Symmetries in the $g_{9/2}$ shell

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We consider symmetries which arise when two-body interaction matrix elements with isospin $T = 0$ are set equal to a constant in a single-j-shell calculation. The nucleus ^{96}Cd is used as an example.

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The recent discovery of a $J = 16^+$ isomer in ^{96}Cd by Nara Singh *et al*. [1] was noted in a work on symmetries in the $g_{9/2}$ shell by Zamick and Robinson [2]. In a previous work by these authors [3], the main emphasis was on the $f_{7/2}$ shell, although the equations were written in a general way so as to apply to any shell. In this work we elaborate on the work in Ref. [2] by giving detailed wave functions and energies.

First, however, we would like to point out that the $g_{9/2}$ shell has, in recent years, been a beehive of activity both experimentally and theoretically. For example, in contrast to the $f_{7/2}$ shell, where the spectrum of three identical particles would be identical to that of five particles (or three holes) in the single-j-shell approximation, one can have different spectra in the $g_{9/2}$ shell. It was noted by Escuderos and Zamick [4] that, with a seniority-conserving interaction, the $(J = 21/2^+)$–$(J = 3/2^+)$ splitting (maximum and minimum spins for three identical particles) was the same for five identical particles as for three. However, with a $Q \cdot Q$ interaction (which does not conserve seniority), the splittings were equal in magnitude but opposite in sign.

Even more interesting, it was noted in Ref. [4] that for four identical particles (or holes, e.g., ^{96}Pd) there are three $J = 4^+$ states—two with seniority 4 and one with seniority 2. In general, seniority is not conserved in the $g_{9/2}$ shell. Despite this fact, it was found that no matter what interaction was used, one eigenstate emerged that was always the same for all interactions—this was a seniority-4 state. The other two states were a mix of seniorities 2 and 4. A unique state also emerged for $J = 6^+$. This observation led to considerable activity with an intent to explain this behavior as seen in Refs. [5–11].

Another recently emerging topic has to do with $T = 0$ pairing in which the pairs are coupled to the maximum angular momentum, which for the $g_{9/2}$ shell is $J = 9^+$(see Refs. [12–15]). In the *Nature* article [12], an experiment is presented in which an almost equal-spaced spectrum is found in $N = Z$ ^{92}Pd for $J = 0^+, 2^+, 4^+$, and 6^+. This is well reproduced by the shell model but also by $T = 0$ pairing with maximum alignment of pairs. The 2^+ state is lower in ^{92}Pd than in ^{96}Pd, which is not surprising because the latter is semi-magic. An interesting observation in Ref. [13] is that, although the spectrum looks vibrational, the $B(E2)$s might obey a rotational formula rather than vibrational. It would be of interest to have experimental verification.

Recently the current authors addressed a different problem [16], but one that has some implications for the above topic. We studied the question of isomerism for systems of three protons and one neutron, e.g., ^{96}Ag. We found that for the upper half of a j shell, be it $f_{7/2}$ or $g_{9/2}$ shell, the $J = J_{\max}$ two-body matrix element is much more attractive than in the lower half. The states in question have $J = 7^+$ and 9^+, respectively. This was necessary to explain the isomeric behaviors of states in various nuclei, e.g., why the lifetime of the $J = 12^+$ state in ^{52}Fe is much longer than in ^{44}Ti, or why the $J = 16^+$ states in ^{96}Cd and the $J = 15^+$ states in ^{96}Ag are isomeric. In that work, we also used the previously determined two-body matrix elements of Coraggio *et al*. [17] as well as our own. With regards to the works of Refs. [12–15], this suggests the $J = J_{\max}$ pairing is a better approximation in the upper half of a j shell rather than the lower half.

It should be added that one of us had previously studied the problem of the effects of varying two-body matrix elements in the $f_{7/2}$ region [18] and that one can learn a lot by studying the explicit $f_{7/2}$ wave functions in Ref. [6], which are based on previous works in Refs. [19,20].

