Nuclear quasielastic electron scattering limits nucleon off-mass shell properties

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The use of quasielastic electron nucleus scattering is shown to provide significant constraints on models of the proton electromagnetic form factor of off-shell nucleons. Such models can be constructed to be consistent with constraints from current conservation and low-energy theorems, while also providing a contribution to the Lamb shift that might potentially resolve the proton radius puzzle in muonic hydrogen. However, observations of quasielastic scattering limit the overall strength of the off-shell form factors to values that correspond to small contributions to the Lamb shift.

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I. INTRODUCTION

The structure of nucleons bound in the nucleus is different than that of free nucleons. A prominent example is the European Muon Collaboration (EMC) effect in which the influence of the medium is known to modify the quark distribution functions of nucleons (see the reviews [1]). The existence of medium effects on the structure of nucleons cannot be denied, but elucidating all of the possible effects and the relations between them is a task for ongoing research. We focus here on medium modifications of proton electromagnetic form factors [2], which potentially affect quasielastic scattering [3,4] and may contribute to solving the proton radius puzzle [5].

A prominent example of medium modifications is the work of Ref. [2] which involves measuring the double ratio of proton-recoil polarization-transfer coefficients of the quasielastic $^4\mathrm{He}(e, e'p)^3\mathrm{H}$ reaction with respect to the elastic $^1\mathrm{H}(e, e'p)$ reaction which is sensitive to possible medium modifications of the proton form factor in ^{4}He. Measurements of this double ratio at four-momentum transfers squared between 0.4 and 2.6 GeV^2 performed at both Mainz and Jefferson Lab find a reduction of about 10% in the double ratio, which corresponds to a similar reduction in the ratio of electric to magnetic form factors, G_E/G_M. Models which treat the nucleon as a bound state of three quarks which move under the influence of quarks in other nucleons, consistent with the EMC effect and much nuclear phenomenology, predict such a reduction [6]. Alternative explanations involving final state interactions are possible (see the discussion in Ref. [2] and the references therein), but seem to be incompatible with the totality of relevant data.

It is noteworthy that measurements of quasielastic scattering are related to one of the most perplexing physics issues of recent times—the proton radius puzzle. This puzzle originates in the extremely precise extraction of the proton radius [7] from the measured energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states of muonic hydrogen (H). This Lamb shift depends on the finite size of the proton's electromagnetic field. The extreme precision of the measurement leads to an extracted value of the proton radius smaller than the CODATA [8] value (extracted mainly from electronic H) by about 4% or 5.0 standard deviations. This implies [7] that either the Rydberg constant has to be shifted by 4.9 standard deviations or that present QED calculations for hydrogen are insufficient. Because the Rydberg constant is extremely well measured, and the QED calculations seem to be very extensive and highly accurate, the muonic H finding has presented a significant puzzle to the entire physics community.

We need a brief discussion of the relevant phenomenology. Pohl *et al.* [7] show, perturbatively, that the energy difference between the $2P_{3/2}^{F=2}$ and $2S_{1/2}^{F=1}$ states, $\Delta\widetilde{E}$, is given by

$$\Delta\widetilde{E} = 209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ meV}, \quad (1)$$

where r_p [r_p^2 is related to the slope of $G_E(Q^2)$ at $Q^2 = 0$] is given in units of fm. Using this equation and the experimentally measured value, $\Delta\widetilde{E} = 206.2949$ meV, one can see that the difference between the Pohl *et al.* [7] and the CODATA [8] values of the proton radius would be removed by an increase of the first term on the right-hand side of Eq. (1) by just $0.31 \text{ meV} = 3.1 \times 10^{-10}$ MeV, but an effect of even half that much would be large enough to dissipate the puzzle.

This proton radius puzzle has been attacked from many different directions [5,9–23]. The present communication is intended to investigate the hypothesis [5] that the off-mass shell dependence of the proton electromagnetic form factor that occurs in the lepton-proton two-photon exchange interaction can account for the 0.31 meV. This idea is attractive because the computed effect is proportional to the lepton mass to the fourth power and so is capable of being relevant for muonic atoms, but irrelevant for electronic atoms. To make a calculation one needs to postulate a specific dependence of the electromagnetic form factor as a function of the proton's virtuality (difference between the square of the proton's four-momentum vector and the square of the proton mass). Many different functional forms are possible and the authors of Ref. [5] chose one that accounted for the difference between the muonic and electronic hydrogen measurements.

