# Tensor and tensor-isospin terms in the effective Gogny interaction

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We discuss the need of including tensor terms in the effective Gogny interaction used in mean-field calculations. We show in one illustrative case that, with the usual tensor term that is employed in the Skyrme interaction (and that allows us to separate the like–nucleon and the neutron-proton tensor contributions), we can describe the evolution of the $N = 28$ neutron gap in calcium isotopes. We propose to include a tensor and a tensor-isospin term in finite-range interactions of Gogny type. The parameters of the two tensor terms allow us to treat separately the like-nucleon and the neutron-proton contributions. Two parametrizations of the tensor terms have been chosen to reproduce different neutron single-particle properties in the $^{48}$Ca nucleus and the energy of the first $0^-$ state in the $^{16}$O nucleus. By employing these two parametrizations we analyze the evolution of the $N = 14$, 28, and 90 neutron energy gaps in oxygen, calcium, and tin isotopes, respectively. We show that the combination of the parameters governing the like-nucleon contribution is crucial to correctly reproduce the experimental (where available) or shell-model trends for the evolution of the three neutron gaps under study.

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## I. INTRODUCTION

The presence of an electric quadrupole moment in the ground state of the deuteron [1,2] can be explained by including static tensor terms in the microscopic nucleon-nucleon force as first suggested by Rarita *et al.* [3–5]. This procedure is today commonly adopted by all the modern microscopic nucleon-nucleon interactions [6–10]. In a description of the nucleon-nucleon interaction based on a meson-exchange picture [6], the strongest of the tensor components, the tensor-isospin term, is dominated by the exchange of a single pion. Since the pion is the lightest meson, this means that the interaction range of the tensor-isospin term is the longest one inside the nucleon-nucleon interaction.

For many years, tensor terms have not been considered in effective interactions used in mean-field theories such as Hartree-Fock (HF) and random-phase approximation (RPA), commonly used to describe medium and heavy nuclei. An exception to this is represented by the semirealistic M3Y-P interactions [11,12]. These interactions are based on the effective M3Y-Paris interaction [13] which has been constructed to describe inelastic nucleon-nucleus processes. The new M3Y-P interactions are obtained by including a density-dependent zero-range term and by modifying some of the force parameters. The tensor and tensor-isospin terms of the M3Y-P3 and of the M3Y-P5 interactions are the same as those used in the original M3Y-Paris interaction.

In almost all the existing parametrizations of the most used effective interaction in HF and RPA self-consistent calculations, the Skyrme interaction, the tensor term is neglected, even though a zero-range tensor term was proposed in the original formulation of the force [14,15]. In the last years, tensor terms have been included either on top of existing Skyrme parametrizations like the SIII [16] and the SLy5 [17–19] forces (see Refs. [20,21]) or by inserting them in the global fit procedure producing new parametrizations of Skyrme interactions [22–24]. In the following we shall indicate as SLy5$_\mathrm{T}$ the parametrization introduced in Ref. [21] where tensor terms have been added on top of the SLy5 interaction.

Finite-range effective Gogny-like interactions are less used in HF and RPA calculations than the Skyrme ones. As far as the tensor terms are concerned, we find in the literature only few works where they have been introduced in this type of forces. A first effort in this direction was done by Onishi and Negele [25] who added a tensor term to an effective force of finite range which was taken of Gaussian form. After the introduction of the Gogny force in 1980 [26], some further attempts of including a finite-range tensor term have been done. Otsuka *et al.* [27] proposed the GT2 force obtained by adding to the standard central channels of the original Gogny force [26] a finite-range tensor-isospin term of Gaussian form, and by refitting all the parameters. An alternative procedure was proposed in Ref. [28] where the D1ST and the D1MT forces were constructed by adding on top of the D1S [29] and D1M [30] parametrizations a finite-range tensor-isospin term chosen to reproduce the energy of the first $0^-$ state in $^{16}$O in self-consistent HF plus RPA calculations.

The inclusion of tensor terms in effective interactions allows us to have the same operator structure of the microscopic nucleon-nucleon interactions, and, moreover, it is necessary to describe observables related to both single particle (s.p.) [21,31,32] and collective properties of medium and heavy nuclei [28,33–37].

