)$$

with $q_v^\pi(x)$ denoting the valence parton distribution of charged pion. In Fig. 1 we show the pion distributions with nucleon

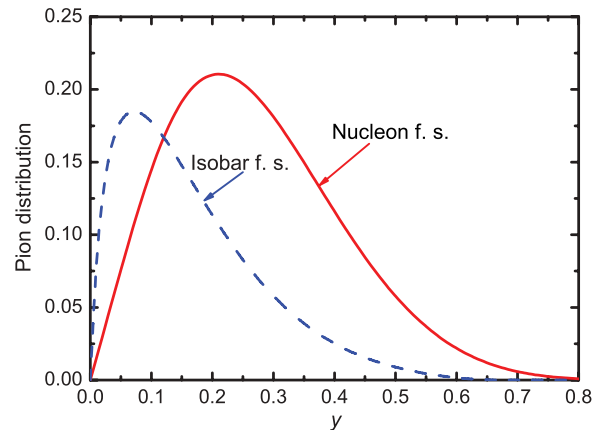


FIG. 1. (Color online) Pion distribution in the free proton: $f^{\pi^0 NN}(y)$ is shown by the solid line and $f^{\pi^- \Delta/N}(y)$ is shown by the dashed line.

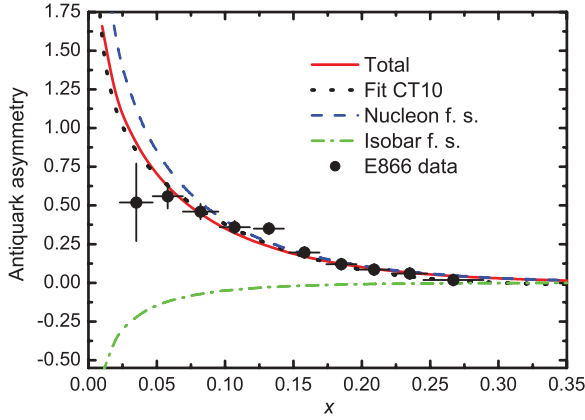


FIG. 2. (Color online) The pion-cloud result for the $\bar{d} - \bar{u}$ asymmetry in free proton (solid line) compared to the difference of the proton \bar{d} and \bar{u} distributions (dot line) from the fit CT10 [17] and data points by the Fermilab E866/NuSea Collaboration [18]. The isobar contribution (dash-dotted line) is negative and much smaller than the nucleon term (dashed line). Used parameter values: $\Lambda_{\pi N}^{(\pi)} = 0.95$ GeV and $\Lambda_{\pi\Delta}^{(\pi)} = 0.75$ GeV.

final state (solid line) and isobar final state (dashed line). For the form-factor cutoff we used the following values: $\Lambda_{\pi N}^{(\pi)} = 0.95$ GeV and $\Lambda_{\pi\Delta}^{(\pi)} = 0.75$ GeV. The bare-nucleon probability then takes the value $Z = 0.69$, which suggests that higher order terms with more than one pion do not contribute significantly.

The calculated value for the $\bar{d} - \bar{u}$ asymmetry in the free proton is shown in Fig. 2 by solid line and compared to the result using the \bar{u} and \bar{d} fits CT10 [17] (dotted line). Also shown separately are the contributions from the nucleon final state (dashed line) and isobar final state (dash-dotted line). These results are quite similar to the $\bar{d} - \bar{u}$ obtained in Ref. [19], although there the infinite-momentum-frame formalism was used with suitably adjusted values of the πN and $\pi\Delta$ form factors.

III. NUCLEAR EFFECTS

First calculations of the nuclear Drell-Yan process [20] suggested an enhancement coming from the medium modification of the pion cloud. However, the experimental data [8] did not show that enhancement within a 10% uncertainty. Later on other groups reported more detailed calculations of the Drell-Yan ratio with a large variation in results as shown in Refs. [2–4]. Here we consider the ratio of the cross sections of proton-nucleus and proton-deuteron scattering,

$$R_{A/d} = \frac{2}{A} \frac{d\sigma^{pA}/dx_1 dx_2}{d\sigma^{pd}/dx_1 dx_2}, \quad (9)$$

where A denotes the nucleus and its nucleon number. We specialize the case of isoscalar targets for which the cross-section ratio becomes

$$R_{A/d} = \frac{\sum_f e_f^2 \{q_f^p(x_1) [\bar{q}_f^{p/A}(x_2) + \bar{q}_f^{n/A}(x_2)] + \bar{q}_f^p(x_1) [q_f^{p/A}(x_2) + q_f^{n/A}(x_2)]\}}{\sum_f e_f^2 \{q_f^p(x_1) [\bar{q}_f^p(x_2) + \bar{q}_f^n(x_2)] + \bar{q}_f^p(x_1) [q_f^p(x_2) + q_f^n(x_2)]\}}. \quad (10)$$

In the case when x_1 is large, say, $x_1 > 0.3$, the second term in the numerator becomes negligible and only a medium effect on the antiquarks plays a role.

The (anti)quark distribution in the medium can be modified in two ways: (i) through Fermi motion and binding of the nucleon and (ii) through modification of the nucleon's pion cloud. To establish the connection to the (anti)quark distribution of the free nucleon we use Eq. (2), which for the free proton gives (with isobar terms not written out for brevity)

$$q_f^p(x) = Z q_{f,\text{bare}}^p(x) + \frac{1}{3} \int_x^1 \frac{dy}{y} f^{N/N}(y) [q_{f,\text{bare}}^p(x/y) + 2q_{f,\text{bare}}^n(x/y)] + \int_x^1 \frac{dy}{y} f^{\pi^0 N/N}(y) [q_f^{\pi^0}(x/y) + 2q_f^{\pi^+}(x/y)], \quad (11)$$

with

$$Z \equiv 1 - \int_0^1 dy f^{N/N}(y) = 1 - 3 \int_0^1 dy f^{\pi^0 N/N}(y), \quad (12)$$

where the last equality expresses flavor-charge conservation. Similarly, the quark distribution for the nuclear proton can be written

$$\tilde{q}_f^p(x) = Z_A q_{f,\text{bare}}^p(x) + \frac{1}{3} \int_x^A \frac{dy}{y} f^{N/A}(y) [q_{f,\text{bare}}^p(x/y) + 2q_{f,\text{bare}}^n(x/y)] + \int_x^A \frac{dy}{y} f^{\pi^0 N/A}(y) [q_f^{\pi^0}(x/y) + 2q_f^{\pi^+}(x/y)], \quad (13)$$