Reference [24] uses a dispersion analysis of the two-photon exchange term [25,26] to provide limits on the size of the allowed off-shell effect of the specific chosen form of Ref. [5]. This is done by expressing the virtual Compton scattering amplitude implied in Ref. [5] in terms of the invariant $T_{1,2}$ and relating those amplitudes to electric and magnetic polarizabilities. The ones used in Ref. [5] are shown in Ref. [24] to be far larger than the experimentally measured ones. The accuracy of such dispersion relation approaches may be less than previously thought [27]. Nevertheless, we construct a new model of the off-shell form factor that is consistent with all of the conditions mentioned in Ref. [24]. These conditions are derived using the constraints of the second order in chiral perturbation theory. Very recently Birse and McGovern [28] evaluated the constraints to the fourth order in chiral perturbation theory. We show how to develop off-shell models that are consistent with any order of chiral perturbation theory. Moreover, these kinds of off-shell models are testable in a variety of arenas, and in particular it is of interest to examine the consequences of the proposed off-shell model of Ref. [5] for electron nucleus scattering.

The idea we consider is that a bound nucleon can be taken off its mass shell by virtue of its interactions with other nucleons and that the consequences of using the model [5] can therefore be tested. We use off-mass shell here in the sense of the Bethe-Salpeter equation [29,30] in which an integration over all four components of the internal momentum is needed. One may reduce this four-dimensional equation to a three-dimensional equation by putting one (Gross equation [31]) or both (Blankenbecler-Sugar reduction [32]) of the particles on their mass shell. In the present case, the two particles in the Bethe-Salpeter equation are the struck nucleon (which is off its mass shell) and the spectator $(A-1)$ nucleus which is on its mass shell.

We make an explicit calculation of how the ratio of proton electromagnetic form factors, G_E/G_M, is changed in the nuclear medium according to the model of Ref. [5], and we confront the ensuing predictions with the data of Ref. [2], Sec. II. The model of Ref. [5] is shown to yield medium modifications of the ratio G_E/G_M far in excess of the observed effects. A new model is constructed in Sec. III that leaves the ratio G_E/G_M unmodified in the medium. This model also is constructed to be consistent with the restrictions of any finite order in chiral perturbation theory. The off-shell modification depends on an overall strength parameter denoted as λ, which is limited by quasielastic scattering. The corresponding change in the Lamb shift is computed in Sec. IV. We find that the use of the largest values of λ allowed by quasielastic scattering leads to changes in the Lamb shift that are far too small to account for the proton radius puzzle.

II. OFF-SHELL PROTON FORM FACTOR IN QUASIELASTIC ELECTRON SCATTERING

The version of the Dirac form of the electromagnetic vertex operator for an interaction between one on-mass shell nucleon and one off-mass shell nucleon used in Ref. [5] can be expressed as

$$\Gamma^{\mu}_{\rm med}(p',p) = \gamma^\mu F_1(q^2) + \frac{(p+p')^\mu}{2M}\frac{\not p - M}{M}\frac{\frac{-\lambda q^2}{b^2}}{1-q^2/\Lambda^2}F_1(q^2) \quad (2)$$

$$= \gamma^\mu F_1(q^2) + \delta\Gamma^\mu, \quad (3)$$

$$\delta\Gamma^\mu \equiv \frac{(p+p')^\mu}{2M}\frac{\not p - M}{M}F_1(q^2)F(q^2), \quad (4)$$

$$F(q^2) \equiv \frac{\frac{-\lambda q^2}{b^2}}{1-q^2/\Lambda^2}, \quad (5)$$

for a photon interacting with a proton (of momentum p) that is initially off its mass shell:

$$p^2 \neq M^2, \quad (6)$$

where M is the nucleon mass, and $p' = p + q$, with q being the four-momentum of the virtual photon. Note that Eq. (2) is one of three possible forms $(O_{a,b,c})$ of operators proposed in Ref. [5]. Other forms are possible. The values

$$\frac{\lambda}{b^2} = \frac{2}{(79\ \text{MeV})^2} \quad \text{and} \quad \Lambda = 841\ \text{MeV} \quad (7)$$

were used in Ref. [5] to give a contribution to the Lamb shift large enough to allow the CODATA value [8] of the proton radius to be consistent with that of the Pohl *et al.* experiment [7]. We note that the use of Eq. (2) in Ref. [5] is consistent with current conservation because replacing $(p+p')^\mu$ by $(p+p')^\mu - (p+p')qq^\mu/q^2$ gives no change to the computed shift in the atomic binding energy.

