Ratio of neutron and proton entropy excess in ^{121}Sn compared to ^{122}Sn

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(Received 19 June 2012; revised manuscript received 13 August 2012; published 3 October 2012)

Level densities and entropies in ^{121}Sn and ^{122}Sn nuclei have been extracted within a BCS theory that includes nuclear pairing interaction based on the modified harmonic oscillator according to Nilsson potential. The entropy of the ^{121}Sn nucleus follows closely the entropy for the ^{122}Sn, but the even-odd system has an entropy excess at low temperatures. The neutron and proton entropy excess ratio has been calculated as a function of nuclear temperature. The proton system at low temperatures plays a minor role in entropy excess and neutron entropy excess ratio is about 1, as expected.

DOI: 10.1103/PhysRevC.86.047303 PACS number(s): 21.10.Ma, 21.30.−x, 21.60.−n, 27.60.+j

Introduction. Experimentalists have developed a new method to extract level densities from measured Î3 spectra at the Oslo cyclotron laboratory [1]. Recently, this method has been applied to obtain level densities in ^{121}Sn and ^{122}Sn using (^{3}He, ^{4}He) and (^{3}He, ^{3}He) reactions [2], respectively.

On the other hand, most of the theoretical research in the area of nuclear level densities has been based on the traditional Fermi gas model first introduced by Bethe [3]. The level density as a function of excitation energy is the starting point to deduce the thermodynamic quantities. Pairing correlations are one of the fundamental properties [4,5] of nuclei and have been successfully described by Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity [6].

In this work, the nuclear level densities have been computed for ^{121,122}Sn nuclei using superconducting theory with the inclusion of pairing effects. The results are compared with their corresponding experimental values. The entropy for even-odd (^{121}Sn) and even-even (^{122}Sn) nuclei have been calculated and the results are compared. Then the ratio of proton as well as neutron entropy excess have been determined and discussed.

Summary of the theory. This section is a brief review of the microscopic model used to calculate the state and level density as well as the entropy of the system. The calculation procedure is outlined in our previous publications [7–10]. In the framework of statistical mechanics, the state density is defined as [11,12]

$$\omega(N,E)=\frac{\exp(S)}{2\pi|D|^{1/2}}. \quad (1)$$

D is the determinant of the second derivations of the grand partition function taken at the saddle point. At this stationary point, the entropy S is given by Ref. [13]

$$S=2\sum_k \mathrm{Ln}[1+\exp(-\beta E_k)]+2\beta\sum_k\frac{E_k}{1+\exp(\beta E_k)}, \quad (2)$$

where β is the inverse of nuclear temperature and λ is related to the chemical potential. $E_k=[(\varepsilon_k-\lambda)^2+\Delta^2]^{1/2}$ is the quasiparticle energy, where ε_k is the energy of single-particle k and Δ is the gap parameter that is a measure of nuclear pairing. The saddle point conditions that must be satisfied are

$$N=\sum_k n_k, \quad (3)$$

$$E=\sum_k n_k E_k. \quad (4)$$

The occupational probability of level k is Ref. [10]

$$n_k=1-\frac{\varepsilon_k-\lambda}{E_k}\tanh\frac{\beta E_k}{2}. \quad (5)$$

For a system of N neutrons and Z protons, the total energy is given by

$$E=E_n+E_p, \quad (6)$$

and the total entropy is given by

$$S=S_n+S_p. \quad (7)$$

The total level density for a system of N neutrons and Z protons at an excitation energy of $U=U_n+U_p$ is Refs. [14–16]

$$\rho(N,Z,U)=\frac{\omega(N,Z,U)}{(2\pi\sigma^2)^{1/2}}, \quad (8)$$

where σ^2 is the spin cut-off parameter defined as

$$\sigma^2=\sigma_n^2+\sigma_p^2, \quad (9)$$

with [17]

$$\sigma_n^2=\frac{1}{2}\sum_k m_k^2\mathrm{sech}^2\left(\frac{\beta E_k}{2}\right) \quad (10)$$

and a similar equation for σ_p^2. Here, m_k is the magnetic momentum spin quantum number of the state k.