where isospin-symmetric nuclear medium was assumed. Adding the difference of the left-hand side and right-hand side of Eq. (11) to the right-hand side of Eq. (13) and repeating the same procedure for the neutron, we obtain

$$\begin{aligned} \tilde{q}_f^p(x) + \tilde{q}_f^n(x) &= q_f^p(x) + q_{f,\text{bare}}^p(x) \int_0^1 f^{N/N}(y) dy - \int_x^1 \frac{dy}{y} f^{N/N}(y) q_{f,\text{bare}}^p(x/y) - q_{f,\text{bare}}^p(x) \int_0^A f^{N/A}(y) dy \\ &+ \int_x^A \frac{dy}{y} f^{N/A}(y) q_{f,\text{bare}}^p(x/y) + (p \rightarrow n) + 2 \int_x^A \frac{dy}{y} [f^{\pi^0 N/A}(y) - f^{\pi^0 N/N}(y)] \\ &\times [q_{\pi^0}^f(x/y) + q_{\pi^+}^f(x/y) + q_{\pi^-}^f(x/y)], \end{aligned} \quad (14)$$

where we used that

$$Z_A \equiv 1 - \int_0^1 dy f^{N/A}(y).$$

To proceed with the above expression one needs the bare antiquark distributions, which could be determined from Eq. (11), and their analogs for the neutron. A much simpler, though approximate, procedure is just to subtract the meson-cloud contribution from the antiquark distribution of the physical nucleon. Indeed, using the fact that antiquark distributions at small x behave as $1/x$, one can confirm that the first and second terms on the right-hand side of Eq. (11) combine to give the bare distribution if one adds to Eq. (11) its neutron analog. Using the same argument about the small x behavior of antiquark distributions, one can establish an approximate cancellation of the second and third as well as the fourth and fifth terms on the right-hand side of Eq. (14) and of the corresponding terms involving the neutron. This simplification was used in our previous work [6], but in the present calculation we take into account these contributions with the bare antiquark distributions determined by the abovementioned subtraction of the pion contribution from the physical distribution. In Fig. 3 we show the proton-neutron average of the sum of the second, third, fourth, and fifth terms in Eq. (14) divided by the antiquark distribution of the free “isoscalar” nucleon for two used pion parameter sets. For the distribution of the bare nucleon in the free nucleon we use the relation $f^{N/N}(y) = 3f^{\pi^0 N/N}(1-y)$, which follows from the probabilistic interpretation of these functions [19] and its analog for the in-medium case, $f^{N/A}(y) = 3f^{\pi^0 N/A}(1-y)$. We remark that the latter relationship can be only approximate since the support of the pion in-medium distribution is not strictly limited by value one, but in view of the similarity of the pion distributions in the two cases it should be a reasonable approximation for the estimate of a small effect. We observe that this contribution is indeed quite small, as expected from the form of the nucleon antiquark distribution and pion (as well as related nucleon) light-cone-momentum distributions. The parameter set (1) is $M_* = 0.89$ GeV, $\Sigma_N^v = 0$, $\Sigma_\Delta^s = -0.1$ GeV, $\Sigma_\Delta^v =$

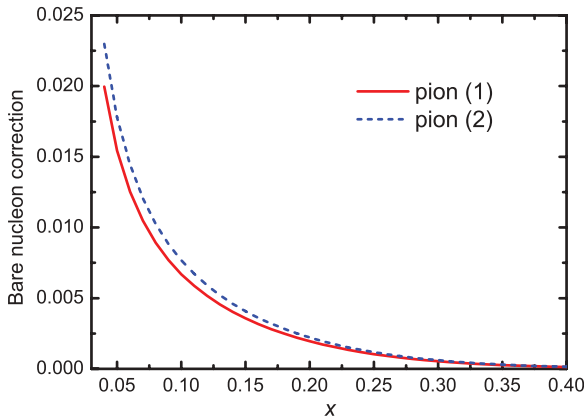


FIG. 3. (Color online) Sum of the second, third, fourth, and fifth terms in Eq. (14) divided by the antiquark distribution of the free “isoscalar” nucleon. The solid line is for the pion parameter set (1) and the dashed line is for parameter set (2).

0 , $g'_{11} = 0.9$, $g'_{12} = 0.3$, $g'_{22} = 0.3$, while the set (2) is given by $M_* = 0.89$ GeV, $\Sigma_N^v = 0$, $\Sigma_\Delta^s = -0.05$ GeV, $\Sigma_\Delta^v = 0$, $g'_{11} = 1.0$, $g'_{12} = 0.4$, $g'_{22} = 0.3$. M_* is the mean-field shifted nucleon mass, Σ_N^v the energy shift of the nucleon, Σ_Δ^s and Σ_Δ^v are the δ mean-field shifts, and g'_{ij} are the Migdal four-fermion interaction parameters.

By introducing a shorter notation for the bare nucleon contribution to the in-medium antiquark distribution,

$$\begin{aligned} q_{f,\text{bare}}^{p+n}(x) &= q_f^p(x) + q_{f,\text{bare}}^p(x) \int_0^1 f^{N/N}(y) dy \\ &\quad - \int_x^1 \frac{dy}{y} f^{N/N}(y) q_{f,\text{bare}}^p(x/y) \\ &\quad - q_{f,\text{bare}}^p(x) \int_0^A f^{N/A}(y) dy \\ &\quad + \int_x^A \frac{dy}{y} f^{N/A}(y) q_{f,\text{bare}}^p(x/y) + (p \rightarrow n), \end{aligned} \quad (15)$$

we can finally write the sum of in-medium proton and neutron antiquark distribution as

$$\begin{aligned} q_f^{p/A}(x) + q_f^{n/A}(x) &= \int_x^A \frac{dy}{y} f_{Fb}^N(y) q_{f,\text{bare}}^{p+n}(x/y) \\ &\quad + 2 \int_x^A \frac{dy}{y} [f^{\pi^0/A}(y) - f^{\pi^0/N}(y)] \\ &\quad \times [q_{\pi^0}^f(x/y) + q_{\pi^+}^f(x/y) + q_{\pi^-}^f(x/y)]. \end{aligned} \quad (16)$$

