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Hybrid quark-diquark baryon model

Giuseppe Galatà

INFN and Università di Genova, via Dodecaneso 33, 16146 Genova, Italy and Instituto de Ciencias Nucleares, Universidad Nacional Autonoma de Mexico, Apdo. Postal 70-543 04510 Mexico, DF, Mexico

Elena Santopinto*

INFN, via Dodecaneso 33, 16146 Genova, Italy

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A simple hybrid quark-diquark model for the baryons is constructed as a partial solution to the well-known "missing resonances" problem. In this model, the quark-diquark approach is merged with the usual constituent three-quark model. The underlying idea is that the quark-diquark approach describes the excited states while the three-quark model the ground states. The spectrum is calculated through a mass formula, a generalization of the Gürsey-Radicati formula, built to reproduce the rotational and vibrational Regge trajectories. Using the quark-diquark scheme, we were also able to describe the spin-flavor degrees of freedom in the framework of an algebraic model. Moreover, we have constructed a complete classifications of the possible quark-diquark states only based on group theory and thus it can be useful both for other model builders and for experimentalists.

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I. INTRODUCTION

Since the introduction of the quarks, the baryons have always been thought of as made up of three constituent confined quarks. The light baryons, in particular, have been ordered according to the approximate $SU(3)_f$ symmetry, which requires that the baryons belong to the multiplets $[1]_A \oplus [8]_M \oplus [8]_M \oplus [10]_S$. However, when we consider the spatially excited resonances, many more states are predicted than observed; on the other hand, states with certain quantum numbers appear in the spectrum at excitation energies much lower than predicted [1]. Considering only the nonstrange sector up to an excitation energy of 2.41 GeV, on average about 45 N states are predicted, but only 12 have been established (four- or three-star) and seven are tentative (two- or one-star) [1]. This is the so-called "missing resonances" problem. One possible solution to this problem is to describe two correlated quarks inside the baryons by means of the diquark effective degree of freedom. In this case the number of states predicted is considerably lower. The concept of the diquark was first proposed by Gell-Mann [2] and soon afterward constituent quark-diquark models for the baryons were proposed by Ida and Kobayashi [3] and Lichtenberg and Tassie [4]. More recently, several studies, ranging from one-gluon exchange models to lattice QCD calculations, have investigated the possibility of diquark correlations and found that they are indeed attractive (see for example [5–9]). In this article we construct all the allowed states in the framework of the constituent quark-diquark model and we try to assign all known light baryons (with masses smaller than ≈2 GeV) to the appropriate multiplet. If we think of the quark-diquark system as a stringlike object with an $o(4)$ dynamical symmetry analogous to the quark-antiquark mesons [10,11], we can, moreover, write a simple mass formula constructed with the aim of reproducing both rotational and vibrational Regge trajectories.

II. A QUARK-DIQUARK MODEL FOR BARYONS

In this model, we hypothesize that the baryons are a bound state of two elements, a constituent quark and a constituent diquark. We regard the diquark as two correlated quarks with no internal spatial excitations, or at least we hypothesize that their internal spatial excitations will be of higher energy than the scale of masses of the resonances we will consider, i.e., light resonances up to 2 GeV masses. Calculations in a rainbow-ladder DSE model [7,8,12] have now confirmed that the first spatially excited diquark, the vector diquark, has a mass much higher than the ground states, the scalar, and the axial-vector diquarks. Diquarks are made up of two identical fermions and so they have to satisfy the Pauli principle. Since we consider diquarks with no spatial excitations, their color-spin-flavor wave functions must be antisymmetric. This limits the possible color-spin-flavor representations to

$$\text{color in } [\bar{3}] \text{ (AS)}, \ \text{spin-flavor in } [21]_{sf} \text{ (S)}, \tag{1a}$$

$$\text{color in } [6] \text{ (S)}, \ \text{spin-flavor in } [15]_{sf} \text{ (AS)}. \tag{1b}$$

The decomposition of these $SU_{sf}(6)$ representations in terms of $SU(3)_f \otimes SU(2)_s$ is (in the notation [flavor repr., spin])

$$[21]_{sf} = [\bar{3}, 0] \oplus [6, 1], \tag{2a}$$

$$[15]_{sf} = [\bar{3}, 1] \oplus [6, 0]. \tag{2b}$$

Since the baryons must be colorless, we can allow only the diquark states in color $[\bar{3}]_c$:

$$|[\bar{3}]_c, [\bar{3}]_f, 0\rangle, \quad |[\bar{3}]_c, [6]_f, 1\rangle. \tag{3}$$

The first of the above states is the scalar (or good) diquark; the second is the axial-vector (or bad) diquark. In the following, we will represent scalar diquarks by their constituent quarks (denoted by s if strange, n otherwise) in a square bracket,

*Corresponding author: santopinto@ge.infn.it

while axial-vector diquarks are indicated in a brace bracket. This choice is not random, because the explicit expression of diquarks is the commutator of the constituent quarks for the scalar diquarks and the anticommutator for the axial-vector ones.

III. BARYONS AND THE PAULI PRINCIPLE

The Pauli principle implies that the baryons must be antisymmetric to exchange each couple of quarks. First, we describe the application of this principle to the baryons in the three-quark model, in order, then, to underline the differences from the quark-diquark model.

In the three-quark model we can have the spin-flavor states

$$[6] \otimes [6] \otimes [6] = [56]_S \oplus [70]_M \oplus [70]_M \oplus [20]_A, \quad (4)$$

where the subscripts indicate the symmetry of the state. These multiplets can be decomposed in terms of $SU(3)_f \otimes SU(2)_s$ as: $[56]_S = [10, 3/2] \oplus [8, 1/2]$, $[70]_M = [10, 1/2] \oplus [8, 3/2] \oplus [8, 1/2] \oplus [1, 1/2]$, $[20]_A = [1, 3/2] \oplus [8, 1/2]$.

Since we have two different relative angular momenta, we can have symmetric, mixed and antisymmetric spatial parts, independently from the spatial model adopted.

In order to obtain an antisymmetric baryon, we have to combine the spin-flavor-spatial part, with the antisymmetric color part. Thus, we need a symmetric spin-flavor-spatial part which can be obtained only through the combinations reported on the left side of Table I.

In the quark-diquark model we can have only the spin-flavor states

$$[21] \otimes [6] = [56]_S \oplus [70]_M. \quad (5)$$

Since, in the quark-diquark model, we freeze one spatial degree of freedom, thus setting one of the two relative angular momenta to zero and letting the other vary, we can have only symmetric (if the relative orbital angular momentum L is even) or mixed (if L is odd) spatial parts. On the right side of Table I, we report the allowed spin-flavor-space combinations. Hence the sequence of states would be

$$(SU(6)_{sf}, L^P) = ([56], 0^+), ([70], 1^-), ([56], 2^+) \quad (6)$$

and so on.

It is important to underline the absence of the [20] spin-flavor multiplet, which features the state with the unique quantum numbers [1, 3/2] of flavor and spin. In the standard three-quark model, the resonance present in this multiplet would be allowed to have quantum numbers $L^P = 1^+$ [13,14], and thus would violate the sequence permitted [Eq. (6)] in the quark-diquark scheme. Thus the presence or the absence of a flavor singlet with $L^P = 1^+$ (i.e., a Λ with positive parity) is an unambiguous signature of the three-quark or the quark-diquark model, respectively.

TABLE I. Allowed spin-flavor and spatial combinations in the three-quark model (left) and in the quark-diquark model (right).

