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Refinement of the n-α and p-α fish-bone potential

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The fish-bone potential of composite particles simulates the Pauli effect by nonlocal terms. We determined the n-α and p-α fish-bone potential by fitting to the experimental phase shifts. We found that with a double Gaussian parametrization of the local potential we can describe the n-α and p-α phase shifts simultaneously for all partial waves.

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I. Introduction. In nature, we can hardly find true elementary particles. Basically most of them are composite particles made of even more elementary constituents. These constituents are fermions. Fermions obey the Pauli principle; i.e., they cannot occupy the same quantum state. On the other hand, when we try to place two composite particles at the same location we try to force the constituents to occupy the same quantum state. The quantum system tries to prevent this and we observe the phenomena of Pauli blocking.

The simplest way to model the Pauli blocking is to use a local repulsive short-range potential. This suppresses the wave function at short distances and reduces the probability of finding the constituents there. Because the Pauli blocking depends strongly on quantum numbers these types of potentials exhibit a strong dependence on partial waves.

In fact, the Pauli blocking is a restriction on the Hilbert space. For composite particles the available Hilbert space is not the same as for structureless particles. Those states, which are occupied by the constituents, are absent or suppressed in the relative motion.

There are several models for composite particle interaction, based on the cluster model, which, more or less, follow this philosophy. Probably the most elaborated is the fish-bone model by Schmid [1,2]. In the fish-bone potential the fully Pauli forbidden states are removed from the Hilbert space. This model also uses the concept of partly Pauli forbidden states, whose contribution is suppressed. As a result, the fish-bone potential is a combination of local and nonlocal terms. The structure of nonlocality is determined by the internal structure of the composite particles and the local potential can be determined from a fitting procedure.

In the fish-bone model we put all the information about the internal structure of the composite particles in their mutual interaction and we hoped that we would achieve some simplification in parameters. Unfortunately, this was only partly true. In the case of the α-α fish-bone potential, the parametrization of Ref. [3] provided a good description of the two-body experimental phase shift, but it needed quite a sizable three-body potential to get the correct binding energy for the three-α system [4].

In a recent study we reexamined the α-α fish-bone potential [5]. We fitted the fish-bone potential to the $l = 0$, $l = 2$, and $l = 4$ experimental phase shifts, the $l = 0$ two-α resonant state, and the low-energy three-α ground and excited states. We found that a single Gaussian term in the local part of the fish-bone potential provides a reasonably good description of all these data. There is no need for any explicit angular momentum dependence and there is no need for a three-body potential. If the composite structure of the particles is properly built into the nonlocality of the interaction, the fitted local part of the potential becomes really simple.

The n-α and p-α fish-bone potentials have been determined in Ref. [6]. For the local part of the potentials, the authors adopted a Wood-Saxon shape with a spin-orbit term. Although they observed an excellent fit to the experimental phase shifts, the agreement was achieved by using independent parameters for each partial wave and different strength parameters for the spin-orbit term for partial waves $^2P_{1/2}$ and $^2P_{3/2}$.

However, some of the parameters are rather close to each other. So, it may be possible to find a better parametrization of these potentials which are, in the spirit of the fish-bone model, simple and do not have explicit angular momentum dependence. Maybe, the angular momentum dependence we observe in nature comes entirely from the composite structure of the α particles.

In Sec. II we outline the fish-bone optical model. In Sec. III we specialize it to the n-α and p-α cases and present our results. In Sec. IV we summarize and draw some conclusions.