The convolution with $f_{Fb}^N(z)$ takes into account Fermi motion and binding effects on the in-medium nucleons. For the function $f_{Fb}^N(z)$ we take the result of Birse [21]:

$$f_{Fb}^N(z) = \frac{3}{4\epsilon^3} [\epsilon^2 - (z - \eta)^2] \Theta(\epsilon - |z - \eta|), \quad (17)$$

where $\epsilon \equiv p_F/M$, η is a parameter with value slightly below one which takes into account the nuclear binding, and $\Theta(x)$ is the unit step function. In Fig. 4 we show the ratio of the $F_2(x)$

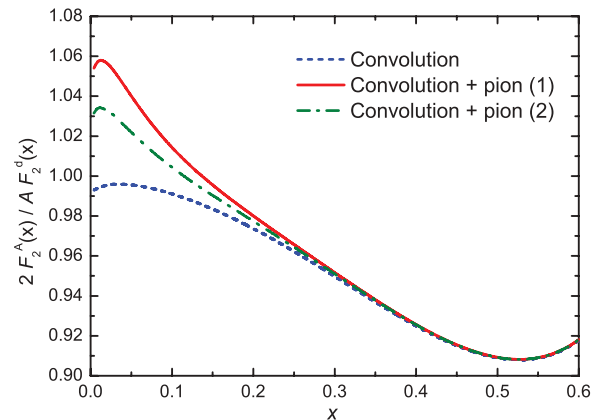


FIG. 4. (Color online) The ratio of $F_2(x)$ per nucleon for isospin symmetric nuclear medium with the following parameters: Fermi momentum $p_F = 250$ MeV, $\eta = 0.97$, and deuteron. Pion contributions with parameter sets (1) and (2) are included in the results shown by solid and dash-dotted lines.

structure functions for the isospin symmetric nuclear matter and the deuteron. We assume negligible medium effects in the deuteron and for the nuclear medium use the convolution model with light-cone distribution (17) and parameters $p_F = 250$ MeV and $\eta = 0.97$. In this way one can reproduce the negative slope in the classical EMC effect for $0.1 < x < 0.5$ as shown, for example, in Ref. [22]. The experimental enhancement observed around $x = 0.1$ can be attributed to the pion enhancement as shown in the figure for the two parameter sets (1) and (2) used also for the plots in Fig. 3 and given above. The pion enhancement term was calculated by the convolution of the in-medium pion light-cone distribution enhancement relative to the free nucleon and the pion F_2 distribution. Note that expected shadowing effects would lead to decrease of the nuclear cross section for $x \leq 0.05$.

IV. ANTIQUARKS IN BOUND NUCLEONS: PION CONTRIBUTION

We now turn to consideration of the pion contribution to antiquark distributions in nucleons bound in large nuclei, which we model by an infinite system with appropriate average nuclear density.

For corresponding pion properties in the nuclear matter, we use the results of a fully covariant self-consistent model developed in Ref. [23]. Compared to the case of the free nucleon, the nuclear environment changes the pion propagator appearing in the Sullivan formula (4) and renormalizes the πNN as well as the $\pi N\Delta$ vertices through nucleon-nucleon correlations modeled by the Migdal four-fermion interactions [24]. Nucleon properties are also affected, and we take into account the binding effects through mean-field mass and energy shifts consistent with the approach in Ref. [23].

The inclusion of the dressed pion propagator is straightforward but the dressing of the πNN and the $\pi N\Delta$ vertices requires summation of nucleon-hole and δ -hole bubbles. The types of relevant diagrams are shown in Fig. 5. For resummation of these diagrams we use the relativistically covariant formalism introduced in Ref. [25] and applied for pion and δ self-energy calculation in Ref. [23]. In the present case it concerns a different type of contribution which for the nucleon in the final state can be written as

$$K_N = \bar{u}(p')\gamma_5\gamma^\mu u(p)\Pi_{\mu\nu}(q)q^\nu, \quad (18)$$

where $q = p - p'$ is the pion four-momentum, $u(p)$ is the nucleon in-medium spinor (with mean-field shifts of mass and energy), and $\Pi_{\mu\nu}(q)$ is the resummed contribution of nucleon-hole and δ -hole loops. Using the decomposition of nucleon-hole and δ -hole loops [23],

$$\begin{aligned} \Pi_{\mu\nu}^{(Nh)}(q) &= \sum_{i,j=1}^2 \Pi_{ij}^{(Nh)}(q) L_{\mu\nu}^{(ij)}(q) + \Pi_T^{(Nh)}(q) T_{\mu\nu}(q), \\ \Pi_{\mu\nu}^{(\Delta h)}(q) &= \sum_{i,j=1}^2 \Pi_{ij}^{(\Delta h)}(q) L_{\mu\nu}^{(ij)}(q) + \Pi_T^{(\Delta h)}(q) T_{\mu\nu}(q) \end{aligned} \quad (19)$$

where $L_{\mu\nu}^{(ij)}(q)$ and $T_{\mu\nu}(q)$ have projector properties. The nonzero contribution involving one particle-hole (nucleon-

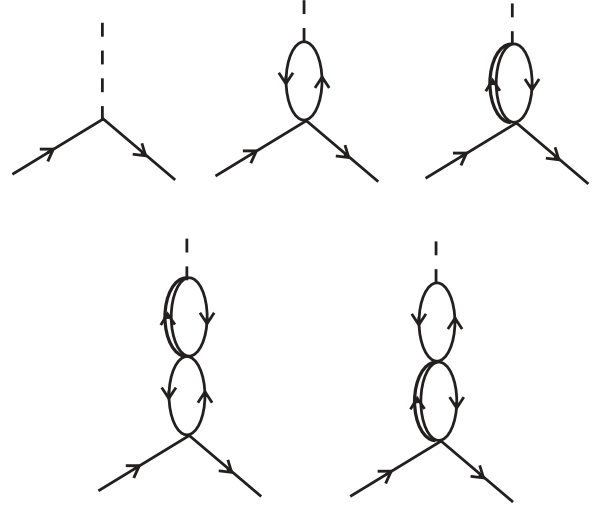


FIG. 5. Types of diagrams contributing to the in-medium pion distribution with outgoing nucleon. The same types appear also with outgoing Δ baryon. The dashed line denotes the dressed pion propagator, while the solid and the double lines correspond to nucleon and δ .

