Shell model analysis of competing contributions to the double-β decay of ^{48}Ca

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Background: Neutrinoless double-β decay, if observed, would reveal physics beyond the standard model of particle physics; namely, it would prove that neutrinos are Majorana fermions and that the lepton number is not conserved.

Purpose: The analysis of the results of neutrinoless double-β decay observations requires an accurate knowledge of several nuclear matrix elements (NME) for different mechanisms that may contribute to the decay. We provide a complete analysis of these NME for the decay of the ground state (g.s.) of ^{48}Ca to the g.s. 0_1^+ and first excited 0_2^+ state of ^{48}Ti.

Method: For the analysis we used the nuclear shell model with effective two-body interactions that were fine-tuned to describe the low-energy spectroscopy of pf-shell nuclei. We checked our model by calculating the two-neutrino transition probability to the g.s. of ^{48}Ti. We also make predictions for the transition to the first excited 0_2^+ state of ^{48}Ti.

Results: We present results for all NME relevant for the neutrinoless transitions to the 0_1^+ and 0_2^+ states, and using the lower experimental limit for the g.s. to g.s. half-life, we extract upper limits for the neutrino physics parameters.

Conclusions: We provide accurate NME for the two-neutrino and neutrinoless double-β decay transitions in the $A = 48$ system, which can be further used to analyze the experimental results of double-β decay experiments when they become available.

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I. INTRODUCTION

If observed, neutrinoless double-β ($0\nu\beta\beta$) decay, which can only occur by violating the conservation of the total lepton number, will reveal physics beyond the standard model, and it will represent a major milestone in the study of the fundamental properties of neutrinos [1–7]. Indeed, its discovery would decide if neutrinos are their own antiparticles [8] and would provide a hint about the scale of their absolute masses. That is why there are intensive investigations of this process, both theoretical and experimental. Recent results from neutrino oscillation experiments have demonstrated that neutrinos have mass and they can mix [9–11]. However, the neutrino oscillation experiments cannot be used to determine the neutrino-mass hierarchy and the lowest neutrino mass. Neutrinoless double-β decay is viewed as one of the best routes to decide these unknowns. A key ingredient for extracting the absolute neutrino masses from $0\nu\beta\beta$ decay experiments is a precise knowledge of the nuclear matrix elements (NME) for this process.

There are potentially many mechanisms that could contribute to the neutrinoless double-β decay process that will be briefly reviewed below. Several of these mechanisms do not provide contributions to the decay rate that explicitly depend on the neutrino masses, but their effect would vanish if the neutrinos are not massive Majorana particles [8]. In all cases the half-lives depend on the nuclear matrix elements that need to be accurately calculated using low-energy nuclear structure models. In particular, if the exchange of light left-handed neutrinos is proven to be the dominant mechanism, one could be able to use the experimental results and the associated NME to extract the neutrino mass hierarchy and the lowest neutrino mass [7]. The two-neutrino double-β ($2\nu\beta\beta$) decay is an associate process that is allowed by the standard model, and it was observed in about ten isotopes. Therefore, a good but not sufficient test of nuclear structure models would be a reliable description of the $2\nu\beta\beta$ half-lives.

Since most of the $\beta\beta$ decay emitters are open-shell nuclei, many calculations of the NME have been performed within the proton-neutron quasiparticle random-phase approximation (pnQRPA) approach and its extensions [12–23]. However, the pnQRPA calculations of the more common two-neutrino double-β decay half-lives, which were measured for about ten cases [24], are very sensitive to the variation of the so-called g_{pp} parameter (the strength of the particle-particle interactions in the 1^+ channel) [12–14], and this drawback still persists in spite of various improvements brought by its extensions [15–20], including higher-order QRPA approaches [21–23]. The outcome of these attempts was that the calculations became more stable against g_{pp} variation, but at present there are still large differences between the values of the NME calculated with different QRPA-based methods, which do not yet provide a reliable determination of the two-neutrino double-β decay half-life. Therefore, although the QRPA methods do not seem to be suited to predict the $2\nu\beta\beta$ decay half-lives, one can use the measured $2\nu\beta\beta$ decay half-lives to calibrate the g_{pp} parameters, which are further used to calculate the $0\nu\beta\beta$ decay NME [25]. Other methods that were recently used to provide NME for most $0\nu\beta\beta$ decay cases of interest are the interacting boson model (IBM-2) [26,27], the projected Hartree-Fock Bogoliubov

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model (PHFB) [28], and the generator coordinate method (GCM) [29].

