PHYSICAL REVIEW C **86**, 055204 (2012)

Δ-resonance contribution to the two-photon exchange amplitude

Dmitry Borisyuk[1] and Alexander Kobushkin[1,2]
[1]*Bogolyubov Institute for Theoretical Physics, 14-B Metrologicheskaya street, Kiev 03680, Ukraine*
[2]*National Technical University of Ukraine "KPI", 37 Prospect Peremogy, Kiev 03056, Ukraine*
(Received 22 June 2012; published 29 November 2012)

We consider two-photon exchange (TPE) in the elastic electron-proton scattering and study the contribution arising from the production of $\Delta(1232)$ resonance in the intermediate state. We calculate all three TPE amplitudes (generalized form factors), and find that the Δ contribution mainly influences the generalized electric form factor (contrary to the elastic contribution, which affects the magnetic form factor), and the effect grows with Q^2. If the corresponding correction is applied to the recent polarization transfer measurements of proton form factors, their results will change markedly. Thus we suggest that TPE corrections due to inelastic intermediate states are important to polarization experiments at high Q^2, and should not be neglected.

DOI: 10.1103/PhysRevC.86.055204 PACS number(s): 25.30.Bf, 13.40.Gp, 14.20.Gk, 14.20.Dh

I. INTRODUCTION

Due to smallness of the fine structure constant $\alpha \approx \frac{1}{137}$, the elastic electron-proton scattering amplitude is dominated by the first-order term, corresponding to the exchange of a single photon, Fig. 1, left. The one-photon exchange (OPE) amplitude has a specific structure, which allows, e.g., for the Rosenbluth separation of form factors. In the next (second) order the only nontrivial diagram is two-photon exchange (TPE), Fig. 1, right. Despite its smallness, in some cases TPE correction is very important, because it changes qualitatively the structure of the scattering amplitude. Thus, TPE influence naturally explains the discrepancy between Rosenbluth and polarization methods in proton form factor (FF) measurements. For a further review and up-to-date bibliography see, e.g., Ref. [1].

The full TPE amplitude may be split into separate contributions according to the hadronic intermediate state (IS), which depicted as the blob in TPE diagram. We will have the elastic contribution (pure proton IS), and inelastic ones, which come from multiparticle states such as $p\pi$, $p\pi\pi$, $p\eta$, and so on. We may also distinguish the contributions of resonances, such as Δ, Roper, etc. (This is so-called "hadronic approach". There are also QCD-style calculations [2,3], which assume IS to be a set of quarks. In this paper we resort to the former approach.)

At first, the TPE amplitude was approximated by the elastic contribution, which is the most well-studied one. Then, it was proposed to study contributions of different hadronic resonances as ISs. Kondratyuk *et al.* performed calculations with Δ [4] and several other light resonances [5]. They studied the TPE correction to the cross section and concluded that Δ contribution is the largest among all resonances, though still much smaller than the elastic one. Similar results were obtained for target normal spin asymmetry, the observable that is related to imaginary part of the TPE amplitude [6].

In this paper we present new results for the Δ-resonance contribution to the TPE amplitude in the elastic ep scattering. To calculate it we employ the dispersion method, which was developed in Ref. [7] for ep scattering and applied to $e\pi$ scattering in Ref. [8]. The method is described in detail in those papers, here we just recall that it

(i) ensures correct behavior of the TPE amplitude at $\varepsilon \to 1$ (the amplitude goes to zero),
(ii) eliminates a need for off-shell FFs.

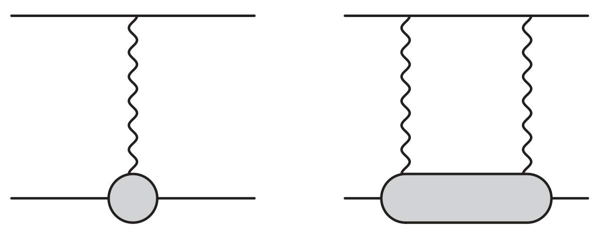

FIG. 1. One- and two-photon exchange diagrams.

In comparison to previous works of Kondratyuk *et al.* [4,5], we consider not just cross-section correction, but all three generalized FFs (TPE amplitudes) and discuss corrections to polarization transfer (PT) experiments.

