

Resonance width distribution for open quantum systems

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(Received 20 December 2011; revised manuscript received 26 August 2012; published 1 October 2012)

Recent measurements of resonance widths for low-energy neutron scattering off heavy nuclei show large deviations from the Porter-Thomas distribution. We propose a “standard” width distribution based on the random matrix theory for a chaotic quantum system with a single open decay channel. Two methods of derivation lead to a single analytical expression that recovers, in the limit of very weak continuum coupling, the Porter-Thomas distribution. The parameter defining the result is the ratio of typical widths Γ to the energy level spacing D . Compared to the Porter-Thomas distribution, the new distribution suppresses small widths and increases the probabilities of larger widths. We show also that it is necessary to take into account the γ channels.

DOI: [10.1103/PhysRevC.86.044602](https://doi.org/10.1103/PhysRevC.86.044602)

PACS number(s): 24.60.Lz, 28.20.Cz, 05.45.Mt, 21.10.Tg

I. INTRODUCTION

Random matrix theory as a statistical approach for exploring properties of complex quantum systems was pioneered by Wigner and Dyson half a century ago [1]. This theory was successfully applied to excited states of complex nuclei and other mesoscopic systems [2–5], evaluating statistical fluctuations and correlations of energy levels and corresponding wave functions supposedly of “chaotic” nature.

The standard random matrix approach based on the Gaussian orthogonal ensemble (GOE) for systems with time-reversal invariance, and on the Gaussian unitary ensemble (GUE) if this invariance is violated, was formulated originally for *closed* systems with no coupling to the outside world. Although practical studies of complex nuclei, atoms, disordered solids, or microwave cavities always require the use of reactions produced by external sources, the typical assumption was that such a probe at the resonance is sensitive to the specific components of the exceedingly complicated intrinsic wave function, one for each open reaction channel, and the resonance widths are measuring the weights of these components [6]. With the Gaussian distribution of independent amplitudes in a chaotic intrinsic wave function, the widths under this assumption are proportional to the squares of the amplitudes and as such can be described, for ν independent open channels, by the chi-square distribution with ν degrees of freedom. For low-energy elastic scattering of neutrons off heavy nuclei, where the interactions can be considered time-reversal invariant, one expects $\nu = 1$, that is, the usually called Porter-Thomas distribution (PTD) [7].

From recent measurements [8,9] it has been claimed that the neutron width distributions in low-energy neutron resonances on certain heavy nuclei are different from the PTD. As a rule, the fraction of greater widths is increased, while the fraction of narrow resonances is reduced, which, being approximately presented with the aid of the same standard class of functions,

would require $\nu \neq 1$. The literature discussing the scattering and decay processes in chaotic systems (see, for example, [10–13] and references therein) does not provide a detailed description of the width distribution for the region of relatively small widths as observed in low-energy neutron resonances.

There are various reasons for possible deviations from the simple statistical predictions [14–16]. First of all, the intrinsic dynamics, even in heavy nuclei, can be different from that in the GOE limit of many-body quantum chaos. If so, a detailed analysis of specific nuclei is required. As an example we can mention ^{232}Th , where for a long time a sign problem exists [17] concerning the resonances with strong enhancement of parity nonconservation in scattering of longitudinally polarized neutrons. The observed predominance of a certain sign of parity-violating asymmetry contradicts the statistical mechanism of the effect and may be related to the nonrandom coupling between quadrupole and octupole degrees of freedom [18]. The width distribution in the same nucleus reveals noticeable deviations from the PTD. The presence of a shell-model single-particle resonance serving as a doorway to the compound nucleus can also make its footprint, distorting the statistical pattern. Another (maybe related to the doorway resonance) effect can come from the changed energy dependence of the widths that is usually assumed to be proportional to $E^{\ell+1/2}$ for neutrons with orbital momentum ℓ . Finally, the situation is not strictly one channel, since, along with elastic neutron scattering, many γ channels are open as well. However, apart from structural effects, even in the one-channel approximation, there exists a generic cause for the deviations from the PTD, since the applicability of the GOE is anyway violated by the *open* character of the system [19]. The appropriate modification of the GOE and PTD predictions, which should be applied before making specific conclusions, is our goal below.

The resonances are not the eigenstates of a Hermitian Hamiltonian; they are poles of the scattering matrix in the complex plane. Their complex energies $\mathcal{E} = E - i\Gamma/2$ can be rigorously described as eigenvalues of the effective non-Hermitian Hamiltonian [20]. As shown long ago, even for a single open channel, the statistical properties of the complex energies cannot be described by the GOE. The new dynamics

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is related to the interaction of intrinsic states through the continuum. In the limit of strong coupling this leads to overlapping resonances, Ericson fluctuations of cross sections, and a sharp redistribution of widths similar to the phenomenon of superradiance (see the review [21] and references therein). The control parameter of such restructuring is the ratio

$$\kappa = \frac{\pi\Gamma}{2D} \quad (1)$$

of typical widths, Γ , to the mean spacing between the resonances, D . In the region of low-energy neutron resonances, κ is still small, but in order to correctly separate the general statistical effects from peculiar properties of individual nuclei we need to have at our disposal a generic width distribution that differs from the PTD as a function of the degree of openness.