Recent progress in computer power, numerical algorithms, and improved nucleon-nucleon effective interactions made possible large-scale shell model calculations (LSSM) of the $2\nu\beta\beta$ and $0\nu\beta\beta$ decay NME [30–32]. The main advantage of the large-scale shell model calculations is that they seem to be less dependent on the effective interaction used, as far as these interactions are consistent with the general spectroscopy of the nuclei involved in the decay. Their main drawback is the limitation imposed by the exploding shell model dimensions with the size of the valence spaces that can be used. The most important success of the large-scale shell model calculations was the correct prediction of the $2\nu\beta\beta$ decay half-life for ^{48}Ca [30,33]. In addition, these calculations did not have to adjust any additional parameter; i.e., given the effective interaction and the Gamow-Teller (GT) quenching factor extracted from the overall spectroscopy in the mass region (including β decay probabilities and charge-exchange strength functions), one can reliably predict the $2\nu\beta\beta$ decay half-life of ^{48}Ca.

Clearly, there is a need to further check and refine these calculations and to provide more details on the analysis of the NME that could be validated by experiments. We have recently revisited [34] the $2\nu\beta\beta$ decay of ^{48}Ca using two recently proposed effective interactions for this mass region, GXPF1 and GXPF1A, calculating the NME and half-lives for the transition of the ^{48}Ca ground state (g.s.) to the g.s. and the first excited 2^+ state of ^{48}Ti.

In this paper we add the $2\nu\beta\beta$ transition to the first excited 0_2^+ state of ^{48}Ti to the analysis. We also extend our analysis [35] of the $0\nu\beta\beta$ decay of ^{48}Ca by providing the NME associated with the most important $0\nu\beta\beta$ mechanisms for transitions to the g.s. 0_1^+ and first excited 0_2^+ state of ^{48}Ti. Future experiments on double-β decay of ^{48}Ca (CANDLES [36] and CARVEL [37]) may reach the required sensitivity of measuring such transitions, and our results could also be useful for planning these experiments.

II. TWO-NEUTRINO DOUBLE-β DECAY

LSSM calculations of $2\nu\beta\beta$ decay NME can now be carried out rather accurately for many nuclei [38]. In the case of ^{48}Ca, Ref. [30] reported for the first time a full pf-shell calculation of the NME for the $2\nu\beta\beta$ decay mode for transitions both to the g.s. and to the 2_1^+ excited state of ^{48}Ti. As an effective interaction, the Kuo-Brown G matrix [39] with minimal monopole modifications (KB3 [40]) was used. In Ref. [34] we used the recently proposed GXPF1A two-body effective interaction, which has been successfully tested for the pf shell [41–43], to perform $2\nu\beta\beta$ decay calculations for ^{48}Ca. Our goal was to obtain the values of NME for this decay mode for transitions both to the g.s. and to the 2_1^+ state of ^{48}Ti, with increased degree of confidence, which would allow us to consider similar calculations for the $0\nu\beta\beta$ decay mode of this nucleus [32]. The $2\nu\beta\beta$ transitions to excited states have longer half-lives compared with the transitions to the g.s. due to the reduced values of the corresponding phase-space factors, but they were measured in some cases, such as ^{100}Mo [44].

For the $2\nu\beta\beta$ decay mode the relevant NME are of the Gamow-Teller type and have the following expression for decays to states in the granddaughter that have angular momentum $J = 0, 2$ [1–6]:

$$M_{GT}^{2\nu}(J^+) = \frac{1}{\sqrt{J+1}} \sum_k \frac{\langle J_f^+||\sigma\tau^-||1_k^+\rangle\langle 1_k^+||\sigma\tau^-||0_i^+\rangle}{(E_k + E_J)^{J+1}}. \tag{1}$$

Here E_k is the excitation energy of the 1_k^+ state of the intermediate odd-odd nucleus, and $E_J = \frac{1}{2}Q_{\beta\beta}(J^+) + \Delta M$. $Q_{\beta\beta}(J^+)$ is the Q value corresponding to the $\beta\beta$ decay to the final J_f^+ state of the granddaughter nucleus, and ΔM is the mass difference between the parent and the intermediate nucleus ^{48}Sc. The most common case is the decay to the 0_1^+ g.s. of the granddaughter, but decays to the first excited 0_2^+ and 2_1^+ states are also investigated.