The s.p. proton (neutron) energy gaps at $Z$ ($N$) = 8, 20, and 28 have been investigated in Refs. [32,38] by using both nonrelativistic and relativistic HF techniques. These studies showed that nuclear systems corresponding to $Z$ or $N = 8$ and 20 are particularly suitable to study the neutron-proton tensor component of the effective interaction.

 

On the other hand, the s.p. proton (neutron) energy gaps along isotonic (isotopic) chains may be sensitive to the like-nucleon component of the tensor interaction. Experimental observations indicate that the $N = 14$ neutron gap in oxygen isotopes increases when going from $^{16}$O to $^{22}$O. A similar behavior is found in calcium isotopes for the $N = 28$ neutron s.p. energy gap which increases from $^{40}$Ca to $^{48}$Ca. The shell-model calculations of Ref. [39] describe this behavior and predict an analogous increase of the $N = 90$ neutron gap from $^{132}$Sn to $^{140}$Sn. In that work, the previous effect has been attributed to the three-body terms of the interaction. Here we show that, in the framework of mean-field HF theory, the evolution of these three neutron gaps strongly depends on the presence of the like-nucleon component of the tensor term.

For the Skyrme interaction, several works exist in the literature where both the neutron-proton and the like-nucleon tensor contributions have been analyzed. An extensive study of the effects generated by these two contributions has been carried out in Ref. [23]. In the Gogny case, a detailed analysis where the two components are studied separately is still missing. As already mentioned, in some recent works a finite-range tensor term has been introduced only in the isospin dependent channel with a single parameter to be chosen [27,28]. As it will be discussed in the next section, this implies that the neutron-proton and the like-nucleon contributions are proportional and have the same sign, that is, they are both attractive or repulsive. Considering that realistic and semirealistic nucleon-nucleon forces include both types of tensor terms (pure tensor and tensor isospin), we propose here to take into account both terms also for the effective Gogny interaction. This implies the introduction of a second parameter which allows us to separately tune the neutron-proton and the like-nucleon tensor contributions of the effective interaction.

The work is organized as follows. In Sec. II we present the physics case of the $N = 28$ neutron gap in calcium isotopes. We describe the experimental energies by using the Gogny D1ST and the Skyrme SLy5$_\mathrm{T}$ interactions, and we show the need of including both tensor and tensor-isospin terms in the Gogny interaction. In Sec. III we discuss the implementation of the two finite-range tensor terms in the Gogny interaction, and we propose two possibilities for the choice of the parameters. In Sec. IV we apply these two parametrizations of the tensor terms in the Gogny interaction to describe the evolution of the neutron gaps in oxygen, calcium, and tin isotopes. We compare our results with experimental data (where available) and with the results of the HF calculations carried on with the Skyrme interaction. Finally, we draw in Sec. V our conclusions and we discuss the perspectives of future applications of the Gogny plus tensor interaction.

## II. NEUTRON $N = 28$ ENERGY GAP IN CALCIUM ISOTOPES

It has been experimentally established that the $N = 28$ neutron energy gap, that is, the difference between the s.p. energies of the $2p_{3/2}$ and $1f_{7/2}$ neutron levels, increases when going from $^{40}$Ca to $^{48}$Ca. The experimental situation is summarized in Fig. 7 of Ref. [40] and indicates a change from a value of about 2.2 MeV in $^{40}$Ca to 4.8 MeV in $^{48}$Ca.

