Matter radii of $^{29-35}$Mg

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We have computed matter radii for the ground states of $^{29-35}$Mg for a variety of reasonable assumptions about the structure of the relevant states. For cases in which the dominant configuration is generally agreed, our computed radii are in good agreement with experimental ones. For cases in which the dominant configuration is unknown or ambiguous, comparisons between the calculated and experimental R_m do not allow a decision as to the preferred configuration.

I. INTRODUCTION

In Mg nuclei, above about $A = 30$, the structure of low-lying states changes rapidly with changing neutron number. The heaviest Mg isotope whose ground state (g.s.) is primarily within the sd shell is ^{30}Mg, even though it has $N = 18$, not 20. Both ^{31}Mg and ^{32}Mg exhibit characteristics of excitations into the fp shell. In ^{32}Mg, some workers insist the g.s. is mostly an $(fp)^2$ intruder, while others are able to explain the observations without such intruder dominance (but, still with some intruder component). In ^{31}Mg, the evidence is more convincing that the close-lying $1/2^+$ (g.s.) and $3/2^+$ (at 50 keV) involve core excitation. The history is summarized in several recent papers [1–5].

Matter radii R_m provide a convenient meeting ground between experiment and theory. Experimenters typically measure interaction and/or reaction cross sections and analyze them in a Glauber-type model to extract matter radii. Theorists calculate wave functions and/or densities from which they compute matter radii. A simple expression [6–10] relates the square of the matter radius R_m^2 of a neutron-excess nucleus A to that of an $A - 1$ core R_c^2 and R_v^2, the expectation value of r^2 computed for the last neutron. The formula is

$$R_m^2 = [(A-1)/A]\left(R_c^2 + R_v^2/A\right). \tag{1}$$

The quantity R_v^2 is the expectation value of r^2 computed with the wave function of the last neutron, assumed to be a single-particle neutron radial wave function calculated with a Woods-Saxon potential well having $r_0, a = 1.25, 0.65$ fm. The well depth is adjusted to reproduce the neutron separation energy. In these calculations a neutron radial wave function is obtained by solving the Schrödinger equation for given A, ℓ, and binding energy B_n. The expectation value of r^2 is then computed with that wave function by integrating from zero to a very large maximum radius $R_{\max}$.

The strength of our procedure lies in its simplicity. For a given configuration and binding energy, our calculated R_v is exact—subject to the choice of potential parameters mentioned above. Our method of using this R_v to compute R_m is basically equivalent, numerically, to modeling the density as the sum of a core density and the density of a valence neutron, a procedure which is very common in this field. One difference is that in some cases (for example, in Refs. [11,12]) a cutoff radius is introduced to separate interior and exterior regions. Variation of this cutoff radius can be used to artificially enhance the contribution of the neutron tail. We have no such parameter. Once the geometrical parameters of the Woods-Saxon well are chosen (and we always use the same ones), our method is parameter free.

For a core plus valence neutron(s), the computed R_m depends on the binding energy and ℓ value of the valence neutron(s). This dependence is slight except for cases of weak binding and/or low ℓ. Otherwise, rather large variations in the assumed configuration of the valence neutron produce small changes in the computed values of R_m. For a chain of neutron-rich C nuclei, a recent comparison [13] of our calculations with those of one of these more sophisticated models [14] gave excellent agreement—except for the case of ^{17}C, for which the two calculations used different configurations. Our calculations also agreed reasonably well with measured values [15] of R_m for these $^{15-20}$C nuclei.

For even-N nuclei, we can also use the $2n$ procedure of Ref. [7], in which the matter radius is computed from the expression

$$R_m^2 = [(A-2)/A]\left(R_c^2 + 2R_v^2/A\right). \tag{2}$$

This $2n$ procedure was applied to several nuclei in Ref. [7]. It was later proposed by Bhagwat *et al.* [16] for use in a much more sophisticated model. The $2n$ procedure and the approximation of $B_n = B_{2n}/2$ has become a common feature of work in this field [11,14,17–19]. In order to make a 0^+ state, the last two neutrons must be identical. So, having them share the binding energy equally is reasonable. The $2n$ equation is identical to that of Ref. [16], but slightly different from that of Ref. [18]. It is a special case of the generalized expression in Ref. [8].

Suzuki *et al.* [20] used $950A$ MeV beams of several Mg nuclei on a carbon target and measured interaction cross sections. They extracted matter radii from these cross sections with the aid of a Glauber-type model. Kanungo *et al.* [21] performed a similar experiment and analysis for $^{32-35}$Mg. Here, we investigate whether these matter radii can be used to infer dominant configurations in heavy isotopes of Mg.

TABLE I. Binding energies (MeV) and matter radii (fm) for $^{29-34}$Mg.