But let us not lose sight of what this Brief Report is about—partial dynamical symmetries (PDS) in ^{96}Cd. One obvious distinction of our work relative to that of Refs. [12–15] is that we are dealing with higher spins than the ones that they consider. In the *Nature* article [12], they measure $J = 0, 2, 4$, and 6 and also discuss $J = 8$, whereas the PDS that we consider occur for $J = 11^+$ and beyond. Thus, our work may be regarded as different but complimentary to theirs.

At the time of this writing, the 16^+ isomer is the only known excited state of ^{96}Cd [1]. It decays to a 15^+ isomer in ^{96}Ag. In previous work by Escuderos and Zamick [4], it was noted that the single-j approximation for a few holes relative to $Z = 50$, $N = 50$ works fairly well. However, they cautioned that this approximation does not work at all for a few particles relative to $Z = 40$, $N = 40$. The relevant shell is, of course, $g_{9/2}$.

The symmetry in question mentioned above comes from setting two-body interaction matrix elements with isospin $T = 0$ to be constant, whilst keeping the $T = 1$ matrix elements unchanged [2]. It does not matter what the $T = 0$ constant is as far as symmetries are concerned, but it does affect the relative energies of states of different isospins. Briefly stated, for the four-hole system ^{96}Cd, we then have a PDS, one which involves $T = 0$ states with angular momenta that do not exist for $T = 2$ states in a pure $g_{9/2}$ configuration. That is to say, the PDS will not occur for states with angular momenta which can occur in the $(g_{9/2})^4$ configuration of four identical particles (^{96}Pd). Although many 6-j and 9-j relations were used in Refs. [2,3] to describe why these symmetries are partial, one simple argument is illuminating. If a given angular momentum can occur for, say, four $g_{9/2}$ neutron holes ($T = 2$), then there is a constraint on the $T = 0$ states with the same angular momentum—namely their wave functions have to be orthogonal to the $T = 2$ states. This constraint prevents the occurrence of a PDS.

For qualifying $T = 0$ states, the PDS consists of J_p and J_n being good dual quantum numbers. That is to say, the wave function of a state will have only one (J_p, J_n) and (J_n, J_p). Another way of saying this is that $J_p \cdot J_n$ is a good quantum number. Furthermore, states with different total angular momentum J but with the same (J_p, J_n) will be degenerate.

We have given a physical argument for the PDS. We can also explain it mathematically. There are both off-diagonal and diagonal conditions. The former is needed to explain why (J_p, J_n) are good dual quantum numbers. The reason is the vanishing of the $9j$-symbol:

$$\begin{Bmatrix} j & j & (2j-1) \\ j & j & (2j-1) \\ (2j-1) & (2j-3) & (4j-4) \end{Bmatrix} = 0 \tag{1}$$

Next we need diagonal conditions to explain why states with the same (J_p, J_n) are degenerate. These are given by

$$\begin{Bmatrix} j & j & (2j-3) \\ j & j & (2j-1) \\ (2j-3) & (2j-1) & I \end{Bmatrix} = \frac{1}{4(4j-5)(4j-1)} \tag{2}$$

TABLE I. Wave functions and energies (in MeV, at the top) of selected states of ^{96}Cd calculated with the INTd interaction (see text).