The use of the vertex function of Eq. (2) in computing virtual photon-proton Compton scattering leads to new contributions at low values of $Q^2 \equiv -q^2$. It is worthwhile to compare these effects with those of standard formulations in which two invariant amplitudes, T_1 and T_2, appear. Given the model of Eq. (2) there is a new contribution to T_2 (but not to T_1), which has been found to be [24]

$$T_2^{\rm off} \approx -\frac{2\lambda}{\pi M b^2}Q^2, \quad (8)$$

for small values of Q^2. In standard formulations the Q^2 term of T_2 is given in terms of the electric α_E and magnetic β_M polarizability of the proton as $Q^2/e^2(\alpha_E + \beta_M)$. This means that using the constraints imposed by the known electromagnetic polarizabilities along with Eq. (8) yields a value of λ/b^2 that is about 55 smaller than and of the opposite sign to the value given in Eq. (7). See Ref [24] for details. The other models mentioned in Ref. [5] were not used to compute the Lamb shift, but would correspond to different values of λ/b^2 which are of the same order of magnitude as that of Eq. (7), and those models would therefore fare equally poorly. Thus the model of Ref. [5] is not consistent with known features of the virtual photon-proton Compton scattering amplitude. Nevertheless, it is worthwhile to examine the consequences of such a model for other processes to illustrate the connections between different areas of physics. Moreover, the conditions found in Ref. [24] can be simply satisfied, for example, by postulating that the off-shell effects of Eq. (2) be proportional to q^4 instead of q^2.

We therefore consider quasielastic electron nuclear scattering. The aim is to provide a simple evaluation of the consequences of using the vertex function $\delta\Gamma^\mu$ of Eq. (3) in nuclear physics. It is not necessary to provide a complete calculation of quasielastic scattering to achieve this aim. At this stage, all that is needed is a simple estimate of the size of the effects of $\delta\Gamma^\mu$ of Eq. (3). We show that the computed effects

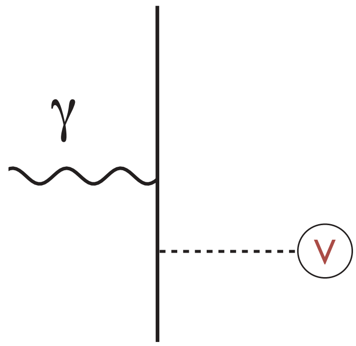

FIG. 1. (Color online) A photon (wiggly line) interacts with a bound nucleon that is off its mass shell because of the interaction V.

are indeed huge, large enough that no technical improvements can remove the vast disagreements with experiment.

The basic idea is that a proton bound in the nucleus is slightly off its mass shell. The average binding energy of a nucleon is less than 1% of its mass. But even this very small binding corresponds to an off-shell effect that contradicts experiment if the model of Eq. (2) is used. Consider a proton bound via a Dirac mean-field Hamiltonian. The bound-state wave function $|\psi\rangle$ obeys the equation

$$|\psi\rangle = \frac{1}{\not{p} - M} V|\psi\rangle, \tag{9}$$

as in Fig. 1, in which the presence of the residual spectator nucleus is represented by the interaction V. The appearance of the interaction V in Eq. (9) causes $p^2 \neq M^2$ and the proton is off its mass shell. The change in the scattering amplitude $\delta\mathcal{M}^\mu$ caused by the off-shell term of Eq. (2) is then given by

$$\delta\mathcal{M}^\mu = \langle\psi'|\delta\Gamma^\mu|\psi\rangle, \tag{10}$$

in which the final state single-particle wave function is represented by $|\psi'\rangle$. Use Eq. (2) and Eq. (9) in Eq. (10) to find

$$\begin{aligned}\delta\mathcal{M}^\mu &= F_1 F\langle\psi'|\frac{(p+p')^\mu}{2M}\frac{\not{p}-M}{M}\frac{1}{\not{p}-M}V|\psi\rangle\\ &= F_1 F\langle\psi'|\frac{(p+p')^\mu}{2M}\frac{V}{M}|\psi\rangle.\end{aligned} \tag{11}$$

We explain the relationship between the amplitude of Eq. (10) (inherent in Fig. 1) and the analysis of Ref. [5]. There are two interaction vertices in Fig. 1, one involving the photon and one involving the strong interaction field. The intermediate virtual proton propagator appears between these two vertices. This propagator is canceled by the inverse propagator in the term $\delta\Gamma^\mu$, so that effectively one sees a contact interaction between the virtual photon, the struck proton, and the residual nucleus. The interaction between the photon and the virtual proton converts the off-shell proton into its final state, $|\psi'\rangle$. Thus the very same off-shell interaction $\delta\Gamma^\mu$ of Ref. [5] enters here. Here one sees the combination of $\delta\Gamma^\mu$ and V; in the two-photon exchange term we use a combination of $\delta\Gamma^\mu$ and Γ^ν.