The $2\nu\beta\beta$ decay half-life expression is given by

$$\left[T_{1/2}^{2\nu,J}\right]^{-1} = G_J^{2\nu}\left|M_{GT}^{2\nu}(J)\right|^2, \tag{2}$$

where $G_J^{2\nu}$ are $2\nu\beta\beta$ phase-space factors. Specific values of $G_J^{2\nu}$ for different $2\nu\beta\beta$ decay cases can be found in different reviews, such as Ref. [3]. For a recent analysis of $G_J^{2\nu}$ see Ref. [45]. In Ref. [34] we explicitly analyzed the dependence of the double-Gamow-Teller sum entering the NME equation (1) vs the excitation energy of the 1^+ states in the intermediate nucleus ^{48}Sc. This sum was recently investigated experimentally [46], and it was shown that, indeed, the incoherent sum (using only absolute values of the Gamow-Teller matrix elements) would provide an incorrect NME, thus validating our prediction. We have also corrected by several orders of magnitude the probability of the transition of the g.s. of ^{48}Ca to the first excited 2^+ state of ^{48}Ti reported in Ref. [30].

In Ref. [34] we fully diagonalized 250 1^+ states in the intermediate nucleus to calculate the $2\nu\beta\beta$ decay NME for ^{48}Ca. This procedure can be used for somewhat heavier nuclei using the J-scheme shell model code NUSHELLX [47], but for cases with large dimensions one needs an alternative method. The pioneering work on ^{48}Ca [30] used a strength-function approach that converges after a small number of Lanczos iterations, but it requires large-scale shell model diagonalizations when one wants to check the convergence. Reference [48] proposed an alternative method, which converges very quickly, but it did not provide a complete recipe for all its ingredients, and it was never used in practical calculations. Recently [49], we proposed a simple numerical scheme to calculate all coefficients of the expansion proposed in Ref. [48]. Following Ref. [48], we choose as a starting Lanczos vector $L_1^{\pm}$ either the initial or final state in the decay (only 0^+ to 0^+ transitions are considered) to which we apply the Gamow-Teller operator. This approach is very efficient for large model spaces, for example, the $jj55$ space (consisting of the $0g_{7/2}$, $1d$, $2s$, and $h_{11/2}$ orbits), which for the ^{128}Te decay leads to m-scheme dimensions of the order of 10 billion necessary to calculate the g.s. of ^{128}Xe. In the calculation of ^{48}Ca decay we use the standard quenching factor, $qf = 0.77$, for the Gamow-Teller operator $\sigma\tau$. We checked the result reported in Ref. [34] using

TABLE I. Matrix elements and half-lives for 2ν decay calculated using GXPF1A interaction and two quenching factors. Matrix elements are in MeV^{-1} for transitions to 0^+ states and in MeV^{-3} for transitions to 2^+ states.

	$qf = 0.77$		$qf = 0.74$	
J_n^π	$M^{2\nu}$	$T_{1/2}^{2\nu}$ (yr)	$M^{2\nu}$	$T_{1/2}^{2\nu}$ (yr)
0_1^+	0.054	3.3×10^{19}	0.050	3.9×10^{19}
2_1^+	0.012	8.5×10^{23}	0.010	1.0×10^{24}
0_2^+	0.050	1.6×10^{24}	0.043	1.9×10^{24}

this alternative method, and we found the same result. The novel result reported here for the first time is for the transition to the first excited 0^+ state in ^{48}Ti at 2.997 MeV. The matrix element when using GXPF1A interactions is 0.050, very close to that for the transition to the g.s. Using the phase-space factor $G_{0_2^+}^{2\nu} = 2.43 \times 10^{-22}$ MeV^{-1} from Ref. [3] (a new set of phase-space factors was recently proposed [45], but for $2\nu\beta\beta$ decays they differ only by 4% from those of Ref. [3]), we found that the half-life for this transition is 1.6×10^{24} yr. We recall here that our results reported in [34] for the half-lives of the transitions to g.s. and to the first 2^+ excited state are 3.3×10^{19} and 8.5×10^{23} yr, respectively. One can see that the transition to the first excited 0_2^+ state at 2.997 MeV is predicted to compete with the transition to the first excited 2_1^+ state at 0.994 MeV.

