PHYSICAL REVIEW C **87**, 014609 (2013)

Synthesis of thin, long heavy nuclei in ternary collisions

Yoritaka Iwata,[1] Kei Iida,[2] and Naoyuki Itagaki[3]

[1]*GSI Helmholtzzentrum für Schwerionenforschung, D-64291 Darmstadt, Germany*

[2]*Department of Natural Science, Kochi University, Akebono-cho, Kochi 780-8520, Japan*

[3]*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

(Received 6 November 2012; published 16 January 2013)

We illustrate the formation of a thin, long structure of heavy nuclei by three-nucleus simultaneous collisions within time-dependent density functional theory. The impact parameter dependence for such a formation is systematically demonstrated through clarifications of the difference between binary and ternary collision events. A new method for producing thin, long heavy nuclei in the laboratory is suggested, as well as the possible formation of the thin, long structure in hot dense matter such as that encountered in core collapse supernovae.

DOI: 10.1103/PhysRevC.87.014609 PACS number(s): 25.70.Jj, 26.20.−f, 27.90.+b

I. INTRODUCTION

Generally, synthesis of superheavy elements and many other exotic nuclei comes from collisions between two nuclei, whereas simultaneous collisions among more than two nuclei, which can occur in principle, have not been taken seriously. In experiments using accelerators, it seems difficult to construct a setup to make three nuclei collide simultaneously. Nevertheless it is reasonable to expect that a fixed target can be bombarded by two beams moving in the opposite directions. In astrophysical circumstances, on the other hand, there seem to be many chances to see three-body reactions such as triple-α reactions [1,2]. The triple-α reactions, if occurring through two-body resonances as in the Hoyle picture, are similar to radioactive ion beam experiments (for example, see [3]) in the sense that two reactions occur nonsimultaneously in terms of a time scale of the strong interaction. Even in stars and supernova cores, however, simultaneous three-nucleus collisions within the strong interaction time scale are usually presumed to be rather hard to encounter, let alone more than three nucleus collisions. How hard they are remains to be examined thoroughly, although the possible influence of simultaneous triple-α fusion reactions on carbon production in stars has attracted much attention [4].

Two-nucleus fusion reactions in stars and supernova cores are often investigated in two steps [5,6]. One starts with fundamental two-nucleus fusion reactions in a vacuum and then incorporates medium (plasma) effects. For example, in the famous Gamow approach, one solves the elementary tunneling problem for a given relative kinetic energy and then obtains the temperature-dependent reaction rate by averaging the resulting kinetic-energy-dependent reaction rate over the Boltzmann distribution of the kinetic energy. In this work, we concentrate on the fundamental binary and ternary collisions within the time-dependent density functional theory (TDDFT). Among multinucleus collisions, ternary collisions are particularly worth investigating in the sense that there always exists one plane containing all the three center-of-mass coordinates of colliding pairs. Such a geometric restriction facilitates the formation of a low-dimensional quantum system. As we shall see, a thin, long structure of the fusion products can be stabilized by rotation in the case of noncentral binary and ternary collisions.

The TDDFT approach, which was originally proposed by Dirac [7], is useful for describing nuclear collisions at low energies except for the reactions below the Coulomb barrier. In the present case, we perform three-dimensional TDDFT calculations with Skyrme-type effective nucleon-nucleon interactions (SLy6 [8] and SKI3 [9]). One can then derive various quantities from the many-nucleon wave function self-consistently obtained in the form of the Slater determinant. We remark that the colliding system would break into a few fragments with various neutron to proton ratios [10] for the collision energy per nucleon of the order of the nucleon Fermi energy, and multifragmentation takes place for rather higher energies. Multifragmentation, in which two-nucleon collisions are usually expected to play an important role [11], is out of our scope.

II. PRODUCTION OF THIN, LONG HEAVY NUCLEI

Since ^{56}Fe can be abundant in dense stellar matter, we consider heavy element synthesis due to multiple ^{56}Fe collisions within $\sim 33.3 \times 10^{-22}$ s, which corresponds to the typical duration time of low-energy heavy-ion reactions. We first show an example of ternary collisions with the initial condition shown in Fig. 1 and the incident energy of 1.47 MeV per nucleon in the center-of-mass frame, which is illustrated in Fig. 2. In these cases, the fusion product is ^{168}Pt. In contrast with reactions producing light elements, the corresponding reactions are endothermic and hence require additional energy even after the contact. For the impact parameters that allow nuclei I and II as well as nuclei II and III to touch with each other, corresponding to the cases of $|\boldsymbol{b}| = 1, 3, 5, 7$ fm, the final products have a thin, long structure of length 25 to 30 fm, which is stabilized by rotation. For example, in the case of $|\boldsymbol{b}| = 7$ fm, rotation up to π rad is achieved in a few 10^{-21} s, which corresponds to the total rotational energy of the order of 2.10 MeV. Note that thin, long nuclei can occur at optimal values of the incident energy, below and above which fusion is drastically suppressed. Note also that the stabilization of such a thin, long structure by rotation can be seen in exotic clustering of light nuclei [12].