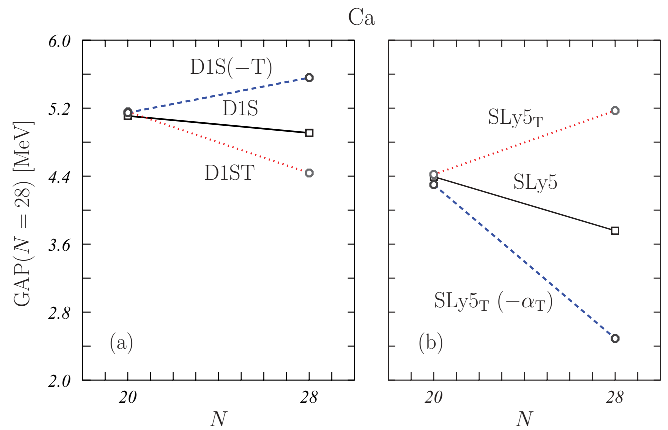


FIG. 1. (Color online) (a) Neutron energy gap for $N = 28$ in $^{40}$Ca and $^{48}$Ca nuclei obtained with the Gogny interactions D1S (solid line) and D1ST (dotted line). The result obtained by changing the sign of the tensor term in the D1ST case is shown by the dashed line. (b) Neutron energy gap for $N = 28$ obtained with the Skyrme interactions SLy5 (solid line) and SLy5$_\mathrm{T}$ (dotted line). The result obtained by changing the sign of the parameter $\alpha_\mathrm{T}$ in the SLy5$_\mathrm{T}$ case is shown by the dashed line.

We have calculated the evolution of this energy gap in the HF framework by using Skyrme and Gogny interactions. Our results are presented in Fig. 1. In panel (b) we show with black solid and red dotted lines, respectively, the results obtained with the Skyrme SLy5 and SLy5$_\mathrm{T}$ interactions. The values of the energy gaps we have obtained are, in general, larger than the experimental ones. Despite this deficiency, we observe that the interaction without tensor terms, the SLy5, does not describe the trend of the energy gap, which is slightly decreasing in this calculation. On the other hand, the result obtained with the SLy5$_\mathrm{T}$ force, which includes tensor terms, shows an increasing behavior of the energy gap.

The behavior of the energy gap is controlled by the like-nucleon term of the tensor force. In the Skyrme interaction the contribution of tensor components to the energy density of the system can be written as [21,32]

$$\Delta E_\mathrm{T}(r) = \frac{1}{2}\alpha_\mathrm{T}\left[J_p^2(r) + J_n^2(r)\right] + \beta_\mathrm{T} J_p(r) J_n(r) \ , \qquad (1)$$

where $J_p(r)$ and $J_n(r)$ are the proton and neutron spin-orbit densities. The parameters $\alpha_\mathrm{T}$ and $\beta_\mathrm{T}$ rule, respectively, the like-nucleon and the proton-neutron terms of the tensor interaction. In the SLy5$_\mathrm{T}$ force [21] these parameters assume the values $-170$ MeV fm$^5$ and 100 MeV fm$^5$, respectively.

It is easy to show that the effect of the tensor interaction is almost zero in spin-saturated nuclear systems, since the effect on the $j = l + 1/2$ s.p. level is canceled by that on the $j = l - 1/2$ one. The global effect would be exactly zero if the radial wave functions of the two levels were the same. Since calcium isotopes are spin-saturated in protons, the like-nucleon tensor term does not act on protons, and the neutron-proton contribution is not active in the evolution of the neutron gap. The consequences of this in the excitation of magnetic states in calcium isotopes have been widely discussed in [37].

The sensitivity of our results to the like-nucleon tensor term is shown in panel (b) of Fig. 1 by the dashed line, obtained by changing the sign of the parameter $\alpha_T$. This modification leads to a decreasing energy gap going from $^{40}$Ca to $^{48}$Ca.

In panel (a) of Fig. 1 we show the results of HF calculations carried out with the Gogny interaction. The black solid line indicates the result obtained with the D1S force [26] which does not contain tensor terms. The behavior of the energy gap is analogous to that obtained with the Skyrme interaction without tensor term. The red dotted line shows the result obtained with the D1ST interaction. In this case the behavior of the energy gap is opposite with respect to the experimental one, and also with respect to that obtained with the SLy5$_T$ interaction. If the sign of the parameter that determines the strength of the tensor term in the D1ST interaction is changed, the results indicated by the blue dashed line in panel (a) of Fig. 1 are obtained. We remark that this operation on the D1ST force acts only on the tensor-isospin dependent term and, therefore, changes both the like-nucleon and unlike components of the tensor force. In this way, nuclear properties depending on the neutron-proton tensor interaction that are well described by the D1ST force are not any more reproduced. For example, the energy of the first $0^-$ state in $^{16}$O whose experimental value of 10.94 MeV was used to tune the tensor force term in the D1ST interaction, appears at 14.48 MeV when the sign of the total strength is changed. Evidently, a unique tensor-isospin term in the D1S force is not able to reproduce simultaneously both nuclear properties.