The half-life for the transition to the g.s. 0_1^+ was measured by several groups with increased precision (see, e.g., [24]). The most recent result from NEMO-3 collaboration (see [24] and references therein) is $T_{1/2}^{2\nu} = 4.4^{+0.5}_{-0.4}(\text{stat.}) \pm 0.4(\text{syst.})$. Our GXPF1A result is marginally out of the recently reduced error bars. However, a recent publication [50] found a quenching factor of 0.74 for the pf-shell nuclei using the GXPF1A interaction. The same quenching factor was proposed some time ago [51] using a different effective interaction. The smaller quenching factor of 0.74 brings the calculated half-life within the experimental limits. A comparison of the matrix elements and the associated half-lives for the two quenching factors used here is given in Table I. Potential observation of the $2\nu\beta\beta$ transitions to the excited states of ^{48}Ti could shed some light on the variation of the quenching factor for the Gamow-Teller operator in this nucleus. One should also mention that the excitation energy of the 0_2^+ state in ^{48}Ti calculated with the GXPF1A interaction is about 1 MeV higher than the experimental value, while it is about right for ^{48}Ca. Other available effective interactions do no provide a better description of this state. This result may raise concerns about the validity of the nuclear structure description of this state within the pf shell. An experimental observation of the $2\nu\beta\beta$ transition to this state could be used to validate (or not) our result.

III. NEUTRINOLESS DOUBLE-β DECAY

The $0\nu\beta\beta$ decay, $(Z, A) \to (Z+2, A) + 2e^-$, requires the neutrino to be a massive Majorana fermion; i.e., it is identical to the antineutrino [8]. We already know from the neutrino oscillation experiments that some of the neutrinos participating in the weak interaction have mass and that the mass eigenstates are mixed by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U_{lk}, where l is the lepton flavor and k is the mass eigenstate number (see, e.g., Ref. [52]). However, the neutrino oscillation experiments cannot decide the mass hierarchy, the mass of the lightest neutrino, and some of the CP-nonconserving phases of the PMNS matrix (assuming that neutrinos are Majorana particles).

Considering only contributions from the exchange of light, left-handed(chirality), Majorana neutrinos [7], the $0\nu\beta\beta$ decay half-live is given by

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M_\nu^{0\nu}\right|^2 \left(\frac{|\langle m_{\beta\beta}\rangle|}{m_e}\right)^2. \tag{3}$$

Here, $G^{0\nu}$ is the phase-space factor, which depends on the $0\nu\beta\beta$ decay energy $Q_{\beta\beta}$, the charge of the decaying nucleus Z, and the nuclear radius [3,45]. The effective neutrino mass $\langle m_{\beta\beta}\rangle$ is related to the neutrino mass eigenstates m_k via the left-handed lepton mixing matrix U_{ek},

$$\langle m_{\beta\beta}\rangle/m_e \equiv \eta_{\nu L} = \sum_{k=\text{light}} m_k U_{ek}^2\,/m_e. \tag{4}$$

Here, m_e is the electron mass. The NME $M_\nu^{0\nu}$ is given by

$$M_\nu^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} - M_T^{0\nu}, \tag{5}$$

where $M_{GT}^{0\nu}$, $M_F^{0\nu}$, and $M_T^{0\nu}$ are the Gamow-Teller (GT), Fermi (F), and tensor (T) matrix elements, respectively. Using closure approximation, these matrix elements are defined as follows:

$$\begin{aligned} M_\alpha^{0\nu} &= \langle 0_f^+|\sum_{m,n}\tau_{-m}\tau_{-n}O_{mn}^\alpha|0_i^+\rangle \\ &= \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\big(j_p j_{p'}, j_n j_{n'}; J^\pi\big) \\ &\quad\times \langle j_p j_{p'}; J^\pi T|\tau_{-1}\tau_{-2}O_{12}^\alpha|j_n j_{n'}; J^\pi T\rangle_a, \end{aligned} \tag{6}$$

where O_{mn}^α are $0\nu\beta\beta$ transition operators, $\alpha = (GT,\ F,\ T)$, $|0_i^+\rangle$ is the g.s. of the parent nucleus, and $|0_f^+\rangle$ is the final 0^+ state of the granddaughter nucleus. The two-body transition densities (TBTD) can be obtained from LSSM calculations [35]. Expressions for the antisymmetrized two-body matrix elements (TBME) $\langle j_p j_{p'}; J^\pi T|\tau_{-1}\tau_{-2}O_{12}^\alpha|j_n j_{n'}; J^\pi T\rangle_a$ can be found elsewhere, e.g., Refs. [35,53]. Assuming that one can unambiguously measures a $0\nu\beta\beta$ half-life and one can reliably calculate the NME for that nucleus, one could use Eqs. (3) and (4) to extract information about the lightest neutrino mass and the neutrino mass hierarchy [52]. In addition, one could consider the contribution from the right-handed currents to the effective Hamiltonian, which can mix light and heavy neutrinos of both chiralities [left (L) and right (R)]:

