

## Excited states of $^{19}\text{Mg}$

H. T. Fortune

*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*

R. Sherr

*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*

(Received 31 May 2012; published 15 January 2013)

We have calculated energies of the first two excited states of  $^{19}\text{Mg}$  by using a model that was previously successful for the ground state. Computed excitation energies are 1.12 and 1.54 MeV for  $(3/2^-)$  and  $(5/2^-)$ , respectively—somewhat in disagreement with values of 1.38 and 2.14 MeV from a recent experiment.

DOI: [10.1103/PhysRevC.87.014316](https://doi.org/10.1103/PhysRevC.87.014316)

PACS number(s): 21.10.Dr, 21.10.Sf, 23.50.+z, 27.20.+n

### I. INTRODUCTION AND HISTORY

Earlier [1], we used a simple potential model, together with mirror symmetry of  $^{18}\text{N}/^{18}\text{Na}$  and  $^{19}\text{N}/^{19}\text{Mg}$  and shell-model (sm) spectroscopic factors, to compute the expected energy of the ground state (g.s.) of  $^{19}\text{Mg}$ . Our prediction was  $E_{2p} = 0.87(7)$  MeV. A later experiment [2] found  $E_{2p} = 0.75(5)$  MeV, just at the  $1\sigma$  limit of the combined uncertainties. A very recent experiment [3] has reported additional states in  $^{19}\text{Mg}$ , and we address them here.

But first, we briefly review the history of relevant calculations and experimental information. We computed energies of several states in  $^{17}\text{Ne}$  and used them, together with sm spectroscopic factors, to calculate the g.s. mass excess of  $^{18}\text{Na}$  [4]. The result was a mass excess of 25.132 MeV. That paper did not assign an uncertainty to this calculated number, but a later paper [1] estimated the uncertainty to be  $\pm 80$  keV. An experiment [5] had suggested the g.s. mass excess to be either 25.04(17) or 24.19(16) MeV. We had reanalyzed those data and had concluded [6] that the higher value was the g.s. and the other peak arose from excited state to excited-state decays. We also demonstrated that the g.s. width in Ref. [5] was not the width of any state but rather the result of two unresolved narrow states. Our analysis gave 25.06(13) for the g.s. With a mass excess of 16.461(17) MeV for  $^{17}\text{Ne}$ (g.s.) [7], these correspond to  $E_p = 1.38(8)$  MeV for our calculation and 1.31(13) MeV for our analysis of the data (1.29(17) MeV for the number in Ref. [5]). The difference was calc – exp = 70(150) keV.

Later, we calculated the energies of the six lowest states of  $^{18}\text{Na}$  and used them, together with shell-model spectroscopic factors, to compute the g.s. energy of  $^{19}\text{Mg}$  [1] as mentioned above. In a brief update [8], we reported new calculations of the  $^{19}\text{Mg}$ (g.s.) width at the experimental energy.

Very recently, we reported new results [9] for the g.s. and lowest  $2^+$  states of  $^{20}\text{F}$  and  $^{20}\text{Mg}$ , updating an earlier calculation [10] of only the g.s. (Ref. [9] contains a list of some of our work on other nuclei). This new calculation used sm spectroscopic factors from a full  $(sd)^4$  calculation. But, we found that a severely truncated calculation gave nearly identical results—because the first three states of  $^{19}\text{O}$  account for nearly all of the summed strength. Agreement with experimental results was excellent.

Even though our prediction for the energy of  $^{19}\text{Mg}$ (g.s.) agreed with the experimental value at the  $1\sigma$  limit of the combined uncertainties, we had been seeking improvements in our calculation by investigating dependence on various components of our model—such as potential-model parameters and source of spectroscopic factors. Results of these efforts are summarized later herein.

Almost simultaneously, results appeared from an experiment [11] to measure energies in  $^{18}\text{Na}$ . We used these to recalculate the g.s. energy of  $^{19}\text{Mg}$ (g.s.) and the sequential  $2p$  decay width [12]. (Of course, the simultaneous  $2p$  decay width does not depend on the  $^{18}\text{Na}$  energies.) The very good agreement between our calculations and the new experimental energies for states in  $^{18}\text{Na}$ , and the apparent robustness of our calculations for  $^{19}\text{Mg}$ (g.s.), gave us confidence to attempt to compute energies of excited states of  $^{19}\text{Mg}$ .

