Pairing interactions and one-nucleon separation energies

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Empirical proton-neutron, proton-proton, and neutron-neutron paring and so-called nonpairing interactions are extracted by using experimental data of binding energies. Nonpairing interactions, $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$, are very sensitive to the shell and subshell evolution, the phase transition, and the Wigner effect. Odd-even staggerings of one-nucleon separation energies are discussed in terms of empirical pairing interactions.

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I. INTRODUCTION

Nuclear binding energy B and its derivatives, such as one-proton separation energy S_p, one-neutron separation energy S_n, and α-decay energy Q_α, are of great importance not only in nuclear physics but also in astrophysics. On the one hand, there are many theoretical efforts in describing and predicting the values of B, e.g., mean field approaches [1,2], macroscopic-microscopic approaches [3,4], and liquid drop models [4,5]. On the other hand, the values of B exhibit various simplicities which provide us with a number of approaches to predict unknown binding energies by extrapolations. Good examples are the Garvey-Kelson mass relations [6] and the recent work by using systematics of empirical proton-neutron interactions [7–10]. See Refs. [11,12] for comprehensive reviews.

In Ref. [13] Streletz *et al*. noticed an interesting pattern. Let N_p and N_n denote the valence proton number and valence neutron number, respectively. Streletz *et al*. found that

$$S_p = C_1 N_p + C_2 N_n + C_3, \quad S_n = C_1' N_p + C_2' N_n + C_3', \tag{1}$$

where C_1, C_2, C_3, C_1', C_2', and C_3' are constants for all even-even nuclei in the same shell. This behavior was discussed by using the Weizsäcker binding energy formula in Ref. [5] and by the major shell lowest seniority mass equation in Ref. [14].

In Ref. [15] Jiang *et al*. showed that the relation in Eq. (1) holds not only for even-even but also for even-odd, odd-even, and odd-odd nuclei. For S_p (S_n) the main difference between these four types is the value of C_3 (C_3'): The value of C_3 (C_3') is the largest for even-even cases, the second largest for proton-even-neutron-odd (proton-odd-neutron-even) cases, the third for odd-odd cases, and the smallest for proton-odd-neutron-even (proton-even-neutron-odd) cases. The values of C_1 and C_2 (C_1' and C_2') remain approximately the same for S_p (S_n) of these four types. These features were studied in terms of the symmetry energy of Ref. [4] and the pairing interaction of Ref. [16].

In this paper we study the origin of the features in Refs. [13,15], in terms of pairing interaction and so-called nonpairing interactions. We suggest that the linear correlation in S_p (S_n) is given by the smooth and slow change of these interactions, and that the odd-even staggering is originated from the paring interactions.

The one-nucleon separation energies are defined as follows.

$$S_p(Z,N) = B(Z,N) - B(Z-1,N),$$
$$S_n(Z,N) = B(Z,N) - B(Z,N-1),$$

where Z and N present the proton and neutron numbers of a given nucleus, respectively. B, S_p, and S_n are taken to be positive in this paper. All experimental data in this paper are taken from the AME2011-preview database [17].

This paper is organized as follows. In Sec. II we discuss the systematics of pairing interactions (and the nonpairing part) between valence nucleons. In Sec. III we discuss S_p and S_n in terms of the systematics of pairing and nonpairing interactions between nucleons, and in Sec. IV we summarize our results obtained in this paper.

II. THE EMPIRICAL PAIRING INTERACTIONS

In this section we discuss the empirical pairing interactions of proton-neutron, proton-proton, and neutron-neuron types. Similar to Refs. [18,19], we define

$$-\delta V_{pp}^{(1)}(Z-1,Z;N) = \begin{cases} B(Z,N) - 2B(Z-1,N) + B(Z-2,N) + \delta V_{pp}^{(2)}(Z-1,Z;N) & \text{for even } Z \text{ and even } N \\ B(Z,N-1) - 2B(Z-1,N-1) + B(Z-2,N-1) + \delta V_{pp}^{(2)}(Z-1,Z;N) & \text{for even } Z \text{ and odd } N \\ 0 & \text{for odd } Z, \end{cases} \tag{2}$$

$$-\delta V_{nn}^{(1)}(N-1,N;Z) = \begin{cases} B(Z,N) - 2B(Z,N-1) + B(Z,N-2) + \delta V_{nn}^{(2)}(N-1,N;Z) & \text{for even } N \text{ and even } Z \\ B(Z-1,N) - 2B(Z-1,N-1) + B(Z-1,N-2) + \delta V_{nn}^{(2)}(N-1,N;Z) & \text{for even } N \text{ and odd } Z \\ 0 & \text{for odd } N, \end{cases} \tag{3}$$

