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Fine structure in α decay of even-even trans-lead nuclei: An insufficiently exploited spectroscopic tool

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The experimental values of both branching ratios and hindrance factors for the alpha decay of even-even trans-lead nuclei, corresponding to the 2_1^+, 4_1^+, and 6_1^+ excited states in the daughter nucleus, are examined within the valence correlation scheme. Existing calculations do not reproduce certain conspicuous features of these data in the deformed nuclei region, thus leaving open the question of what details of nuclear structure in this region are responsible for these effects.

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Although it is one of the oldest observed nuclear structure phenomena, alpha decay remains a very important experimental tool for the investigation of unstable nuclei, especially the superheavy ones. The fine structure of the alpha decay was experimentally observed by Salomon Rosenblum in 1929 [1], and, soon after that, explained by Gamow [2] as due to the population of the ground and excited states in the daughter nucleus.

The population of the excited states in the residual nucleus is usually much weaker than that of the ground state, mainly due to the different Q values, reflecting the strong energy dependence of the penetrability of the alpha particle through the nuclear and Coulomb potential barrier. On the other hand, the population intensity of the excited states also contains important information on their structure. However, the theoretical calculation of the absolute values of the alpha-decay rates is a problem that is not fully solved yet. The best studied alpha-decay processes are those implying the unhindered transitions (with $L = 0$), that have mostly been measured for the ground state to ground state transition of even-even nuclei, whereas the population of the excited states in the daughter nuclei (the fine structure of α decay), especially for nonzero spin states, has been much less studied. In this work we show that the experimental fine structure data for the yrast 2^+, 4^+, and 6^+ states in even-even trans-lead daughter nuclei are not fully understood by the current theoretical models of alpha decay, implying that certain details of the structure of these states are neglected, or even not understood. To this end, we review the systematics of the experimental data [3–5] on the α-decay fine structure measured for the even-even trans-lead nuclei.

The quantities experimentally determined in alpha decay are the Q values (related to the excitation energies of the daughter nucleus states), the branching ratios $B_{r,i}$ (for excited states denoted by i), and the half-life $T_{1/2}$ of the alpha-decaying state in the parent nucleus (in our case, the 0^+ ground state). For each state i one defines a partial half-life $T_{1/2,i} = T_{1/2}/B_{r,i}$ and a partial width $\Gamma_i = \hbar \ln 2/T_{1/2,i}$. One usually factorizes the widths as $\Gamma_i = \delta_i^2 P_i$ where δ^2 is called reduced width [6] and mainly contains the nuclear structure information, while P is the penetrability of the alpha particle through the barrier. For comparison with theory one also defines the hindrance factors (HFs) as

$$HF_i = \frac{{\delta_{gs}}^2}{{\delta_i}^2} = \frac{\Gamma_{gs}}{\Gamma_i}\frac{P_i}{P_{gs}} = \frac{B_{r,gs}}{B_{r,i}}\frac{P_i}{P_{gs}}$$

Thus, for the even-even nuclei the hindrance factor defined for the ground state to ground state transition is, by definition, $HF_{gs} = 1$. As ratios of reduced widths, the HFs also contain information on the structure of the states implied in the decay process. On the other hand, they are model dependent quantities because they contain the penetrabilities that can be calculated in various ways. One should also emphasize that the structure information obtained from the alpha decay (the reduced widths) can be compared to similar spectroscopic information obtained from the analysis of the α-pickup (d, ^{6}Li) direct reaction. However, there are reported only a few such experiments for nuclei in our set, and their results corroborated with those from alpha decay [7–9]. An extension of such studies to other (strongly radioactive) targets of interest in the region discussed here is undoubtedly a very difficult task, leaving the alpha decay as the main spectroscopic tool for these nuclei.