It has been shown here that thin, long structures are produced by noncentral ternary collisions for a broad range of

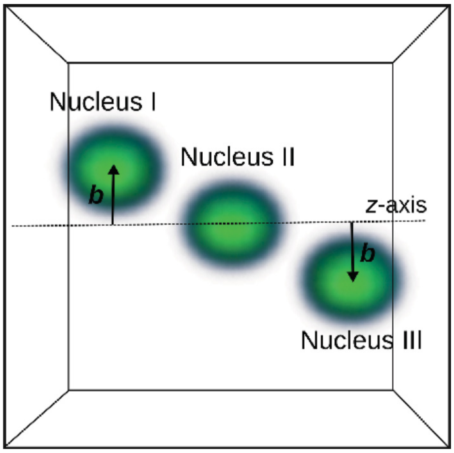

FIG. 1. (Color online) The initial positions of three identical ^{56}Fe nuclei, nucleus I, nucleus II, and nucleus III, which are set at $(|\boldsymbol{b}|, 0, 15\,\mathrm{fm})$, $(0, 0, 0)$, and $(-|\boldsymbol{b}|, 0, -15\,\mathrm{fm})$, respectively. The velocity vector of each nucleus is given as $(0, 0, |\boldsymbol{v}|)$, $(0, 0, 0)$, and $(0, 0, -|\boldsymbol{v}|)$, respectively. The volume of a box inside which the simulation is performed is $48 \times 48 \times 48\ \mathrm{fm}^3$.

the impact parameter and that the rotation speed becomes faster for larger $|\boldsymbol{b}|$ as it should. Thin, long structures turn into more spherical shapes within 33.3×10^{-22} s if there are no rotations, which suggests that the rotation plays an indispensable role in keeping the fusion products thin and long. We remark in passing that for triple-α reactions, a similar thin, long structure is predicted to occur for a considerably longer time even in the case of head-on collisions [2].

For comparison, we consider two-nucleus collisions starting with the same initial configurations as in Fig. 1 except for the absence of nucleus II. The corresponding incident energy is 2.20 MeV per nucleon in the center-of-mass frame, and the fusion product is ^{112}Te. We can observe from Fig. 3 that a similar but shorter structure occurs in the case of binary collisions. The degree of deformation from a spherical case is significantly larger in the case of ternary collisions (see Table I). We have thus quantitatively confirmed that ternary collisions are more efficient than binary collisions in producing thin, long structures.

Fusion is not only one of the most efficient ways of producing heavy elements in stars, but also the main method for producing superheavy nuclei in the laboratory. The present TDDFT calculations suggest that fusion reactions of two or three identical ^{56}Fe, which lead to the synthesis of chemical elements heavier than iron, can be easier to occur with the help of rotations.

III. STABILITY OF THIN, LONG HEAVY NUCLEI

The efficiency of multinucleus collisions in the production of thin, long heavy nuclei can be essentially understood by the competition between the surface tension and the Coulomb repulsion. Let us describe this competition within the framework of an incompressible liquid-drop model, which can characterize a specific geometry of the early stage of multinucleus collisions [see Fig. 4(a)]. In this model we restrict