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where

$$-\delta V_{pp}^{(2)}(Z-1,Z;N) = \tfrac{1}{2}[B(Z,N) - B(Z-1,N) - B(Z-2,N) + B(Z-3,N)], \tag{4}$$

$$-\delta V_{nn}^{(2)}(N-1,N;Z) = \tfrac{1}{2}[B(Z,N) - B(Z,N-1) - B(Z,N-2) + B(Z,N-3)]. \tag{5}$$

$\delta V_{pp}^{(1)}$ ($\delta V_{nn}^{(1)}$) is the proton-proton (neutron-neutron) pairing interaction, and $\delta V_{pp}^{(2)}$ ($\delta V_{nn}^{(2)}$) is our "nonpairing" interaction between the last two protons (neutrons). The study of $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$ has been initiated in Refs. [18,20].

In Figs. 1(a) and 1(b) we show $\delta V_{pp}^{(2)}$ versus Z and $\delta V_{nn}^{(2)}$ versus N, respectively. One sees that most of the values are positive, which means that the nonpairing part of interactions between like particles are repulsive (and very small). This feature has been already pointed out and discussed in Refs. [18,21]. From Figs. 1(a) and 1(b) one also sees the absolute values of $\delta V_{pp}^{(2)}$ for nuclei with $Z-N=1$ and $Z-N=2$ and those of $\delta V_{nn}^{(2)}$ for nuclei with $N-Z=1$ and $N-Z=2$ are exceptionally large. These "anomalies" are given mainly by the Wigner effect. In this paper, however, we are interested in anomalies irrelevant of the Wigner effect, i.e., solid diamonds in red which largely deviate from the compact trajectory of solid circles in black. One sees that these anomalous results (diamonds in red) include all well-known magic numbers (8, 20, 28, 50, 82, 126). The anomaly at magic numbers is easily understood. For example, when protons fill all the orbits of the closed shell, adding one additional proton would lead to a reduction of the attractive force between the last proton and other nucleons. Thus one obtains a large value of $\delta V_{pp}^{(2)}$ in Eq. (4) for nuclei with one or two extra nucleons outside the magic nucleus core. The same situation holds for $\delta V_{nn}^{(2)}$. A similar argument of this phenomenon was also suggested in Ref. [19].

In Fig. 1 the results of $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$ which are not affected by the Wigner effect, the shell or subshell closure, or the phase transition (see solid circles in black), change smoothly with Z and N, respectively. We denote their average values by using $\overline{\delta V_{pp}^{(2)}}$ and $\overline{\delta V_{nn}^{(2)}}$ and obtain their empirical formulas:

$$\overline{\delta V_{pp}^{(2)}}(Z,Z-1) = 442 + 1924\exp(-0.0520Z)\ \text{keV} \tag{6}$$

and

$$\overline{\delta V_{nn}^{(2)}}(N,N-1) = 154 + 1840\exp(-0.0626N)\ \text{keV}. \tag{7}$$

One sees $\delta V_{pp}^{(2)}$ is more repulsive than $\delta V_{nn}^{(2)}$; for heavy nuclei the difference is ~290 keV.

As pointed out in Refs. [18,20], the anomalous results of $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$ are fingerprints of shells and subshells, as well as phase transitions. Below we summarize these anomalous $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$ and list the corresponding shells and/or subshells and phase transitions.

(i) New magic numbers $Z=16$ and $N=16$ discovered in Refs. [22,23]. $\delta V_{pp}^{(2)}$ with $Z=16$ and $\delta V_{nn}^{(2)}$ with $N=16$ are anomalous [see Figs. 1(a) and 1(b)].