## III. TENSOR TERMS AND THE GOGNY INTERACTION

The D1ST and D1MT interactions have been constructed by adding a tensor-isospin term to the Gogny D1S and D1M interactions, respectively [28]. The radial part of this term was based on the analogous one in the microscopic Argonne V18 interaction [10]. Specifically, we have considered

$$v_{\mathrm{Tt}}(r) = v_{\mathrm{Tt,AV18}}(r)\,[1 - \exp(-b\,r^2)]\,, \tag{2}$$

where the radial part of the Argonne V18 tensor isospin term [10], $v_{\mathrm{Tt,AV18}}(r)$, has been multiplied by a function that simulates the effect of the short-range correlations [41]. Here $b$ is a free parameter. The inclusion of this tensor-isospin term was done without changing the values of the other force parameters but the strength of the spin-orbit term. The values of the two free parameters, one for the tensor and the other one for the spin-orbit term, have been chosen to reproduce, in an iterative HF plus RPA calculation chain, the energy of the first $0^-$ state and the s.p. energy gap between the $1p_{3/2}$ and $1p_{1/2}$ neutron states, in $^{16}$O [36].

In the present work we use an expression for the tensor interaction similar to that proposed by Onishi and Negele [25]

$$\begin{aligned} V_{\mathrm{tensor}}(r_1,r_2) &= \left(V_{\mathrm{T1}} + V_{\mathrm{T2}}\,P^\tau_{12}\right)\, S_{12}\, \exp\left[-(r_1-r_2)^2/\mu_{\mathrm{T}}^2\right] \\ &= \left[\left(V_{\mathrm{T1}} + \frac{1}{2}V_{\mathrm{T2}}\right) + \frac{1}{2}V_{\mathrm{T2}}\,\boldsymbol{\tau}(1)\cdot\boldsymbol{\tau}(2)\right] \\ &\quad\times S_{12}\, \exp\left[-(r_1-r_2)^2/\mu_{\mathrm{T}}^2\right], \end{aligned} \tag{3}$$

where we have indicated with $P^\tau$ the usual isospin exchange where $S_{12}$ and $\boldsymbol{\tau}$ represent the usual tensor and isospin Pauli operators. In the second line we have separated the pure tensor and tensor-isospin terms. In this approach the radial part of the two independent tensor terms is identical, and it has been chosen of Gaussian form. In our calculations we used $\mu_{\mathrm{T}} = 1.2$ fm, corresponding to the longest range of the D1S interaction.

In this approach, the strength of the full tensor force is ruled by the two parameters $V_{\mathrm{T1}}$ and $V_{\mathrm{T2}}$. A calculation of the isospin matrix elements for the interaction (3) indicates that the strength of the force acting in like-nucleon pairs is given by $V_{\mathrm{T1}} + V_{\mathrm{T2}}$, while that between proton-neutron pairs is $V_{\mathrm{T2}}$. These combinations of the parameters are, respectively, analogous to the $\alpha_T$ and $\beta_T$ parameters of the Skyrme interaction given in Eq. (1).

The two tensor terms in Eq. (3) have been added to the D1S force without changing any other parameter value, including the strength of the spin-orbit. In this way we are able to analyze exclusively the effect of the tensor force.

In order to choose the values of the two free parameters, $V_{\mathrm{T1}}$ and $V_{\mathrm{T2}}$, we have used two observables. The first one is the energy difference between the $1f_{5/2}$ and $1f_{7/2}$ s.p. neutron states in $^{48}$Ca. As already discussed in Sec. II, this observable depends only on the like-nucleon tensor contribution and therefore is ruled by $V_{\mathrm{T1}} + V_{\mathrm{T2}}$. We show in Table I the energy difference between these two s.p. states obtained for various values of $V_{\mathrm{T1}} + V_{\mathrm{T2}}$. We have verified that by changing $V_{\mathrm{T1}}$ and $V_{\mathrm{T2}}$ the result is the same if the sum does not change. The experimental value of the energy difference is 8.8 MeV [42], therefore we have chosen $V_{\mathrm{T1}} + V_{\mathrm{T2}} = -20$ MeV.