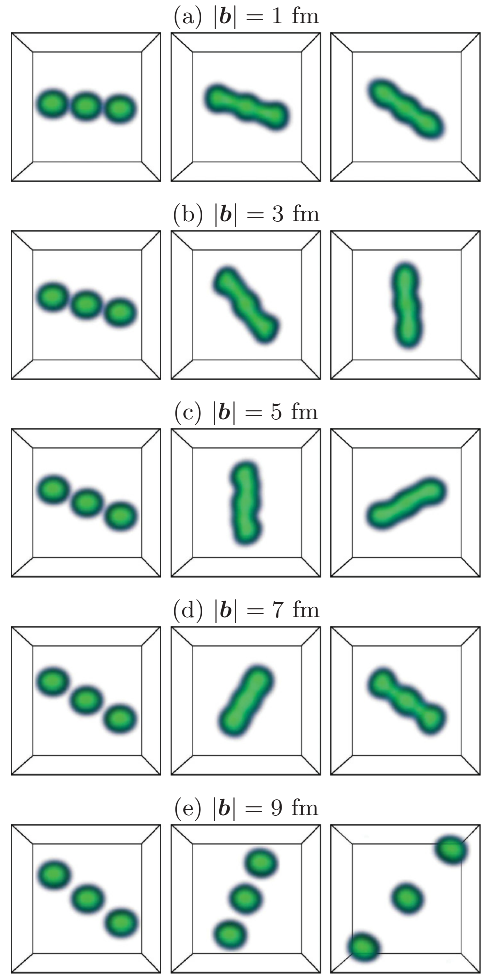

FIG. 2. (Color online) Ternary collisions with different impact parameters that start with the initial condition shown in Fig. 1 (the SLy6 parameter set is taken). The velocity is set to be 1.95×10^{22} fm/s. Snapshots at 1.33×10^{-22} s (left), 13.3×10^{-22} s (middle), and 26.6×10^{-22} s (right) are shown for $|\boldsymbol{b}| = 1, 3, 5, 7$ fm, while those at 1.33×10^{-22} s, 13.3×10^{-22} s, and 20.0×10^{-22} s are shown for $|\boldsymbol{b}| = 9$ fm.

ourselves to the situation in which the total volume of the system is fixed to $4\pi r_0^3 A/3$ with $r_0 = 1.20$ fm and the number of nucleons A.

Let us now consider a string of n identical nuclei of radius $R = r_0(A/n)^{1/3}$, which we regard as a precursor of a thin, long heavy nucleus as long as $n > 1$ [Fig. 4(a)]. Then, we obtain the length R_z of this string as

$$R_z = 2nR = 2r_0 n^{2/3} A^{1/3}. \tag{1}$$

The liquid-drop model allows one to quantify the competition between the surface tension and the Coulomb repulsion, which is essential to the formation of thin, long heavy nuclei. The surface and Coulomb energies of the string can be written as

$$E_{\mathrm{Surf}} = a_s \frac{4n\pi R^2}{4\pi r_0^2} = a_s n^{1/3} A^{2/3} \tag{2}$$

and

$$E_{\mathrm{Coul}} = \sum_{i>j} a_c \frac{(A/\kappa n)^2}{r_{ij}} + \frac{3}{5} n a_c \frac{(A/\kappa n)^2}{R} = \frac{2a_c}{\kappa^2 r_0}\left[\frac{4}{5} n^{-2/3} + (2^{-n} - 1) n^{-5/3}\right] A^{5/3}, \quad (3)$$

where $\kappa = A/Z$ with the number of protons Z, r_{ij} is the distance between the centers of the ith and jth nuclei that constitute the string, and a_s and a_c are set to 17.0 MeV and 1.38 MeV fm, respectively, in a manner that is consistent with empirical masses of stable nuclei. Accordingly, the competition is characterized by the sum

$$E(\kappa, n, A) \equiv E_{\mathrm{Surf}} + E_{\mathrm{Coul}} = 17.0\, n^{1/3} A^{2/3} + \frac{2.30}{\kappa^2}\left[\frac{4}{5} n^{-2/3} + (2^{-n} - 1)\, n^{-5/3}\right] A^{5/3} \text{ MeV}. \quad (4)$$

Although n and A are integers, for optimization we take n and A as real numbers. The extremal condition for given κ and A can then be obtained from

$$\partial_n E(\kappa, n, A) = \frac{850\,\kappa^2\, n^2\, 2^n - [(184\, n - 575)\, 2^n + 239\, n + 575]\, A}{150\,\kappa^2\, n^{8/3}\, 2^n A^{-2/3}} \text{ MeV}. \quad (5)$$

The mass number satisfying $\partial_n E(\kappa, n, A_{\mathrm{crit}}) = 0$ corresponds to

$$A_{\mathrm{crit}}(\kappa, n) = \frac{850\kappa^2 n^2 2^n}{(184n - 575)2^n + 239n + 575}, \quad (6)$$

where the denominator is positive for any n. At least one set of the numbers (κ, n, A) satisfying $\partial_n E(\kappa, n, A) = 0$ exists only when $A > A_{\mathrm{crit}}(\kappa, n)$. Otherwise, $\partial_n E(\kappa, n, A) > 0$ holds. Since, for any given κ, $A_{\mathrm{crit}}(\kappa, n)$ has the global minimum value $A_0(\kappa) = 212(\kappa/2)^2$ at $n = 4$, we can conclude that the value of $A_0(\kappa)$ gives a rough criterion for the occurrence of the stability transition from the state with $n = 1$ to the state with $n \geqslant 2$. In fact, for $A < A_0(\kappa)$ where $\partial_n E(\kappa, n, A) > 0$ is satisfied for any n, a single sphere ($n = 1$) is energetically preferred, while, as shown in Fig. 4(b), there can be more than one value of n that fulfills $\partial_n E(\kappa, n, A) = 0$ for $A > A_0(\kappa)$.