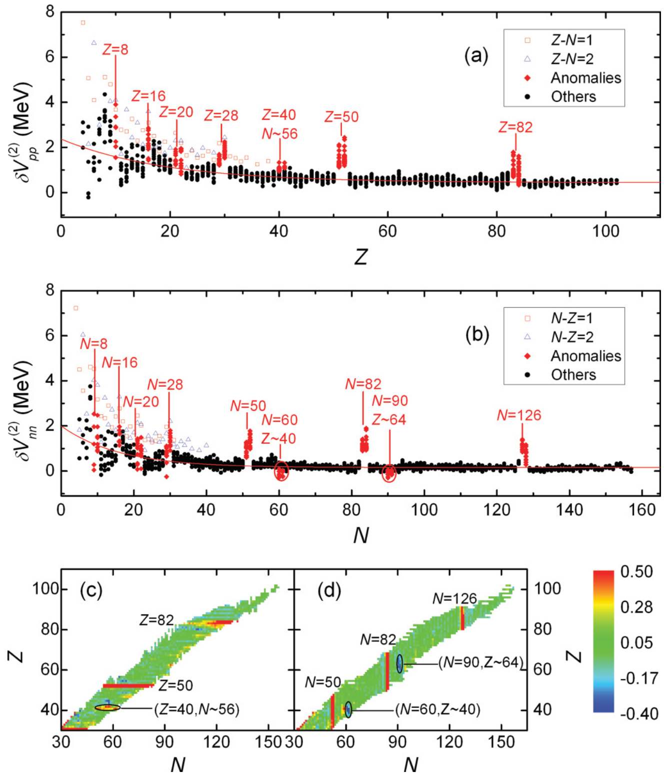

FIG. 1. (Color online) Nonpairing interactions between the last two protons and that between the last two neutrons. (a) $\delta V_{pp}^{(2)}$ versus Z. (b) $\delta V_{nn}^{(2)}$ versus N. (c) Two-dimensional contour of ($\delta V_{pp}^{(2)} - \overline{\delta V_{pp}^{(2)}}$) in the $Z-N$ chart. (d) Contour of ($\delta V_{nn}^{(2)} - \overline{\delta V_{nn}^{(2)}}$) in the $Z-N$ chart. In panels (a) and (b) open squares and triangles correspond to those with $|Z-N| = 1$ or 2; solid diamonds in red correspond to those affected by shell or subshell effects, or phase transitions; and solid circles in black correspond to the usual cases. The curves in panels (a) and (b) are plotted to best fit their average values (denoted by $\overline{\delta V_{pp}^{(2)}}$ and $\overline{\delta V_{nn}^{(2)}}$) by using empirical formulas: $\overline{\delta V_{pp}^{(2)}} = 442 + 1924\exp(-0.052Z)$, and $\overline{\delta V_{nn}^{(2)}} = 154 + 1840\exp(-0.0626N)$ (both in kev). One sees large deviations arise for nuclei with either $N \simeq Z$, shell or subshell closures, or phase transitions.

(ii) The $Z=40$ subshell (which disappears if neutron number $N \geqslant 60$) [24–26]. Here $\delta V_{pp}^{(2)}$ with $Z=40$, $N \sim 56$ are anomalous [see Fig. 1(c)], indicating the $Z=40$ subshell.

(iii) The $Z=64$ subshell (which disappears if neutron number $N \geqslant 90$) [26–28]. Here $\delta V_{nn}^{(2)}$ with $N=90$, $Z \sim 64$, are anomalous [see Fig. 1(d)]. However we do not see sharp anomalies for $\delta V_{pp}^{(2)}$ with $Z=64$, $N<90$ in Fig. 1(c).

(iv) The phase transition at $N=60$, $Z \sim 40$ [see Fig. 1(d)]. These $\delta V_{nn}^{(2)}$ are much smaller than $\overline{\delta V_{nn}^{(2)}}$, indicating the phase transitions for low-lying states of nuclei with $N=60$, $Z \sim 40$.

The pairing interactions, $\delta V_{pp}^{(1)}$ and $\delta V_{nn}^{(1)}$, are strongly attractive. In Fig. 2 we show $\delta V_{pp}^{(1)}$ of even-Z nuclei and $\delta V_{nn}^{(1)}$ of even-N nuclei in the nuclide charts, separately. Their values range from -4 to -1 MeV as mass number A increases. The systematics of $\delta V_{pp}^{(1)}$ and $\delta V_{nn}^{(1)}$ are complicated; their values are strongly influenced by shell structures.

Now let us come to the proton-neutron interaction between the last proton and the last neutron,

$$-\delta V_{1p-1n}(Z,N) = B(Z,N) + B(Z-1,N-1) - B(Z-1,N) - B(Z,N-1). \quad (8)$$

Let us suppose the odd-even staggering of δV_{1p-1n} in Refs. [9,29,30] is given by the pairing interaction, denoted as $\delta V_{pn}^{(1)}$, and the residual part, the nonpairing interaction, is denoted as $\delta V_{pn}^{(2)}$. We have

$$\delta V_{pn}^{(1)}(Z,N) = \begin{cases} 0 & \text{if N + Z is odd} \\ \delta V_{1p-1n}(Z,N) - \delta V_{pn}^{(2)}(Z,N) & \text{if N + Z is even,} \end{cases} \quad (9)$$

where

$$\delta V_{pn}^{(2)}(Z,N) = \begin{cases} \delta V_{1p-1n}(Z,N) & \text{if N + Z is odd} \\ \frac{1}{4}[\delta V_{1p-1n}(Z,N-1) + \delta V_{1p-1n}(Z,N+1) + \delta V_{1p-1n}(Z-1,N) + \delta V_{1p-1n}(Z+1,N)] & \text{if N + Z is even.} \end{cases} \quad (10)$$