Three-quark baryons		Quark-diquark baryons	
spin-flavor	space	spin-flavor	space
$[56]_S$	S	$[56]_S$	S
$[70]_M$	M	$[70]_M$	M
$[20]_A$	A		

IV. DIFFERENT TYPES OF DIQUARK MODELS

In the present paper we will consider several ways to implement the diquarks inside the baryons in a not completely broken $SU(3)$ scheme; the completely broken scheme has already been studied by one of the authors [13]. Thus, this is the only other possibility that remains to be investigated systematically and will be the main subject of this paper. We think of the baryon as being composed of a constituent quark and a constituent diquark. We have already described the possible spin and flavor quantum numbers and the symmetry properties of the diquark alone and of the baryon in the quark-diquark scheme in detail in Secs. II and III. In a quark-diquark model, we consider every baryon as a bound state of a quark (strange or nonstrange) and a diquark (scalar or axial-vector, with all the possible strangenesses). In a first and elementary approach, which we will henceforth call the "pure" quark-diquark model, we discard the possibility of a superposition of the two types of diquarks in some baryons. Thus, for example, only scalar diquarks are considered for the nucleon and all the particles in the same flavor octet. All the known baryons are classified according to their quantum numbers and their diquark content. While all the currently known resonances fit well into this scheme, the main problem of this model is that it still predicts several missing resonances. In particular, as can be seen by applying Table II in the $L^P = 0^+$ case, we expect the presence of two ground state

TABLE II. General classification of the baryon multiplets in the quark-diquark model. m is an integer $\geqslant 0$; S_D is the diquark spin (0 is the scalar diquark, 1 the axial-vector diquark). For $J = \frac{1}{2}$ the states $[8, \frac{3}{2}]$ with $L^P = (2m-1)^-$ and $[10, \frac{3}{2}]$ with $L^P = (2m)^+$ are not allowed. The energy splittings and the actual ordering of the various multiplets will obviously depend on the details of the particular model used.

J	L^P	S_D	multiplets ($[SU(3)_f$, Spin])
$2m + \frac{1}{2}$	$(2m)^+$	0	$[8, \frac{1}{2}]$
	$(2m+1)^-$	0	$[8, \frac{1}{2}]$,$[1, \frac{1}{2}]$
	$(2m)^+$	1	$[8, \frac{1}{2}]$
	$(2m+1)^-$	1	$[8, \frac{1}{2}]$,$[10, \frac{1}{2}]$
	$(2m-1)^-$	1	$[8, \frac{3}{2}]$
	$(2m)^+$	1	$[10, \frac{3}{2}]$
	$(2m+1)^-$	1	$[8, \frac{3}{2}]$
	$(2m+2)^+$	1	$[10, \frac{3}{2}]$
$2m + \frac{3}{2}$	$(2m+1)^-$	0	$[8, \frac{1}{2}]$,$[1, \frac{1}{2}]$
	$(2m+2)^+$	0	$[8, \frac{1}{2}]$
	$(2m+1)^-$	1	$[8, \frac{1}{2}]$,$[10, \frac{1}{2}]$
	$(2m+2)^+$	1	$[8, \frac{1}{2}]$
	$(2m)^+$	1	$[10, \frac{3}{2}]$
	$(2m+1)^-$	1	$[8, \frac{3}{2}]$
	$(2m+2)^+$	1	$[10, \frac{3}{2}]$
	$(2m+3)^-$	1	$[8, \frac{3}{2}]$

$J^P = \frac{1}{2}^+$ octets, one with a scalar and the other with an axial-vector diquark. The former can be identified with the octet featuring the nucleon, but for the latter no particles with a suitable energy (estimated around 1.1–1.3 GeV for the N of the octet) are known with certainty. Anisovich *et al.* [15] have suggested the possibility that these $J^P = \frac{1}{2}^+$ octet states with axial-vector diquarks actually could overlap somewhat with the radially excited $\frac{1}{2}^+$ octet formed by $N(1440)$, $\Sigma(1660)$, and $\Lambda(1600)$. In this hypothesis, a double-pole structure of the partial amplitude in the regions of masses of these resonances should be present. Recently, Morsch and Zupranski [16] and Arndt *et al.* [17] have seen traces of a second Roper resonance in the energy range 1.35–1.40 GeV that could agree with our predictions. A discussion of the Roper resonance and the possibility of this controversial second pole can be found in Ref. [18].

Alternatively, we could modify the "pure" quark-diquark model in a way that does not require the presence of a second $\frac{1}{2}^+$ octet. In this case, the double pole structure discussed above would be only a different manifestation of the same resonance in different channels. Anisovich [15] suggested that the two kinds of diquark could be degenerate. In this case, the missing octet would simply collapse to the energy of the nucleon octet and would become identical with it. However, the large majority of studies have found the axial-vector diquark to be about 200 MeV heavier than the scalar one (see Table VIII) and therefore very far from a degenerate condition. Another variation, here called the "mixed" quark-diquark model, can be introduced, in which the nucleon results from the superposition of the scalar diquark and of the axial-vector diquark. The name "mixed" has been chosen to underline the fact that the nucleon is now a superposition of both types of nondegenerate diquark. In the hypothesis that the weight of the two diquarks inside the nucleon (and in the rest of its octet) is approximately 1/2, the two ground state octets would have the same composition and thus would again overlap and solve the problem of the second "embarrassing" octet. While both of these modifications to the "pure" quark-diquark model would be able to solve the problem of the missing octet, they are, as yet, only theoretical speculations without solid evidence. Recent calculations by Roberts *et al.* [12] show that the effective model calculations based on DSE equations could confirm the superposition of the two types of diquark, which is what we have called the "mixed" point of view.

In the present paper we have also considered a further possibility, which we have called the "hybrid" model. Theoretical calculations [19,20] could support the supposition that the diquarks do not form inside the ground-state, nonexcited baryons: these baryons would be compressed to provide the usual three-quark $SU(6)$-symmetry structure, while the excited states would be spatially separated to give rise to the quark-diquark string-like structure, as shown in Fig. 1. In this case we would have only one ground-state three-quark octet, and the quark-diquark scheme would be valid only starting from the spatially excited resonances. In brief, this model results from the deformation of a three-quark model for the ground states into a quark-diquark model for the excited states; hence the name "hybrid". Santopinto has already hinted at this idea in Ref. [13]. In Tables III– VI we present a classification of all the known low-energy resonances according to the "pure" (valid if the detection of the second Roper resonance is considered credible), "hybrid" and "mixed" models.

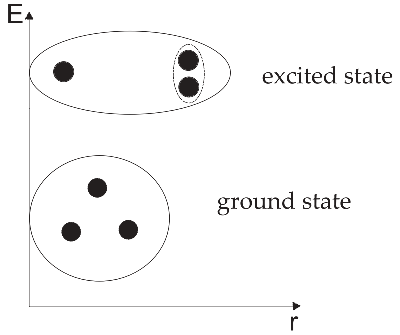

FIG. 1. Ground three quarks and excited quark-diquark baryons in the "hybrid" model.

V. THE MASS FORMULA

In order to apply the models described in Sec. IV, we need to develop a mass formula able to reproduce the baryon spectrum. The basic elements on which we choose to construct our mass formula are the Regge trajectories, which have become an essential feature of hadronic phenomenology, and we follow the methodology developed in the algebraic models for mesons and baryons [10,11,21–24] . Actually, we need two mass formulas, one to be applied to the "pure" and "mixed" models and the other to the "hybrid" model. In the former case, we write Eq. (7), which implements the rotational and vibrational Regge trajectories for a string-like object and presents explicit dependence on the masses of the diquarks M_{diq} and the isolated quark M_q. The mass formula is completed by spin $SU(2)$, flavor $SU(3)$, and spin-flavor $SU(6)$ quark-diquark interaction terms. These are inspired by similar terms introduced in potential models [25,26], but here they are used in the framework of algebraic models. Still, these latter terms account only for fine-tuning corrections:

$$
\begin{aligned}
M^2 &= b\,L + n\,\nu + (M_q + M_{\text{diq}})^2 \\
&\quad + d\,J + e\,I(I+1) + \frac{c}{M_q M_{\text{diq}}}\mathbf{S}_q\cdot\mathbf{S}_{\text{diq}} \\
&\quad + \frac{g}{M_q M_{\text{diq}}}\mathbf{F}_q\cdot\mathbf{F}_{\text{diq}} + \frac{h}{M_q M_{\text{diq}}}\mathbf{G}_q\cdot\mathbf{G}_{\text{diq}} + \Lambda, \qquad (7)
\end{aligned}
$$