The next step is to evaluate the matrix element appearing in Eq. (11). Prior to this, we provide a simple estimate to understand that, with Eq. (7), the size of the effect is huge. Consider the regime in which $Q^2 = -q^2 > 0$ is small enough so that the dominant component of $\mathcal{M}^\mu$ and $\delta\Gamma^\mu$ is $\mu = 0$. In that case, the ratio

$$\frac{\delta M^0}{\mathcal{M}^0} \approx \frac{(\lambda/b^2)Q^2}{1+Q^2/\Lambda^2}\frac{\langle\psi'|\frac{V}{M}|\psi\rangle}{\langle\psi'|\psi\rangle}. \tag{12}$$

The largest effects of the interaction V occur at the center of the nucleus, where the density and the mean field can be regarded as constants. Thus we take V to be a number (representing an average nuclear interaction) not an operator. It is sufficient to show that the lower limit of the magnitude of the ratio of Eq. (12) is unacceptably large. To do this, we take V/M to be as small as possible. Therefore we take V/M to be the ratio of the average nuclear binding divided by the nucleon mass (7 MeV for ^{4}He), or $V/M \approx 0.007$. Thus we find

$$\left|\frac{\delta M^0}{\mathcal{M}^0}\right| > \frac{0.007\frac{\lambda}{b^2}Q^2}{1+Q^2/\Lambda^2}. \tag{13}$$

Using Eq. (7), in the above equation, we find that the magnitude of the ratio monotonically increases, reaching 0.1 for $Q \approx 250$ MeV/c and 0.2 for $Q \approx 400$ MeV/c. These lower limits correspond to about a 20% or 36% decrease in the computed cross section. Including such large changes would contrast violently with the knowledge that the (e, e', p) nuclear reaction is a quasielastic scattering process.

The next step is to provide a slightly more detailed evaluation of Eq. (11) so as to allow a comparison with the data of Ref. [2]. To do this, we first examine the single-particle wave functions $|\psi\rangle$ and $|\psi'\rangle$. We treat the final state wave function as a plane wave represented by a Dirac spinor. This is reasonable for the present purpose because the effects of final state interactions are included and removed in the experimental analyses [2]. We aim to consider quasielastic electron scattering in the kinematic regime in which the impulse approximation is valid and thus use a relativistic Fermi gas model to approximate the initial nuclear wave function. This often-used approximation [33] is accurate enough for the schematic estimate that is the present aim. In using this approximation, we neglect the influence of the binding potential on the Dirac spinor of the bound nucleon, which also is of order V/M. This is a separate effect which would enter when using the on-shell form factors. As indicated above, the effects of using our particular off-shell form factors are very dramatic and far outweigh the influence of increasing the precision of the impulse approximation. Under the stated assumptions we find

$$\delta\mathcal{M}^\mu \approx F_1 F\bar{u}(p')\frac{(p+p')^\mu}{2M}\frac{V}{M}u(p). \tag{14}$$

As stated above, the largest effects of the interaction V occur at the center of the nucleus, where the density and the mean field can be regarded as constants. Thus we take V to be a number (representing an average nuclear interaction) not an operator, define $\epsilon \equiv \frac{V}{M}$, and use $-q^2 = Q^2 > 0$ to obtain

$$\delta\mathcal{M}^\mu \approx F_1\bar{u}(p')\left(\frac{(p+p')^\mu}{2M}f(Q^2)\right)u(p), \tag{15}$$

where

$$f(Q^2) \equiv \frac{\epsilon \frac{\lambda Q^2}{b^2}}{1 + Q^2/\Lambda^2}. \tag{16}$$

Under the stated approximations, the present calculation is consistent with current conservation. Replacing $(p + p')^\mu$ by $(p + p')^\mu - (p + p')qq^\mu/q^2$ gives no change to the matrix element of Γ^μ appearing in Eq. (14) because the operator is evaluated between on-shell spinors so that $(p + p') \cdot q = 0$.

We gain insight by using the Gordon identity to make the following replacement: $\frac{(p+p')^\mu}{2M} \to \gamma^u - i\frac{\sigma^{\mu\nu}q_\nu}{2M}$, so that

$$\delta\mathcal{M}^\mu = F_1\bar{u}(p')\left[\gamma^\mu[1 + f(Q^2)] - i\frac{\sigma^{\mu\nu}q_\nu}{2M}f(Q^2)\right]u(p), \tag{17}$$

which shows that the nuclear medium modifies both F_1 and F_2:

$$\delta F_1(Q^2) = F_1(Q^2)f(Q^2),\ \delta F_2(Q^2) = -F_1(Q^2)f(Q^2), \tag{18}$$

so that the change in F_1 is the negative of the change in F_2.