Thus δV_{1p-1n} is rewritten by

$$\delta V_{1p-1n}(Z,N) = \frac{1+(-)^{Z+N}}{2}\delta V_{pn}^{(1)}(Z,N) + \delta V_{pn}^{(2)}(Z,N). \quad (11)$$

In Fig. 3 we present $\delta V_{pn}^{(1)}$ and $\delta V_{pn}^{(2)}$ extracted by experimental binding energies versus mass number A. One sees the abrupt enhancement of empirical proton-neutron interactions δV_{1p-1n} with $Z = N$ (open triangles in red) given by the Wigner energy. $\delta V_{pn}^{(1)}$ for $Z \neq N$ and $\delta V_{pn}^{(2)}$ follow compact trajectories. For $A \geqslant 70$, we assume two simple formulas:

$$\delta V_{pn}^{(1)}(Z,N) = 3.512A - 1042 \text{ keV} \quad (12)$$

and

$$\delta V_{pn}^{(2)}(Z,N) = -82 \text{ keV}. \quad (13)$$

$\delta V_{pn}^{(2)}$ is strong (the absolute value equals a few MeV) for $A < 20$, and drops down rapidly with A and saturates at ~ -82 keV when $A \geqslant 70$. $\delta V_{pn}^{(1)}$ changes smoothly from -1 MeV to -200 keV as A increases from 5 to 200. Comparing panel (a) with panel (b) in Fig. 3 very carefully, one sees $\delta V_{pn}^{(1)}$ stronger than $\delta V_{pn}^{(2)}$ for $A > 20$. The anomaly of shell closures is not as striking in $\delta V_{pn}^{(1)}$ or $\delta V_{pn}^{(2)}$ as in $\delta V_{pp}^{(2)}$ and $\delta V_{nn}^{(2)}$.

We note that $\delta V_{pn}^{(1)}$ (the pairing interaction of a proton-neutron type) presents an additionally attractive interaction in odd-odd nuclei which is usually neglected in the conventional pairing force. This effect was discussed in terms of configuration mixing in Ref. [31], and in terms of proton-neutron correlation in Ref. [32]. However, further investigation of this effect is warranted.

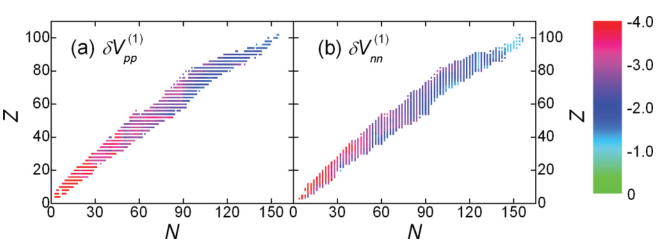

FIG. 2. (Color online) Pairing interactions $\delta V_{pp}^{(1)}$ (even-Z) and $\delta V_{nn}^{(1)}$ (even-N) in units of MeV.

It is also worthy to note that the root-mean-square deviations of $\delta V_{pp}^{(2)}$ (with $Z > 30$), $\delta V_{nn}^{(2)}$ (with $N > 40$), $\delta V_{pn}^{(1)}$ (with $A > 60$), and $\delta V_{pn}^{(2)}$ (with $A > 60$) obtained by using the empirical formulas in Eqs. (6), (7), (12), and (13) from those extracted from experimental data by using Eqs. (4), (5), (9), and (10) are very small (116, 85, 228, and 128 keV, respectively, for these four quantities), if we exclude those affected by the Wigner effect or shells. Therefore they would be very useful in constructing local mass relations.

III. ONE-NUCLEON SEPARATION ENERGIES

In this section we discuss one-nucleon separation energies, S_p and S_n. We begin our discussion with the definition of

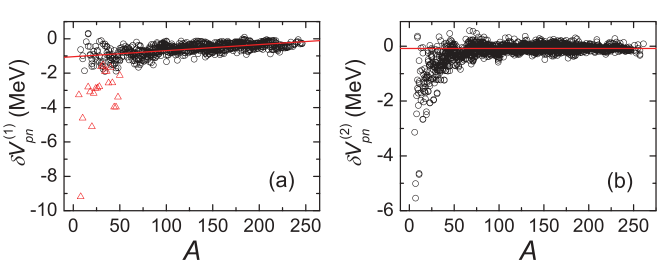

FIG. 3. (Color online) Pairing interaction $\delta V_{pn}^{(1)}$ and nonpairing interaction $\delta V_{pn}^{(2)}$ versus mass number A. (a) Open circles in black correspond to $\delta V_{pn}^{(1)}$ with $Z \neq N$ and open triangles in red correspond to $\delta V_{pn}^{(1)}$ with $Z = N$; (b) open circles in black correspond to $\delta V_{pn}^{(2)}$ of all nuclei. Line in panel (a): $\delta V_{pn}^{(1)} = 3.512A - 1042$ keV; line in panel (b): $\delta V_{pn}^{(2)} = -82$keV.