### II. CALCULATIONS AND RESULTS

We treat the first  $3/2^-$  and  $5/2^-$  states of  $^{19}\text{N}$  as a  $p_{1/2}$  proton hole in the first-excited  $2^+$  state of  $^{20}\text{O}$ . Similarly, the  $1/2^-$  g.s. can be thought of as a  $p_{1/2}$  hole in  $^{20}\text{O}$ (g.s.). Likewise, the first six states of  $^{18}\text{N}$  can be considered as a  $p_{1/2}$  hole in the first three states of  $^{19}\text{O}$  with  $J^\pi = 5/2^+, 3/2^+$ , and  $1/2^+$ . Thus, we can construct  $s$  and  $d$  spectroscopic factors for  $^{19}\text{N} \rightarrow ^{18}\text{N}$  by applying weak-coupling (wc) formulas to  $(sd)^4$  spectroscopic factors for  $^{20}\text{O} \rightarrow ^{19}\text{O}$ . The relationship is

$$\begin{aligned} S[^{19}\text{N}(J_{19}) \rightarrow ^{18}\text{N}(J_{18})] \\ = (2J_{18} + 1)(2J + 1)W^2(1/2jJ_{19}J_x; J_{18}J) \\ \times S[^{20}\text{O}(J) \rightarrow ^{19}\text{O}(j)], \end{aligned}$$

where  $W$  is a Racah coefficient and  $J^\pi$  is  $0^+$  or  $2^+$  in  $^{20}\text{O}$ ;  $J_x$  is the single-particle transfer  $1/2, 3/2$ , or  $5/2$ ;  $j^\pi$  is  $5/2^+, 3/2^+$ , or  $1/2^+$  in  $^{19}\text{O}$ ;  $J_{19}^\pi$  is  $1/2^-, 3/2^-,$  or  $5/2^-$  in  $^{19}\text{N}$ ; and  $J_{18}^\pi$  runs from  $0^-$  to  $3^-$  in  $^{18}\text{N}$  (with two each of  $1^-$  and  $2^-$ ).

We take the  $^{20}\text{O} \rightarrow ^{19}\text{O}$  spectroscopic factors from an  $(sd)^4$  sm calculation [9] by using the universal  $sd$ -shell interaction [13]. In a recent treatment of the g.s. and  $2^+$  states of  $^{20}\text{O}/^{20}\text{Mg}$ , we demonstrated [9] that a calculation that included only the first three core states of  $^{19}\text{O}/^{19}\text{Na}$  gave results that were virtually identical to those of a complete calculation. And, in our earlier paper [1] on  $^{19}\text{Mg}$ (g.s.), the first six states of

TABLE I. Comparison of calculations for  $^{19}\text{Mg}(\text{g.s.})$ .

Source of $S$	Potential	$^{18}\text{Na}$ energies	$E_{2p}$ (g.s.) (MeV) <sup>d</sup>
sm <sup>a</sup>	Set 1 <sup>a</sup>	Calc. <sup>a</sup>	0.87 <sup>a</sup>
sm	Set 1	Exp. <sup>c</sup>	0.84 <sup>e</sup>
sm	Set 2 <sup>b</sup>	Calc.	0.80 <sup>b</sup>
sm	Set 2	Exp.	0.76 <sup>b</sup>
smwc <sup>b</sup>	Set 2	Calc.	0.83 <sup>b</sup>
smwc	Set 2	Exp.	0.80 <sup>b</sup>

<sup>a</sup>Reference [1].<sup>b</sup>Present paper.<sup>c</sup>Reference [11].<sup>d</sup>Estimated uncertainty is 70 keV.<sup>e</sup>Reference [12].