We aim to see whether such modifications are consistent with present observations. Strauch *et al.* [2] measured the ratio of polarization transfer in the ^{4}He nucleus to that of a nucleon for $0.4 < Q^2 < 2.6$ GeV^2. They observed a decrease of about 10%. If final state interactions are properly accounted for, this is a measurement of how the ratio G_E/G_M is changed in the medium. We therefore study the variation of that ratio. Recall the definitions

$$G_E = F_1 - \frac{Q^2}{4M^2}F_2 \quad \text{and} \quad G_M = F_1 + F_2. \tag{19}$$

The medium-modified form factors $\tilde{G}_{E,M}$ are given by adding the changes in $F_{1,2}$ indicated by Eq. (17) and Eq. (18). Note that $\tilde{G}_M = G_M$.

The medium-modified ratio is given by

$$\frac{\tilde{G}_E}{\tilde{G}_M} = \frac{G_E + F_1 f\left(1 + \frac{Q^2}{4M^2}\right)}{G_M} = \frac{G_E}{G_M}\left[1 + \frac{F_1}{G_E}f\left(1 + \frac{Q^2}{4M^2}\right)\right]. \tag{20}$$

We now evaluate the function f. Our aim is to see if the smallest possible values of f are consistent with observations. Therefore we take ϵ to be the ratio of the average nuclear binding divided by the nucleon mass (7 MeV for ^{4}He), so $\epsilon \approx -0.007$. Using Eq. (7) we find

$$f(Q^2) \approx -1.8\frac{\frac{Q^2}{\Lambda^2}}{1 + Q^2/\Lambda^2}, \tag{21}$$

which ranges between -0.6 and -1.3 as Q^2 varies between 0.4 and 2.6 GeV^2. This is between 6 and 25 times the effect observed by Ref. [2], if one asserts that the entire 10% reduction of the double ratio of polarization observables is a true medium modification. Otherwise, the discrepancy would be larger.

One could argue that the model used to evaluate the nuclear effect, taking V/M to be a constant, is too simple to be used. The interaction V represents the nuclear mean field and has a spatial extent corresponding to the size of the entire nucleus. Treating this as a constant means that we are computing form factor modifications near the center of the nucleus. This is appropriate because the experimental analyses of Ref. [2] to which we compare include corrections so as to approximate the situation near the center of the nucleus. The most evident improvement would allow V to have an attractive scalar term and a repulsive vector term. Using this would lead to a larger computed effect because the cancellation between these terms that lead to the small average binding energy of 7 MeV per nucleon would be somewhat disrupted. Using $V/M = -0.007$ minimizes the size of the effect of using Eq. (2) in the nuclear medium. Even with this minimization, the predicted modification of the ratio of electric to magnetic form factors is too large.

Another issue is our use of the Fermi gas model to evaluate the matrix element appearing in Eq. (11). This is not necessary to establish the large effects caused by using $\delta\Gamma^\mu$. Using a bound state spinor (obtained in infinite nuclear matter) causes detailed changes in the numerical results, but does not change the overall huge nature of the computed change in the medium-modified ratio defined in Eq. (20).

Note that the modified ratios that we compute do not show up in full strength in the $(e, e'p)$ experiment. This is because the reaction may occur at the edge of the nucleus. But such effects are far too small to account for the order of magnitude problems we encounter.

The model embodied in Eq. (2) can be regarded as ruled out by the data of Ref. [2]. The next section is concerned with deriving a new model.

III. NEW MODELS THAT DO NOT MODIFY RATIOS OF FORM FACTORS

An alternate approach is to consider the Strauch *et al.* [2] data as a constraint on the effects of medium modifications. In that case, we state

$$\frac{\tilde{G}_E}{\tilde{G}_M} \approx \frac{G_E}{G_M}, \tag{22}$$

where the approximation means within about 10%. We are able to satisfy an even stronger constraint in which the approximation symbol is replaced by an equal sign. We express the constraint in terms of $F_{1,2}$ and $\delta F_{1,2}$, with $\tilde{F}_i = F_i + \delta F_i$, where δF_i is the change in F_i induced by the medium. Using the definitions in Eq. (19) allows us to reexpress the constraint Eq. (22) as

$$\frac{\delta F_1}{F_1} = \frac{\delta F_2}{F_2}. \tag{23}$$

The medium modification of the ratio F_2/F_1 is experimentally accessible [34]. The use of Eq. (23) leads to

$$\frac{F_2 + \delta F_2}{F_1 + \delta F_1} = \frac{F_2}{F_1}. \tag{24}$$

The results, Eqs. (23) and (24), show why our medium modification is so large. Equation (18) shows that $\delta F_1 = -\delta F_2$.