$J = 11$							
		5.5640	5.6482	6.4693	6.6384	6.9319	8.1822
J_p	J_n			$T = 1$	$T = 1$	$T = 1$	$T = 1$
4	8	0.4709	−0.6359	−0.2463	0.3092	−0.4544	0.1051
6	6	0.2229	0.0000	0.8712	0.0000	−0.3121	−0.3065
6	8	0.4607	−0.3092	−0.0631	−0.6359	0.4432	−0.2956
8	4	0.4709	0.6359	−0.2463	−0.3092	−0.4544	0.1051
8	6	0.4607	0.3092	−0.0631	0.6359	0.4432	−0.2956
8	8	0.2869	0.0000	0.3343	0.0000	0.3110	0.8421
$J = 12$							
		5.0303	5.8274	6.1835	6.7289	6.8648	9.0079
J_p	J_n				$T = 1$	$T = 1$	$T = 2$
4	8	0.4364	0.3052	−0.3894	−0.3592	−0.5903	0.2957
6	6	0.7797	−0.4079	0.0000	0.2927	0.0000	−0.3742
6	8	0.0344	0.5602	−0.5903	0.2078	0.3894	−0.3766
8	4	0.4364	0.3052	0.3894	−0.3592	0.5903	0.2957
8	6	0.0344	0.5602	0.5903	0.2078	−0.3894	−0.3766
8	8	0.0940	0.1402	0.0000	0.7550	0.0000	0.6337
$J = 13$							
		5.8951	6.1898	7.5023			
J_p	J_n		$T = 1$	$T = 1$			
6	8	0.7071	0.6097	−0.3581			
8	6	−0.7071	0.6097	−0.3581			
8	8	0.0000	0.5065	0.8623			
$J = 14$							
		5.1098	6.4980	6.7036			
J_p	J_n			$T = 1$			
6	8	0.6943	−0.1339	−0.7071			
8	6	0.6943	−0.1339	0.7071			
8	8	0.1894	0.9819	0.0000			
$J = 15$							
		6.2789					
J_p	J_n	$T = 1$					
8	8	1.0000					
$J = 16$							
		4.9371					
J_p	J_n						
8	8	1.0000					

TABLE II. Wave functions and energies (in MeV, at the top) of selected states of ^{96}Cd calculated with the INTd interaction (see text) with $T = 0$ matrix elements set to zero.

$J = 11$							
		5.0829	5.3798	6.8295	7.4699	7.5178	7.8842
J_p	J_n			$T = 1$	$T = 1$	$T = 1$	$T = 1$
4	8	0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
6	6	0.0000	0.0000	0.2913	0.5605	0.6482	−0.4253
6	8	0.0000	0.7071	0.5350	0.0396	−0.4111	−0.2079
8	4	−0.7071	0.0000	0.2933	−0.5491	0.3351	−0.0121
8	6	0.0000	−0.7071	0.5350	0.0396	−0.4111	−0.2079
8	8	0.0000	0.0000	0.4130	0.2822	0.1319	0.8558
$J = 12$							
		5.1165	5.2336	5.4865	7.5293	7.5959	12.4531
J_p	J_n				$T = 1$	$T = 1$	$T = 2$
4	8	0.5699	0.2803	−0.0961	−0.4783	0.5208	0.2957
6	6	0.5712	−0.7151	0.1498	0.0000	0.0000	−0.3742
6	8	0.0925	0.3679	0.4629	−0.5208	−0.4783	−0.3766
8	4	0.5699	0.2803	−0.0961	0.4783	−0.5208	0.2957
8	6	0.0925	0.3679	0.4629	0.5208	0.4783	−0.3766
8	8	−0.0846	−0.2465	0.7284	0.0000	0.0000	0.6337
$J = 13$							
		5.3798	7.6143	7.8873			
J_p	J_n		$T = 1$	$T = 1$			
6	8	0.7071	0.5265	−0.4721			
8	6	−0.7071	0.5265	−0.4721			
8	8	0.0000	0.6676	0.7445			
$J = 14$							
		5.3798	5.6007	7.8515			
J_p	J_n			$T = 1$			
6	8	0.7071	0.0000	−0.7071			
8	6	0.7071	0.0000	0.7071			
8	8	0.0000	1.0000	0.0000			
$J = 15$							
		7.9251					
J_p	J_n	$T = 1$					
8	8	1.0000					
$J = 16$							
		5.6007					
J_p	J_n						
8	8	1.0000					

for $I = (4j-4), (4j-5), (4j-7)$ and

$$\begin{Bmatrix} j & j & (2j-1) \\ j & j & (2j-1) \\ (2j-1) & (2j-1) & I \end{Bmatrix} = \frac{1}{2(4j-1)^2} \qquad (3)$$

for $I = (4j-4), (4j-2)$.

How the partial dynamical symmetry manifests itself is best illustrated by examining Tables I and II. Here we use the two-body INTd matrix elements from Zamick and Escuderos [16] to perform single-j-shell calculations of the energies and wave functions of ^{96}Cd. Actually, it does not matter what charge-independent interaction is used to illustrate the symmetry that will emerge.