An examination of the HFs known for parent nuclei with $Z \geqslant 82$ was recently presented by us [10]. Most of the nuclei in our set, i.e., those with mass number larger than about 220, present collective features. It is well known that for collective nuclei many properties, such as level energies, deformations, $B(E2)$ values, etc., scale very well within the so-called valence correlation scheme (VCS), that is, they present rather compact trends when represented as a function of such quantities as N_pN_n [11], or $P = N_pN_n/(N_p + N_n)$ [12], which represent good measures of the strength of the residual neutron-proton interaction, and of the ratio between the strengths of the neutron-proton and pairing interactions, respectively. Here, N_p (N_n) represent the numbers of active protons (neutrons) counted with respect to the nearest magic number. In Ref. [10] it was argued that these VCS representations should be useful for the HF values because these are quantities that depend only on the nuclear structure. Indeed, it was found that the VCS representations display much more compact trajectories for the HF values, than, e.g., the usual representation as a function of the mass number A [10]. The advantages of

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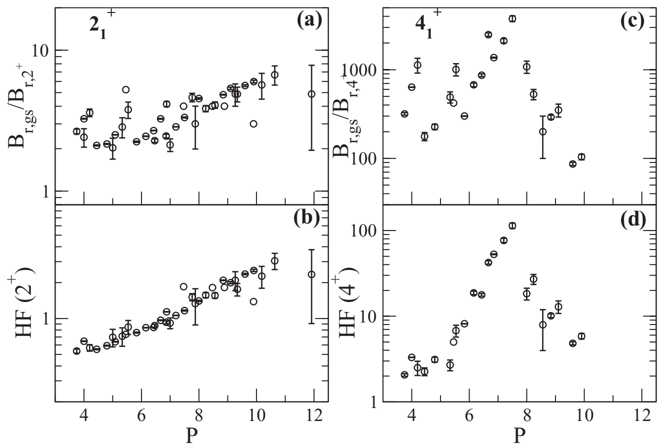

FIG. 1. Relative branching ratios and hindrance factors (as calculated in Ref. [15]) for the 2_1^+ and 4_1^+ states of collective nuclei, represented as a function of the VCS P factor.

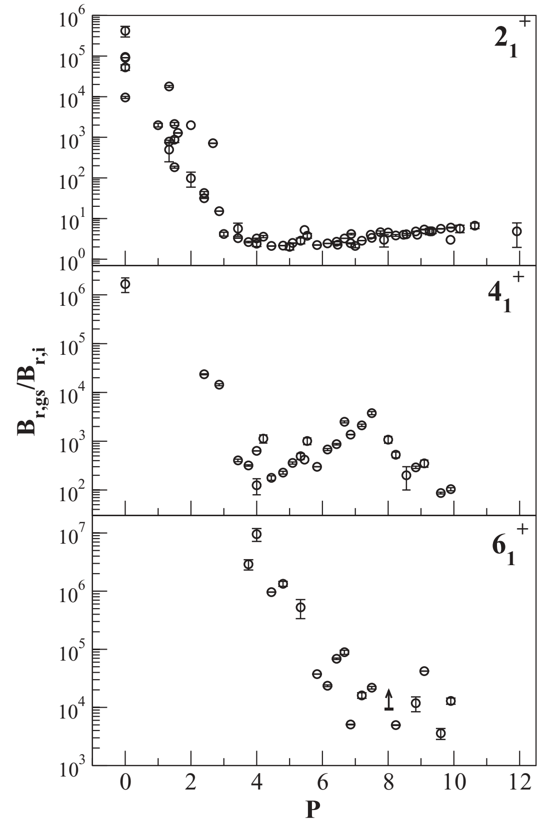

FIG. 2. Experimental relative branching ratios for the first three excited states of the ground state (quasi)band, as a function of the P factor, where $P = N_pN_n/(N_p + N_n)$. Note the scale differences for the three states.