The next step is to see if one can construct a model of off-shell form factors that satisfies the constraints of Eqs. (22)–(24). This can be done if we include an effect that changes F_2 so that Eq. (23) is satisfied. We can do this by adding a new off-shell term of the form $\frac{\sigma^{\mu\nu}q_\nu}{2M}(\not{p} - M)\cdots$. In particular, we postulate a new version of the off-shell vertex intended to replace the ruled-out term $\delta\Gamma^\mu$ of Eq. (2). Defining this operator as $\mathcal{O}^\mu$, we use

$$\mathcal{O}^\mu = \lambda F(Q^2)\left[F_1(Q^2)\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) + i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2(Q^2)\right]\frac{(\not{p} - M)}{M}, \tag{25}$$

which is intended to replace $\delta\Gamma^\mu$ of Eq. (3). The aim is simply to find an off-shell modification that satisfies all of the constraints. Current conservation is explicitly satisfied by both terms. When one calculates the diagram of Fig. 1, the term $\not{q}q^\mu/q^2$ does not contribute because the lepton current is conserved. With Eq. (25) we have

$$\delta F_1 = \lambda F F_1, \quad \text{and} \quad \delta F_2 = \lambda F F_2, \tag{26}$$

so that Eq. (23) is satisfied.

The use of this model in the diagram of Fig. 1 leads to an extremely simple evaluation of the modified quasielastic cross section. The effect of the medium modification is to simply multiply the computed quasielastic scattering cross section by a factor of $[1 + \epsilon\lambda F(Q^2)]^2$. We thus are able to obtain a constraint on the product $\epsilon\lambda F(Q^2)$ without specifying any of the individual factors. The form of $F(Q^2)$ is needed to compute the contribution to the Lamb shift and is discussed below.

If we assume that a 10% change in the cross section (which is the typical uncertainty in the computation of a cross section) can be tolerated, we find that $|\epsilon\lambda| F(Q^2) < 0.05$, or

$$|\lambda| F(Q^2) < 7, \tag{27}$$

for $Q^2 < 10\ \text{GeV}^2$. Quasielastic experiments have not been performed for larger values of Q^2. We take $\epsilon = -0.007$ to obtain the above constraint. We note that this number is the smallest conceivable magnitude that one could extract from nuclear physics. A more detailed analysis would lead to a number that is larger in magnitude and an even stronger constraint on the value of λ. However, the limit, Eq. (27), leads to a very small contribution to the Lamb shift.

IV. LAMB SHIFT CALCULATION

The invariant lepton-proton scattering amplitude arising from two-photon exchange is given by

$$\mathcal{M} = \frac{e^4}{(2\pi)^4}\int d^4k \frac{L_{\mu\nu}(k)T^{\mu\nu}(k,p)}{(k^2 + i\epsilon)^2}, \tag{28}$$

where p is the proton momentum and is evaluated in the common rest frame. The spin-averaged lepton tensor $L_{\mu\nu}$ is given by

$$L_{\mu\nu} = \frac{1}{4m}\text{Tr}\left[(\not{l} + m)\frac{\gamma_\mu(\not{l} - \not{k} + m)\gamma_\nu}{(k^2 - 2l)(k + i\epsilon)}\right], \tag{29}$$

where m is the lepton mass. The term $T^{\mu\nu}$ is the virtual photon nucleon forward scattering amplitude. We use the definition

$$T^{\mu\nu}(k,p) = -\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)T_1(k^0,k^2) + \frac{1}{M^2}\left(p^\mu - \frac{pk}{k^2}k^\mu\right)\left(p^\nu - \frac{pk}{k^2}k^\nu\right)T_2(k^0,k^2), \tag{30}$$

with $k^0 \equiv k\cdot p/M$. Then the use of Eq. (29) in Eq. (28) leads to the following result:

$$\mathcal{M} = \frac{2m\, e^4}{(2\pi)^4}\int \frac{d^4k}{[k^4 - (2lk)^2](k^2 + i\epsilon)^2} \times\left[-\left(2k_0^2 + k^2\right)T_1 + \left(k^2 - k_0^2\right)T_2\right]. \tag{31}$$