binding energy:

$$\begin{aligned}-B(Z,N) = &\sum_{i=1}^{[Z/2]} \delta V_{pp}^{(1)}(2i-1,2i) + \sum_{1\leqslant i<j\leqslant Z} \delta V_{pp}^{(2)}(i,j)\\ &+ \sum_{i=1}^{[N/2]} \delta V_{nn}^{(1)}(2i-1,2i) + \sum_{1\leqslant i<j\leqslant N} \delta V_{nn}^{(2)}(i,j)\\ &+ \sum_{i=1}^{Z}\sum_{j=1}^{N} \frac{1+(-)^{i+j}}{2}\delta V_{pn}^{(1)}(i,j)\\ &+ \sum_{i=1}^{Z}\sum_{j=1}^{N} \delta V_{pn}^{(2)}(i,j), \end{aligned} \tag{14}$$

where $[Z/2]$ is, respectively, $Z/2$ or $(Z-1)/2$ for even or odd Z. However, it is not known how to extract the values of $\delta V_{pp}^{(1,2)}(2i-1,2i)$, $\delta V_{nn}^{(1,2)}(2i-1,2i)$ with small i, or $\delta V_{pn}^{(1,2)}(i,j)$ with small i and j with a reasonable precision for heavy nuclei. Therefore Eq. (14) is not useful in practice.

Now let us treat an arbitrary nucleus with Z protons and N neutrons as an even-even "core" (with Z_0 protons and N_0 neutrons) and additional $(Z-Z_0)$ protons and $(N-N_0)$ neutrons outside the core. Z_0 and N_0 are not necessarily magic numbers. The "binding energy" of the core is as follows:

$$\begin{aligned}V(Z_0,N_0) \equiv &V_{pp}^{(1)}(Z_0) + V_{pp}^{(2)}(Z_0) + V_{nn}^{(1)}(N_0) + V_{nn}^{(2)}(N_0)\\ &+ V_{pn}^{(1)}(Z_0,N_0) + V_{pn}^{(2)}(Z_0,N_0),\end{aligned}$$

where

$$\begin{aligned}V_{pp}^{(1)}(Z_0) &= \sum_{i=1}^{Z_0/2} \delta V_{pp}^{(1)}(2i-1,2i),\\ V_{pp}^{(2)}(Z_0) &= \sum_{1\leqslant i<j\leqslant Z_0} \delta V_{pp}^{(2)}(i,j),\\ V_{nn}^{(1)}(N_0) &= \sum_{i=1}^{N_0/2} \delta V_{nn}^{(1)}(2i-1,2i),\\ V_{nn}^{(2)}(N_0) &= \sum_{1\leqslant i<j\leqslant N_0} \delta V_{nn}^{(2)}(i,j),\\ V_{pn}^{(1)}(Z_0,N_0) &= \sum_{i=1}^{Z_0}\sum_{j=1}^{N_0} \frac{1+(-)^{i+j}}{2}\delta V_{pn}^{(1)}(i,j),\\ V_{pn}^{(2)}(Z_0,N_0) &= \sum_{i=1}^{Z_0}\sum_{j=1}^{N_0} \delta V_{pn}^{(2)}(i,j).\end{aligned}$$

Equation (14) is rewritten in the following form:

$$\begin{aligned}-B(Z,N) = &V(Z_0,N_0) + \sum_{i=Z_0+1}^{Z} V_p^{(0)}(i) + \sum_{i=Z_0/2+1}^{[Z/2]} \delta V_{pp}^{(1)}(2i-1,2i) + \sum_{Z_0+1\leqslant i<j\leqslant Z} \delta V_{pp}^{(2)}(i,j) + \sum_{i=N_0+1}^{N} V_n^{(0)}(i)\\ &+ \sum_{i=N_0/2+1}^{[N/2]} \delta V_{nn}^{(1)}(2i-1,2i) + \sum_{N_0+1\leqslant i<j\leqslant N} \delta V_{nn}^{(2)}(i,j) + \sum_{i=Z_0+1}^{Z}\sum_{j=N_0+1}^{N} \frac{1+(-)^{i+j}}{2}\delta V_{pn}^{(1)}(i,j)\\ &+ \sum_{i=Z_0+1}^{Z}\sum_{j=N_0+1}^{N} \delta V_{pn}^{(2)}(i,j),\end{aligned} \tag{15}$$

where

$$\begin{aligned}V_p^{(0)}(i) &= \sum_{k=1}^{Z_0} \delta V_{pp}^{(2)}(k,i) + \sum_{k=1}^{N_0}\left[\delta V_{pn}^{(1)}(i,k) + \delta V_{pn}^{(2)}(i,k)\right],\\ V_n^{(0)}(i) &= \sum_{k=1}^{N_0} \delta V_{nn}^{(2)}(k,i) + \sum_{k=1}^{Z_0}\left[\delta V_{pn}^{(1)}(k,i) + \delta V_{pn}^{(2)}(k,i)\right].\end{aligned}$$