As a result of detailed analyses of the energy landscape, we find that a state with $n = 1$ remains energetically optimal for a mass number up to a value that is a little bit larger than $A_0(\kappa)$. Above this value, the optimal state is not preferably realized in two-nucleus collisions ($n = 2$) but in multinucleus collisions ($n \geqslant 3$). This implies that for superheavy synthesis, production of thin, long nuclei by ternary collisions ($n = 3$) can be more efficient than the production by the usual binary collisions leading to the states with $n = 1$ or $n = 2$ (Table II).

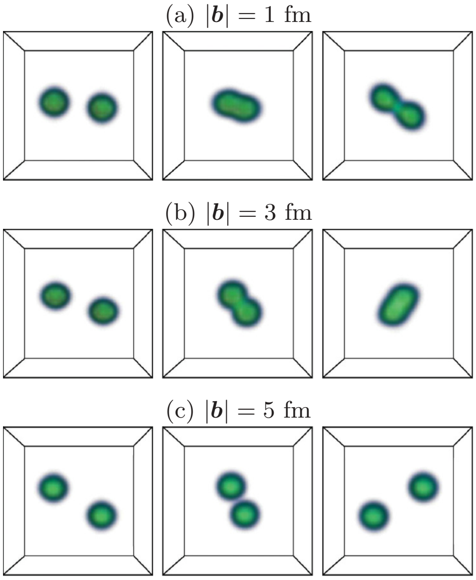

FIG. 3. (Color online) Two-nucleus collisions with different impact parameters. The difference from Fig. 2 lies solely in the absence of nucleus II in the initial configurations illustrated by Fig. 1. Snapshots at 3.33×10^{-22} s (left), 6.66×10^{-22} s (middle), and 13.3×10^{-22} s (right) are shown for $|\boldsymbol{b}| = 1,3,5$ fm.

IV. CONCLUSION

In summary we have found a thin, long structure of heavy nuclei as a result of simultaneous three-nucleus collisions within the TDDFT approach. The validity of the calculations using the SLy6 interaction has been confirmed by using the other interaction (SKI3 [9]) in terms of the realization and rotational stabilization of thin, long heavy nuclei. This study is expected to provide a motivation for designing a new accelerator and detector system for superheavy synthesis in which three-nucleus simultaneous collisions can take place. In fact, in addition to the existing methods for superheavy synthesis that are based on "binary" fusion and multinucleon transfer reactions, "ternary" fusion reactions and subsequent

TABLE I. Lengths of the thin, long products produced by a ternary collision at 26.6×10^{-22} s and by a binary collision at 13.3×10^{-22} s that are shown in Figs. 2 and 3, respectively. The diameter for a minimum sphere that contains each product is calculated and then averaged over all the cases in which the product is fused. The resultant length, denoted by $\mathcal{R}$, is divided by $A^{1/3}$. For comparison, a typical diameter for a spherical nucleus is also given.