We are concerned with the change in $T^{\mu\nu}$ caused by off-shell form factors and denote the corresponding contribution to the Lamb shift, ΔE_{off}. We use the standard procedure in which the zero-energy, constant amplitude $\mathcal{M}$ is treated as a δ function at the origin in coordinate space so that

$$\Delta E_{\text{off}} = -i\mathcal{M}_{\text{off}}\frac{(m_r\alpha)^3}{8\pi}, \tag{32}$$

where the factor appearing to the right of $\mathcal{M}_{\text{off}}$ is the square of the $2S$ muonic hydrogen wave function at the origin. The change in Compton scattering by our postulated off-shell effects is obtained by computing Compton scattering in the Born approximation. Define the conventional electromagnetic vertex operator for the absorption of a photon of momentum k as $\Gamma^\mu(k)$. Then

$$T^{\mu\nu} = T^{\mu\nu}_{\text{on}} + T^{\mu\nu}_{\text{off}} \tag{33}$$

$$= [\Gamma^\mu(-k) + \mathcal{O}^\mu(-k)]\frac{1}{(\not{p} + \not{k} - M)}[\Gamma^\nu(k) + \mathcal{O}^\nu(k)] + [\mu \to \nu, \nu \to \mu, k \to -k] \tag{34}$$

$$= T^{\mu\nu}_{\text{on}} + [\Gamma^\mu(-k)\mathcal{O}^\nu(k) + \mathcal{O}^\mu(-k)\Gamma^\nu(k) + \Gamma^\nu(k)\mathcal{O}^\mu(-k) + \mathcal{O}^\nu(k)\Gamma^\mu(-k)] + [\mathcal{O}^\mu(-k)(\not{p} + \not{k} + M)\mathcal{O}^\nu(k) + \mathcal{O}^\nu(k)(\not{p} - \not{k} + M)\mathcal{O}^\mu(-k)] \tag{35}$$

We need the spin average, obtained by multiplying the above by $(\not{p} + M)/(4M)$ and taking the trace. In the following text all quantities $X^{\mu\nu}$ are spin averaged.

We proceed by breaking up $T^{\mu\nu}$ as a sum of three terms:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{on}} + \delta T^{\mu\nu} + \delta Z^{\mu\nu}, \tag{36}$$

respectively, of order λ^0, λ^1, and λ^2. Then we find

$$T_{1\mathrm{on}} = -\frac{{F_1}^2(2p\cdot k)^2 + 2F_1F_2k^4 + {F_2}^2k^4}{M[k^4-(2pk)^2]}, \quad (37)$$

$$T_{2\mathrm{on}} = \frac{-4F_1^2M^2k^2 + F_2^2k^4}{M[(2pk)^2-k^4]}, \quad (38)$$

which are standard results, and

$$\delta T^{\mu\nu} \equiv \mathrm{Tr}\left[\frac{(\not p + M)}{4M}[\Gamma^\mu(-k)\mathcal{O}^\nu(k) + \mathcal{O}^\mu(-k)\Gamma^\nu(k) + \Gamma^\nu(k)\mathcal{O}^\mu(-k) + \mathcal{O}^\nu(k)\Gamma^\mu(-k)]\right]. \quad (39)$$

We find

$$\delta T_1 = \frac{-\lambda F(-k^2)}{M}\frac{F_2^2k^2 + 4F_1^2M^2}{M^2}, \quad \delta T_2 = 0. \quad (40)$$

The second-order terms are obtained to be

$$\delta Z^{\mu\nu} = \mathrm{Tr}\left[\frac{(\not p + M)}{4M}[\mathcal{O}^\mu(-k)(\not p + \not k + M)\mathcal{O}^\nu(k) + \mathcal{O}^\nu(k)(\not p - \not k + M)\mathcal{O}^\mu(-k)]\right], \quad (41)$$

$$\delta Z_1 = \lambda^2F^2\frac{F_2[F_2(k_0^2 - k^2) - 2F_1k^2]}{M^3},$$
$$\delta Z_2 = \lambda^2F^2\frac{(4F_1^2M^2 - F_2^2k^2)}{M^3}. \quad (42)$$

The low-energy theorem and constraints of chiral perturbation theory constrain $T_i(\nu, Q^2)$ for small values of ν and Q^2. Those constraints, as applied in Ref. [24] and earlier works, are not modified if we choose $F(-k^2) \sim k^4$ for small values of k^2. Thus we use

$$F(-k^2) = \frac{(k^2/\Lambda^2)^2}{[1 + (-k^2)/\Lambda^2)]^2}. \quad (43)$$

Birse and McGovern [28] have provided constraints to the fourth order in chiral perturbation theory. In general, one can satisfy the constraints to the nth order by using a more general version of $F(-k^2)$, $F_n(-k^2)$:

$$F_n(-k^2) = \frac{(-k^2/\Lambda^2)^n}{[1 + (-k^2)/\Lambda^2)]^n}. \quad (44)$$

Now evaluate the integral by Wick rotation:

$$k_0 \to iK_0, \quad \vec{k} \to \vec{K}, \quad k^2 \to -K_0^2 - \vec{K}^2 = -K^2,$$
$$K_0 = K\cos\psi, \quad |\vec{K}| = K\sin\psi. \quad (45)$$