The steps necessary to calculate level density and system entropy are as follows: The set of single-particle levels and a particular choice of temperature T, the parameter λ and Δ are

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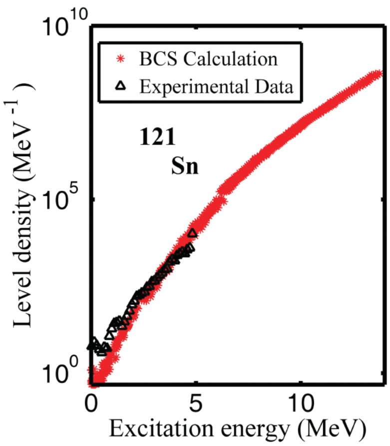

FIG. 1. (Color online) The experimental [2] and the calculated level density as a function of excitation energy in ^{121}Sn nucleus. The open triangles are data from (^{3}He, ^{4}He) reaction. The filled stars denote the BCS calculations.

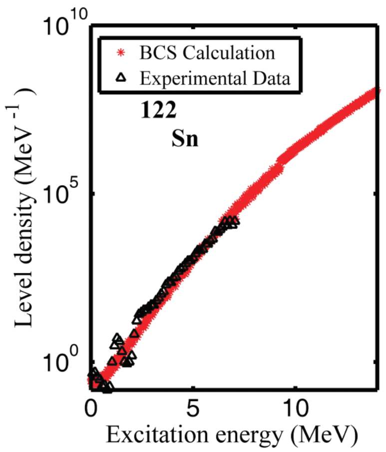

FIG. 2. (Color online) The experimental [2] and the calculated level density as a function of excitation energy in ^{122}Sn nucleus. The open triangles are data from (^{3}He, ^{3}He) reactions. The filled stars denote the BCS calculations.

estimated, and a set of occupational probabilities is calculated using Eq. (5). Next, the saddle point conditions are checked for a given nucleon number using Eq. (3). If the conditions are not met, the values of λ and Δ are adjusted and the procedure is repeated until the saddle point conditions are satisfied. Once the proper set of n_k values are computed the entropy S_n is computed from Eq. (2). The energy E_n is calculated by applying Eq. (4) at a particular temperature T. The excitation energy U_n is then determined by subtracting the energy at $T = 0$. The quantities σ_n^2 and $\omega(N, E)$ are determined using Eqs. (1)–(10). A similar set of calculations is used to calculate U_p and S_p for protons. Total entropy S, at excitation energy $U = U_n + U_p$, is then determined from Eq. (7), and the total level density is calculated using Eq. (8).

Summary and results. In performing calculations of the level densities for deformed nuclei ^{121,122}Sn with particular angular momentum I, we have used Eqs. (1)–(10). However, the single particle energies and spins were first calculated for specified deformation. Calculations have been based on the modified harmonic oscillator potential due to Nilsson [18]. The oscillator quantum number $\hbar\omega_0$ has been assigned the value of $41A^{-\frac{1}{3}}$ MeV. The quantities μ and χ, which enter in the Nilsson potential, are taken from Ref. [18]. The state and level densities as a function of excitation energy were then evaluated as outlined above. Results of the theoretical calculations for prolate deformation ($\epsilon = 0.2$) for ^{122}Sn nucleus is plotted in Fig. 1. Their corresponding experimental values taken from Ref. [2] are also plotted for comparison. Similar calculations are carried out for the odd A nucleus, ^{121}Sn. In doing so, the statistical functions were calculated for the even A nucleus and the energy scale was shifted by an energy equivalent to that required to produce one quasiparticle [16]. The results are plotted in Fig. 2 and are compared with experimental values [2].

Although our results are in general agreement with previous experiments, a slight discrepancy at lower energies is apparent. The experimental values which are used in our analysis are relatively large compared to general excitations, according to Toft *et al.* [2]. According to them, more experimental information is needed in order to answer this question. Examination of Figs. 1 and 2 reveals that at intermediate energies, slope of the level density is slightly different. This behavior has also been reported by the Oslo group [2]. The step-like structures interpreted as a signature of neutron pair breaking accordance with the finding in $^{116-119}$Sn nuclei [5,19].