$^{18}\text{N}/^{18}\text{Na}$  were enough. In all our papers, we assume that mirror nuclei have the same nuclear structure, which differ only by the Coulomb interaction. Therefore, we can use the  $^{19}\text{N} \rightarrow ^{18}\text{N}$  spectroscopic factors for  $^{19}\text{Mg} \rightarrow ^{18}\text{Na}$ . The model is explained fully in Ref. [9]. Isospin mixing should not be a problem here. In an isospin multiplet, the “interior” nuclei are susceptible to  $T$  mixing but not the ones with minimum and maximum  $T_z$ 's.

Most of our previous papers on Coulomb energies have used a Woods-Saxon potential with geometrical parameters  $r_0 = r_{0c} = 1.25$ ,  $a = 0.65$  fm. We call this potential Set 1. Here, we compare results by using a slightly different Set 2:  $r_0 = 1.26$ ,  $a = 0.60$ , and  $r_{0c} = 1.40$  fm. This set has long been used for the bound (and unbound) state potential in analysis of proton transfer reactions.

Our first item of business is to recalculate the energy of  $^{19}\text{Mg}(\text{g.s.})$  for a number of different inputs: potential Set 1 vs Set 2,  $S$  from sm vs  $S$  from sm + weak coupling, and calculated energies in  $^{18}\text{Na}$  vs recently reported [11] experimental ones. Results are listed in Table I. Results in the first row are from Ref. [1], those in the second row are reported in Ref. [12]; all others are new here. We noted in Ref. [12] that changing from calculated  $^{18}\text{Na}$  energies to experimental ones lowered  $E_{2p}$  by 30 keV. We note here that changing from Set 1 to Set 2 lowers it by about 70 keV. By using sm + wc  $S$ 's rather than sm  $S$ 's increases  $E_{2p}$  by 30 keV. All these changes are within our estimated [1] uncertainty of 70 keV. The relatively small variation in the g.s. calculations mentioned above indicates the likely robustness of our current predictions for the excited states.

We turn now to the  $3/2^-$  and  $5/2^-$  first- and second-excited states of  $^{19}\text{Mg}$ . Here, we use potential Set 2,  $S$ 's from sm + wc, and experimental  $^{18}\text{Na}$  energies. (Because the authors of Ref. [11] did not see the  $1_1^-$  and  $2_2^-$  resonances, we have used our calculated energies for them.) Results are listed in Table II, along with those suggested from a recent experiment [3]. It can be noted that the calculated excitation energies in  $^{19}\text{Mg}$  are slightly lower than the mirror states in  $^{19}\text{N}$ . A similar small downward shift was observed for  $^{20}\text{O}$ - $^{20}\text{Mg}$  and was correctly accounted for [9] by our calculations (Table III.).

A recent experiment [3] reported candidates for several excited states of  $^{19}\text{Mg}$ , populated via neutron removal from

TABLE II. Energies (MeV) of first- and second- excited states in  $^{19}\text{Na}/^{19}\text{Mg}$ .

$J^\pi$	$E_x$ ( $^{19}\text{N}$ ) <sup>a</sup>	$E_x$ ( $^{19}\text{Mg}$ )	
		Calc. <sup>b</sup>	Exp. <sup>c</sup>
$(3/2^-)$	1.141	1.12	1.38(24)
$(5/2^-)$	1.676	1.54	2.14(21)

<sup>a</sup>Reference [14].<sup>b</sup>Present paper.<sup>c</sup>Reference [3].

$^{20}\text{Mg}$  and observed through their  $2p$  and/or  $p + p$  decays to  $^{17}\text{Ne}$ . They suggest states at excitation energies of 1.38 and 2.14 MeV as possibly the  $(3/2^-)$  and  $(5/2^-)$  mirrors of the first two excited states in  $^{19}\text{N}$  [14]. If those identifications are correct, then the disagreement with our calculated energies of 1.12 and 1.54 MeV are much larger than encountered in any previous application. The difference is only about  $1\sigma$  for the first state but is about  $3\sigma$  for the second one. This difference for the proposed  $(5/2^-)$  state is distressingly large. Mukha *et al.* presented some shell-model predictions (within the *spstdpf* space) for  $^{19}\text{Mg}$  states that should have been strong in neutron removal from  $^{20}\text{Mg}$ . The first  $3/2^-$  state in that calculation had an excitation energy of 1.68 MeV, compared to our prediction of 1.12 MeV and the experimental value of 1.38 MeV. They do not list the sm prediction for the  $5/2^-$  state because it is not expected to be strongly populated in their experiment. The next state that should have been strong is the second  $3/2^-$  state at a calculated excitation energy of 3.59 MeV.