hole or δ -hole) loop comes from

$$\begin{aligned} \Pi_{\mu\nu}^{(1)} &= g'_{11} (\Pi_{11}^{(Nh)}(q) L_{\mu\nu}^{(11)}(q) + \Pi_{21}^{(Nh)}(q) L_{\mu\nu}^{(21)}(q)) \\ &+ g'_{12} (\Pi_{11}^{(\Delta h)}(q) L_{\mu\nu}^{(11)}(q) + \Pi_{21}^{(\Delta h)}(q) L_{\mu\nu}^{(21)}(q)), \end{aligned} \quad (20)$$

where g'_{11} , g'_{12} , g'_{22} are the usual Migdal parameters [23] and where index 1 refers to the nucleon and index 2 to the δ . We took into account that

$$L_{\mu\nu}^{(12)}(q)q^\nu = L_{\mu\nu}^{(22)}(q)q^\nu = T_{\mu\nu}(q)q^\nu = 0. \quad (21)$$

In order to perform the summation involving arbitrary number of nucleon-hole or δ -hole loops it is convenient to introduce the following matrices [25]:

$$\begin{aligned} g^{(L)} &= \begin{pmatrix} g'_{11} & 0 & g'_{12} & 0 \\ 0 & g'_{11} & 0 & g'_{12} \\ g'_{12} & 0 & g'_{22} & 0 \\ 0 & g'_{12} & 0 & g'_{22} \end{pmatrix}, \\ \Pi^{(L)} &= \begin{pmatrix} \Pi_{11}^{(Nh)} & \Pi_{12}^{(Nh)} & 0 & 0 \\ \Pi_{21}^{(Nh)} & \Pi_{22}^{(Nh)} & 0 & 0 \\ 0 & 0 & \Pi_{11}^{(\Delta h)} & \Pi_{12}^{(\Delta h)} \\ 0 & 0 & \Pi_{21}^{(\Delta h)} & \Pi_{22}^{(\Delta h)} \end{pmatrix}. \end{aligned} \quad (22)$$

The lowest order contribution (20) can then be written as

$$\begin{aligned} \Pi_{\mu\nu}^{(1)} &= [(\Pi^{(L)} g^{(L)})_{11} + (\Pi^{(L)} g^{(L)})_{31}] L_{\mu\nu}^{(11)}(q) \\ &+ [(\Pi^{(L)} g^{(L)})_{21} + (\Pi^{(L)} g^{(L)})_{41}] L_{\mu\nu}^{(21)}(q). \end{aligned} \quad (23)$$

Higher order terms are accounted for by taking appropriate matrix elements of products of $(\Pi^{(L)} g^{(L)})$ matrices, and the summation of terms with arbitrary number of loops is simply

achieved by replacing $(\Pi^{(L)}g^{(L)})$ in Eq. (23) by $\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}$, leading to

$$\begin{aligned} \Pi_{\mu\nu} = & ([\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{11} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{31})L_{\mu\nu}^{(11)}(q) \\ & + ([\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{21} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{41})L_{\mu\nu}^{(21)}(q). \end{aligned} \quad (24)$$

Adding $g_{\mu\nu}$ to $\Pi_{\mu\nu}$ in Eq. (18), we obtain the full contribution of the diagram with nucleon final state. By squaring its absolute value, performing summation over the spin projections of the nucleon in the final state, and averaging for the nucleon in the initial state, we obtain

$$\begin{aligned} \frac{1}{2} \sum_{s,s'} |K_N|^2 = & A_{qq}(2(p - \Sigma_N^v u)q(p' - \Sigma_N^v u)q - q^2 \\ & \times [M_*^2 + (p' - \Sigma_N^v u)(p - \Sigma_N^v u)]) \\ & + 2A_{qu}((p - \Sigma_N^v u)u(p' - \Sigma_N^v u)q \\ & + (p' - \Sigma_N^v u)u(p - \Sigma_N^v u)q \\ & - qu[M_*^2 + (p' - \Sigma_N^v u)(p - \Sigma_N^v u)]) \\ & + A_{uu}(2(p - \Sigma_N^v u)u(p' - \Sigma_N^v u)u - M_*^2 \\ & - (p' - \Sigma_N^v u)(p - \Sigma_N^v u)), \end{aligned} \quad (25)$$

where u is the four-velocity of the medium [implicitly present also in Eqs. (18)–(24)] and $M_* = M_N + \Sigma_N^s$, Σ_N^s and Σ_N^v are the nucleon mean-field mass and energy shifts. The factors A_{qq} , A_{qu} , A_{uu} are given by

$$\begin{aligned} A_{qq} = & 2|1 + A + quB/\sqrt{q^2 - (qu)^2}|^2, \\ A_{qu} = & -2\text{Re}[q^2B/\sqrt{q^2 - (qu)^2}(1 + \bar{A} \\ & - qu\bar{B}/\sqrt{q^2 - (qu)^2})] \\ A_{uu} = & 2|q^2B/\sqrt{q^2 - (qu)^2}|^2, \end{aligned} \quad (26)$$

where the bar denotes complex conjugation,

$$\begin{aligned} A \equiv & [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{11} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{31}, \\ B \equiv & [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{21} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{41}, \end{aligned} \quad (27)$$

and we used that

$$\begin{aligned} L_{\mu\nu}^{(11)}(q)q^v = & q_\mu, \\ L_{\mu\nu}^{(21)}(q)q^v = & \frac{qu}{\sqrt{q^2 - (qu)^2}}q_\mu - \frac{q^2}{\sqrt{q^2 - (qu)^2}}u_\mu. \end{aligned} \quad (28)$$

To compute the pion light-cone distribution per nucleon in the medium, we integrate over incoming nucleons in the Fermi sea and outgoing ones above the Fermi sea, restricting

the pion light-cone momentum fraction to the specified value by inserting a δ function and finally divide by the nucleon density. The final expression obtained is

$$\begin{aligned} f_N(y) = & 3My \left(\frac{f_N}{m_\pi} \right)^2 \frac{1}{32\pi^3 p_F^3} \int_{-p_F}^{p_F} dp_3 \int_0^{\sqrt{p_3^2 - p_3^2}} p_\perp dp_\perp \\ & \times \int_{p_\perp^{\min}}^\infty p'_\perp dp'_\perp \int_0^{2\pi} d\vartheta \frac{1}{2b} \sum_{s,s'} |K_N|^2 \\ & \times |F_{\pi NN}^{(\pi)}(-q^2)D_\pi(q)|^2, \end{aligned} \quad (29)$$

where $D_\pi(q)$ is the in-medium dressed pion propagator, $b \equiv My - p_3 - \sqrt{M_*^2 + p_3^2 + p_\perp^2}$, $p_\perp^{\min} = \sqrt{2b\sqrt{M_*^2 + p_F^2} - M_*^2 - b^2}$, ϑ is the angle between \vec{p}_\perp and \vec{p}'_\perp , and the πNN form factor $F_{\pi NN}^{(\pi)}(-q^2)$ was also included.