	$n = 3$	$n = 2$	Spherical case
$\mathcal{R}/A^{1/3}$ (fm)	4.98	3.63	~2.40

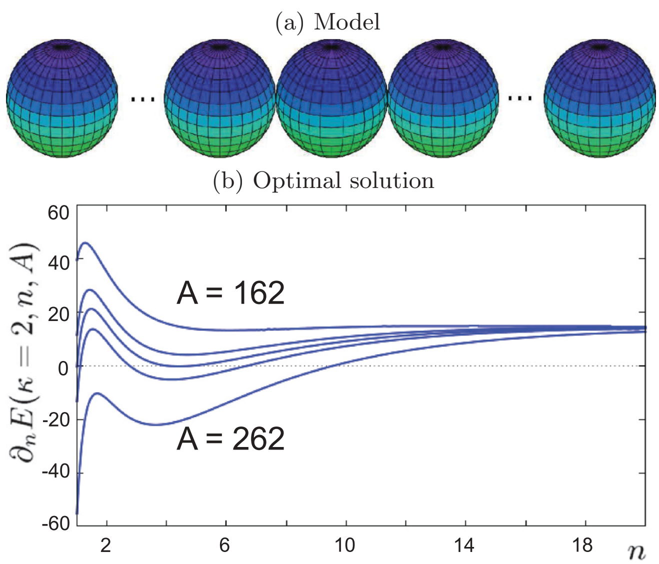

FIG. 4. (Color online) An incompressible liquid-drop model for multinucleus collisions. (a) Linearly connected n spherical nuclei with the total and proton numbers (A, Z). (b) $\partial_n E(\kappa = 2, n, A)$ is depicted as a function of n ($A = 162, 199, 212, 225, 262$). A state with $n = 1$ loses its global stability at $A = 225$, which is a little bit larger than $A_0(\kappa = 2) = 212$, and, for $A > 225$, a state with $n > 2$ is stabilized.

rotational stabilization of compound nuclei could provide a novel method for the superheavy science.

It is also interesting to consider possible astrophysical implications. For example, the three-nucleus fusion rate in stars and supernova cores can be estimated by allowing for the average over the initial configurations through the Boltzmann factor and the plasma effects, i.e., electron screening effects and many-body Coulomb correlation effects between ionic nuclei [5]. The latter effects act to reduce the Coulomb barrier between the colliding nuclei and thus to enhance the fusion rate. If the plasma is relatively dilute and hot as in the Sun, the plasma effects can be safely ignored. In this case, an extension of the usual Gamow rate to three-nucleus fusion applies, leading to the fusion rate as a function of the plasma temperature T. Once the plasma becomes dense and thus strongly coupled as in supernova cores [13], the plasma effects should manifest themselves in the fusion rate through the three-particle static correlation function for electron-screened ions. However, it is a challenging problem to accurately obtain the three-particle correlation function as a function of T and the plasma density ρ [14]. Recall that there are optimal values of the incident energy for three-nucleus fusion in a vacuum. In matter, the optimal values can be lowered by the plasma effects, leading to enhancement of the three-nucleus fusion rate through the Boltzmann factor. If the inverse of this rate at the values of ρ and T relevant for stars and supernova cores is shorter than the corresponding evolutionary time scale, one can expect that thin, long heavy nuclei occur in such celestial objects.

TABLE II. The energy $E(\kappa, n, A)$ calculated from Eq. (4) as function of n. We take $A = 400, 500$ and $\kappa = 2.5$, which are typical values expected for ternary collisions of heavy nuclei in the laboratory.

	$n = 3$	$n = 2$	$n = 1$
$E(2.5, n, 400)$ (GeV)	3.28	3.30	3.32
$E(2.5, n, 500)$ (GeV)	4.38	4.45	4.55

ACKNOWLEDGMENTS

This work was supported in part by the Helmholtz Alliance HA216/EMMI and in part by Grants-in-Aid for Scientific Research on Innovative Areas through No. 24105008 provided by the Ministry of Education, Culture, Sports, Science, and Technology (MEXT) of Japan.

[1] D. D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (The University of Chicago Press, Chicago, 1983).
[2] A. S. Umar, J. A. Maruhn, N. Itagaki, and V. E. Oberacker, Phys. Rev. Lett. **104**, 212503 (2010).
[3] H. Geissel *et al.*, Eur. Phys. J. Special Topics **150**, 109 (2007).
[4] K. Ogata, M. Kan, and M. Kamimura, Prog. Theor. Phys. **122**, 1055 (2009).
[5] S. Ichimaru, Rev. Mod. Phys. **65**, 255 (1993).
[6] L. S. Brown and R. Sawyer, Rev. Mod. Phys. **69**, 411 (1997).
[7] P. A. M. Dirac, Proc. Cambridge Phil. Soc. **26**, 376 (1930).
[8] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, Nucl. Phys. A **635**, 231 (1998); **643**, 441(E) (1998).
[9] P.-G. Reinhard and H. Flocard, Nucl. Phys. A **584**, 467 (1995).
[10] Y. Iwata, T. Otsuka, J. A. Maruhn, and N. Itagaki, Phys. Rev. Lett. **104**, 252501 (2010).
[11] T. Maruyama, G. Watanabe, and S. Chiba, Prog. Theor. Exp. Phys. (2012) 01A201.
[12] T. Ichikawa, J. A. Maruhn, N. Itagaki, and S. Ohkubo, Phys. Rev. Lett. **107**, 112501 (2011).
[13] H. Bethe, Rev. Mod. Phys. **62**, 4 (1990).
[14] S. Ichimaru, *Statistical Plasma Physics* (Addison-Wesley, Reading, 1994), Vol. II.