Integrate on ψ from 0 to π:

$$\int d^4k \cdots \to 4\pi i \int dK\,K^3 \int_0^\pi d\psi\,\sin^2\psi \cdots. \quad (46)$$

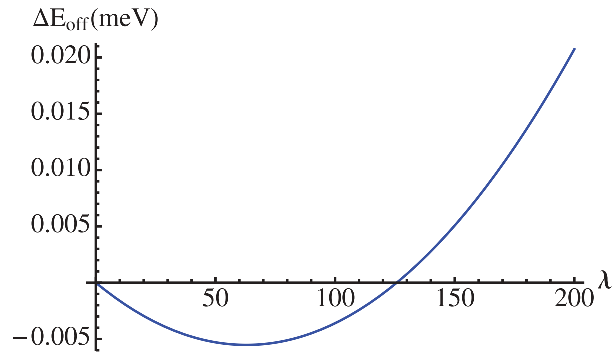

FIG. 2. (Color online) The energy shift ΔE_{off} as a function of the parameter λ, using Eq. (43).

Use $e^2 = 4\pi\alpha$ so

$$\mathcal{M} = i\frac{(4\pi\alpha)^2}{(2\pi)^4}8m\pi\int dK\,K\int_0^\pi d\psi\frac{\sin^2\psi}{K^4 + 4m^2K^2\cos^2\psi} \times [T_1(2\cos^2\psi + 1) - T_2\sin^2\psi)]. \quad (47)$$

Now use Eqs. (40) and (42) in the above to get the off-shell correction. We need

$$\delta T_1 + \delta Z_1 = \frac{-\lambda F(K^2)}{M}\frac{-F_2^2K^2 + 4F_1^2M^2}{M^2} + \lambda^2F^2\frac{F_2[F_2(K^2\sin^2\psi) + 2F_1K^2]}{M^3},$$
$$\delta T_2 + \delta Z_2 = \delta Z_2 = \lambda^2F^2\frac{(4F_1^2M^2 + F_2^2K^2)}{M^3}, \quad (48)$$

$$\delta\mathcal{M}_{\mathrm{off}} = i\frac{8\alpha^2}{\pi}m\int dK\,K\int_0^\pi d\psi\frac{\sin^2\psi}{K^4 + 4m^2K^2\cos^2\psi} \times [(\delta T_1 + \delta Z_1)(2\cos^2\psi + 1) + \delta Z_2\sin^2\psi)]. \quad (49)$$

The above result, along with Eq. (32), determines the value of the computed energy shift arising from the off-shell effect. The principal parameter is λ, constrained to be less than about 7 [Eq. (27)] from quasielastic scattering data. The proton electromagnetic form factors F_1 and F_2 are taken as dipole forms with $\Lambda = 0.841$ GeV, and $F_2(0) = 1.79$. We start by using Eq. (43) and numerical results for values of λ between 0 and 200 are shown in Fig. 2. With $\lambda = 7$, we obtain a shift of -0.001 meV, which is about 100 times too small to significantly affect the Lamb shift calculations. Increasing the value of λ provides a maximal shift of -0.005 meV, but further increases leads to a positive shift in the energy, due to the dominance of the second-order terms δZ_1 and δZ_2 for large values of λ. A positive shift in energy is of the wrong sign to explain the proton radius puzzle.

The requirements of Birse and McGovern [28] can be satisfied by using Eq. (44) with $n = 3$. The use of such a function in calculations of the Lamb shift requires even larger values of λ to explain the proton radius puzzle. The use of our limit $\lambda = 7$ leads again to a very small increase of the Lamb shift: 0.001 meV.

V. DISCUSSION

The principal result we have is that quasielastic electron scattering places significant limits on the off-shell dependence of the nucleon electromagnetic vertex function. While it is possible to construct gauge-invariant models of the off-shell behavior that are consistent with known features of the virtual photon-proton Compton scattering amplitude, these models are incapable of resolving the proton radius puzzle without causing dramatic effects in nuclear quasielastic scattering in disagreement with observed data.

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