The contribution coming from the Δ baryon in the final state is more involved by the complicated structure of the in-medium δ propagator [14]. However, considerable simplification can be achieved by including only the two dominant contributions in the convenient relativistically covariant decomposition since the imaginary part of the other components is typically two orders of magnitude smaller at nuclear saturation density [26]. The dominant contributions come from the $Q_{[11]}^{\mu\nu}$ and $P_{[55]}^{\mu\nu}$ terms, which were degenerate in the free δ case but are different in the medium [14]. Summation of particle-hole loops dressing the $\pi N \Delta$ vertex is analogous to the πNN case, with the only difference being in the relevant matrix elements of $\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}$, replacing the expression (24) with

$$\begin{aligned} \Pi_{\mu\nu}^\Delta = & ([\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{13} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{33})L_{\mu\nu}^{(11)}(q) \\ & + ([\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{23} \\ & + [\Pi^{(L)}g^{(L)}(1 - \Pi^{(L)}g^{(L)})^{-1}]_{43})L_{\mu\nu}^{(21)}(q). \end{aligned} \quad (30)$$

The expression analogous to Eq. (25) in this case takes the form:

$$\begin{aligned} \frac{1}{2} \sum_{s,s'} |K_\Delta|^2 = & \frac{1}{2} \text{Tr}[(\not{p} - \Sigma_N^v \not{u} + M_*)\text{Im}G^{\mu\nu}(p')] \\ & \times (g_{\mu\alpha}g_{\nu\beta} + \Pi_{\mu\alpha}^\Delta \bar{\Pi}_{\nu\beta}^\Delta)q^\alpha q^\beta, \end{aligned} \quad (31)$$

where $\text{Im}G_{\mu\nu}(p')$ denotes the imaginary part of the in-medium Δ propagator for which we take the dominant contribution given in the basis used in Ref. [14] by just two terms:

$$G^{\mu\nu}(p') = Q_{[11]}^{\mu\nu}(p')G_{[11]}^{(Q)}(p') + P_{[55]}^{\mu\nu}(p')G_{[55]}^{(P)}(p'). \quad (32)$$

In this way the expression (31) takes the form

$$\begin{aligned} \frac{1}{2} \sum_{s,s'} |K_\Delta|^2 = & [A_{qq}^{(\Delta)}c_{qq}^{(Q)} + 2A_{qu}^{(\Delta)}c_{qu}^{(Q)} + A_{uu}^{(\Delta)}c_{uu}^{(Q)}]\text{Im}G_{[11]}^{(Q)}(p') \\ & + [A_{qq}^{(\Delta)}c_{qq}^{(P)} + 2A_{qu}^{(\Delta)}c_{qu}^{(P)} + A_{uu}^{(\Delta)}c_{uu}^{(P)}]\text{Im}G_{[55]}^{(P)}(p'), \end{aligned} \quad (33)$$

where the expressions for $A_{qq}^{(\Delta)}$, $A_{qu}^{(\Delta)}$, $A_{uu}^{(\Delta)}$, $c_{qq}^{(Q)}$, $c_{qu}^{(Q)}$, $c_{uu}^{(Q)}$, $c_{qq}^{(P)}$, $c_{qu}^{(P)}$, $c_{uu}^{(P)}$ are given in the Appendix. The pion

light-cone-momentum distribution stemming from the process with the nucleon emitting a pion and a Δ baryon is analogous to expression (29) and reads

$$\begin{aligned}
 f_{\Delta}(y) &= 3M_y \left(\frac{f_{\Delta}}{m_{\pi}} \right)^2 \frac{1}{32\pi^4 p_F^3} \int_{-p_F}^{p_F} dp_3 \int_0^{\sqrt{p_j^2 - p_3^2}} p_{\perp} dp_{\perp} \\
 &\times \int_{-\infty}^{\infty} dp'_3 \int_0^{\infty} p'_{\perp} dp'_{\perp} \int_0^{2\pi} d\vartheta \frac{1}{2} \sum_{s,s'} |K_{\Delta}|^2 \\
 &\times |F_{\pi N \Delta}^{\Delta}(p') F_{\pi N \Delta}^{(\pi)}(-q^2) D_{\pi}(q)|^2. \quad (34)
 \end{aligned}$$

We checked by explicit numerical calculation that both Eqs. (29) and (34) have the correct low-density limit, i.e., reproduce the free nucleon and δ results.

V. NUMERICAL RESULTS AND DISCUSSION

For the computation of in-medium pion and isobar properties we rely on the recently developed relativistically covariant self-consistent model presented in Ref. [23] and used for the nuclear photoabsorption calculation in the isobar region in Ref. [27]. For the medium computation we use the same πNN and $\pi N\Delta$ form factors as in the vacuum one and include them in the model of Ref. [23]. The values of the Migdal g' parameters, which model the short-range nucleon and isobar correlations, were taken in the range preferred by the results of Ref. [27]; in this work a good description of the nuclear photoabsorption cross section in the isobar region was obtained. Binding effects for the nucleon are taken into account by the effective (mean-field) mass M_* and the energy shift Σ_N^v . A consequence of the use of the mean-field approximation is a reduction of the in-medium pion distribution coming from the nucleon final state. The dominant contribution to it is the term proportional to A_{qq} in Eq. (25) with

$$\begin{aligned}
 &2(p - \Sigma_N^v u)q (p' - \Sigma_N^v u)q \\
 &- q^2 [M_*^2 + (p' - \Sigma_N^v u)(p - \Sigma_N^v u)] = -2M_*^2 q^2, \quad (35)
 \end{aligned}$$

which is the same expression as for the free nucleon, except that M_* appears instead of M .