II. The fish-bone model for composite particle interaction. The fish-bone model has been derived from the resonating group model. In the resonating group model the total wave function is an antisymmetrized product of the cluster $|\Phi\rangle$ and the intercluster $|\chi\rangle$ relative states:

$$|\Psi\rangle = |\{\mathcal{A}\Phi\chi\}\rangle. \tag{1}$$

The state $|\Phi\rangle$ describes the internal properties of the clusters, including the spin and isospin structure. The unknown relative motion state $|\chi\rangle$ is determined from the variational ansatz

$$\langle\Phi\delta\chi|\mathcal{A}(H-E)\mathcal{A}|\Phi\chi\rangle = 0. \tag{2}$$

This results in a rather complicated equation for $|\chi\rangle$, which is possible to solve only by using strong approximations on $|\Phi\rangle$ and on the interaction of the particles. In a typical example $|\Phi\rangle$ describes fermions in harmonic oscillator potential wells located at some distance apart and $|\chi\rangle$ is the relative motion of the oscillator wells. It is easy to see that if $|\chi\rangle$ is expressed in terms of harmonic oscillator states, some of the lowest states in the relative motion space are not allowed due to the Pauli principle.

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The fish-bone model is a model for the relative motion $|\chi\rangle$ [1]. It is defined by an effective Hamiltonian,

$$h_p = h_p^0 + v_p - \sum_{i,j} |u_{p,i}\rangle\langle u_{p,i}|\big(h_p^0 + v_p - \epsilon_{p,i}\big) \times |u_{p,j}\rangle \bar{M}_{p,ij}\langle u_{p,j}|, \tag{3}$$

where p refers to the angular momentum channel, h_p^0 is the kinetic energy operator, and v_p is a local potential. Our knowledge about the internal structure $|\Phi\rangle$ and our knowledge about the Pauli principle are incorporated in the last term. The states $|u_{p,i}\rangle$ are eigenstates of the norm operator,

$$\langle \Phi\vec{r}|\mathcal{A}|\Phi u_{p,i}\rangle = (1-\eta_{p,i})\langle \vec{r}|u_{p,i}\rangle, \tag{4}$$

where $\vec{r}$ is the center-of-mass distance of the two clusters. If the relative motion is forbidden by the Pauli principle, then $\langle \Phi\vec{r}|\mathcal{A}|\Phi u_{p,i}\rangle = 0$ and $\eta_{p,i} = 1$. The $\eta_{p,i}$ eigenvalues are ordered such that $|\eta_{p,i}| \geqslant |\eta_{p,i+1}|$. The matrix $\bar{M}$ is then given by

$$\bar{M}_{p,ij} = \begin{cases} 1 - \frac{1-\eta_{p,i}}{[(1-\bar{\eta}_{p,i})(1-\bar{\eta}_{p,i})]^{1/2}}, & \text{if} \quad i \leqslant j, \\ 1 - \frac{1-\eta_{p,j}}{[(1-\bar{\eta}_{p,j})(1-\bar{\eta}_{p,i})]^{1/2}}, & \text{if} \quad i > j, \end{cases} \tag{5}$$

where $\bar{\eta}_{p,i} = 0$ if $\eta_{p,i} = 1$ and $\bar{\eta}_{p,i} = \eta_{p,i}$ otherwise. If the system has only one Pauli-forbidden state, the matrix $\bar{M}$, which exhibits a fish-bone-like structure, is given by

$$\bar{M}_p = \begin{pmatrix} 1 & 1 & 1 & 1 & \dots \\ 1 & 0 & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,3}}} & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,4}}} & \dots \\ 1 & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,3}}} & 0 & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,4}}} & \dots \\ 1 & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,3}}} & 1-\sqrt{\frac{1-\eta_{p,2}}{1-\eta_{p,4}}} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \tag{6}$$

We can see that in the Hamiltonian (3) some matrix elements of $h_p^0 + v_p$ are eliminated or partly suppressed by the fish-bone term. If a state $|u_{p,i}\rangle$ is fully Pauli forbidden, then the corresponding elements of $h_p^0 + v_p$ are removed from h_p. Consequently, $|u_{p,i}\rangle$ becomes a solution of the Schrödinger equation at zero energy. Or, if we take $\epsilon_{p,i}$ nonzero in h_p, for Pauli forbidden states only, the corresponding $|u_{p,i}\rangle$ become solutions at ϵ energy. In the fish-bone model we take ϵ as a large positive number. Then the states at physically accessible energies become orthogonal to the Pauli-forbidden states. If the Pauli-forbidden state is like a ground-state harmonic oscillator wave function, i.e., it is without any node, then the physical state has to be orthogonal to the Pauli forbidden state and must have a node. So, the fish-bone model simulates the Pauli blocking by a node in the wave function at short distances.