$$\begin{aligned} \nu_{eL} &= \sum_{k=\text{light}} U_{ek}\nu_{kL} + \sum_{k=\text{heavy}} U_{ek}N_{kL}, \\ \nu_{eR} &= \sum_{k=\text{light}} V_{ek}\nu_{kR} + \sum_{k=\text{heavy}} V_{ek}N_{kR}, \end{aligned} \tag{7}$$

where N_k are the heavy neutrinos that are predicted by several seesaw mechanisms for neutrino masses [52]. U_{lk} and V_{lk} are the left- and right-handed components of the unitary matrix that diagonalizes the neutrino mass matrix [54]. One should also mention that there are several other mechanisms that could contribute to the $0\nu\beta\beta$ decay, such as the exchange of supersymmetric (SUSY) particles (e.g., gluino and squark exchange [55]), whose effects are not directly related to the neutrino masses but indirectly related via the Schechter-Valle theorem [8]. Assuming that the masses of the light neutrinos are smaller than 1 MeV and the masses of the heavy neutrinos M_k are larger than 1 GeV, the particle physics and nuclear structure parts get separated, and the inverse half-life can be written as

$$\begin{aligned}\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\big|\eta_{\nu L}M_{\nu}^{0\nu} + \langle\lambda\rangle\tilde{X}_{\lambda} \\ + \langle\eta\rangle\tilde{X}_{\eta} + (\eta_{NL}+\eta_{NR})M_N^{0\nu} \\ + \eta_{\lambda'}M_{\lambda'}^{0\nu} + \eta_{\tilde{q}}M_{\tilde{q}}^{0\nu} + \eta_{KK}M_{KK}^{0\nu}\big|^2,\end{aligned} \tag{8}$$

where $\eta_{\nu L}$ was defined in Eq. (4) and

$$\begin{aligned}\eta_{NL} &= \sum_{k=\text{heavy}} U_{ek}^2\frac{m_p}{M_k}, \\ \eta_{NR} &\approx \left(\frac{M_{W_L}}{M_{W_R}}\right)^4 \sum_{k=\text{heavy}} V_{ek}^2\frac{m_p}{M_k}, \\ \langle\lambda\rangle &= \epsilon\sum_{k=\text{light}} U_{ek}V_{ek}, \\ \langle\eta\rangle &= \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sum_{k=\text{light}} U_{ek}V_{ek}.\end{aligned} \tag{9}$$

Here ϵ is the mixing parameter for the right heavy boson W_R and the standard left-handed heavy boson W_L, $W_R \approx \epsilon W_1 + W_2$, M_{WR} and M_{WL} are their respective masses, and m_p is the proton mass. The $\eta_{\lambda'}$ and $\eta_{\tilde{q}}$ are the R-parity violation contributions in SUSY grand unified theories (GUT) related to the long-range gluino exchange and squark-neutrino mechanism, respectively [52]. Finally, the η_{KK} term is due to possible Kaluza-Klein (KK) neutrino exchange in an extradimensional model [56]. The set of nuclear matrix elements $M_{\nu}^{0\nu}$, $\tilde{X}_{\lambda}$, $\tilde{X}_{\eta}$, $M_N^{0\nu}$, $M_{\lambda'}^{0\nu}$, and $M_{\tilde{q}}^{0\nu}$ is discussed in many reviews, e.g., Ref. [52]. The $M_{KK}^{0\nu}$ analysis can be found in Ref. [56]. In particular, using the factorization ansatz [56], one gets

$$\begin{aligned}\eta_{KK}M_{KK}^{0\nu} &= \frac{\langle m\rangle_{SA}}{m_e}M_{\nu}^{0\nu} + m_p\langle m^{-1}\rangle M_N^{0\nu} \\ &\equiv \eta_{lKK}M_{\nu}^{0\nu} + \eta_{hKK}M_N^{0\nu},\end{aligned} \tag{10}$$