II. THEORETICAL BACKGROUND

To describe the elastic scattering amplitude in the presence of TPE, we will use the amplitudes $\mathcal{G}_E$, $\mathcal{G}_M$, and $\mathcal{G}_3$, defined in Ref. [7]:

$$\begin{aligned} \mathcal{G}_E &= \tilde{F}_1 - \tau\tilde{F}_2 + \nu\tilde{F}_3/4M^2, \\ \mathcal{G}_M &= \tilde{F}_1 + \tilde{F}_2 + \varepsilon\nu\tilde{F}_3/4M^2, \\ \mathcal{G}_3 &= \nu\tilde{F}_3/4M^2, \end{aligned} \quad (1)$$

where $\tilde{F}_i$ are related to the scattering amplitude as

$$\mathcal{M}_{fi} = -\frac{4\pi\alpha}{q^2}\bar{u}'\gamma_\mu u \times \bar{U}'\left(\gamma^\mu\tilde{F}_1 - \frac{1}{4M}[\gamma^\mu,\hat{q}]\tilde{F}_2 + \frac{P^\mu}{M^2}\hat{K}\tilde{F}_3\right)U \quad (2)$$

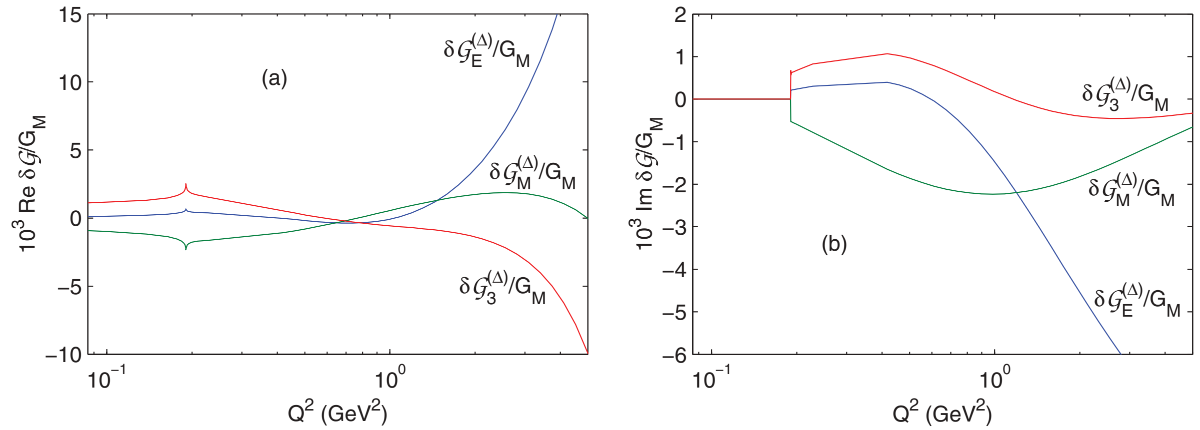

FIG. 2. (Color online) Δ-resonance contribution to TPE amplitudes, $\varepsilon = 0.25$, real part (a) and imaginary part (b).

and all notation is identical to that of Ref. [7]. The TPE contribution will be indicated by the prefix δ, viz.

$$\mathcal{G}_E = G_E + \delta\mathcal{G}_E = G_E + \delta\mathcal{G}_E^{(el)} + \delta\mathcal{G}_E^{(\Delta)} + \dots, \tag{3}$$

where G_E is the usual proton FF, $\delta\mathcal{G}_E^{(el)}$ is elastic contribution, $\delta\mathcal{G}_E^{(\Delta)}$ is the Δ-resonance contribution, and the contributions of other ISs (neglected hereafter) are indicated by the ellipsis. As the amplitude $\mathcal{G}_3$ is absent in OPE approximation, it coincides with the corresponding TPE contribution, $\mathcal{G}_3 = \delta\mathcal{G}_3$.

Recall that the observables are expressed via these amplitudes as follows. The reduced cross-section correction is

$$\delta\sigma_R = 2\,\mathrm{Re}\,(\varepsilon G_E \delta\mathcal{G}_E + \tau G_M \delta\mathcal{G}_M) \tag{4}$$

and the main contribution to it comes from $\delta\mathcal{G}_M$ [9], the correction to the FF ratio[1] $R = G_E/G_M$ is

$$\delta R = R\ \mathrm{Re}\left(\frac{\delta\mathcal{G}_E}{G_E} - \frac{\delta\mathcal{G}_M}{G_M} - \frac{\varepsilon(1-\varepsilon)}{1+\varepsilon}\frac{\delta\mathcal{G}_3}{G_M}\right) \tag{5}$$

and the last term in the brackets has little effect because of small factor $\frac{\varepsilon(1-\varepsilon)}{1+\varepsilon}$.