Nucleus	J^π	Core	n	R_c	B_n [a]	R_v	R_m	R_m (calc)	R_m (exp1)[b]	R_m (exp2)[c]
^{29}Mg(g.s.)	$3/2^+$	^{28}Mg(g.s.)	d	3.00	3.655(12)	3.95	3.04	3.04	3.00(3)	
^{30}Mg(g.s.)	0^+	^{29}Mg(g.s.)	d	3.04	6.361(17)	3.72	3.06	3.06	3.06(2)	
^{30}Mg(exc)[d]	*0^+*	*^{28}Mg(g.s.)*	*f^2*	*3.00*	*8.227/2*	*3.85*	*3.06*	*3.08*		
^{30}Mg(exc)[d]	*0^+*	*^{28}Mg(g.s.)*	*p^2*	*3.00*	*8.227/2*	*4.44*	*3.11*			
^{31}Mg(g.s.)	$1/2^+$	^{30}Mg(exc)	s	3.08	4.159(21)	4.23	3.12	3.12	3.12(6)	
^{31}Mg(exc)[d]	*$3/2^+$*	*^{30}Mg(g.s.)*	*d*	*3.06*	*2.320(21)*	*4.16*	*3.10*			
^{31}Mg(exc)[d]	*$3/2^+$*	*^{30}Mg(exc)*	*d*	*3.08*	*4.109(21)*	*3.94*	*3.11*			
^{32}Mg(g.s.)	0^+	^{31}Mg(exc)	d	3.10	5.843(24)	3.80	3.13	3.13		
^{32}Mg(g.s.)	0^+	^{31}Mg(exc)	s	3.12	5.793(24)	4.00	3.15	3.15		
^{32}Mg(g.s.)	0^+	^{30}Mg(g.s.)	f^2	3.06	8.164/2	3.89	3.12	3.14	3.12(5)	3.17(11)
^{32}Mg(g.s.)	0^+	^{30}Mg(g.s.)	p^2	3.06	8.164/2	4.48	3.17			
^{32}Mg(g.s.)	0^+	^{30}Mg(g.s.)	d^2	3.06	8.164/2	3.96	3.12	3.124		
^{32}Mg(exc)[d]	*2^+*	*^{31}Mg(g.s.)*	*d*	*3.12*	*4.938(24)*	*3.88*	*3.15*			
^{33}Mg(g.s.)	$3/2^-$	^{32}Mg(2$^+$)	f	3.15	3.067(29)	4.05	3.18	3.19		3.19(3)
^{33}Mg(g.s.)	$3/2^-$	^{32}Mg(g.s.)	p	3.14	2.212(29)	5.01	3.20			
^{34}Mg(g.s.)	0^+	^{32}Mg(g.s.)	f^2	3.14	6.670/2	4.00	3.19	3.21		3.23(13)
^{34}Mg(g.s.)	0^+	^{32}Mg(g.s.)	p^2	3.14	6.670/2	4.70	3.25			

[a]Reference [23].
[b]Reference [20].
[c]Reference [21].
[d]Whenever column 1 contains an excited state (exc), that entire row is in italics.

II. CALCULATIONS

We start with ^{30}Mg for two reasons: (1) As mentioned above, it is the heaviest Mg nucleus whose structure is clearly primarily within the sd shell; and (2) Its matter radius has the smallest uncertainty (0.02 fm). We fix $R_m(^{30}$Mg$) = 3.06$ fm. We can then determine the value of R_c that will produce this R_m as ^{29}Mg $+ n$, with $\ell = 2$. This $R_c = 3.03$ fm is then the matter radius of ^{29}Mg, to be compared with the experimental value of 3.00(3) fm.

Our results are summarized and compared with experiment in Table I. Excited states are in italics; g.s. are in roman. Whenever a g.s. has a single dominant configuration, the calculated values in columns 8 and 9 are identical. For mixed configurations, column 9 is the weighted average of values in column 8.

Reference [5], and references therein, argue convincingly that the $1/2^+$ g.s. of ^{31}Mg is of $2p$-$3h$ character, i.e., $(fp)^2(sd)^{-3}$. It is thus presumably an sd-shell neutron coupled to the predominantly $2p$-$4h$ 0^+ excited state of ^{30}Mg. To compute the matter radius of ^{31}Mg(g.s.), we therefore need the radius of this excited 0^+ state in ^{30}Mg. We have used the $2n$ version of Eq. (1) described above and have done the calculation for a mixture of f^2 and p^2, with $f^2/p^2 = 2$, as

suggested by neutron removal experiments [22]. The result is $R_m[^{30}$Mg$($exc $0^+)] = 3.08$ fm, only slightly larger than for the g.s. Coupling an s neutron to this excited 0^+ state produces $R_m[^{31}$Mg$($g.s.$)] = 3.12$ fm, in good agreement with the experimental value of 3.12(6) fm.