where

$$
\begin{aligned}
M_q &= M_n + N_s\,\Delta M_s, \\
M_{\text{diq}} &= M_{[n,n]} + N_{[n,s]}\,\Delta M_{[n,s]} + N_{\{n,n\}}\,\Delta M_{\{n,n\}} \\
&\quad + N_{\{n,s\}}\,\Delta M_{\{n,s\}} + N_{\{s,s\}}\,\Delta M_{\{s,s\}}, \\
\mathbf{S}_q\cdot\mathbf{S}_{\text{diq}} &= \frac{S(S+1) - 3/4 - S_{\text{diq}}(S_{\text{diq}}+1)}{2}, \\
\mathbf{F}_q\cdot\mathbf{F}_{\text{diq}} &= \frac{C_2(SU(3)_f) - 4/3 - C_2(SU(3)_f)_{\text{diq}}}{2}, \\
\mathbf{G}_q\cdot\mathbf{G}_{\text{diq}} &= \frac{C_2(SU(6)_{sf}) - 115/12}{2}.
\end{aligned}
$$

TABLE III. Quark-diquark model assignments for some of the known baryons with no vibrational excitations. Assignments for several states are merely educated guesses. Resonances marked with "no" are forbidden in the quark-diquark scheme by the rules summed up in Sec. VI. Resonances labeled "missing" are the so-called missing resonances. As discussed in Sec. IV, in the "hybrid" model the $L = 0$ resonances are considered in a three-quark scheme; thus they have no diquark spin (S_{diq}) and only one octet is present. These corrections to the "pure" quark-diquark model are shown in *italics* in the Table. The resonance $\Omega(2250)$ has been inserted in the same multiplet as $\Delta(1910)$ even though its quantum numbers are still unknown.

	octet				decuplet				singlet
$J^P, L, S, S_{\text{diq}}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{3}{2}$	1	$\frac{1}{2}$	0	0
$\frac{1}{2}^+, 0, \frac{1}{2}, \mathit{0}$	$N(939)$	$\Sigma(1189)$	$\Lambda(1116)$	$\Xi(1318)$	no	no	no	no	no
$\frac{1}{2}^+, 0, \frac{1}{2}, 1$	*missing*	*missing*	*missing*	*missing*	*no*	*no*	*no*	*no*	*no*
$\frac{1}{2}^+, 2, \frac{3}{2}, 1$	no	no	no	no	$\Delta(1910)$	missing	missing	$\Omega(2250)$ (?)	no
$\frac{1}{2}^-, 1, \frac{1}{2}, 0$	$N(1535)$	missing	$\Lambda(1670)$	missing	no	no	no	no	$\Lambda(1405)$
$\frac{1}{2}^-, 1, \frac{1}{2}, 1$	missing	$\Sigma(1750)$	missing	missing	$\Delta(1620)$	missing	missing	missing	no
$\frac{1}{2}^-, 1, \frac{3}{2}, 1$	$N(1650)$	missing	$\Lambda(1800)$	missing	no	no	no	no	no
$\frac{3}{2}^+, 2, \frac{1}{2}, 0$	$N(1720)$	missing	$\Lambda(1890)$	missing	no	no	no	no	no
$\frac{3}{2}^+, 0, \frac{3}{2}, \mathit{1}$	no	no	no	no	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$	no
$\frac{3}{2}^+, 2, \frac{1}{2}, 1$	missing	missing	missing	missing	no	no	no	no	no
$\frac{3}{2}^+, 2, \frac{3}{2}, 1$	no	no	no	no	$\Delta(1920)$	missing	missing	missing	no
$\frac{3}{2}^-, 1, \frac{1}{2}, 0$	$N(1520)$	$\Sigma(1670)$	$\Lambda(1690)$	$\Xi(1820)$	no	no	no	no	$\Lambda(1520)$
$\frac{3}{2}^-, 1, \frac{1}{2}, 1$	missing	missing	missing	missing	$\Delta(1700)$	missing	missing	missing	no
$\frac{3}{2}^-, 1, \frac{3}{2}, 1$	$N(1700)$	$\Sigma(1940)$	missing	missing	no	no	no	no	no
$\frac{3}{2}^-, 3, \frac{3}{2}, 1$	missing	missing	missing	missing	no	no	no	no	no
$\frac{5}{2}^+, 2, \frac{1}{2}, 0$	$N(1680)$	$\Sigma(1915)$	$\Lambda(1820)$	$\Xi(2030)$	no	no	no	no	no
$\frac{5}{2}^+, 2, \frac{1}{2}, 1$	missing	missing	$\Lambda(2110)$	missing	no	no	no	no	no
$\frac{5}{2}^+, 2, \frac{3}{2}, 1$	no	no	no	no	$\Delta(1905)$	missing	missing	missing	no
$\frac{5}{2}^+, 4, \frac{3}{2}, 1$	no	no	no	no	missing	missing	missing	missing	no
$\frac{5}{2}^-, 3, \frac{1}{2}, 0$	missing	missing	missing	missing	no	no	no	no	missing
$\frac{5}{2}^-, 3, \frac{1}{2}, 1$	missing	missing	missing	missing	$\Delta(1930)$	missing	missing	missing	no
$\frac{5}{2}^-, 1, \frac{3}{2}, 1$	$N(1675)$	$\Sigma(1775)$	$\Lambda(1830)$	missing	no	no	no	no	no
$\frac{5}{2}^-, 3, \frac{3}{2}, 1$	missing	missing	missing	missing	no	no	no	no	no

Λ is an overall constant taken to be equal to -0.55 GeV^2; $M_n = 0.3331$ GeV is the mass of the nonstrange quark as obtained in Ref. [21]; $M_{[n,n]}$ is the mass of the nonstrange scalar diquark $[n,n]$, N_s and ΔM_s are the number of strange quarks and the mass difference between the strange quark and the nonstrange one; $N_{[n,s]}$, $N_{\{n,n\}}$, $N_{\{n,s\}}$, and $N_{\{s,s\}}$ are the number of strange scalar, nonstrange axial-vector, strange axial-vector, and double strange axial-vector diquarks, respectively; $\Delta M_{[n,s]}$, $\Delta M_{\{n,n\}}$, $\Delta M_{\{n,s\}}$ and $\Delta M_{\{s,s\}}$ are the mass difference between the strange scalar, nonstrange axial-vector, strange axial-vector, double strange axial-vector diquarks and the nonstrange scalar diquark $[n,n]$, respectively; $C_2(SU(3)_f)$ and $C_2(SU(6)_{sf})$ are the quadratic Casimirs of flavor SU(3)$_f$ and spin-flavor SU(6)$_{sf}$, respectively, referred to the entire baryon, while $C_2(SU(3)_f)_{\text{diq}}$ refers to the diquark only; L is the relative orbital angular momentum; I the total isospin; S the total spin and S_{diq} the diquark's spin; J the total angular momentum, and ν the vibrational quantum number. This mass formula is suitable for both the "pure" and the "mixed" models, since in the latter only the input numbers $N_{[n,s]}$, $N_{\{n,n\}}$ and $N_{\{n,s\}}$ would change, and only for the ground state octets.