It is interesting to note that the same combination of the amplitudes determines target normal spin asymmetry [6]:

$$A_n = -\sqrt{2\varepsilon(1+\varepsilon)}\frac{2QMR}{Q^2+4M^2R^2\varepsilon} \times \mathrm{Im}\left(\frac{\delta\mathcal{G}_E}{G_E} - \frac{\delta\mathcal{G}_M}{G_M} - \frac{\varepsilon(1-\varepsilon)}{1+\varepsilon}\frac{\delta\mathcal{G}_3}{G_M}\right). \tag{6}$$

The $\Delta N\gamma^*$ vertex, in general, contains three FFs (magnetic, electric, and Coulomb ones). However, it is well known that electric and Coulomb FFs are small and experimental data are described rather well by single magnetic FF. Thus, to simplify the calculation, we assume purely magnetic $\Delta \to N\gamma^*$ transition, which may be described by the following amplitude (see, e.g., [10]):

$$\mathcal{M}_{\Delta N\gamma} = \sqrt{4\pi\alpha}\, i\epsilon^{\mu\alpha\beta\gamma} p_\beta q_\gamma \bar{U} V_\alpha \frac{F_\Delta(q^2)}{2M^2}, \tag{7}$$

where p and q are Δ and photon momenta, respectively (the nucleon momentum thus will be $p-q$), U is the nucleon spinor, and V_α is the Rarita-Schwinger wave function of the Δ resonance, normalized according to $\bar{V}_\alpha V_\alpha = -2M_\Delta$. The

[1] What is measured in polarization experiments is the proton L/T polarization ratio, which, *in OPE approximation*, is proportional to the G_E/G_M ratio. It will be convenient to divide the polarization ratio by the appropriate kinematical factor and define the *experimentally measured* FF ratio R, which equals to G_E/G_M in the OPE approximation but really differs because of TPE corrections.

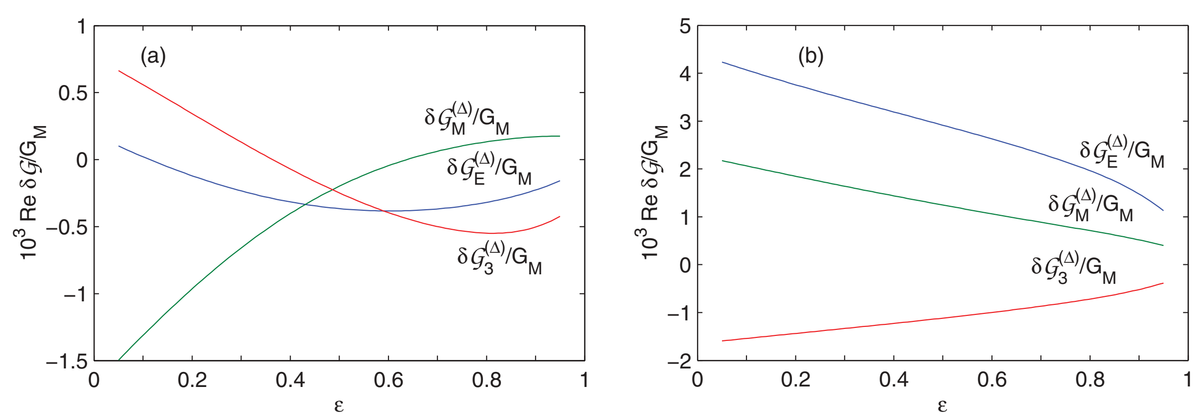

FIG. 3. (Color online) TPE amplitudes at $Q^2 = 0.5$ GeV^2 (a) and 2 GeV^2 (b).

TABLE I. Parameters of $N \to \Delta$ transition form factor, Eq. (8).

m_i	0.0	2.170270	0.660810	0.715202	0.768494
c_i	−3.377428	0.072839	−20.794000	69.497989	−45.399399

transition FF was expressed as

$$F_\Delta(q^2) = \sum_{i=1}^{5} \frac{c_i q^2}{q^2 - m_i^2} \tag{8}$$

with m_i and c_i given in Table I. These values were obtained by fitting experimental data from Ref. [11]. We will neglect the width of the resonance, as it was done in Ref. [4].

For the elastic proton FFs we use parametrization from Ref. [12].