where $\langle m\rangle_{SA}$ and $\langle m^{-1}\rangle$ KK masses depend on the brane shift and bulk radius parameters and are given in Table II of [56]. One can see that the mass parameters $\langle m\rangle_{SA}/m_e$ and $m_p\langle m^{-1}\rangle$ have the effect of modifying $\eta_{\nu L}$ and η_{NR}, respectively. $|m_p\langle m^{-1}\rangle| < 10^{-8}$, and it could, in principle, compete with η_{NR}. $|\langle m\rangle_{SA}/m_e|$ varies significantly with the model parameters, and it could also compete with $\eta_{\nu L}$. One needs to go beyond the factorization ansatz and use information from several nuclei [57] to discern any significant contribution from the KK mechanism.

TABLE II. Matrix elements for 0ν decay using the GXPF1A interaction and two SRC models [61], CD-Bonn (SRC1) and Argonne (SRC2). For comparison, the values labeled (a) are taken from Ref. [27], and the value labeled (b) is taken from Ref. [62] for $g_{pp} = 1$ and no SRC.

	Model	$M_{\nu}^{0\nu}$	$M_N^{0\nu}$	$M_{\lambda'}^{0\nu}$	$M_{\tilde{q}}^{0\nu}$
0_1^+	SRC1	0.90	75.5	618	86.7
	SRC2	0.82	52.9	453	81.8
	others	2.3[(a)]	46.3[(a)]	392[(b)]	
0_2^+	SRC1	0.80	57.2	486	84.2
	SRC2	0.75	40.6	357	80.6

Constraints from collider experiments suggest that terms proportional with the mixing angles ϵ, $U_{ek(\text{heavy})}$, and $V_{ek(\text{light})}$ are very small [54]. The present limits are $|\langle\lambda\rangle| < 10^{-8}$ and $|\langle\eta\rangle| < 10^{-9}$, but they are expected to be smaller. In addition, the contributions from the $\tilde{X}_{\lambda}$ and $\tilde{X}_{\eta}$ terms in Eq. (8) would produce an angular and energy distribution of the outgoing electrons different than that coming from all other terms [2], and these signals are under investigation at SuperNEMO [58]. Here we assume that these contributions are small and can be neglected. In addition, if $\langle\lambda\rangle$ is small, Eq. (9) suggests that η_{NL} is small. Information from colliders also puts some limits on $(M_{W_R}, M_N) \sim (2.5\ \text{GeV}, 1.4\ \text{GeV})$, and these limits will be refined at CERN Large Hadron Collider (LHC) in the coming years. Based on this information and the present limit on the $0\nu\beta\beta$ decay of ^{76}Ge, one can estimate that $|\eta_{\nu L}| < 10^{-6}$, and $|\eta_{NR}| < 10^{-8}$. Then, the half-life can be written as

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|\tilde{\eta}_{\nu L}M_{\nu}^{0\nu} + \tilde{\eta}_N M_N^{0\nu} + \eta_{\lambda'}M_{\lambda'}^{0\nu} + \eta_{\tilde{q}}M_{\tilde{q}}^{0\nu}\right|^2, \tag{11}$$

where we adjusted $\eta_{\nu L}$ and η_{NR} for potential KK contributions, $\tilde{\eta}_{\nu L} = \eta_{\nu L} + \eta_{lKK}$ and $\tilde{\eta}_N = \eta_{NR} + \eta_{hKK}$.

If one neglects the SUSY and KK contributions until a hint of their existence is provided by colliders experiments or future results of $0\nu\beta\beta$ decay experiments show that these contributions are necessary [57], then

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left(\left|M_{\nu}^{0\nu}\right|^2|\eta_{\nu L}|^2 + \left|M_N^{0\nu}\right|^2|\eta_{NR}|^2\right), \tag{12}$$

where we used the fact that the interference between the left-handed terms and the right-handed terms is negligible [52].