Now we make very crude approximations as follows.

$$\begin{aligned}V_p^{(0)}(i) &\approx V_p^{(0)}, & \delta V_{pp}^{(1)}(2i-1,2i) &\approx \delta V_{pp}^{(1)},\\ \delta V_{pp}^{(2)}(i,j) &\approx \delta V_{pp}^{(2)}, & \delta V_{pn}^{(1)}(i,j)|_{i+j \text{ is even}} &\approx \delta V_{pn}^{(1)},\\ V_n^{(0)}(i) &\approx V_n^{(0)}, & \delta V_{nn}^{(1)}(2i-1,2i) &\approx \delta V_{nn}^{(1)},\\ \delta V_{nn}^{(2)}(i,j) &\approx \delta V_{nn}^{(2)}, & \delta V_{pn}^{(2)}(i,j) &\approx \delta V_{pn}^{(2)}.\end{aligned}$$

Some of the above assumptions were taken in Refs. [33,34] in constructing the integrated empirical valence proton-neutron interactions. Under these assumptions, Eq. (15) becomes

$$\begin{aligned}-B(Z,N) \approx &V(Z_0,N_0) + N_p' V_p^{(0)} + \left[\frac{N_p'}{2}\right]\delta V_{pp}^{(1)}\\ &+ \frac{N_p'(N_p'-1)}{2}\delta V_{pp}^{(2)} + N_n' V_n^{(0)} + \left[\frac{N_n'}{2}\right]\delta V_{nn}^{(1)}\\ &+ \frac{N_n'(N_n'-1)}{2}\delta V_{nn}^{(2)} + N_p' N_n'\left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right)\\ &+ \frac{1-(-)^{N_p' N_n'}}{4}\delta V_{pn}^{(1)},\end{aligned} \tag{16}$$

where $N_p' = Z - Z_0$ and $N_n' = N - N_0$. This formula is very similar to Eq. (10) of Ref. [14]. The last term of Eq. (16) indicates the extra binding of odd-odd nuclei. From Eq. (16), we obtain

$$\begin{aligned}S_p(Z,N) \approx &\left(\delta V_{pp}^{(2)} - V_p^{(0)}\right) - N_p'\delta V_{pp}^{(2)}\\ &- N_n'\left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) - \frac{1+(-)^{N_p'}}{2}\delta V_{pp}^{(1)}\end{aligned}$$

$$- \frac{(-)^{N_p' N_n'}}{4}[(-)^{N_n'} - 1]\delta V_{pn}^{(1)} \approx \alpha Z + \beta N + \gamma, \quad (17)$$

$$S_n(Z,N) \approx \left(\delta V_{nn}^{(2)} - V_n^{(0)}\right) - N_n' \delta V_{nn}^{(2)} - N_p'\left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) - \frac{1+(-)^{N_n'}}{2}\delta V_{nn}^{(1)} - \frac{(-)^{N_p' N_n'}}{4}[(-)^{N_p'} - 1]\delta V_{pn}^{(1)} \approx \alpha' Z + \beta' N + \gamma', \quad (18)$$

with

$$\alpha = -\delta V_{pp}^{(2)},$$
$$\beta = -\frac{1}{2}\delta V_{pn}^{(1)} - \delta V_{pn}^{(2)},$$
$$\gamma = \left[\delta V_{pp}^{(2)}(Z_0+1) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) N_0 - V_p^{(0)}\right] - \frac{1+(-)^Z}{2}\delta V_{pp}^{(1)} - \frac{(-)^{Z\times N}}{4}[(-)^N - 1]\delta V_{pn}^{(1)}, \quad (19)$$

and

$$\alpha' = -\frac{1}{2}\delta V_{pn}^{(1)} - \delta V_{pn}^{(2)},$$
$$\beta' = -\delta V_{nn}^{(2)},$$
$$\gamma' = \left[\delta V_{nn}^{(2)}(N_0+1) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) Z_0 - V_n^{(0)}\right] - \frac{1+(-)^N}{2}\delta V_{nn}^{(1)} - \frac{(-)^{Z\times N}}{4}[(-)^Z - 1]\delta V_{pn}^{(1)}. \quad (20)$$