III. RESULTS AND DISCUSSION

The calculated TPE amplitudes are shown in Figs. 2 and 3. Figure 2 displays the Q^2 dependence of the Δ-resonance contribution to the TPE amplitudes at fixed ε ($\varepsilon = 0.25$). As usual, we consider "normalized" TPE amplitudes (divided by the proton magnetic FF). The scale is made logarithmic in Q^2 for better display of low-Q^2 region.

Looking at the real parts of the amplitudes, we see sharp peaks coinciding with the resonance position. It was noted in Ref. [8], that such peaks are artifacts, appearing due to assumed zero resonance width. With a finite width, the curve must become "smeared" and the peaks should disappear. But this means that the TPE amplitude in the close vicinity of the resonance is not adequately described by the present "zero-width" calculation. Further we will mainly concentrate on the high-Q^2 region ($Q^2 > 1$ GeV^2), where we do not hit the resonance and the problem will not emerge.

We see that at high Q^2 Δ contributions grow (in absolute value) with Q^2. Though the contribution $\delta\mathcal{G}_M^{(\Delta)}/G_M$ changes sign at $Q^2 \approx 5$ GeV^2, it still grows beyond this point (not shown in the figure). The elastic contribution has a similar property (Fig. 4). The difference is that the largest contribution goes to the amplitude $\mathcal{G}_E$ (much larger than to $\mathcal{G}_M$). This fact

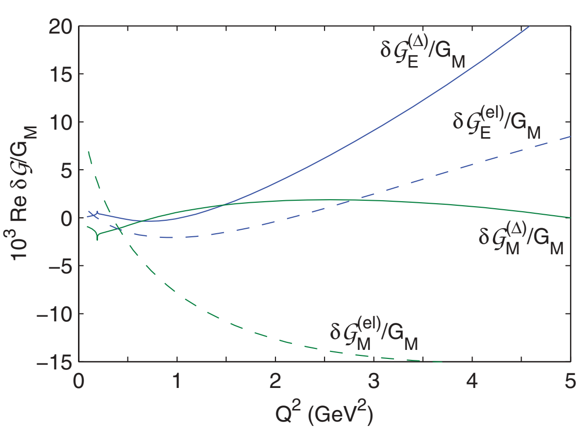

FIG. 4. (Color online) Comparison of Δ (solid) and elastic (dashed) contributions, $\varepsilon = 0.25$.

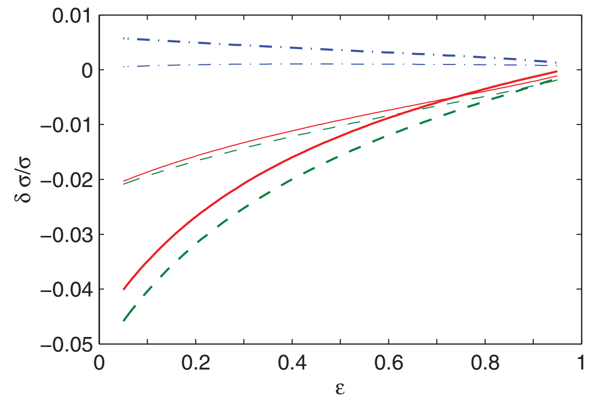

FIG. 5. (Color online) TPE correction to cross section at $Q^2 = 1$ GeV^2 (thin lines) and 3 GeV^2 (thick lines). Proton contribution (dashed lines), Δ contribution (dash-dotted lines), and total (solid lines).

has not much effect on the cross section, but implies relatively large corrections to the polarization ratio (see below).

The imaginary parts [Fig. 2(b)], naturally, have a step-like behavior, i.e., they vanish below the threshold and are nonzero above it. Having obtained the imaginary part of the amplitudes, we can perform some crosschecks of our results. First, we can check the sign of the TPE amplitudes with the help of the optical theorem. It reads

$$\mathrm{Im}\,\mathcal{M}_{ii} = 2|p|\sqrt{s}\,\sigma, \tag{9}$$

where $\mathcal{M}_{ii}$ is forward scattering amplitude, s is c.m. energy squared, and σ is total cross section, $ep \to eX$. The same holds true for contribution of each IS h separately, i.e.,

$$\mathrm{Im}\,\mathcal{M}_{ii}^{(h)} = 2|p|\sqrt{s}\,\sigma^{(h)}. \tag{10}$$

Here $\sigma^{(h)}$ is the cross section for $ep \to eh$. Putting $u' = u$ and $U' = U$ in Eq. (2), we easily get

$$\mathrm{Im}\,\mathcal{M}_{ii} = \frac{4\pi\alpha\nu}{Q^2}\,\mathrm{Im}\,\mathcal{G}_E \tag{11}$$

thus the optical theorem implies

$$\mathrm{Im}\,\delta\mathcal{G}_E^{(h)} > 0 \quad \text{for } Q^2 \to 0 \text{ at fixed } s. \tag{12}$$

Note that for the elastic contribution such a check constrains only infrared divergent part.