The structure of the $M_N^{0\nu}$ is the same as that described in Eqs. (5)–(8), with slightly different neutrino potentials $H_{\alpha}(r)$ (see, e.g., Ref. [52]). A detailed description of the matrix elements of O_{12}^{α} for the jj-coupling scheme consistent with the conventions used by modern shell model effective interactions is given in Ref. [35]. One should also mention that our method [35] of calculating the TBTD, Eq. (6), is different from that used in other shell model calculations [32]. We included in the calculations the recently proposed higher-order terms of the nucleon currents, three old and recent parametrization of the short-range correlations (SRC) effects, finite-size (FS) effects, and intermediate-state energy effects, and we treated carefully a few other parameters entering into the calculations. We found very small variation of the NME with the average energy of the intermediate states and FS cutoff parameters and moderate variation vs the effective interaction and SRC parametrization. We could also show that if the ground-state wave functions of the initial and final nuclei can be accurately described using only the valence space orbitals, the contribution from the core orbitals can be neglected. This situation is different from that of the nuclear parity-nonconservation matrix elements [59], for which the "mean-field"-type contribution from the core orbitals could be significant [60]. Another important result that clearly transpires from our formalism is that in the closure approximation the neutrinoless transition to the first excited 2^+ state is zero. This result is due to the rotational invariance of the TBME entering Eq. (6) (see also the Appendix of Ref. [35]). The structure of the R-parity-breaking SUSY mechanism NME is similar to that of light and heavy neutrino exchange mechanisms, but with no $\alpha = F$ component [55]. The neutrino potentials used here for the $M_N^{0\nu}$ and those used for the most significant contributions to $M_{\lambda'}^{0\nu}$ and $M_{\tilde{q}}^{0\nu}$ NME are given in Ref. [52], but for completeness they are reviewed in the Appendix with the specific parameters included in these calculations.

The results for all NME entering Eq. (11) for the transition to the 0_1^+ g.s. and first excited 0_2^+ state of ^{48}Ti are presented in Table II. Comparisons with results of other models, when available, are also included. For the light neutrino exchange matrix element we choose to compare with the IBM-2 results, which is very different from ours. Other shell model analyses of this particular NME give similar results for both transitions to 0_1^+ and 0_2^+ states [32,63]. To our knowledge, with the exception of the light neutrino exchange NME, no other results of shell model calculations for these matrix elements were reported so far (with the possible exception of Ref. [64], where the NME as a function of neutrino mass is reported, and it could potentially be used to extract the corresponding $M_N^{0\nu}$). Based on these calculations and using the experimental lower limit of the half-life, one can extract the "single-mechanism dominance" upper limits for $|\eta_j|$, where $j = (\nu L),\ N,\ \lambda',\ \tilde{q}$. At present only the lower limit of the half-life for the transition to the g.s. of ^{48}Ti, 1.4×10^{22} yr [52], is available. Using the phase-space factor from Ref. [45], $G^{0\nu} = 61.4 \times 10^{-15}$ yr^{-1} (for $g_A = 1.254$ and $R = 1.2A^{1/3}$ fm), we obtained the upper limits for $|\eta_j|$ shown in Table III. Alternatively, assuming that two or more mechanisms contributing to the half-life in Eq. (11) are competing, one could use the experimental data from several isotopes to assess the contribution of each mechanism [55]. Clearly, this scenario requires as many as possible accurate half-lives and associated NME. For example, in the likely scenario that no more than two mechanisms are competing and they are the light and heavy neutrino exchanges, then Eq. (12) can be used to analyze the data. If the exchange of light neutrinos is determined as the dominant mechanism, then our results could possibly be used to decide the light neutrino mass hierarchy and the lowest neutrino mass [52].

TABLE III. Single-mechanism upper limits for neutrino physics parameters η_j extracted from the lower limit of the half-life for the transition to the ground state of ^{48}Ti [52] and using the matrix elements from Table II.

	Model	$\vert\tilde{\eta}_{\nu L}\vert \times 10^5$	$\vert\tilde{\eta}_N\vert \times 10^7$	$\vert\eta_{\lambda'}\vert \times 10^8$	$\vert\eta_{\tilde{q}}\vert \times 10^7$
0_1^+	SRC1	3.79	4.52	5.52	3.93
	SRC2	4.16	6.45	7.53	4.17

IV. CONCLUSIONS AND OUTLOOK

In conclusion, we analyzed the $2\nu\beta\beta$ and several mechanisms that could compete in the $0\nu\beta\beta$ decays of ^{48}Ca using shell model techniques. We described very efficient techniques to calculate accurate $2\nu\beta\beta$ NME for cases that involve large shell model dimensions. These techniques were tested for the case of ^{48}Ca, and we provided NME and half-lives for $2\nu\beta\beta$ transitions to the g.s. and excited states of ^{48}Ti. They can be used to make predictions for ^{76}Ge and ^{82}Se using the $jj44$ model space ($0f_{5/2},\ 1p,\ 0g_{9/2}$) and for ^{128}Te, ^{130}Te, and ^{136}Xe using the $jj55$ model space.