the VCS representations are (i) compact patterns with small scattering of the data points, that can be used to predict HF values in nuclei where these are not measured; (ii) removal of the isobaric ambiguities (in our set of nuclei there are a number of doublets, or even triplets of nuclei having the same mass); (iii) the evolution as a function of P is easily related to that of the collectivity. For these representations, against N_pN_n or P, the active numbers of nucleons have been counted in the daughter nucleus, considering 82 and 126 as magic numbers for protons, and 126 and 184 as magic numbers for neutrons. In principle, in counting the valence nucleon numbers one could consider possible important subshell closures. Thus, $Z = 114$ may be considered a magic number for protons. However, in our set there are only three nuclei (with $Z = 100$ and 104) past the midshell in this case, their positions shifting downwards in P (e.g., in Fig. 2, data for the 2_1^+ state), but this would not change the present discussion because they remain still close to the systematic pattern. For the neutrons, one could also consider other subshell closures, such as $N = 148$ or $N = 164$ [13]. The graphs analogous to those in Fig. 1 present in these cases a considerable scattering of the data points, without a clear systematic emerging. Therefore, for the present discussion we keep the "classical" magic numbers for this region, as also considered in Refs. [11,12].

The results presented in Ref. [10] will now be briefly discussed in connection with the model dependence of the HFs. In Ref. [10] we have used the adopted HF values listed in the ENSDF database [3], which are calculated according to an old recipe of Preston [14] (spherical case, with a rectangular potential well). Another definition often encountered is that of Rasmussen [6] (also spherical case, but using a more realistic nuclear potential). Deformation dependent penetrabilities, best suited for most of the nuclei in our set, were also recently considered (see, for example, Refs. [15,16]). We verified that hindrance factors determined by using different calculations for the penetrabilities generally differ from each other just in absolute value, but present similar evolutions. Figure 1 shows a comparison between the P representations of both relative branching ratios $B_{r,gs}/B_{r,i}$ and HFs of Ref. [15] for the 2^+ and 4^+ states of collective nuclei ($P \gtrsim 4.0$). One can clearly see that the HFs (quantities that are corrected for the barrier penetrability) present much more compact trajectories in this representation.

Because only HFs were examined in Ref. [10], and they depend on how the penetrabilities are calculated, here we examine also the branching ratios, more exactly the ratios $B_{r,gs}/B_{r,i}$, which are purely experimental quantities. Figure 2 shows their evolution as a function of P. The following three features highlighted in Ref. [10] by the examination of the HF's are equally pronounced and visible in the evolution of the relative branching ratios from Fig. 2: (i) a practically exponential increase observed for the 2^+ state for the collective nuclei (with $P \gtrsim 4.0$); (ii) a marked maximum around $P \approx 7.5$ for the 4^+ state; (iii) a decrease for the 6^+ state, in the range $P \sim 4.0$–7.0, a variation practically out of phase with that of the 4^+ state, after which there is a hint of stagnation or even slight increase. Because these outstanding features appear also in $B_{r,gs}/B_{r,i}$, one can exclude that their occurrence in HF_i may be an artifact introduced by the calculated barrier penetrabilities, and it is very likely that they are related to nuclear structure. The following discussion will concentrate on the present status of the understanding of these fine structure data in the frame of various recent theoretical approaches.

Figure 3 presents a comparison of the data from Fig. 2 with results of two types of theoretical calculations, both based on