We also have reproduced our results for target normal spin asymmetry from Ref. [6], using Eq. (6).

TABLE II. Form factor ratio, measured in PT experiments ($\mu R_{\rm exp}$) and corresponding TPE corrections ($\mu\delta R$). The explicit factor of $\mu = 2.793$ appears here as it is not included in our definition of R.

Expt.	Q^2	ε	$\mu R_{\rm exp} \pm$ stat. $\pm$ syst.	$\mu\delta R$
	3.98	0.71	$0.517 \pm 0.055 \pm 0.009$	0.031
[14]	4.76	0.59	$0.450 \pm 0.052 \pm 0.012$	0.050
	5.56	0.45	$0.354 \pm 0.085 \pm 0.019$	0.078
	5.17	0.37	$0.443 \pm 0.066 \pm 0.018$	0.081
[15]	6.70	0.51	$0.327 \pm 0.105 \pm 0.022$	0.089
	8.49	0.24	$0.138 \pm 0.179 \pm 0.043$	0.221

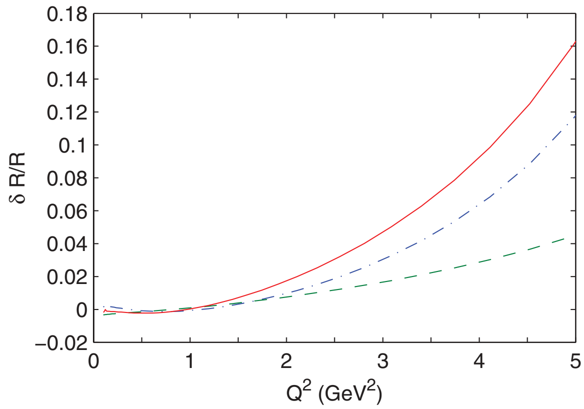

FIG. 6. (Color online) TPE correction to measured form factor ratio at $\varepsilon = 0.5$. Proton contribution (dashed lines), Δ contribution (dash-dotted lines), and total (solid lines).

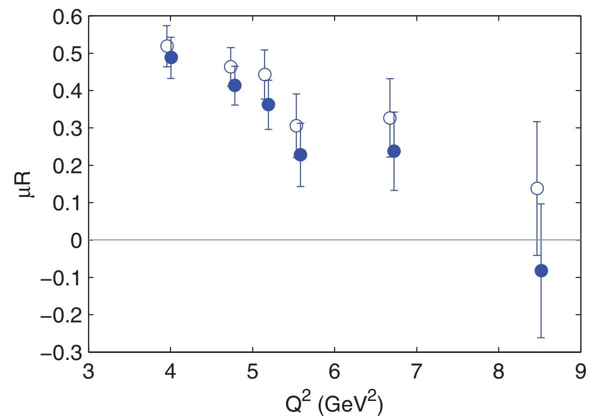

FIG. 7. (Color online) Results of PT experiments, with (solid symbols) and without (hollow symbols) TPE correction. Points are slightly offset in Q^2 for clarity.

The ε dependence of the TPE amplitudes (real parts) is shown in Fig. 3. It is substantially nonlinear for $Q^2 < 1\ \mathrm{GeV}^2$ (a) and becomes almost linear for $Q^2 > 1\ \mathrm{GeV}^2$ (b) for all three amplitudes. Recall that the elastic contribution $\delta\mathcal{G}_M^{(el)}$, is almost linear in ε, and thanks to this fact Rosenbluth plots remain linear even with the corresponding correction taken into account. It is interesting to study how much nonlinearity is introduced by the Δ contribution. We apply the method of Ref. [13], namely, we fit the *calculated* OPE cross section plus TPE correction at fixed Q^2 by the quadratic function of ε:

$$\sigma + \delta\sigma = P_0[1 + P_1(\varepsilon - 0.5) + P_2(\varepsilon - 0.5)^2]. \quad (13)$$

This gives us the nonlinearity coefficient P_2 as a function of Q^2. The Δ contribution to nonlinearity coefficient turns out to be rather small, $|P_2^{(\Delta)}| < 0.014$ for $0.5\ \mathrm{GeV}^2 < Q^2 < 5\ \mathrm{GeV}^2$, whereas the total (elastic + Δ) contribution varies from 0.005 to −0.06. This should be compared to experimental value $P_2 = 0.019 \pm 0.027$ [13].