Let us first focus on the $J = 11^+$ and $J = 12^+$ states. Relative to Table I, we see certain simplicities for the $J = 11^+$ states in Table II (where the $T = 0$ two-body interaction matrix elements are set to a constant). For the lowest state, the only non-zero components are $(J_p, J_n) = (4, 8)$ and $(8, 4)$; for the second state, they are $(6, 8)$ and $(8, 6)$. This confirms what we said above: (J_p, J_n) are good dual quantum numbers. Nothing special happens to $J = 11^+, T = 1$ states.

We show results for $J = 12^+$ as a counterpoint. We see that nothing special happens as we go from Table I to Table II—no PDS. The reason for this is, as discussed above, that four identical $g_{9/2}$ nucleons can have $J = 12^+$, but, because of the Pauli principle, they cannot couple to $J = 11^+$.

The other states with $J = 13, 14, 15, 16$ cannot occur for four identical nucleons and are therefore subject to the PDS. Note certain degeneracies, e.g., $J = 11, 13$, and 14 states, all with $(J_p, J_n) = (6, 8)$ and $(8, 6)$, have the same energy

$E = 5.3798$ MeV. The proof of all these properties are contained in Refs. [2,3].

The $J = 16^+$, which was experimentally discovered by Nara Singh *et al.* [1] is correctly predicted to be isomeric in Table I. It lies below the lowest 15^+ or 14^+ states. In Table II, however, the $J = 16^+$ state lies above the lowest $J = 14^+$ state and is degenerate with the second $J = 14^+$ state ($E = 5.6007$ MeV). Clearly, fluctuations in the $T = 0$ matrix elements are responsible for making the $J = 16^+$ isomeric.

[1] B. S. Nara Singh *et al.*, Phys. Rev. Lett. **107**, 172502 (2011).
[2] L. Zamick and S. J. Q. Robinson, Phys. Rev. C **84**, 044325 (2011).
[3] S. J. Q. Robinson and L. Zamick, Phys. Rev. C **63**, 064316 (2001).
[4] A. Escuderos and L. Zamick, Phys. Rev. C **73**, 044302 (2006).
[5] L. Zamick, Phys. Rev. C **75**, 064305 (2007).
[6] A. Escuderos, L. Zamick, and B. F. Bayman, arXiv:nucl-th/0506050, (2005).
[7] P. Van Isacker and S. Heinze, Phys. Rev. Lett. **100**, 052501 (2008).
[8] L. Zamick and P. Van Isacker, Phys. Rev. C **78**, 044327 (2008).
[9] Chong Qi, Phys. Rev. C **83**, 014307 (2011).
[10] Chong Qi, Z. X. Xu, and R. J. Liotta, Nucl. Phys. A **884–885**, 21 (2012).
[11] I. Talmi, Nucl. Phys. A **846**, 31 (2010).
[12] B. Cederwall *et al.*, Nature (London) **469**, 68 (2011).
[13] S. Zerguine and P. Van Isacker, Phys. Rev. C **83**, 064314 (2011).
[14] C. Qi, J. Blomqvist, T. Bäck, B. Cederwall, A. Johnson, R. J. Liotta, and R. Wyss, Phys. Rev. C **84**, 021301(R) (2011).
[15] Z. X. Xu, C. Qi, J. Blomqvist, R. J. Liotta, and R. Wyss, Nucl. Phys. A **877**, 51 (2012).
[16] L. Zamick and A. Escuderos, Nucl. Phys. A **889**, 8 (2012).
[17] L. Coraggio, A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **85**, 034335 (2012).
[18] L. Zamick and S. J. Q. Robinson, Phys. At. Nucl. **65**, 740 (2002).
[19] B. F. Bayman, J. D. McCullen, and L. Zamick, Phys. Rev. Lett. **11**, 215 (1963).
[20] J. N. Ginocchio and J. B. French, Phys. Lett. **7**, 137 (1963).