We have also extracted the entropy for ^{121,122}Sn nuclei, with the microscopic theory using known values of the single-particle orbital and their corresponding magnetic spin quantum numbers. This we have done from additive property of entropy Eq. (7) with

$$S_n = 2\sum_k \mathrm{Ln}\left[1 + \exp\left(-\beta E_k^n\right)\right] + 2\beta \sum_k \frac{E_k^n}{1 + \exp\left(\beta E_k^n\right)} \tag{11}$$

and similar relation for S_p. The deduced entropies for the case of ^{121}Sn and ^{122}Sn nuclei as a function of nuclear temperature T, up to 1.0 MeV are plotted in Fig. 3. Our results show the entropy does not increase smoothly as expected on the basis of

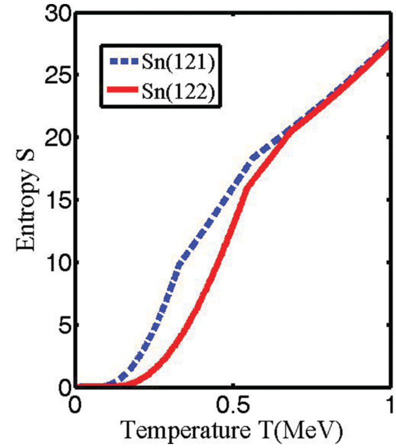

FIG. 3. (Color online) Relation between the entropy and nuclear temperature for ^{121,122}Sn.

macroscopic theory [3]. Instead of it, there is step-like structure at the critical temperature reflecting the angular momenta of the shell model orbital near the Fermi energy. It is interesting to note that at temperatures below critical temperature, the entropy of odd A (^{121}Sn) nucleus is much larger than even A (^{122}Sn) nucleus. This indicates significant role of neutrons on system entropy. We have examined this finding in more detail as follow: If we denote the neutron and proton entropy excess as

$$\Delta S_n = S_n(^{121}\mathrm{Sn}) - S_n(^{122}\mathrm{Sn}), \quad (12)$$

$$\Delta S_p = S_p(^{121}\mathrm{Sn}) - S_p(^{122}\mathrm{Sn}), \quad (13)$$

respectively, and the entropy excess ratio as $R_i(i = p, n)$ is

$$R_i = \frac{\Delta S_i}{\Delta S}, \quad (14)$$

where ΔS is the total entropy excess. In Fig. 4 we have shown the evaluated entropy excess ratio for proton and neutron systems at temperatures below critical temperature, T_c. Examination of this figure reveals that the major contribution to the total entropy at low temperatures comes mainly from the neutrons. The protons make rather small contributions to the total entropy.

In Summary, we have shown that the microscopic model describes well the observed level densities and entropies. Similar conclusions have been reported by S. Goriely *et al.*, using the modified combinatorial method [20]. In addition, we have shown a clear evidence of a phase transition in a finite system for the quenching of pairing correlations as a whole. The mechanism of breaking cooper pairs is believed to be induced by Coriolis forces tending to align single-particle angular momenta along the nuclear rotation axis. The critical temperatures of the pair-breaking process are found at $T_c \sim 0.45$ MeV and $T_c \sim 0.61$ MeV for ^{121}Sn and ^{122}Sn, respectively.

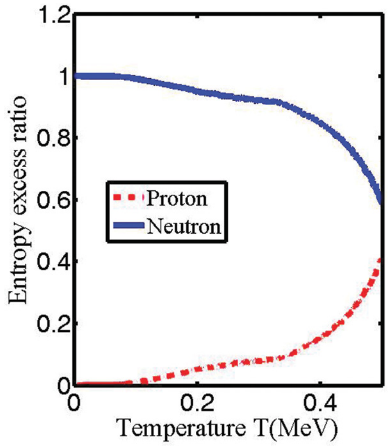

FIG. 4. (Color online) Entropy excess ratio for proton and neutron as a function of nuclear temperature.

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