Since $M_*/M < 1$ a further suppression in addition to that from the Pauli blocking is obtained, depending on the actual value of M_*/M . The latter is difficult to constrain since observables generally are only sensitive to the combination $M_* + \Sigma_N^v$. Since our aim is to make comparisons with experiments on finite nuclei (rather than nuclear matter) with an average density smaller than the saturation density, we assume small values for the energy shift in the range zero to $\Sigma_N^v = 0.04$ GeV, corresponding to effective mass values in the range of 0.85 and 0.89 GeV. These values are close to the ones used in more elaborate treatments of nuclear matter [28,29], where values of 0.8–0.85 GeV at saturation density give good agreement with observables. The mean-field shifts of the isobar mass and energy are chosen in such a way so that they reproduce the isobar-nucleon mass difference used in Ref. [27]. This means $\Sigma_{\Delta}^s = -0.05$ GeV and -0.1 GeV and zero for the energy shift.

In Figs. 6 and 7 we show the pion distributions $f^{\pi^{0N/A}}(y)$ and $f^{\pi^{-\Delta/A}}(y)$ for in-medium nucleons for different parameter sets. For the nucleonic distribution one observes a reduction coming partly from the Pauli blocking of the nucleons in

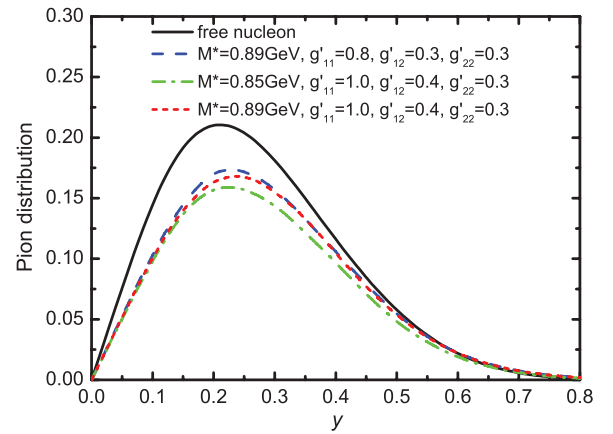


FIG. 6. (Color online) Nucleon contribution to the pion distribution [$f^{\pi^{0N/A}}(y)$] for the in-medium nucleon compared to the case of a free nucleon.

the medium and partly from the M_* effect (which leads to a suppression roughly by the factor $(M_*/M)^2$). The pion broadening in the nuclear medium only partly compensates for these effects and a net reduction is the result. This is not completely surprising since the computations of Ref. [23] do not lead to appreciable softening of the pion spectrum in the medium, which would result in enhanced pion distribution. In this respect the pion dressing of Ref. [23] is not significantly different from an older calculation [30] which used a nonrelativistic treatment of the isobar and a softer pion-nucleon- δ form factor. On the other hand a significant enhancement is observed for the contribution originating from the transition $N \rightarrow \pi \Delta$, which is not Pauli suppressed. These results emphasize the importance of careful treatment of the in-medium isobar self-energy and propagator, which is made possible by the convenient complete basis introduced in this context in Ref. [14]. As a consequence, in the nuclear medium the combined effects from the pion and from the isobar can produce a sizable increase in the pion light-cone distribution. The latter is constrained to smaller light-cone-momentum ratio y values because of kinematical

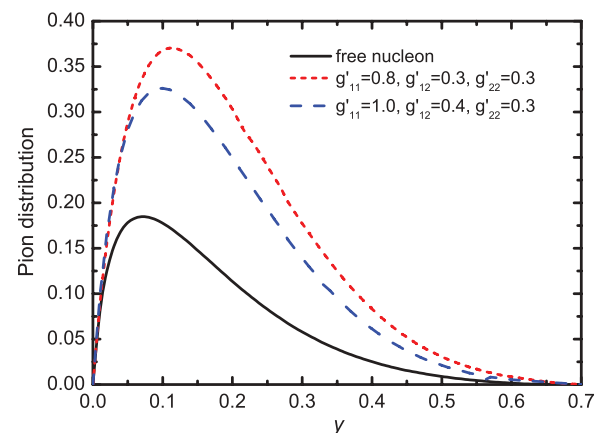


FIG. 7. (Color online) Isobar contribution to the pion distribution [$f^{\pi^{-\Delta/A}}(y)$] in the in-medium nucleon compared to the free nucleon. For both in-medium curves $M_* = 0.89$ GeV and $\Sigma_N^v = 0$.

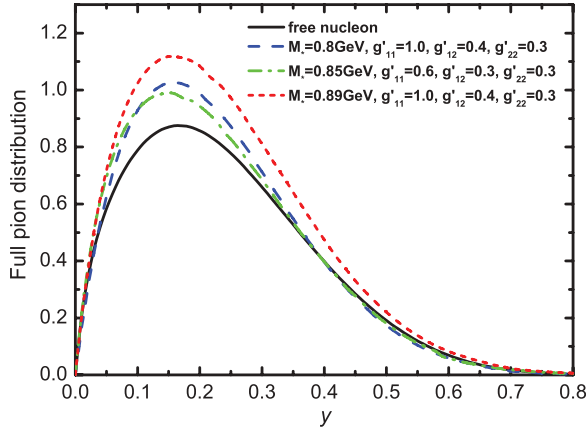


FIG. 8. (Color online) Full pion distribution [$f^{\pi/A}(y)$] in the in-medium nucleon compared to the case of free “isoscalar” nucleon.

effect of the isobar-nucleon mass difference but can still have significant effects on the DY cross-section ratio.

Since we are considering isospin symmetric nuclear medium and make a comparison with the deuteron, it is advantageous to consider the pion distribution in an “isoscalar” nucleon, i.e., to consider a proton-neutron average. Taking into account pions of all charges gives the complete pion distribution of an “isoscalar” nucleon:

$$f^{\pi/A}(y) = 3f^{\pi^0 N/A}(y) + 2f^{\pi^- \Delta/A}(y). \quad (36)$$

In Fig. 8 we show the function $f^{\pi/A}(y)$ for different input parameter values compared to the pion distribution of the free “isoscalar” nucleon. The probability Z_A of the bare nucleon in the medium takes the values from 0.6 to 0.65, i.e., just slightly smaller than in the free nucleon case.