Regarding the "hybrid" model, we need to modify our mass formula to be able to reflect a three-quark structure for all the ground state baryons. Thus, while the quark-diquark part remains largely untouched, we need to insert new terms of quark-quark interaction for the three-quark part. The resulting mass formula is written in Eq. (8):

$$\begin{aligned} M^2 = {} & bL + n\nu + \delta_{L,0}\delta_{\nu,0}\Bigg(\sum_{i<j}\frac{c'\mathbf{S}_i\cdot\mathbf{S}_j + g'\mathbf{F}_i\cdot\mathbf{F}_j}{M_iM_j} \\ & + d'J + e'I(I+1) + M_{3q}^2\Bigg) \\ & + (1-\delta_{L,0}\delta_{\nu,0})\Bigg(\Lambda_1 + \frac{1}{M_qM_{\text{diq}}}(g\mathbf{F}_q\cdot\mathbf{F}_{\text{diq}} \\ & + h\mathbf{G}_q\cdot\mathbf{G}_{\text{diq}} + e\mathbf{I}_q\cdot\mathbf{I}_{\text{diq}}) + (M_q+M_{\text{diq}})^2\Bigg), \end{aligned} \tag{8}$$

TABLE IV. Quantum numbers of the three- or four-star resonances with orbital angular momentum and parity $L^P = 0^+$ in the quark-diquark model. Section (a) lists the resonances used for the fit with our mass formula [Eq. (8)]; section (b) shows the resonances we could not use because either we were not sure of their diquark content—in which case our assignment is marked with a question mark—or they are not well established yet. For the ground-state baryons, the composition is given for the "pure", "mixed", and "hybrid" models. All the masses are in GeV. In the last three columns the theoretical predictions are reported for each model.

Resonances	J	S	composition	$SU(3)_f$	$SU(6)_{sf}$	ν	M(exp)	M(pure)	M(mixed)	M(hybrid)
					(a)					
$N(939)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]n$	8	56	0	0.939 ± 0.005	0.924	–	–
$N(939)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]n + \{n,n\}n$	8	56	0	0.939 ± 0.005	–	0.940	–
$N(939)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	nnn	8	56	0	0.939 ± 0.005	–	–	0.943
$\Sigma(1189)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,s]n$	8	56	0	1.189 ± 0.005	1.189	–	–
$\Sigma(1189)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	nns	8	56	0	1.189 ± 0.005	–	–	1.182
$\Lambda(1116)P_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	nns	8	56	0	1.116 ± 0.005	–	–	1.113
$\Xi(1318)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,s]s$	8	56	0	1.315 ± 0.005	1.349	–	–
$\Xi(1318)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	nss	8	56	0	1.315 ± 0.005	–	–	1.320
$N^*(?)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$\{n,n\}n$	8	56	0	1.390 ± 0.020[a]	1.300	–	–
$\Delta(1232)P_{33}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	0	$1.231 - 1.233$	1.229	1.228	–
$\Delta(1232)P_{33}$	$\frac{3}{2}$	$\frac{3}{2}$	nnn	10	56	0	$1.231 - 1.233$	–	–	1.238
$\Sigma(1385)P_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	nns	10	56	0	1.383 ± 0.005	–	–	1.375
$\Xi(1530)P_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	nss	10	56	0	1.532 ± 0.005	–	–	1.524
$\Omega(1672)$	$\frac{3}{2}$	$\frac{3}{2}$	$\{s,s\}s$	10	56	0	1.672 ± 0.005	1.673	1.675	–
$\Omega(1672)$	$\frac{3}{2}$	$\frac{3}{2}$	sss	10	56	0	1.672 ± 0.005	–	–	1.681
$N(1440)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]n$	8	56	1	$1.420 - 1.470$	1.446	1.478	1.455
$\Sigma(1660)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,s]n$	8	56	1	$1.630 - 1.690$	1.629	1.697	1.683
$N(1710)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$\{n,n\}n$	8	56	1	$1.680 - 1.740$	1.711	1.599	1.654
$\Lambda(1810)P_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$\{n,s\}n$	8	56	1	$1.750 - 1.850$	1.864	1.774	1.820
$\Delta(1600)P_{33}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	1	$1.550 - 1.700$	1.658	1.730	1.658
					(b)					
$\Sigma(1189)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,s]n + \{n,s\}n$ (?)	8	56	0	1.189 ± 0.005	–	1.238	–
$\Lambda(1116)P_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]s$ (?)	8	56	0	1.116 ± 0.005	1.103	–	–
$\Lambda(1116)P_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]s + \{n,s\}n$ (?)	8	56	0	1.116 ± 0.005	–	1.202	–
$\Xi(1318)P_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,s]s + \{s,s\}n$ (?)	8	56	0	1.315 ± 0.005	–	1.393	–
$\Sigma(1385)P_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,n\}s$ (?)	10	56	0	1.383 ± 0.005	1.395	1.419	–
$\Xi(1530)P_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{s,s\}n$ (?)	10	56	0	1.532 ± 0.005	1.531	1.496	–
$\Lambda(1600)P_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]s$ (?)	8	56	1	$1.560 - 1.700$	1.567	1.637	1.580

[a]The datum is taken from Ref. [16].

where

$$M_{3q} = 3M_n + N_s \Delta M_s,$$

$$\mathbf{I}_q \cdot \mathbf{I}_{\text{diq}} = \frac{I(I+1) - 3/4(1-N_s) - I_{\text{diq}}(I_{\text{diq}}+1)}{2},$$

$$\sum_{i<j} \frac{c'\mathbf{S}_i \cdot \mathbf{S}_j}{M_i M_j} \cong \frac{1}{M_n^2}\left(1 - \frac{2}{3}N_s \frac{\Delta M_s}{M_n}\right)\frac{c'}{2}\left(S(S+1) - \frac{9}{4}\right),$$

$$\sum_{i<j} \frac{g'\mathbf{F}_i \cdot \mathbf{F}_j}{M_i M_j} \cong \frac{1}{M_n^2}\left(1 - \frac{2}{3}N_s \frac{\Delta M_s}{M_n}\right)\frac{g'}{2}(C_2(SU(3)_f) - 4),$$

$\Lambda_1 = -0.57$ GeV^2, $M_n = 0.3331$ GeV, M_q, M_{diq}, and all the other terms are the same as defined above.

VI. QUANTUM NUMBERS

In order to use the mass formulas [Eqs. (7) and (8)] it is necessary to assign to every baryon its quantum numbers, in particular those, like L and S, not determined by the experiments. For this purpose, we consider only well-known baryons, namely the three- and four-star baryons. We classify the light baryons by following three guidelines. First of all, we must obviously respect the quantum numbers that can be measured experimentally (like J, P, etc.). Secondly, we must respect the constraint related to the diquark spin-flavor states:

$$\begin{aligned}[21] \otimes [6] &= ([\bar{3}, 0] \oplus [6, 1]) \otimes \left[3, \tfrac{1}{2}\right] \\ &= \left(\left[1, \tfrac{1}{2}\right] \oplus \left[8, \tfrac{1}{2}\right]\right) \oplus \left(\left[8, \tfrac{3}{2}\right] \oplus \left[8, \tfrac{1}{2}\right]\right. \\ &\quad \left.\oplus \left[10, \tfrac{3}{2}\right] \oplus \left[10, \tfrac{1}{2}\right]\right). \end{aligned} \tag{9}$$

TABLE V. Same as Table IV, but for baryons with $L^P = 1^-$. All the masses are in GeV.