TABLE I. Difference between the energies of the $1f_{5/2}$ and $1f_{7/2}$ s.p. neutron states in $^{48}$Ca for some values of $V_{\mathrm{T1}} + V_{\mathrm{T2}}$. The experimental value is 8.8 MeV [42]. All the quantities are expressed in MeV.

| $V_{\mathrm{T1}} + V_{\mathrm{T2}}$ | $\epsilon_\nu(1f_{5/2}) - \epsilon_\nu(1f_{7/2})$ |
|---|---|
| 0.0 | 8.3 |
| −5.0 | 8.5 |
| −10.0 | 8.6 |
| −20.0 | 8.9 |
| −30.0 | 9.3 |
| −40.0 | 9.7 |
| −50.0 | 10.0 |
| −60.0 | 10.4 |
| −70.0 | 10.8 |
| −80.0 | 11.1 |

The second observable we have considered is the energy of the first $0^-$ state in the $^{16}$O nucleus. In Ref. [28] a large sensitivity of the energies of the $0^-$ states in doubly magic nuclei to the tensor-isospin term of the interaction was observed. We show in Fig. 2 the excitation energy of this state calculated in the HF plus RPA approach for different values of $V_{\mathrm{T2}}$. All the calculations shown by the black solid line have been carried out by using $V_{\mathrm{T1}} + V_{\mathrm{T2}} = -20$ MeV. For $V_{\mathrm{T2}} = 115$ MeV we obtain for the energy of the $0^-$ the value of 10.72 MeV, close to the experimental value of 10.96 MeV [43]. This

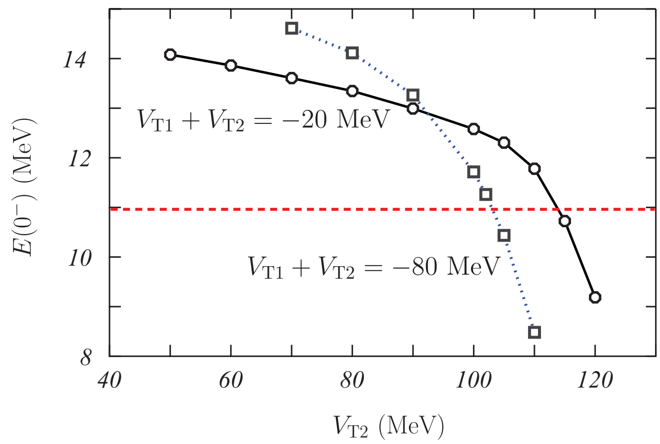


FIG. 2. (Color online) Energy of the first $0^-$ excited in $^{16}$O calculated in RPA as a function of the $V_{\rm T2}$ parameter, Eq. (3), and keeping fixed $V_{\rm T1} + V_{\rm T2}$ at $-20$ MeV (solid line) and $-80$ MeV (dotted line). The horizontal dashed line indicates the experimental value.

choice of $V_{\rm T2}$, together with $V_{\rm T1} + V_{\rm T2} = -20$ MeV, implies $V_{\rm T1} = -135$ MeV. We label this parametrization D1ST2a.

In order to identify the general features of our results we have implemented another parametrization of the tensor terms, which we call D1ST2b. In this case, we selected the like-nucleon part of the tensor force to reproduce the $N = 28$ neutron gap increase from $^{40}$Ca to $^{48}$Ca as obtained in the HF calculation with the SLy5$_{\rm T}$ force. We obtained this results with the value of $V_{\rm T1} + V_{\rm T2} = -80$ MeV. As in the previous case, the other observable we have chosen to select the value of $V_{\rm T2}$ is the excitation energy of the first $0^-$ state in $^{16}$O. The blue dotted line in Fig. 2 indicates the value $V_{\rm T2} = 102$ MeV. In Fig. 3 we compare the two terms of the D1ST2a and D1ST2b tensor force with the analogous ones of the effective M3YP5 [12] and microscopic AV18 [10] interactions. The M3YP5 tensor isospin term is of the same order than that of our interactions. In the case of the $v_T$ term, M3YP5 presents an attractive part for small $q$, that becomes repulsive for $q > 1$ fm$^{-1}$. It is interesting to notice that all the effective interactions have a repulsive $v_T$ term and an attractive $v_{T\tau}$ term. On the contrary, in the microscopic AV18 interaction both terms are attractive. This is an indication of the important role played by both short and long range correlations in modifying the interaction.