Before examining the DY cross section we show the ratio of the antiquark distribution in the in-medium proton and the same distribution in the free proton. The up and down antiquark distributions experience different in-medium modification due to different weights of nucleon and δ contributions even in isospin-symmetric nuclear medium. In Fig. 9 we show the

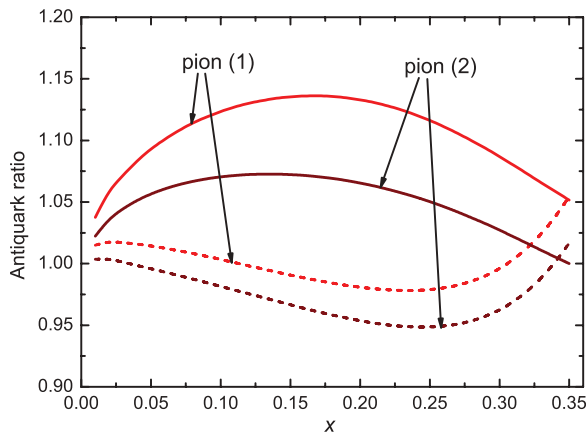


FIG. 9. (Color online) The $\bar{u}_p^{(medium)}/\bar{u}_p^{(free)}$ (solid lines) and $\bar{d}_p^{(medium)}/\bar{d}_p^{(free)}$ (dashed lines) ratios of antiquark distributions of an in-medium proton relative to the free proton for parameter sets (1) and (2).

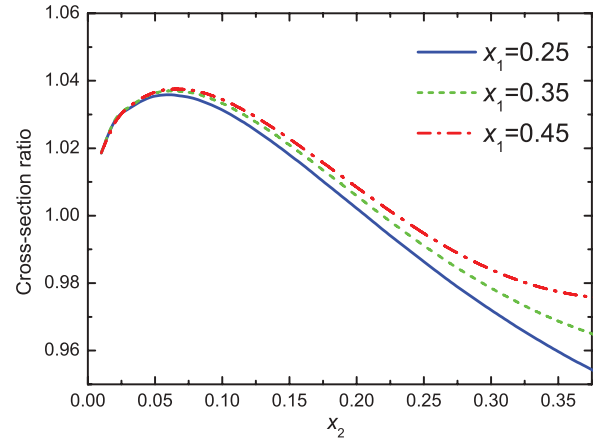


FIG. 10. (Color online) The cross-section ratio (10) as a function of x_2 for fixed values of x_1 . The used parameter values are $M_* = 0.85$ GeV, $\Sigma_N^v = 0.04$ GeV, $\Sigma_\Delta^s = -0.1$ GeV, $\Sigma_\Delta^v = 0$, $g'_{11} = 0.8$, $g'_{12} = 0.3$, and $g'_{22} = 0.3$.

ratios of antiquark distributions for an in-medium proton relative to the free one for two typical parameter sets denoted by “pion (1)” and “pion (2),” already used for plots in Figs. 3 and 4. We observe pronounced enhancement for the up antiquark coming from the substantial pion enhancement due to the Δ -baryon final state as compared to quite modest enhancement and even suppression for the down antiquark as a consequence of the larger weight of the nucleon final state and the smaller weight of the Δ -baryon final state as compared to the up antiquark. This difference points to the possibility of distinguishing between effects coming from the medium modification of the nucleon and Δ baryon by examining observables to which up and down antiquarks contribute with different weights.

We now turn to the DY cross-section ratio (10). In Figs. 10 and 11 we show the cross-section ratio (10) as a function of x_2 for fixed values of x_1 . The input parameter values are given in the figure captions. We observe an enhancement only for small values of x_2 , typically less than 0.2, and for $x_2 > 0.1$ a

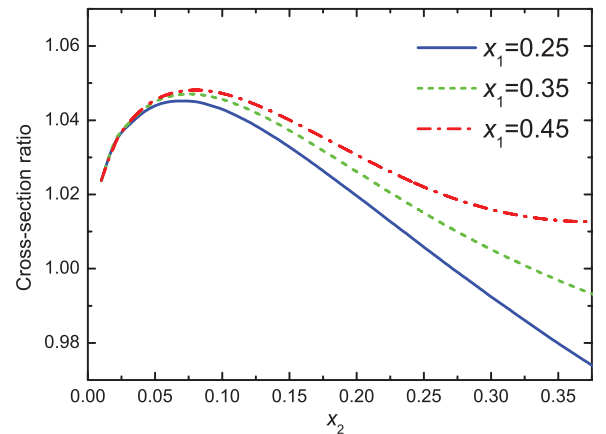


FIG. 11. (Color online) The same as Fig. 10 but with parameter values $M_* = 0.89$ GeV, $\Sigma_N^v = 0$, $\Sigma_\Delta^s = -0.1$ GeV, $\Sigma_\Delta^v = 0$, $g'_{11} = 1$, $g'_{12} = 0.4$, and $g'_{22} = 0.3$.

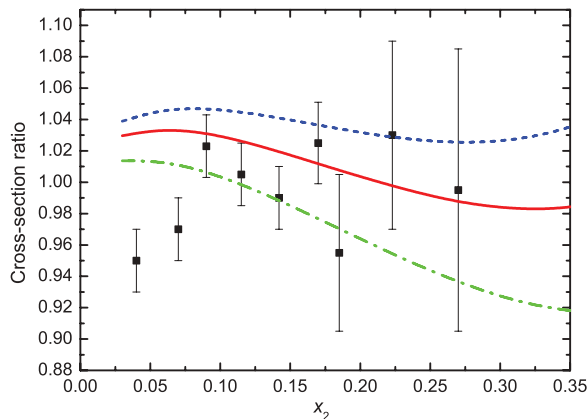


FIG. 12. (Color online) Experimental results from Ref. [8] compared to our calculation for different parameter values. Short dashed line: $M_* = 0.89$ GeV, $\Sigma_N^v = 0$; solid line: $M_* = 0.85$ GeV, $\Sigma_N^v = 0.05$ GeV; and dash-dotted line: $M_* = 0.8$ GeV, $\Sigma_N^v = 0.09$ GeV. For all three curves, $\Sigma_\Delta^s = -0.1$ GeV, $\Sigma_\Delta^v = 0$, $g'_{11} = 1.0$, $g'_{12} = 0.4$, and $g'_{22} = 0.3$.

decreasing trend as a result of the convolution with nucleon distribution (17).