Resonances	J	S	composition	$SU(3)_f$	$SU(6)_{sf}$	ν	M(exp)	M(pure)	M(mixed)	M(hybrid)
					(a)					
$N(1535)S_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]n$	8	70	0	1.525 – 1.545	1.533	1.516	1.520
$N(1520)D_{13}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,n]n$	8	70	0	1.515 – 1.525	1.535	1.517	1.520
$\Sigma(1670)D_{13}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,s]n$	8	70	0	1.665 – 1.685	1.684	1.699	1.698
$\Xi(1820)D_{13}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,s]s$	8	70	0	1.818 – 1.828	1.773	1.819	1.819
$N(1650)S_{11}$	$\frac{1}{2}$	$\frac{3}{2}$	$\{n,n\}n$	8	70	0	1.645 – 1.670	1.676	1.675	1.676
$\Lambda(1800)S_{01}$	$\frac{1}{2}$	$\frac{3}{2}$	$\{n,s\}n$	8	70	0	1.720 – 1.850	1.828	1.819	1.820
$N(1700)D_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,n\}n$	8	70	0	1.650 – 1.750	1.677	1.676	1.676
$N(1675)D_{15}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,n\}n$	8	70	0	1.670 – 1.680	1.679	1.677	1.676
$\Lambda(1830)D_{05}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,s\}n$	8	70	0	1.810 – 1.830	1.831	1.821	1.820
$\Delta(1620)S_{31}$	$\frac{1}{2}$	$\frac{1}{2}$	$\{n,n\}n$	10	70	0	1.600 – 1.660	1.784	1.661	1.680
$\Delta(1700)D_{33}$	$\frac{3}{2}$	$\frac{1}{2}$	$\{n,n\}n$	10	70	0	1.670 – 1.750	1.785	1.662	1.680
					(b)					
$\Lambda(1405)S_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]s$ (?)	1	70	0	1.402 – 1.410	1.563	1.566	1.380
$\Lambda(1520)D_{03}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,n]s + [n,s]n$ (?)	1	70	0	1.520 ± 0.005	1.606	1.604	1.505
$\Lambda(1670)S_{01}$	$\frac{1}{2}$	$\frac{1}{2}$	$[n,n]s + [n,s]n$ (?)	8	70	0	1.660 – 1.680	1.652	1.659	1.699
$\Lambda(1690)D_{03}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,n]s + [n,s]n$ (?)	8	70	0	1.685 – 1.695	1.654	1.660	1.699
$\Sigma(1750)S_{11}$	$\frac{1}{2}$	$\frac{1}{2}$	$\{n,s\}n$ (?)	8	70	0	1.730 – 1.800	1.883	1.759	1.714
$\Sigma(1940)D_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,s\}n$ (?)	8	70	0	1.900 – 1.950	1.818	1.818	1.714
$\Sigma(1775)D_{15}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,s\}n$ (?)	8	70	0	1.770 – 1.780	1.820	1.819	1.714

As we can be seen, only the baryons in a flavor octet and spin $\frac{1}{2}$ can be made up of either the scalar or the axial-vector diquark, while the baryons in a flavor singlet can be composed of only scalar diquarks, and those in a flavor decuplet of only axial-vector diquarks. Finally, the spin-flavor-space part must be symmetric. As we have seen in Sec. III, the

TABLE VI. Same as Table IV, but for baryons with $L^P = 2^+$. All the masses are in GeV.

Resonances	J	S	composition	$SU(3)_f$	$SU(6)_{sf}$	ν	M(exp)	M(pure)	M(mixed)	M(hybrid)
					(a)					
$N(1720)P_{13}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,n]n$	8	56	0	1.700 – 1.750	1.700	1.689	1.693
$N(1680)F_{15}$	$\frac{5}{2}$	$\frac{1}{2}$	$[n,n]n$	8	56	0	1.680 – 1.690	1.702	1.690	1.693
$\Sigma(1915)F_{15}$	$\frac{5}{2}$	$\frac{1}{2}$	$[n,s]n$	8	56	0	1.900 – 1.935	1.859	1.884	1.893
$\Lambda(2110)F_{05}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,s\}n$	8	56	0	2.090 – 2.140	2.008	2.006	2.015
$\Delta(1910)P_{31}$	$\frac{1}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	0	1.870 – 1.920	1.881	1.911	1.870
$\Omega(2250)$	$\frac{1}{2}$(?)	$\frac{3}{2}$	$\{s,s\}s$	10	56	0	2.243 – 2.261	2.196	2.224	2.252
$\Delta(1920)P_{33}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	0	1.900 – 1.970	1.882	1.912	1.870
$\Delta(1905)F_{35}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	0	1.865 – 1.915	1.884	1.912	1.870
$\Delta(1950)F_{37}$	$\frac{7}{2}$	$\frac{3}{2}$	$\{n,n\}n$	10	56	0	1.915 – 1.950	1.885	1.913	1.870
					(b)					
$\Lambda(1890)P_{03}$	$\frac{3}{2}$	$\frac{1}{2}$	$[n,n]s + [n,s]n$ (?)	8	56	0	1.850 – 1.910	1.837	1.858	1.898
$\Lambda(1820)F_{05}$	$\frac{5}{2}$	$\frac{1}{2}$	$[n,n]s$ (?)	8	56	0	1.815 – 1.825	1.805	1.830	1.802
$\Sigma(1880)P_{11}$	$\frac{1}{2}$	$\frac{3}{2}$	$\{n,n\}s$ (?)	8	56	0	1.880	1.963	2.004	1.894
$N(2000)F_{15}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,n\}n$	8	56	0	1817.7[a]	1.858	1.862	1.866
$\Sigma(2080)P_{13}$	$\frac{3}{2}$	$\frac{3}{2}$	$\{n,s\}n$ (?)	10	56	0	2.080	2.031	2.047	2.064
$\Sigma(2070)F_{15}$	$\frac{5}{2}$	$\frac{3}{2}$	$\{n,s\}n$ (?)	10	56	0	2.070	2.032	2.048	2.064
$\Sigma(2030)F_{17}$	$\frac{7}{2}$	$\frac{3}{2}$	$\{n,s\}n$ (?)	10	56	0	2.025 – 2.040	2.034	2.049	2.064

[a]The datum is taken from Ref. [17].

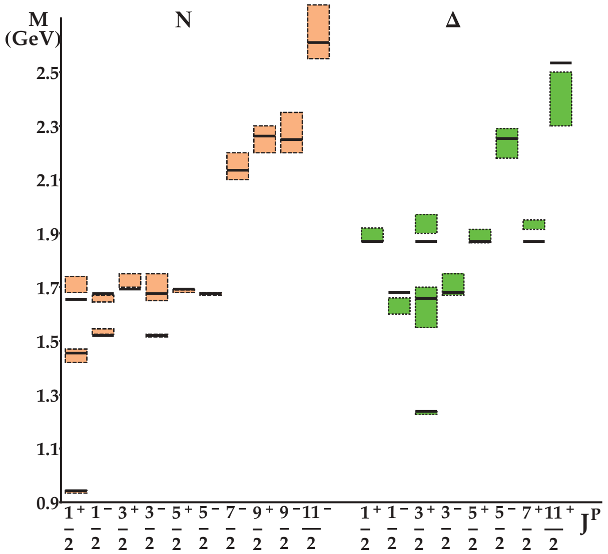

FIG. 2. (Color online) Hybrid model. Comparison between experimental data and theoretical results for nonstrange baryon resonances. The pink boxes with dashed edges and the green ones with dotted edges are the nucleons and the Δ states, respectively; the black lines are the corresponding predicted masses from the mass formula Eq. (8), corresponding to the "hybrid" model (i.e., three quarks for the ground states and quark-diquark for the excited states). For the Δ resonance with $J^P = 5/2^-$ [i.e., the $\Delta(1930)D_{35}$] we used the latest value of the mass from Ref. [17], instead of the very different PDG estimate.

consequence is that we must respect the sequence of states ([56], 0^+), ([70], 1^-), ([56], 2^+), ..., where

$$[56] = \left[10, \tfrac{3}{2}\right] \oplus \left[8, \tfrac{1}{2}\right]$$
$$[70] = \left[10, \tfrac{1}{2}\right] \oplus \left[8, \tfrac{1}{2}\right] \oplus \left[8, \tfrac{3}{2}\right] \oplus \left[1, \tfrac{1}{2}\right].$$

This means, for example, that we cannot have a flavor singlet with $L = 0$.