Throughout we have used separation energies from the 2011 AMDC preprint of Audi and Meng [23]. They are also listed in Table I.

For ^{32}Mg, we will also need the matter radius for the $3/2^+$ first-excited state of ^{31}Mg—just 50 keV above the g.s. If it is $d_{3/2}$ coupled to ^{30}Mg(g.s.), we find that its matter radius is 3.10 fm, whereas if it is based on the excited 0^+ state, as suggested in Ref. [5], it has $R_m = 3.11$ fm (not very different). If ^{32}Mg(g.s.) is primarily d^2 coupled to ^{30}Mg(g.s.) it has $R_m = 3.12$ fm. If it is d coupled to ^{31}Mg($3/2^+$) or s coupled to ^{31}Mg($1/2^+$), the result is 3.13 or 3.15 fm, respectively. Coupling a 2/1 mixture of f^2,p^2 to the g.s. of ^{30}Mg leads to $R_m = 3.14$ fm. Thus, all the likely configurations for ^{32}Mg(g.s.) produce about the same radius, 3.12–3.15 fm, compared to experimental values of 3.12(5) and 3.17(11) fm. For use in subsequent calculations, we adopt 3.13 fm. We also need the matter radius of the first-excited 2^+ state of ^{32}Mg, for which our result is $R_m = 3.15$ fm.
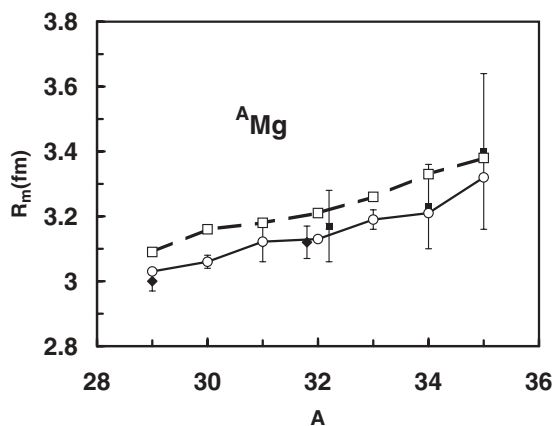
FIG. 1. Matter radii of Mg nuclei from $A = 29$ to 35. Closed symbols with uncertainty bars are experimental (diamonds, Ref. [20]; squares, Ref. [21]). Open symbols are calculated values (circles: present; squares: relativistic mean field from Ref. [25]).

The g.s. of ^{33}Mg is probably $3/2^-$ (Ref. [5], and references therein). We have computed its matter radius as p coupled to ^{32}Mg(g.s.) and as f coupled to ^{32}Mg(2^+). Results are 3.20 and 3.18 fm, respectively—to be compared with the experimental value of 3.19(3) fm.

The most likely structure of ^{34}Mg is $(fp)^2$ coupled to ^{32}Mg(g.s.), for which we compute $R_m = 3.21$ fm. The experimental value is 3.23(13) fm.

We come finally to ^{35}Mg. The J^π of its g.s. is uncertain (likely $3/2^-$ or $5/2^-$ [24]), and the binding energy (0.99(20) MeV [23]) is poorly known. Also, the experimental

R_m has a rather large uncertainty (0.24 fm). For completeness, we have computed R_m for the $2p$ configuration, for which we note agreement, but $1f$ would also barely agree. A more meaningful comparison would require a smaller uncertainty in R_m.

Our results are compared with the experimental values in Fig. 1, where we also compare with results [25] of a relativistic mean-field calculation. The latter are larger than the data for many of the nuclei. Our calculations agree rather well with the matter radii extracted from interaction cross sections [20,21].

III. SUMMARY

In conclusion, we have computed matter radii for the ground states of $^{29-35}$Mg. Whenever the radius of an excited state was needed for use as the core in a subsequent calculation, we have also calculated those. Calculations have been performed for a variety of reasonable assumptions about the structure of the relevant states. For cases in which the dominant configuration is generally agreed, our computed radii are in good agreement with experimental ones. For cases in which the dominant configuration is unknown or ambiguous, in no case does the comparison between the calculated and experimental R_m allow a decision as to the preferred configuration.

Note added. A recent paper [26] has appeared, with calculations of matter radii for Mg and other nuclei. We note that their values are 0.1–0.2 fm larger than ours for the nuclei considered here. They concluded that "…the high-energy cross-section data for O, Ne, and Mg isotopes on a ^{12}C target at $\sim 1000A$ MeV cannot be reproduced…".

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