In Fig. 5 we plot the TPE correction to the cross section at $Q^2 = 1\ \mathrm{GeV}^2$ and $Q^2 = 3\ \mathrm{GeV}^2$. This is the same quantity as in Fig. 2 of Ref. [4], and is in qualitative agreement with the latter.

As long as now we have individual TPE amplitudes, we may easily obtain the TPE correction to FF ratio, Eq. (5). It is plotted in Fig. 6, for $\varepsilon = 0.5$. Of course, one must keep in mind that this correction is also ε dependent, as the TPE amplitudes are. The correction grows rapidly as $Q^2 \to \infty$ due to

(i) large amplitude $\delta\mathcal{G}_E$, growing with Q^2,
(ii) smallness of FF ratio R itself, as it tends to zero near $Q^2 \sim 10\ \mathrm{GeV}^2$.

In Table II, the total correction, $\delta R = \delta R^{(el)} + \delta R^{(\Delta)}$, is shown for the kinematical conditions of experiments [14,15]. As the correction is always much larger than the quoted systematic error, it clearly needs to be taken into account in polarization measurements at high Q^2. With this TPE correction applied, the FF ratio becomes negative already at $Q^2 = 8.5\ \mathrm{GeV}^2$ (Fig. 7).

But is our scheme of calculation (elastic + Δ ISs) perfectly adequate for $Q^2 \sim 5$–$10\ \mathrm{GeV}^2$? We think that at least one needs to estimate the contributions of other prominent resonances as well as multiparticle states before we may apply the correction to data. Whether these contributions are small? It is not quite clear. Even if they are, there are many ISs that contribute, and it is not clear what will be the total effect.

Of course, it would be nice to have a QCD-style calculation for this observable. Unfortunately, this is a hard task. Leading-twist QCD calculation yields only two amplitudes, $\mathcal{G}_M$ and $\mathcal{G}_3$, since a virtual photon (gluon) cannot flip quark spin. The calculation of electric FF G_E or TPE amplitude $\delta\mathcal{G}_E$ requires, at least, knowledge of quark transverse momenta distribution.

Summarizing, we believe that our results give a strong indication that TPE corrections coming from the inelastic intermediate states may be of great importance to polarization measurements at high Q^2, and thus deserve further thorough investigation.

[1] J. Arrington, P. G. Blunden, and W. Melnitchouk, Prog. Part. Nucl. Phys. **66**, 782 (2011).
[2] Y. C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, Phys. Rev. Lett. **93**, 122301 (2004).
[3] D. Borisyuk and A. Kobushkin, Phys. Rev. D **79**, 034001 (2009).
[4] S. Kondratyuk, P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. Lett. **95**, 172503 (2005).
[5] S. Kondratyuk and P. G. Blunden, Phys. Rev. C **75**, 038201 (2007).
[6] D. Borisyuk and A. Kobushkin, Phys. Rev. C **72**, 035207 (2005).
[7] D. Borisyuk and A. Kobushkin, Phys. Rev. C **78**, 025208 (2008).
[8] D. Borisyuk and A. Kobushkin, Phys. Rev. C **83**, 025203 (2011).

[9] D. Borisyuk and A. Kobushkin, Phys. Rev. C **76**, 022201(R) (2007).
[10] H. F. Jones and M. D. Scadron, Ann. Phys. (NY) **81**, 1 (1973).
[11] P. Stoler, Phys. Rev. D **44**, 73 (1991).
[12] J. Arrington, W. Melnitchouk, and J. A. Tjon, Phys. Rev. C **76**, 035205 (2007).
[13] V. Tvaskis, J. Arrington, M. E. Christy, R. Ent, C. E. Keppel, Y. Liang, and G. Vittorini, Phys. Rev. C **73**, 025206 (2006).
[14] A. J. R. Puckett *et al.*, Phys. Rev. C **85**, 045203 (2012).
[15] A. J. R. Puckett *et al.*, Phys. Rev. Lett. **104**, 242301 (2010).