A puzzling feature for the supposed  $(5/2^-)$  state is, as noted in Ref. [3], the surprising strength in neutron knockout from  $^{20}\text{Mg}(\text{g.s.})$ , which should contain virtually no  $f_{5/2}$  neutrons. Thus, it would have to be populated through some second-order process, such as inelastic scattering followed (or preceded) by  $n$  removal. We know of no indication that such processes are important. One paper [15], which concerned  $n$  removal from  $^{12}\text{Be}$  to a  $3/2^-$  state, claimed that the nearby  $5/2^-$  state had no observable strength. The analysis of Ref. [3] is sufficiently complicated that we have no suggestions for an alternative explanation of their proposed  $(5/2^-)$  state, but we expect a future paper will provide a different interpretation. Mukha *et al.* do note that they assumed that all measured decay channels feed only the g.s. of  $^{17}\text{Ne}$  and that none go to the first-excited state at 1.29 MeV.

TABLE III. Excited-state energies (MeV) in  $A = 19, 20$ .

$A = 20, T = 2, 2^+$		$A = 19, T = 5/2$ centroid	
Nucl.	$E_x$	Nucl.	$E_x$
$^{20}\text{O}$ exp.	1.674	$^{19}\text{N}$ exp.	1.462
$^{20}\text{Mg}$ calc.	1.603	$^{19}\text{Mg}$ calc.	1.37
$^{20}\text{Mg}$ exp.	1.598(10)	$^{19}\text{Mg}$ exp.	1.84(16)

### III. SUMMARY

To summarize, we have used mirror symmetry in a simple potential model to calculate expected energies of the first  $3/2^-$  and  $5/2^-$  states of  $^{19}\text{Mg}$ . A similar calculation was previously successful for  $^{19}\text{Mg}(\text{g.s.})$ . Usually, excited-state energies are slightly more reliably calculated in our model than absolute g.s. energies. Our new results are not in agreement with results of a recent experiment, especially for the proposed ( $5/2^-$ )

state. We suspect that this peak will eventually turn out to have a different explanation.

### ACKNOWLEDGMENT

We thank Bradley Berman for his help with some of the calculations.

- 
- [1] H. T. Fortune and R. Sherr, *Phys. Rev. C* **76**, 014313 (2007).
  - [2] I. Mukha *et al.*, *Phys. Rev. Lett.* **99**, 182501 (2007).
  - [3] I. Mukha *et al.*, *Phys. Rev. C* **85**, 044325 (2012).
  - [4] H. T. Fortune, R. Sherr, and B. A. Brown, *Phys. Rev. C* **73**, 064310 (2006).
  - [5] T. Zerguerras *et al.*, *Eur. Phys. J. A* **20**, 389 (2004).
  - [6] H. T. Fortune and R. Sherr, *Phys. Rev. C* **72**, 034304 (2005).
  - [7] G. Audi, O. Bersillon, J. Blachet, and A. H. Wapstra, *Nucl. Phys. A* **729**, 3 (2003).
  - [8] H. T. Fortune and R. Sherr, *Phys. Rev. C* **83**, 057301 (2011).
  - [9] H. T. Fortune, R. Sherr, and B. A. Brown, *Phys. Rev. C* **85**, 054304 (2012).
  - [10] R. Sherr, H. T. Fortune, and B. A. Brown, *Eur. Phys. J. A* **5**, 371 (1999).
  - [11] M. Assie *et al.*, *Phys. Lett. B* **712**, 198(2012).
  - [12] H. T. Fortune and R. Sherr, *Phys. Rev. C* **85**, 051302 (2012).
  - [13] B. H. Wildenthal, *Prog. Part. Nucl. Phys.* **11**, 5 (1984).
  - [14] D. Sohler *et al.*, *Phys. Rev. C* **77**, 044303 (2008).
  - [15] W. A. Peters *et al.*, *Phys. Rev. C* **83**, 057304 (2011).