We reviewed the main contributing mechanisms to the $0\nu\beta\beta$ decay, and we showed that, based on the present constraints from colliders, one could reduce the contributions to the $0\nu\beta\beta$ half-life to the relevant terms described in Eq. (11). A reliable analysis of the $0\nu\beta\beta$ decay experimental data requires accurate calculations of the associated NME. We extended our recent analysis [35] of the $0\nu\beta\beta$ NME for ^{48}Ca to include the heavy neutrino exchange NME, the long-range gluino exchange NME, and the squark-neutrino mechanism NME. We also presented for the first time shell model results of these new NME for the $0\nu\beta\beta$ transitions to the g.s. and the first excited 0_2^+ state in ^{48}Ti.

To extend this analysis to the $A > 48$ cases, more efforts have to be done to include all spin-orbit partners in the valence space and to satisfy the Ikeda sum rule, reduce the center-of-mass spurious contributions, and better understand the changes in the effective $0\nu\beta\beta$ transition operators [65,66]. In addition, the closure approximation used to calculate the NME within the shell model approach and by other methods (e.g., IBM-2 [26], PHFB [28], and GCM [29]) needs to be further checked for accuracy, especially for the heavy neutrino exchange, the long-range gluino exchange, and the squark-neutrino mechanism. An analysis of the double-β decay of ^{136}Xe that addresses some of these issues is in preparation.

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APPENDIX

The matrix elements for the light and heavy neutrino exchanges in Eq. (11) have the same structure as that described in Eqs. (3)–(6) of Ref. [35]. For $M_\nu^{0\nu}$ the neutrino potential is the same as in Eq. (7) of [35]:

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty f_\alpha(qr) \frac{h_\alpha(q^2)}{q + \langle E \rangle} G_\alpha(q^2) q dq, \quad \text{(A1)}$$

with the same ingredients described in Eqs. (9)–(12) of [35]. Here we corrected the $(\mu_p - \mu_n)$ value to 4.71, an error that seems to have been propagating for some time through the literature [7]. This correction explains the small difference between the $M_\nu^{0\nu}$ values of Table II and the corresponding ones reported in Ref. [35]. Fortunately, this correction only changes the matrix elements by a few percent. All other constants are the same as in Ref. [35]. In particular, we used $g_A = 1.254$ and $R = 1.2A^{1/3}$ fm. For $M_N^{0\nu}$ there is a slight change in the neutrino potentials,

$$H_\alpha(r) = \frac{2R}{\pi m_e m_p} \int_0^\infty f_\alpha(qr) h_\alpha(q^2) G_\alpha(q^2) q^2 dq, \quad \text{(A2)}$$

where m_e and m_p are the electron and proton masses, respectively.

The most significant contributions to $M_{\lambda'}^{0\nu}$ and $M_{\tilde{q}}^{0\nu}$ have a structure similar to $M_\nu^{0\nu}$ and $M_N^{0\nu}$; however, only the $\alpha = GT, T$ terms in Eq. (5) are contributing. The radial neutrino potentials for $M_{\lambda'}^{0\nu}$ have the same form as those used for $M_N^{0\nu}$, Eq. (A2), but with different h_α:

$$h_{GT,T} = -(c^{1\pi} + c^{2\pi}) \left[\frac{m_e m_p q^2/m_\pi^4}{1 + q^2/m_\pi^2} + \frac{2 m_e m_p q^2/m_\pi^4}{(1 + q^2/m_\pi^2)^2} \right], \quad \text{(A3)}$$

where m_π is the charged-pion mass, 139 MeV. Expressions for $c^{1\pi}$ and $c^{2\pi}$ are given in Ref. [52]. The numerical values we used are $c^{1\pi} = -85.23$ and $c^{2\pi} = 368.0$.

The radial neutrino potentials for $M_{\tilde{q}}^{0\nu}$ have the same form as those used for $M_\nu^{0\nu}$, Eq. (A1), but with different h_α:

$$h_{GT,T} = -\frac{1}{6} \frac{m_\pi^2}{m_e(m_u + m_d)} \frac{q^2/m_\pi^2}{(1 + q^2/m_\pi^2)^2}, \quad \text{(A4)}$$

where m_u and m_d are the current up and down quark masses. In the calculation we used $m_u + m_d = 11.6$ MeV.

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