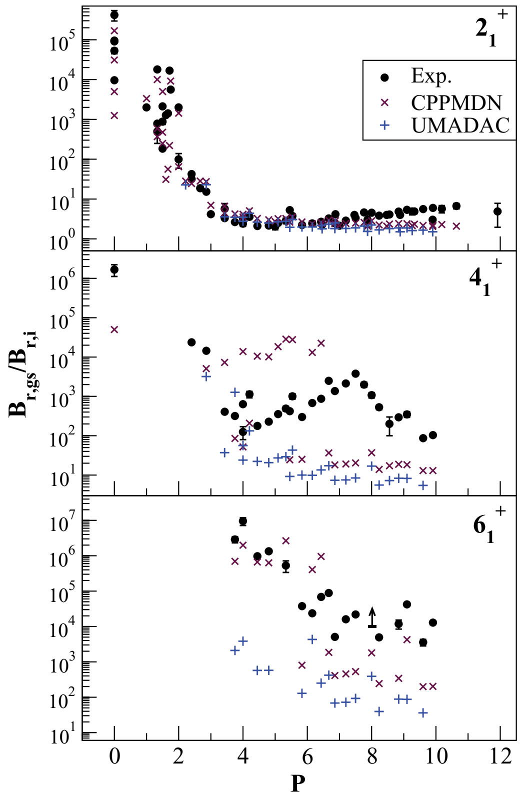

FIG. 3. (Color online) Comparison of experimental relative branching ratios for the three excited states with theoretical model calculations. CPPMDN: Coulomb and proximity potential model for deformed nuclei [18]; UMADAC unified model for alpha decay and alpha capture [17]. Calculated data from the tables of Refs. [17,18].

the semiclassical WKB method: a unified model for α decay and α capture (UMADAC) [17], and a Coulomb and proximity potential model for deformed nuclei (CPPMDN) [18] (results given in the tables of the papers). Very similar results are provided by other calculations, such as those based on the generalized liquid drop model [19], or those based on the Gamow theory with a square well potential barrier penetration [20]. One can see that a reasonable description is obtained with CPPMDN for the vibrational and transitional nuclei (P below $\sim$4.0), while for the rest of the nuclei, especially the well deformed ones (with large P), none of these calculations is able to explain the three main experimental features mentioned above.

Figure 4 shows the same type of comparison as that from Fig. 3, with results of stationary coupled channels calculations [15,21]. Again, while for the vibrational and transitional nuclei [21] a rather good agreement is achieved, for the deformed nuclei [15] the exponential increase for the 2^+ state and the maximum observed for the 4^+ state are not accounted for. A similar disagreement persists for the HF values calculated in Ref. [15].

More recent coupled channels calculations of Ni and Ren [22], that take into account couplings between up to five channels, report an improvement in the description of the fine structure data. The results, essentially contained in their Fig. 1, show that there is still some underestimation of the branchings

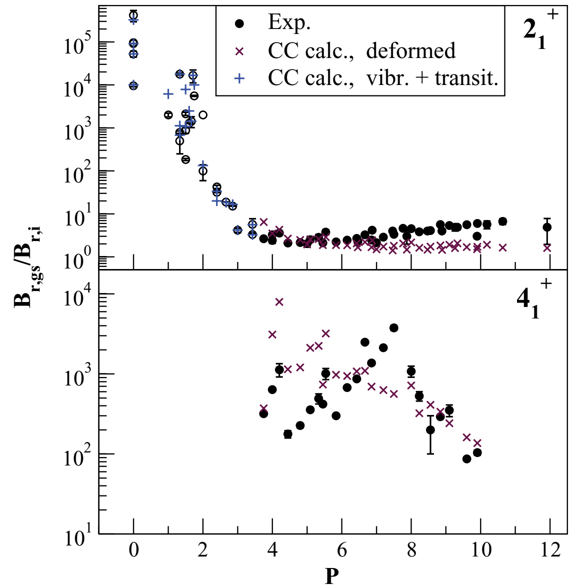

FIG. 4. (Color online) Same as Fig. 3 for the 2^+ and 4^+ states, but for calculations performed with stationary coupled channels calculations for vibrational and transitional [21], and deformed [15] nuclei, respectively.

for the 2^+ state, that increases with the mass number (or P, in our case). For the 4^+ and 6^+ states, although good agreement is obtained for some particular nuclei, the scattering of the points is relatively large, and the striking maximum of the 4^+ state data (Fig. 2) is not described.

All theoretical calculations discussed above do not consider details of the structure of the nuclear states implied in the decay. Only in the coupled channels calculations a rigid rotor model was employed for the deformed nuclei. Therefore it looks interesting to find out whether the observed α-decay fine structure effects, especially those in the region of the deformed nuclei, may be correlated with the evolution of different structure indicators.