## IV. NEUTRON GAPS

The results we discuss in this section have been obtained in the HF framework. Pairing correlations are not included in our calculations since the nuclei we have considered, $^{16}$O, $^{22}$O, $^{40}$Ca, $^{48}$Ca, $^{132}$Sn, and $^{140}$Sn, have a well defined closed-shell character. The study has been conducted by comparing results obtained by using interactions with (D1ST2a, D1ST2b, and SLy5$_{\rm T}$ ) and without (D1S and Sly5) tensor terms.

### A. $N = 28$ and $N = 90$

The gap evolution in calcium and tin isotopes is rather similar. The case $N = 28$ involves the $2p_{3/2}$ and $1f_{7/2}$ neutron s.p. levels in $^{40}$Ca and $^{48}$Ca. The s.p. energies of these states are shown in Fig. 4, for the Gogny (a) and Skyrme (b) interactions.

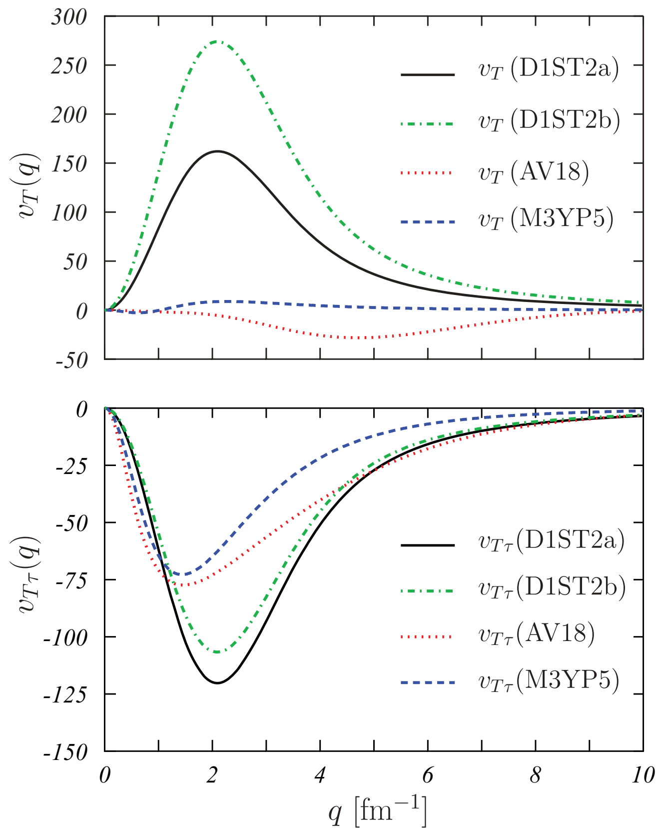


FIG. 3. (Color online) Tensor (a) and tensor-isospin (b) terms of the D1ST2a and D1ST2b parametrizations used in this work, compared with the analogous terms in the effective M3YP5 interaction and in the realistic one AV18.

The corresponding gap values are shown in panels (c) and (d) of the same figure.

The effects of the tensor on the energies of the $2p_{3/2}$ state are rather small, while those on the $1f_{7/2}$ state are more evident, producing a lowering of the energy value in $^{48}$Ca, much pronounced in the case of the Skyrme and D1ST2b interactions. In both types of calculations (Skyrme and Gogny) only the presence of the tensor terms produces an increase of the gap, in agreement with the experimental evidence [40]. In the shell-model calculations of Ref. [39] the energy of the $1f_{7/2}$ level is lowered and that of the $2p_{3/2}$ level is increased. This last effect is not present in our calculations.

The case $N = 90$ involves similar s.p. states which differ from the $N = 28$ case only for the principal quantum numbers. The results obtained are presented in Fig. 5 and show behaviours similar to those shown in Fig. 4. In this case, we found an increase of the energy gap already in the D1S calculation. This effect is enhanced by the inclusion of the tensor term and is more evident for the D1ST2b force. No experimental data are available for the $N = 90$ gap, however shell model calculations carried out with microscopic interactions indicate an increase of the $N = 90$ gap [40].