III. The fish-bone model for the n-α and p-α interactions. We adopt a model that in the α particles the nucleons are in $0s$ states in a harmonic oscillator well of width parameter a. The norm kernel eigenvalues $|u_{p,i}\rangle$ are also harmonic oscillator states with the same width parameter [7]. The eigenvalues are given by the recursion relation $\eta_{l,i+1} = \eta_{l,i}/16$, where $\eta_{l,0} = (-1/4)^l$ and l denotes the orbital angular momentum of the relative motion. So, we have only one fully Pauli-forbidden state in the $^2S_{1/2}$ channel. The other states are partly Pauli forbidden. We used the experimental phase shift compilation from Ref. [8].

The fish-bone model results in a Coulomb-like potential with nonlocal terms. We solved the equations by using the method of Ref. [9]. In this method the problem is written in a Lippmann-Schwinger integral equation form and the short-range terms are expanded in a Coulomb-Sturmian basis. For the ϵ parameter of the fish-bone model, which aims to remove the Pauli-forbidden states, we took $\epsilon = 60\,000$ MeV. In this range of ϵ, the dependence of the results was beyond the fifth significant digit.

The proton and neutron are spin-1/2 particles and the proton has a charge. Therefore we seek the local part of the potential as a sum of a smeared Coulomb, a central, and a spin-orbit term,

$$v(r) = \frac{Ze^2}{r}\mathrm{erf}\left(\sqrt{\frac{2a}{3}}r\right) + V(r) + [j(j+1) - s(s+1) - l(l+1)]\frac{v_{\mathrm{so}}}{r}\frac{d}{dr}V(r), \tag{7}$$

where s is the spin, l is the orbital angular momentum, j is the total angular momentum, and $Z = 0$ for the n-α system and $Z = 2$ for the p-α system.

We took the harmonic oscillator width parameter a and the spin-orbit coupling term v_{so} as fitting parameters and tried out several forms for $V(r)$. We achieved an excellent fit to the phase shift values with a double Gaussian potential:

$$V(r) = v_1\exp(-\alpha_1 r^2) + v_2\exp(-\alpha_2 r^2). \tag{8}$$

The best fit parameters are $a = 0.216\ \mathrm{fm}^{-2}$, $v_1 = -125.241$ MeV, $\alpha_1 = 0.3756\ \mathrm{fm}^{-2}$, $v_2 = 93.625$ MeV, $\alpha_2 = 0.8449\ \mathrm{fm}^{-2}$, and $v_{\mathrm{so}} = -0.2592$. Our results for the n-α and p-α scattering are given in Figs. 1 and 2, respectively, and $V(r)$ is shown in Fig. 3.

We can see that the $^2S_{1/2}$ phase shifts start at 180°, although neither the n-α nor the p-α system has any bound state. This

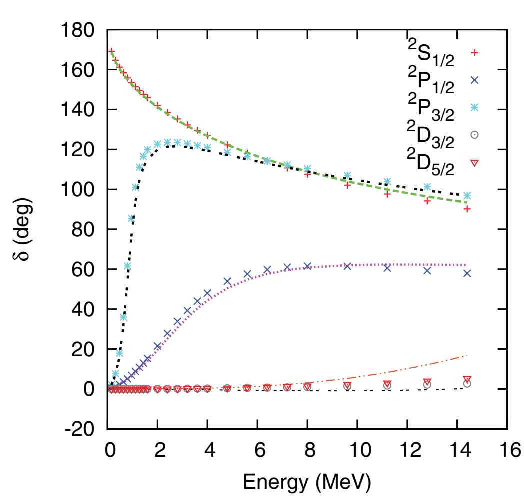

FIG. 1. (Color online) Fit to the experimental $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, and $^2D_{5/2}$ n-α phase shifts.