First we discuss S_p and S_n versus Z and N. α and β in Eq. (19) and α' and β' in Eq. (20) are given in terms of $\delta V_{pp}^{(2)}$, $\delta V_{nn}^{(2)}$, $\delta V_{pn}^{(1)}$, and $\delta V_{pn}^{(2)}$. All these quantities are assumed invariable in a local region (unless there are the Wigner effect, the shell or subshell effect, and the phase transition). On first sight, the first term of γ, $[\delta V_{pp}^{(2)}(Z_0+1) + (\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)})N_0 - V_p^{(0)}]$, and the first term of γ', $[\delta V_{nn}^{(2)}(N_0+1) + (\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)})Z_0 - V_n^{(0)}]$, seem sensitive to Z_0 and N_0; it seems to suggest that the values of γ and γ' are highly relevant to the choice of the core (with Z_0 protons and N_0 neutrons). However, this is not the case in a local region. For example, if we replace Z_0 and N_0 by Z_0' and N_0' in the first term of γ, then we have

$$\begin{aligned}
&\delta V_{pp}^{(2)}(Z_0'+1) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) N_0' - V_p^{(0)\prime} \\
&\approx \delta V_{pp}^{(2)}(Z_0'+1) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) N_0' - \left\{\sum_{k=1}^{Z_0'}\delta V_{pp}^{(2)}(k,i) + \sum_{k=1}^{N_0'}\left[\delta V_{pn}^{(1)}(i,k) + \delta V_{pn}^{(2)}(i,k)\right]\right\} \\
&\approx \delta V_{pp}^{(2)}(Z_0+1) + \delta V_{pp}^{(2)}(Z_0'-Z_0) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) N_0 + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right)(N_0'-N_0) \\
&\quad - \left\{\sum_{k=1}^{Z_0}\delta V_{pp}^{(2)}(k,i) + \delta V_{pp}^{(2)}(Z_0'-Z_0) + \sum_{k=1}^{N_0}\left[\delta V_{pn}^{(1)}(i,k) + \delta V_{pn}^{(2)}(i,k)\right] + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right)(N_0'-N_0)\right\} \\
&= \delta V_{pp}^{(2)}(Z_0+1) + \left(\frac{1}{2}\delta V_{pn}^{(1)} + \delta V_{pn}^{(2)}\right) N_0 - V_p^{(0)}. \quad (21)
\end{aligned}$$

Thus in a local region, γ is approximately invariable for nuclei of each of the even-even, even-odd, odd-even, and odd-odd types. Similarly one sees that γ' is approximately invariable too.

Second, there are two types of odd-even staggering reported in Ref. [15]. For convenience we exemplify them by using S_p and S_n of nuclei in the $A \sim 130$ region in Fig. 4. Nice linear correlations are seen for even-even, odd-even, even-odd, and odd-odd nuclei, with odd-even staggerings.

Let us concentrate our attention on S_p. One sees first that S_p of even-Z nuclei are much larger than those of odd-Z nuclei. This is easily explained by $(-\frac{1+(-)^Z}{2}\delta V_{pp}^{(1)})$ in γ of Eq. (19). For an even-Z value, $(-\frac{1+(-)^Z}{2}\delta V_{pp}^{(1)}) = -\delta V_{pp}^{(1)} \sim 2.875$ MeV in this region; and for an odd Z this term is zero. Second, S_p of even-even nuclei is slightly larger than those of even-odd nuclei, and S_p of odd-odd nuclei is slightly larger than those of odd-even nuclei. These two small differences are given by $(-\frac{(-)^{Z\times N}}{4}[(-)^N - 1]\delta V_{pn}^{(1)})$ in γ of Eq. (19). It is equal to 0 for

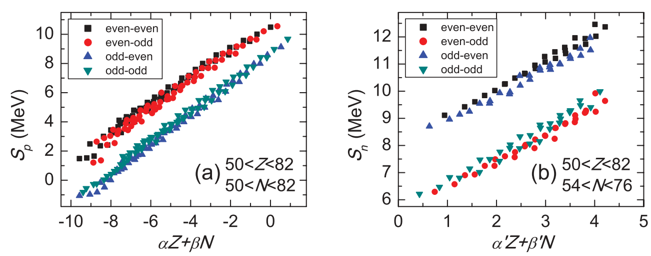

FIG. 4. (Color online) Experimental data of (a) S_p versus $\alpha Z + \beta N$, and (b) S_n versus $\alpha' Z + \beta' N$ for nuclei with $A \sim 130$. See the first row of Table I for values of α, β, α', and β'. Here we use squares in black, circles in red, triangles in blue, and nablas in green to represent even-even, even-odd, odd-even, and odd-odd nuclei, respectively.