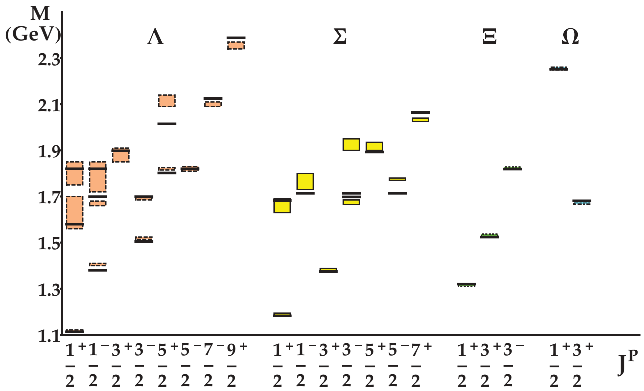

FIG. 3. (Color online) Hybrid model. Comparison between experimental data and theoretical results for strange baryon resonances. The Λ's, Ξ's, Ω's, and Σ resonances are represented, respectively, by pink boxes with dashed edges, green boxes with dotted edges, light blue boxes with dash-dotted edges, and yellow boxes with solid edges; the solid black lines are the resonances calculated with the mass formula Eq. (8), corresponding to the "hybrid" model.

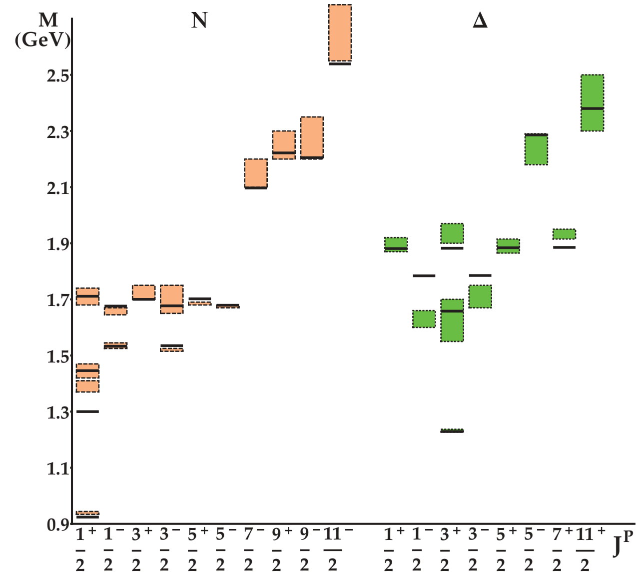

FIG. 4. (Color online) "Pure" model. Comparison between experimental data and theoretical results for nonstrange baryon resonances. Same as Fig. 2, using the mass formula Eq. (7), corresponding to the "pure" model (i.e., quark-diquark states with no diquark superposition).

In Table II we report a general classification, valid for all quantum numbers, of the baryon multiplets in the quark-diquark scheme, while in Table III we assign the known light baryons to each multiplet. The missing and the not allowed states are reported in the table. These tables are in part based on the analogous tables compiled by Bijker, Iachello, and Leviatan [24] and the Particle Data Group [1] for a three-quark model, and Selem and Wilczek [27] for a quark-diquark model.

It is important to underline that we lack a sure criterion for assigning the diquark content to the baryons (i.e., we cannot say whether a particular baryon should be made up of a scalar, an axial-vector or even a mix of the two diquarks). We found only two sure elements on which the choice can be based:

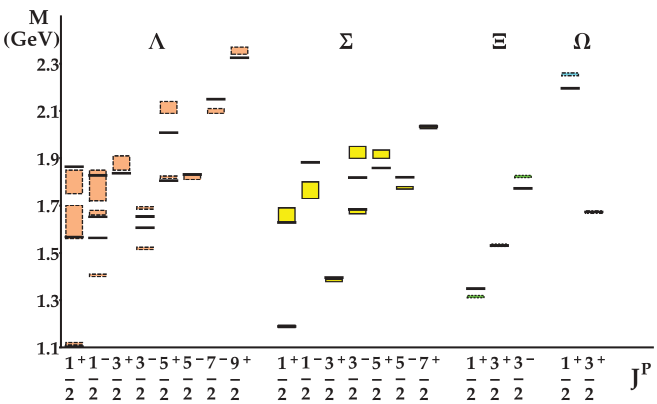

FIG. 5. (Color online) "Pure" model. Comparison between experimental data and theoretical results for strange baryon resonances. Same as Fig. 3, using the mass formula Eq. (7), corresponding to the "pure" model.

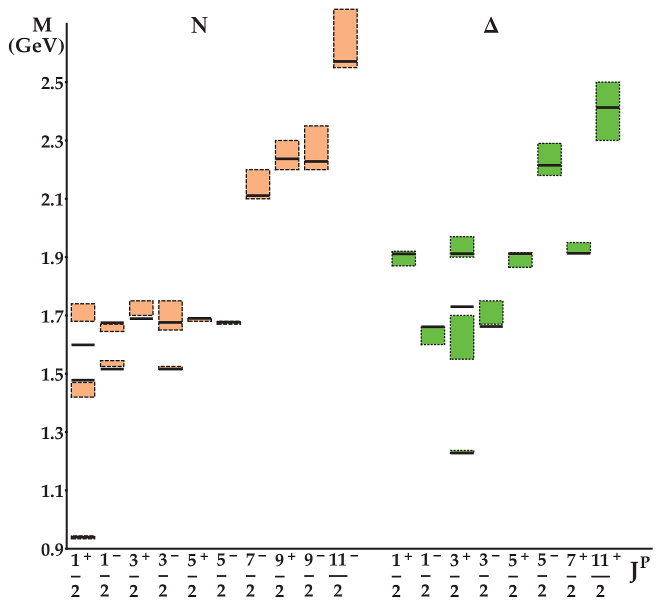

FIG. 6. (Color online) "Mixed" model. Comparison between experimental data and theoretical results for nonstrange baryon resonances. Same as Fig. 2 using the mass formula Eq. (7), corresponding to the "mixed" model (i.e., quark-diquark states with diquark superposition for the ground states).

(i) Isospin and strangeness:
We must remember that every baryon family has a definite isospin and strangeness: N has isospin $I = \frac{1}{2}$ and strangeness $\mathcal{S} = 0$; Δ has $I = \frac{3}{2}$ and $\mathcal{S} = 0$; Λ has $I = 0$ and $\mathcal{S} = -1$; Σ has $I = 1$ and $\mathcal{S} = -1$; Ξ has $I = \frac{1}{2}$ and $\mathcal{S} = -2$; Ω has $I = 0$ and $\mathcal{S} = -3$. Thus, we must combine the diquark and the quark to reproduce the isospin and strangeness of the baryon. But we can easily see that $[n, n]$ has $I = 0$ and $\mathcal{S} = 0$, $[n, s]$ has $I = \frac{1}{2}$ and $\mathcal{S} = -1$, $\{n, n\}$ has $I = 1$ and $\mathcal{S} = 0$, $\{n, s\}$ has $I = \frac{1}{2}$ and $\mathcal{S} = -1$ and $\{s, s\}$ has $I = 0$ and $\mathcal{S} = -2$. On combining the quark and the diquark together, we find the possible diquark content.

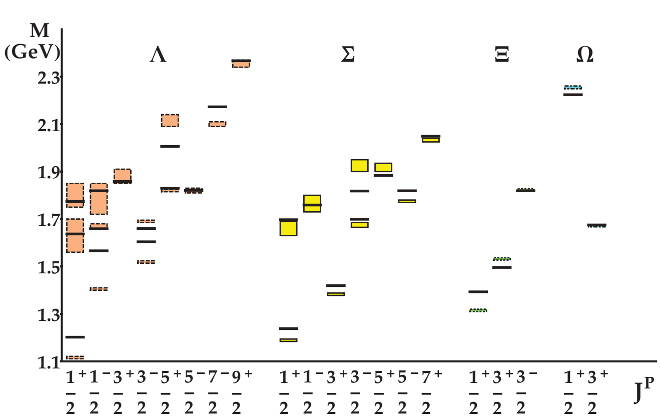

FIG. 7. (Color online) "Mixed" model. Comparison between experimental data and theoretical results for strange baryon resonances. Same as Fig. 3, using the mass formula Eq. (7), corresponding to the "mixed" model.