### B. $N = 14$

We discuss another case, in a different region of the nuclear chart, where the experimental values of the s.p. energies are

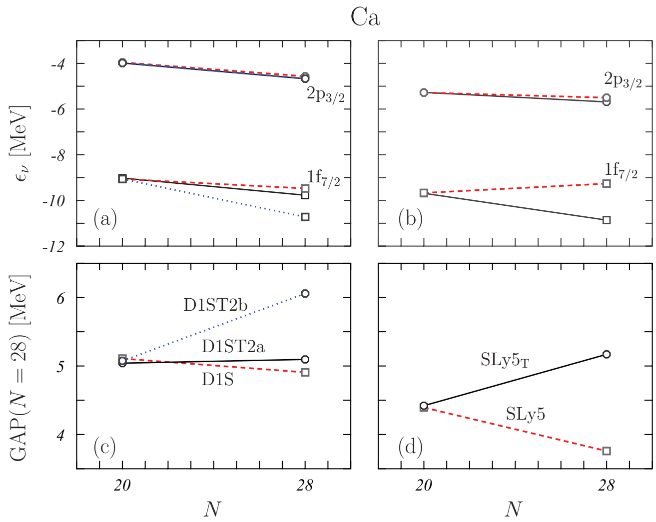


FIG. 4. (Color online) (a) Energies of the neutron s.p. levels around $N = 28$ for the nuclei $^{40}$Ca and $^{48}$Ca obtained with the D1S (dashed lines), D1ST2a (solid lines), and D1ST2b (dotted lines) forces. (c) Evolution of the energy gap for $N = 28$ obtained with the D1S (dashed line), D1ST2a (solid lines), and D1ST2b (dotted lines) forces. (b) The same as in (a) but for the SLy5 (dashed lines) and SLy5$_\mathrm{T}$ (solid lines). (d) The same as (c) for the two Skyrme interactions.

known. We consider the energy gap $N = 14$ between the $2s_{1/2}$ and the $1d_{5/2}$ neutron states in oxygen isotopes. The experimental value of this gap in $^{16}$O is 0.87 MeV [44]. From the study of the excited states in $^{21-23}$O nuclei through their $\gamma$ decay, Stanoiu *et al.* [45] deduced a value of the energy gap of 4.11 MeV in $^{22}$O. This value is relatively large and, for this reason, $^{22}$O can be considered a doubly magic nucleus. This is also supported by the observation that the value of the excitation energy of the first $2^+$ state in $^{22}$O is almost twice that observed in the neighboring even-even nuclei.

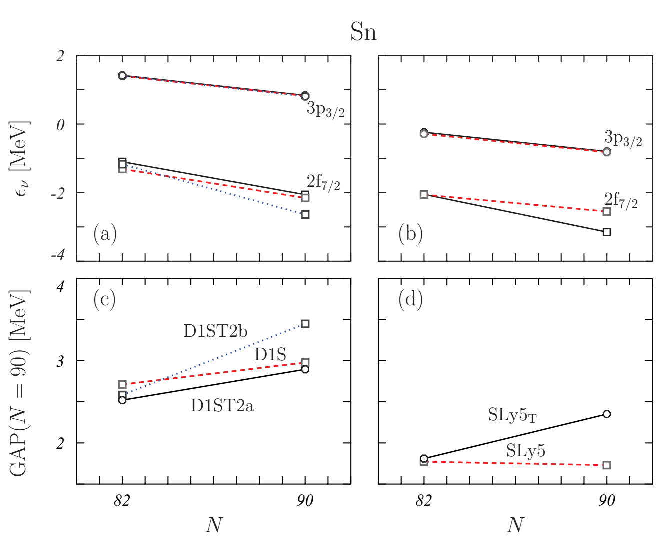


FIG. 5. (Color online) The same as Fig. 4 for the case of the $N = 90$ neutron gap for the nuclei $^{132}$Sn and $^{140}$Sn. The meaning of the lines is analogous to that of Fig. 4 with the obvious changes.