TABLE I. Comparison of two sets of parameters α, β, α', β', γ, and γ', one set obtained by χ^2 fitting of experimental data and denoted by "(1)", and the other by using Eqs. (19) and (20) and empirical pairing and nonpairing interactions, denoted by "(2)", for nuclei with $A \sim 130$ (the same set of nuclei as in Fig. 4). Because the first terms in γ and γ' are unknown constants, we calculate the relative values for even-even, even-odd, odd-even, and odd-odd nuclei, $\Delta\gamma = \gamma - \gamma_{\text{o-e}}$ and $\Delta\gamma' = \gamma' - \gamma'_{\text{e-o}}$. We use abbreviations "e-e," "e-o," "o-e," and "o-o" to represent even-even, even-odd, odd-even, and odd-odd, respectively. One sees that the two sets of parameters are in good consistency.

	α	β	α'	β'
(1)	−0.523	0.340	0.308	−0.204
(2)	−0.527	0.374	0.374	−0.177
	e-e	e-o	o-e	o-o
$\Delta\gamma$ (1)	2.687	2.385	0	0.269
$\Delta\gamma$ (2)	2.875	2.583	0	0.292
$\Delta\gamma'$ (1)	2.715	0	2.410	0.260
$\Delta\gamma'$ (2)	2.755	0	2.463	0.292

even-even nuclei, $\delta V_{pn}^{(1)}/2$ for even-odd, $-\delta V_{pn}^{(1)}/2$ for odd-odd, and 0 for odd-even, where $\delta V_{pn}^{(1)} \approx -0.585$ MeV in this region. We note without details that the odd-even staggering of S_n in Fig. 4(b) can be explained in the same way.

It is interesting to compare two sets of coefficients (α, β, γ, α', β', and γ'), one obtained by the χ^2 fitting to experimental data in Fig. 4 for $A \sim 130$ and the other calculated by Eqs. (19) and (20) for the same set of nuclei. Our parameters in Eqs. (19) and (20) are fixed as follows. The four parameters $\delta V_{pp}^{(2)}$, $\delta V_{nn}^{(2)}$, $\delta V_{pn}^{(1)}$, and $\delta V_{pn}^{(2)}$ are calculated by using Eqs. (6), (7), (12), and (13). For $Z \approx 60$, $N \approx 70$, and $A \approx 130$, $\delta V_{pp}^{(2)}$, $\delta V_{nn}^{(2)}$, $\delta V_{pn}^{(1)}$, and $\delta V_{pn}^{(2)}$ are 0.527, 0.177, −0.585, and −0.082 MeV, respectively. The other two parameters are obtained by using average values of experimental data in Fig. 2: $\delta V_{pp}^{(1)} \sim -2.875$ MeV, and $\delta V_{nn}^{(1)} \sim -2.755$ MeV. A comparison of α, β, γ, α', β', and γ' calculated by using Eqs. (19) and (20) and those extracted by experimental data of Fig. 4 are given in Table I, where good consistency is easily seen.

Finally we note that β in Eq. (19) is equal to α' in Eq. (20). This relation holds reasonably well for realistic nuclei. For example, $\beta = 0.340$ MeV and $\alpha' = 0.308$ MeV for nuclei with $A \sim 130$, according to Table I.

IV. SUMMARY

In this paper we studied proton-neutron, proton-proton, and neutron-neutron paring interactions, and nonpairing interactions, extracted from experimental binding energies. The anomalies in the nonpairing part of proton-proton and neutron-neutron interactions provide us very useful fingerprints of the shell evolution and the phase transition, as pointed out in Refs. [18–20]. In Fig 1, the anomalies correspond to nuclei with conventional magic numbers (8, 20, 28, 50, 82, and 126), with $Z = N$, with the $Z = 40$ and 64 subshells, or phase transitions near $N = 60$ and 90.

We construct formulas of one-neutron and one-proton separation energies in terms of these empirical interactions. In a given local region, separation energies, $S_p \approx \alpha Z + \beta N + \gamma$, and $S_n \approx \alpha' Z + \beta' N + \gamma'$, with two types of odd-even staggering. The coefficients α, β, α', and β' are given in terms of these empirical interactions, whose strength changes very slowly. γ and γ' show the odd-even staggerings, which correspond very well to those presented by experimental data. In Ref. [15] the staggerings were attributed to pairing interaction between like nucleons and symmetry energy in the modified Bethe-Weizsäcker formula; in this paper they were further explained in terms of three types of pairing interactions, i.e., proton-proton, neutron-neutron, and proton-neutron pairing interactions.

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