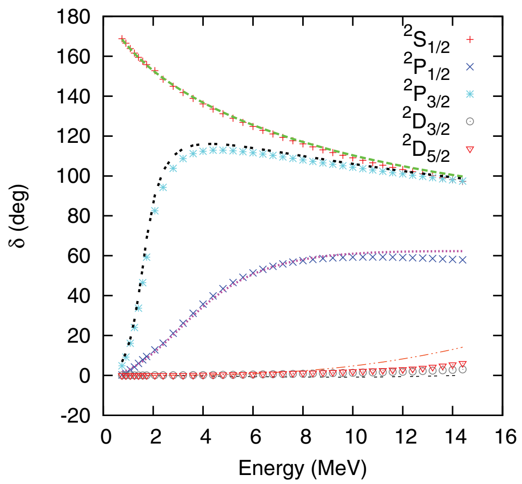

FIG. 2. (Color online) Fit to the experimental $^2S_{1/2}$, $^2P_{1/2}$, $^2P_{3/2}$, $^2D_{3/2}$, and $^2D_{5/2}$ p-α phase shifts.

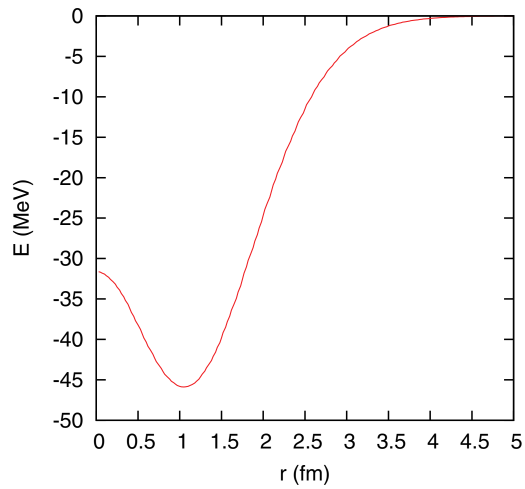

FIG. 3. (Color online) The local potential $V(r)$ of Eq. (8).

seems to be in contradiction to the Levinson theorem, which connects the zero-energy phase shift to the number of bound states. The bound state, which is required by the low-energy behavior of the $^2S_{1/2}$ phase shift, is forbidden by the Pauli principle. Although the Pauli-forbidden states are absent from the spectrum, their effects are clearly visible in the phase shift.

IV. Summary and conclusion. In this work we proposed a new parametrization for the n-α and p-α fish-bone potentials. We determined the parameters of the potential by fitting to the experimental phase shifts. We found that if we incorporate our knowledge on the charge, the spin, and the composite structure into the form of the potential, then the potential to be fitted is really very simple. With the same set of parameters, six parameters altogether, and without any angular momentum dependence in V, we achieved a very good description for the n-α and p-α low-energy scattering data for all the relevant partial waves. In a forthcoming work we will study this potential in the α-α-n and α-α-p systems.

Here we assumed a 0s cluster model for the α particle. This assumption is certainly not valid at higher energies when the α particle is broken up into its constituents. However, the model certainly has relevance at low energies below the α disintegration threshold. It could be very interesting for studying astrophysical processes that generally occur at very low energies.

We believe that the fish-bone model itself deserves further attention. We can see that the fish-bone model really provides a good account of the composite structure of the constituents and of the Pauli principle. Then the local potential, which is to be determined by a fitting procedure, becomes very simple. We can also conclude that the strong angular dependence of the potentials may be mainly due to inadequate treatment of the internal structure of the composite particles. The fish-bone model uses the concept of partly Pauli forbidden states as well. So, it can model the Pauli effect even if there is no complete Pauli blocking, like in the case of nucleon-nucleon interaction.

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