For comparison with the measurements of Ref. [8] we computed the ratio of the nuclear and deuteron cross sections for given x_2 and integrating over x_1 , satisfying the condition $x_1 > x_2 + 0.2$ that corresponds to the experimental cutoff. Figure 12 shows the measured values with error bars and the calculated curves for different input parameters. We consider the lowest curve in Fig. 12 with $M_* = 0.8$ GeV and corresponding rather pronounced suppression of the order $(M_*/M)^2$, probably exaggerating the effect of the nucleon mean-field approximation, and regard the other two curves as representing better our results based on the preferred parameter sets.

VI. SUMMARY

In this work we presented an analysis of nuclear effects on the Drell-Yan process. The approach is based on the pion-cloud model of the nucleon and a relativistically covariant self-consistent in-medium calculation of the pion and Δ baryon propagators, taking into account nuclear effects in the mean-field approximation. Starting with the free nucleon we showed that the observed $\bar{d} - \bar{u}$ antiquark distribution can be well reproduced by suitable choice of the πNN and $\pi N\Delta$ form factors with δ vacuum propagator taking into account its free width.

Using the same values for the form factors, we computed the pion light-cone-momentum distribution for nucleons in an isospin symmetric medium with density corresponding to average densities of medium mass nuclei. We took into account the change of the pion cloud originating from both the pion-nucleon and pion- δ states and a small correction (neglected in previous work) attributed to the binding effect of the bare nucleon.

Fermi motion and binding of nuclear nucleons were accounted for by the two-parameter light-cone-momentum

distribution (17), which reproduces the negative slope of the classical EMC effect in the region $0.1 < x < 0.5$ as shown in Fig. 4. Taking into account the pion enhancement, which comes from the pion- δ state of the nucleon (which is significant only for small light-cone-momentum ($y \approx 0.2$) values), leads to some enhancement of the $F_2(x)$ ratio for $x \leq 0.2$.

Pion and δ properties in the nuclear medium are calculated in a recently developed fully covariant self-consistent model [23], which consistently takes into account the πNN and $\pi N\Delta$ vertex corrections due to Migdal short-range correlations. Pronounced softening of the in-medium pion spectrum present in simpler models does not appear in this approach and consequently Pauli blocking causes some suppression of the pion distribution coming from the pion-nucleon state for an in-medium nucleon. However, enhancement results from a careful treatment of the pion- δ state as a consequence of pion broadening and δ shift and broadening.

The net effect for preferred parameter values is a modest enhancement of pion light-cone-momentum distribution mostly concentrated around the $y \approx 0.2$ value. As a consequence the DY cross-section ratio exceeds 1 for small values, typically less than 0.2, of the x_2 variable for fixed x_1 values or integration over it, corresponding to some experimental cuts. The convolution with distribution (17) acts qualitatively on the antiquarks in the same way as on the quarks, producing a negative slope which is less pronounced for the integrated cross section.

As one can see in Fig. 12 the large error bars of the measured cross-section ratio do not allow a sensitive comparison with calculated results in order to determine more precisely the preferred parameter values and consequently the in-medium properties of the pion and Δ baryon. It would be very desirable to achieve measurements with considerably smaller uncertainties, which could contribute to the resolution of some decades-old issues of nuclear physics. We remark that another interesting possibility for studying sea quark distributions in nuclei would be the use of an electron-ion collider, as detailed in the joint report of the Brookhaven National Laboratory, the Institute for Nuclear Theory (Seattle, WA), and the Thomas Jefferson National Accelerator Facility [31]. The proposed semi-inclusive deep-inelastic electron-nucleus scattering would provide new information about the structure of nuclei and quantum chromodynamics of nuclear matter and extend possibilities for studying effects of the transverse momentum distribution of partons [32].

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APPENDIX

The terms of expression (33) necessary to calculate the contribution of the in-medium delta are given as

follows:

$$\begin{aligned}
A_{qq}^{(\Delta)} &= 2|1 + A_{\Delta} + qu B_{\Delta}/\sqrt{q^2 - (qu)^2}|^2, & A_{qu}^{(\Delta)} &= -2 \operatorname{Re}[q^2 B_{\Delta}/\sqrt{q^2 - (qu)^2}[1 + \bar{A}_{\Delta} - qu \bar{B}_{\Delta}/\sqrt{q^2 - (qu)^2}]], \\
A_{uu}^{(\Delta)} &= 2|q^2 B_{\Delta}/\sqrt{q^2 - (qu)^2}|^2, & c_{qq}^{(Q)} &= [M_* + (p - \Sigma_N^v u) \hat{p}'] [t + (q \hat{p}')^2 - [q X(p')]^2], \\
c_{qq}^{(P)} &= \frac{1}{3} [M_* + (p - \Sigma_N^v u) \hat{p}'] [t + (q \hat{p}')^2 + 3[q X(p')]^2], & c_{qu}^{(Q)} &= -[M_* + (p - \Sigma_N^v u) \hat{p}'] [qu - q \hat{p}' u \hat{p}' + q X(p') u X(p')], \\
c_{qu}^{(P)} &= -\frac{1}{3} [M_* + (p - \Sigma_N^v u) \hat{p}'] [qu - q \hat{p}' u \hat{p}' - 3q X(p') u X(p')], & c_{uu}^{(Q)} &= [M_* + (p - \Sigma_N^v u) \hat{p}'] [-1 + (u \hat{p}')^2 - [u X(p')]^2], \\
c_{uu}^{(P)} &= \frac{1}{3} [M_* + (p - \Sigma_N^v u) \hat{p}'] [-1 + (u \hat{p}')^2 + 3[u \cdot X(p')]^2], & &
\end{aligned} \tag{A1}$$

with

$$\begin{aligned}
A_{\Delta} &\equiv [\Pi^{(L)} g^{(L)} (1 - \Pi^{(L)} g^{(L)})^{-1}]_{13} + [\Pi^{(L)} g^{(L)} (1 - \Pi^{(L)} g^{(L)})^{-1}]_{33}, \\
B_{\Delta} &\equiv [\Pi^{(L)} g^{(L)} (1 - \Pi^{(L)} g^{(L)})^{-1}]_{23} + [\Pi^{(L)} g^{(L)} (1 - \Pi^{(L)} g^{(L)})^{-1}]_{43}, & X(p) &\equiv \frac{(pu) p_{\mu} - p^2 u_{\mu}}{p^2 \sqrt{(pu)^2/p^2 - 1}},
\end{aligned} \tag{A2}$$

$$t = -q^2 \text{ and } \hat{p}_{\mu} = p_{\mu}/\sqrt{p^2}.$$

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