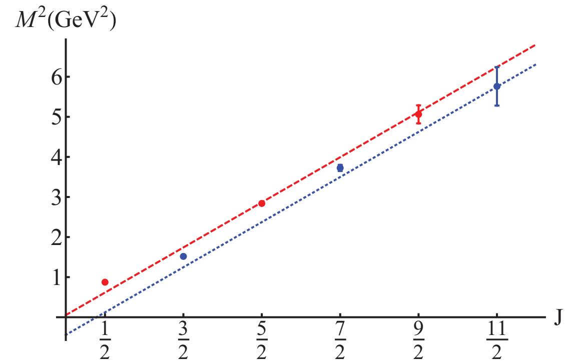

FIG. 8. (Color online) Graph of the $N(939)$ (red dashed line) and the $\Delta(1232)$ (blue dotted line) theoretical Regge trajectories, from Eq. (8), compared with the experimental data.

N can be either $[n, n]n$ or $\{n, n\}n$; Δ can be only $\{n, n\}n$; Λ can be $[n, n]s$ or $[n, s]n$ if they are in a flavor singlet, otherwise they can be $[n, n]s$, $[n, s]n$ or $\{n, s\}n$ if they belong to a flavor octet; Σ can be $[n, s]n$; $\{n, n\}s$ or $\{n, s\}n$; Ξ can be $[n, s]s$; $\{n, s\}s$ or $\{s, s\}n$, and finally Ω can be only $\{s, s\}s$.

(ii) Diquark masses:
According to all the previous studies on the diquarks (as for example Refs. [5,6,9]), we can say that the axial-vector diquark should be heavier than the scalar one. Thus, if we have two baryons with similar quantum numbers but different masses, we will assign the axial-vector diquark to the heavier one.

In this first attempt, we have chosen to assign an analogous diquark content to all the baryons that are part of the same flavor multiplet (i.e., if, for example, we establish that a baryon should have a scalar diquark, then all the other baryons of the same multiplet will have a scalar diquark). In this way, we think that all the mass differences inside a baryon multiplet should be attributed to the different strangeness of the various baryons.

We now determine the parameters of the two mass formulas [Eqs. (7) and (8)] through a fit. We have excluded from the fit the states for which their diquark content cannot be determined by applying the criterion described in Sec. VI. These states have a question mark next to their diquark content in Tables IV–VI. The results of the fit for the parameters are reported in Table VII and the predicted states are confronted to the experimental data in Figs. 2–7. In Fig. 8 a graph of two Regge trajectories is given.

VII. CRITICAL DISCUSSION OF THE RESULTS

We have constructed a complete classification of the quark-diquark states that is general and does not depend on the particular form chosen for the dynamical interaction. Thus, it may also be useful for other model builders. Since we underline the differences from the three-quark case and the signatures in favor or against one or the other model, this work could also be useful to experimentalists. The construction of the quark-diquark states is done by using only group theory techniques; it is therefore evident that the quark-diquark states are a subset of

TABLE VII. Parameters resulting from the fit for the "pure" and "mixed" models, both represented by Eq. (7), and the "hybrid" model [Eq. (8)].

Parameter	"pure"	"mixed"	"hybrid"
$M_{[n,n]}$	0.956 ± 0.005 GeV	0.898 ± 0.013 GeV	0.852 ± 0.040 GeV
ΔM_s	0.107 ± 0.015 GeV	0.156 ± 0.021 GeV	0.128 ± 0.003 GeV
$\Delta M_{[n,s]}$	0.197 ± 0.010 GeV	0.238 ± 0.023 GeV	0.288 ± 0.014 GeV
$\Delta M_{\{n,n\}}$	0.250 ± 0.010 GeV	0.228 ± 0.023 GeV	0.224 ± 0.010 GeV
$\Delta M_{\{n,s\}}$	0.409 ± 0.013 GeV	0.397 ± 0.004 GeV	0.436 ± 0.022 GeV
$\Delta M_{\{s,s\}}$	0.472 ± 0.007 GeV	0.453 ± 0.004 GeV	0.520 ± 0.050 GeV
b	1.016 ± 0.016 GeV^2	1.073 ± 0.017 GeV^2	1.13 ± 0.07 GeV^2
c	-0.075 ± 0.011 GeV^4	0.060 ± 0.013 GeV^4	0 GeV^4
d	0.005 ± 0.015 GeV^2	0.003 ± 0.016 GeV^2	0 GeV^2
e	-0.021 ± 0.012 GeV^2	-0.004 ± 0.04 GeV^2	-0.16 ± 0.04 GeV^4
n	1.24 ± 0.05 GeV^2	1.48 ± 0.05 GeV^2	1.50 ± 0.13 GeV^2
g	0.043 ± 0.011 GeV^4	0.051 ± 0.029 GeV^4	0.16 ± 0.04 GeV^4
h	-0.102 ± 0.003 GeV^4	-0.105 ± 0.003 GeV^4	-0.108 ± 0.009 GeV^4
c'	–	–	0.291 ± 0.020 GeV^4
g'	–	–	-0.316 ± 0.025 GeV^4
e'	–	–	0.079 ± 0.008 GeV^2
d'	–	–	0.75 ± 0.05 GeV^2

the three-quark ones. We have developed three models based on the quark-diquark degrees of freedom. In the first, which we have called the "pure" model, all the baryons are in the quark-diquark configuration and each baryon has a definite type of diquark (scalar or axial-vector). The second, called the "mixed" diquark model, is a variation of the first; in the mixed model, the ground state octet is regarded as a superposition of the scalar and axial-vector types of diquark with equal weight. For the excited states, one type of diquark becomes dominant as a "confinement" effect and there is no longer, as a first approximation, superposition. In the third model, called the "hybrid" model, the baryons in the ground state, i.e., with no spatial excitations, have a standard three-quark structure, while the quark-diquark configuration is applied only to the excited states. The three-quark algebraic model by Bijker, Iachello, and Leviatan [24] works well in calculating both the masses and the decay amplitudes for the ground states. However, it predicts too many excited states. While our "hybrid" model shares the good characteristics of the three-quark scheme for the ground states, it avoids the downside of the too many excited resonances by introducing the diquark degree of freedom. In the case of the "hybrid" model, it should be possible to find a signature of a string-like structure for the excited resonances and a three-dimensional structure for the ground states in particular from the study of their electromagnetic transition form factors, to be compared with the experimental data by Jefferson Lab (JLab), Mainzer Mikrotron (MAMI), etc.

The "pure", "mixed", and "hybrid" models exhaust all the current possibilities in the framework of the constituent quark-diquark baryons, apart from the case already investigated in Ref. [13]. A future development could be to introduce the superposition of the two types of diquark for all the resonances. However, this will become possible only when a model is developed in which the weights of the diquark superposition for each baryon can be calculated theoretically or can be related unambiguously to experimental data, such as decay rates, etc. The baryon spectrum resulting from each model has been calculated through the use of an algebraic mass formula. Actually, the first two models were able to be described by the same mass formula, while the third one needed the introduction of a modified version. All these mass formulas can be regarded as generalizations of the Gürsey-Radicati mass formula, but they offer the advantage that the diquark mass differences are parameters that can be extracted from the phenomenology and compared with other diquark calculations. In a subsequent paper we will discuss in detail the link between them.