Some structure indicators, such as $R(4/2) = E(4_1^+)/E(2_1^+)$, the moment of inertia (as deduced from the energy of the 2_1^+ state), the β_2 (quadrupole) and β_4 (hexadecapole) deformation parameters, were examined in Ref. [10]. Their evolution indicates a region with a normal, smooth transition from vibrational to strongly deformed nuclei [12] [e.g., $R(4/2)$ reaches values larger than 3.25 already at $P \approx 5.0$, thereupon smoothly increasing towards 3.33 with increasing P], and does not correlate with any of the fine structure evolutions of the branching ratios in α decay.

The deformed nuclei (with $P \gtrsim 5.0$) were also examined within the variable moment of inertia (VMI) model [23], by fitting their yrast levels with the formula (also known as the Harris parametrization)

$$E = \frac{1}{2}\omega^2 \left(J_0 + \frac{3}{2}\omega^2 J_1 \right).$$

While J_0 is almost equal to the moment of inertia derived with the rotational model formula from the energy of the 2_1^+ state, the J_1 parameter has an interesting evolution, namely it decreases with increasing P [10]. Within the VMI [23], the

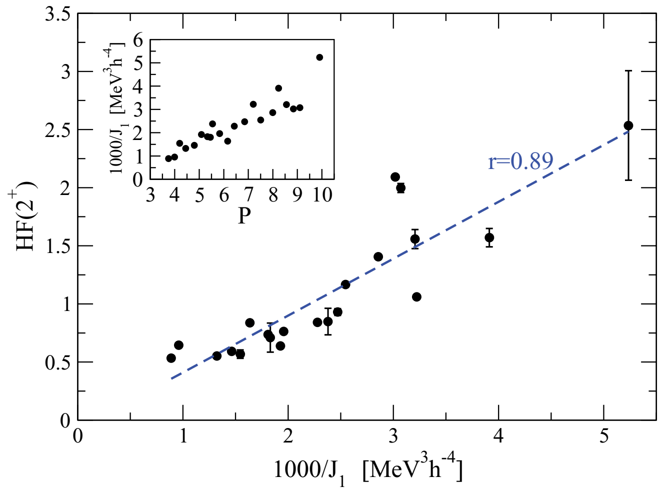

FIG. 5. (Color online) Correlation between the hindrance factors of the 2^+_1 state (as calculated in Ref. [15]) and the inverse of the J_1 Harris parameter which is proportional (see text) with the rigidity parameter. The linear fit to the data gives a correlation coefficient $r = 0.89$.

inverse of J_1 is twice the value of a stiffness parameter C that measures the rigidity of the nucleus. Thus, with increasing P the deformed nuclei become more rigid, that is, their behavior becomes closer to that of a rigid rotor (in the sense that the intrinsic structure of the nucleus changes little with increasing excitation energy, or rotation).

The inset of Fig. 5 shows the steady increase with P of the inverse of J_1. Figure 5 also shows that the hindrance factors of the 2^+_1 states are rather well correlated with the stiffness coefficient, as the relation between the two quantities is rather linear, with a correlation coefficient close to 0.9. This indicates that for a good description of the HF of the 2^+_1 states one should take into account a variable rigidity of the deformed nuclei (those with $P \geqslant 4.0$). The evolution of HF(4^+_1) with the pronounced maximum at $P \approx 7.5$, that is, within the region of well deformed nuclei, could not be correlated with any of these simple structure indicators.

To summarize, a reexamination of the experimental alpha-decay fine structure quantities (both branching ratios and hindrance factors) for the 2^+, 4^+, and 6^+ yrast states in trans-lead nuclei has been presented within the valence correlation scheme, which appears a rather suitable representation especially for the hindrance factors. While the observed patterns are roughly described by coupled channels calculations, some striking details are not described by any of the current theoretical calculations. One of these, namely the exponential increase with P in the region of the deformed nuclei for the HF of the 2^+ state appears to be well correlated with the variation of the nuclear rigidity in the same region. Another outstanding effect, a pronounced maximum of both relative branching ratios and hindrance factors for the 4^+ state in the same region of deformed nuclei, remains a very intriguing, unexplained feature. Thus, it is clear that the alpha-decay fine structure observables contain rather intricate information on the structure of the excited states, that could be revealed by employing more realistic nuclear structure models in the calculations.

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