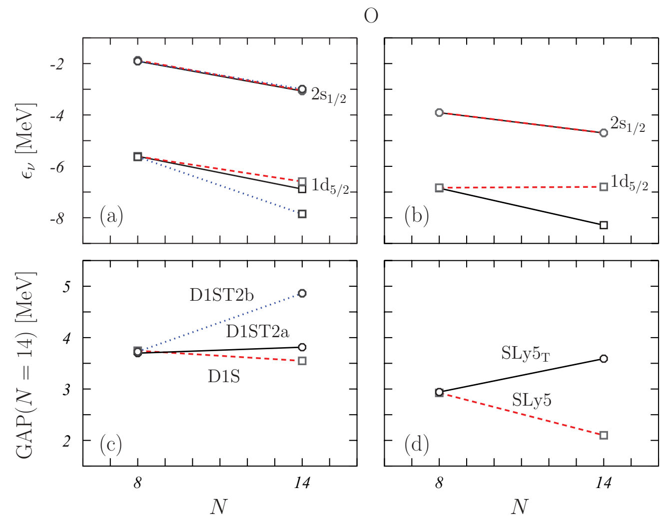


FIG. 6. (Color online) The same as Fig. 4 for the case of the $N = 14$ neutron gap for the nuclei $^{16}$O and $^{22}$O. The meaning of the lines is analogous to that of Fig. 4 with the obvious changes.

The results of our calculations are shown in Fig. 6. Also in this case the behavior found for the two types of interactions, Gogny and Skyrme, is rather similar. The major effects of the tensor terms of the force are present on the neutron $1d_{5/2}$ s.p. energies which, in $^{22}$O are remarkably lower than those obtained without tensor, mainly for the Skyrme interaction and in the case of the D1ST2b force. This effect produces an increase of the energy gap, even though the energies of the $2s_{1/2}$ states remain unchanged. The results obtained with the D1S and SLy5 force show a decreasing gap.

## V. CONCLUSIONS

In this work, we have first pointed out the need of including in the effective Gogny interaction two independent tensor terms acting separately on like-nucleon and proton-neutron pairs. We have included these two independent terms under the form of tensor and tensor-isospin components to be added on top of effective Gogny forces. To the best of our knowledge, only tensor-isospin terms have been considered up to now for these finite-range interactions. We have proposed two different parametrizations of the tensor force. In a first one, the strength of the like-nucleon part of the tensor force has been chosen to reproduce the experimental value of the splitting between the $1f_{7/2}$ and $1f_{5/2}$ neutron s.p. energies in $^{48}$Ca. In the second parametrization, the strength of this part of the interaction has been chosen to reproduce the neutron gap increase in $N = 28$ going from $^{40}$Ca to $^{48}$Ca as obtained with the SLy5$_\mathrm{T}$ interaction. In both parametrizations the remaining term ruling the proton-neutron term has been selected to the energy of the first $0^-$ excited state in $^{16}$O

Using these parametrizations, we have calculated the neutron energy gap for $N =$ 14, 28, and 90, in oxygen, calcium, and tin isotopes, respectively. Our results show that both parametrizations reproduce the trend for the neutron gaps obtained with the Skyrme SLy5$_\mathrm{T}$ interaction, better in the case of the D1ST2b fit. This trend is in agreement with the experimental behavior in oxygen and calcium isotopes, and with the results of shell-model calculations in tin isotopes.

The inclusion of two tensor terms allows us to reproduce the experimental trends of the neutron energy gaps in the isotope chains we have investigated. This is our main result. From the quantitative point of view it is evident that the two observables related to the like-nucleon term of the tensor interaction are not compatible in HF calculations. The parametrization D1ST2a built to reproduce the s.p. splitting of the $f$ states in $^{48}$Ca produces the correct behaviour of the neutron energy gap, but its value is quantitative too small. Probably, a good quantitative description of these two quantities requires to go beyond mean-field calculations, and to consider explicitly the effects of the coupling between s.p. and collective degrees of freedom.

We consider the present work as a step forward in the direction of constructing a new parametrization of the Gogny interaction which includes tensor terms. We have the perspective of validating these new tensor terms by using them in the description of observables where the particle-like contribution of the tensor force is expected to play a role, for example, in the excitation of unnatural parity states in nuclei with neutron excess and with closed proton shells [36]. Of course, a more accurate fit would require to simultaneously modify all the parameters of the Gogny force, especially the spin-orbit strength which has a strong interplay with the tensor force, in both the s.p. energies and the excitation of magnetic states.

## ACKNOWLEDGMENTS

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