The principal feature of all quark-diquark models is the drastic reduction in the number of baryonic states. Indeed, while all the well-established baryon resonances, i.e., the ones labeled with three and four stars in Ref. [1], still fit well in our scheme, we have far fewer missing states than a normal three-quark constituent model. Some of the remaining missing states can be ascribed to one- or two-star resonances. For example the states $\Sigma(1620)S_{11}$, $\Sigma(1770)P_{11}$, $\Sigma(1840)P_{13}$, $\Sigma(2080)P_{13}$, and $\Sigma(2070)F_{15}$ fit quite well in the multiplets of the well-known $N(1535)S_{11}$, $N(1710)P_{11}$, $N(1720)P_{13}$, $\Delta(1920)P_{33}$, and $\Delta(1905)F_{35}$, respectively. Other resonances that, if confirmed, could be inserted easily in our scheme are $N(1890)$ [18] [also called $N(2000)F_{15}$ in the PDG] which can be placed in the same multiplet of $\Lambda(2110)F_{05}$, $N(1880)$ can be identified with the state with quantum numbers $J^P = 1/2^+$, $L = 0$, and $\nu = 2$ for which we predict with the "hybrid" model a mass of 1.903 GeV, and $N(1900)P_{13}$ to whom we assign quantum numbers $J^P = 3/2^+$, $L = 2$, and $\nu = 0$ and predict a mass of 1.866 GeV. On the other hand there are some resonances that fit less than ideally in our scheme. $\Xi(1690)$ (with unknown J^P quantum numbers) is best included in the multiplet of $N(1440)P_{11}$ with a predicted mass of 1.807 GeV. $\Delta(1900)S_{31}$ and $\Delta(1940)D_{33}$ are two resonances that we can assign $L = 1$ and $\nu = 1$ quantum numbers and predict a mass around 2.080 GeV. However the latest George Washington University (GWU) analysis [17] finds no evidence for these resonances. Thus we suggest to look for them at higher energies. $\Delta(1750)P_{31}$ is redundant in our quark-diquark scheme but, again, the existence of this resonance is very doubtful and has not been confirmed by the latest analysis [17]. Finally we assign $L = 0$ and $\nu = 2$ quantum numbers to the two-star $\Sigma(1880)P_{11}$ resonance and we calculate a mass around 2 GeV. With our model we can as well investigate the higher spin resonances. This can be done more easily with the non-strange baryons, in particular the Δs, since there are less doubts in their attribution to the right multiplets. The resonances $N(2220)H_{19}$ and $\Delta(2420)H_{3,11}$ are well established. Our assignments are $L = 4$, $\nu = 0$ and $S = 1/2$ for the former and $L = 4$, $\nu = 0$, and $S = 3/2$ for the latter, and the predicted masses are, respectively, 2.262 and 2.534 GeV. $\Delta(2750)I_{3,13}$, $\Delta(2390)F_{37}$, $\Delta(2300)H_{39}$, and $\Delta(2350)D_{35}$ are one- or two-star resonances which have not been seen by the latest analysis of Ref. [17]. The most probable quantum numbers in our scheme are $L = 7$, $\nu = 0$, and $S = 1/2$ for $\Delta(2750)I_{3,13}$; $L = 4$, $\nu = 0$, and $S = 3/2$ for $\Delta(2390)F_{37}$; $L = 4$, $\nu = 0$, and $S = 3/2$ for $\Delta(2300)H_{39}$ and $L = 3$, $\nu = 1$, and $S = 1/2$ for $\Delta(2350)D_{35}$. With these quantum numbers the resulting masses are, respectively, 3.095, 2.490, 2.490, and 2.565 GeV, somewhat higher than the experimental measurements. Since these measures are very old, we urge new experiments and data analysis to confirm the existence of the resonances and to determine precisely their masses. Finally we have the two-star resonances $\Delta(2950)K_{3,15}$ ($L = 6$, $\nu = 0$, and $S = 3/2$ our assignment) and $\Delta(2400)G_{39}$ ($L = 5$, $\nu = 0$, and $S = 1/2$), for which we predict masses around 2.907 and 2.706 GeV, respectively. $\Delta(2400)G_{39}$ has been seen in Ref. [17] with a mass of 2.643 ± 0.141 GeV.

All these states, together with the missing ones with no known correspondences as yet, should be further investigated, both theoretically and experimentally, since they are surely the missing part of the puzzle. The search for these resonances should be one of the main focuses of the baryon programs at JLab, the Beijing Spectrometer (BES), the Electron Stretcher and Accelerator (ELSA) facility (the Crystal Barrel collaboration), and the Two Arms Photon Spectrometer (TAPS). It should also be mentioned the big effort for the extraction of the resonances in analysis project like EBAC, Jülich, SAID, MAID, etc.

We have managed to describe the spectrum reasonably well by means of both the mass formulas used, and particularly well with the "hybrid" model. The resulting orbital and vibrational Regge trajectory slopes, $\alpha(\text{pure}) = b + d = 1.021$ GeV^2, $n(\text{pure}) = 1.24$ GeV^2, $\alpha(\text{mixed}) = b + d = 1.075$ GeV^2, $n(\text{mixed}) = 1.48$ GeV^2, $\alpha(\text{hybrid}) = b = 1.13$ GeV^2, and $n(\text{hybrid}) = 1.50$ GeV^2 for the "pure", "mixed", and "hybrid" models, respectively, agree quite well with the theoretical expectations ($\alpha_{\text{theo}} = 1.15$ GeV^2 and $n_{\text{theo}} = 1.36$ GeV^2) in a string model [10,11,21,32,33]. We note that the values of the baryon Regge trajectories slopes found in this work are compatible with the corresponding values found for the mesons in Refs. [10,11,21]. This can be due to a fundamental baryon-meson supersymmetry.

Important parameters of constituent models are the mass differences between the constituents, rather than the absolute masses of the constituents, largely dependent on the model used. These mass differences tend to be more stable and can be compared with results yielded by both constituent and other models, such as QCD-inspired and lattice ones. In each model, our values of the mass difference between the strange and the nonstrange quark ΔM_s are compatible with the estimates of the constituent quark models and with the PDG value for the current quark mass difference [1]. The difference $\Delta M_{\{n,n\}} = M_{\{n,n\}} - M_{[n,n]}$, as well as the mass differences between $[n, s]$ and $[n, n]$, between $\{n, s\}$ and $\{n, n\}$ and between $\{n, s\}$ and $[n, s]$, have been compared with the predictions made through the other main models for the constituent diquark (see Table VIII). We find that all the mass differences lie in the same range of values as in the other studies.

We have managed to obtain a sufficiently satisfactory description of the baryon spectrum by means of two very simple mass formulas, based essentially on only three elements: the constituent quark-diquark structure of the baryons, the Regge trajectories and a generalized Gürsey-Radicati–like form of the mass formulas. Thus, we can conclude that these elements should be the basis of future, more advanced investigations. We do not have elements which indicate a definite preference for one of the models used in this work over the other, as these can come only from

TABLE VIII. Mass differences (in GeV) between scalar and axial-vector diquarks according to some important studies, compared with the results obtained in the present study.

$M_{[n,n]}$	$M_{\{n,n\}} - M_{[n,n]}$	$M_{[n,s]} - M_{[n,n]}$	$M_{\{n,s\}} - M_{[n,s]}$	$M_{\{n,s\}} - M_{\{n,n\}}$	$M_{\{s,s\}} - M_{\{n,s\}}$	Source
0.688	0.202	0.272	–	–	–	Maris [28,29]
–	0.29	–	0.11	–	–	Wilczek [6]
–	0.210	–	0.150	–	–	Jaffe [5]
0.595	0.205	0.240	0.140	0.175	–	Lichtenberg [30]
0.74	0.21	0.14	0.17	0.10	0.08	Hecht *et al.* [8]
0.78	0.28	–	–	–	–	Roberts *et al.* [12]
0.60	0.35	–	–	–	–	Ferretti *et al.* [31]
0.50	0.30	–	–	–	–	Santopinto [13]
–	0.183	0.218	0.176	0.211	–	Chakrabarti [26]
0.956	0.250	0.197	0.212	0.159	0.063	"pure" model
0.898	0.228	0.238	0.159	0.169	0.056	"mixed" model
0.852	0.224	0.288	0.148	0.212	0.084	"hybrid" model

new experimental data or new theoretical insights about the internal structure of the baryons. However, the "hybrid" model has the very good characteristic of being able to exploit the advantages of the three-quark model, which has seen several decades of success, in the description of the ground states, while making up for its defects by introducing the dominance of the quark-diquark features for the excited states. The "mixed" model, while displaying only quark-diquark degrees of freedom, is also able to work in a similar way.

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