II. RESONANCE WIDTH DISTRIBUTION

The goal of the paper is to provide a practical tool that would allow one to compare the experimental output for an unstable quantum system with predictions of random matrix theory. We propose a distribution function that is based, similar to the GOE, on the chaotic character of time-reversal-invariant internal dynamics and corresponding decay amplitudes but properly accounts for the continuum coupling through the effective non-Hermitian Hamiltonian. The numerical simulations for this Hamiltonian were described earlier [15,22] but here we derive the analytical expression. We limit ourselves here by the situation typical for nuclear applications, namely $\kappa < 1$. The superradiant regime, $\kappa \geq 1$, can be of special interest (including systems such as microwave cavities), and in the considered framework formal symmetry exists, $\kappa \rightarrow 1/\kappa$. At a large number of resonances and fixed number of open channels, after the superradiant transition the broad state becomes a part of the background while the remaining “trapped” states return into the nonoverlap regime. However, in heavy nuclei this transition hardly can be observed because earlier many new channels can be opened; in the modification of the PTD we see only precursors of this transition.

Our arguments will follow two different routes which lead to equivalent results. The final formula for the statistical width distribution can be presented as

$$P(\Gamma) = C \frac{\exp\left[-\frac{N}{2\sigma^2}\Gamma(\eta - \Gamma)\right]}{\sqrt{\Gamma(\eta - \Gamma)}} \left(\frac{\sinh\left[\frac{\pi\Gamma}{2D}\frac{(\eta - \Gamma)}{\eta}\right]}{\frac{\pi\Gamma}{2D}\frac{(\eta - \Gamma)}{\eta}} \right)^{1/2} \quad (2)$$

Here we consider $N \gg 1$ intrinsic states coupled to a single decay channel, for example, s -wave elastic neutron scattering. The parameter D is the mean energy spacing between the resonances, κ is a new dimensionless combination [Eq. (1)], and C is a normalization constant. The quantity η is the total sum (the trace of the imaginary part of the effective non-Hermitian Hamiltonian that remains invariant in the transition to the biorthogonal set of its eigenfunctions) of all N widths; it appears as a parameter that fixes the starting ensemble distribution [see Eqs. (8) and (9)]. The possible values of widths are restricted from both sides, $0 < \Gamma < \eta$. The above-mentioned symmetry $\kappa \rightarrow 1/\kappa$ is reflected in the symmetry $\Gamma \rightarrow \eta - \Gamma$ of a factor in Eq. (2) but, as was already

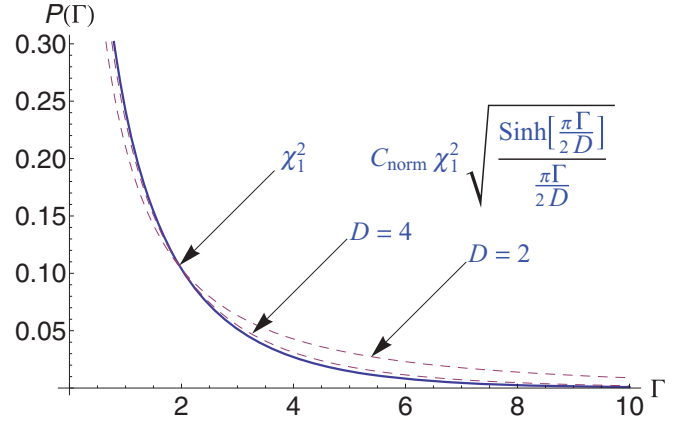


FIG. 1. (Color online) The proposed resonance width distribution according to Eq. (2) with a single neutron channel in the practically important case $\eta \gg \Gamma$. The width Γ and mean level spacing D are measured in units of the mean value $\langle \Gamma \rangle$.

stated, our region of interest is at $\Gamma \ll \eta$. Another parameter, σ , determines the standard deviation of variable Γ evaluated consistently with the distribution of Eq. (2). In the practical region far away from the superradiance we obtain

$$P(\Gamma) = C \chi_1^2(\Gamma) \left(\frac{\sinh \kappa}{\kappa} \right)^{1/2}. \quad (3)$$

The PTD is recovered in the limiting case $\kappa \ll 1$ that corresponds to the approximation of an open quantum system by a closed one. The new element is the factor explicitly determined by the coupling strength κ . With growing continuum coupling the probability of larger widths increases. The distribution (2) for different ratios $\langle \Gamma \rangle / D$ is shown in Fig. 1.

The origin of the square root in the new factor is the linear energy level repulsion typical for GOE spectral statistics. Indeed, in the complex plane, $\mathcal{E} = E - i\Gamma/2$, the distance between two poles \mathcal{E}_m and \mathcal{E}_n is $\sqrt{(E_m - E_n)^2 + (\Gamma_m - \Gamma_n)^2/4}$; after integration over all variables of other states we obtain a characteristic square root in the level repulsion [see below Eq. (12) and the discussion after Eq. (21)].

III. EFFECTIVE NON-HERMITIAN HAMILTONIAN AND SCATTERING MATRIX

In order to come to the result (2), we start with the general description of complex energies $\mathcal{E} = E - i\Gamma/2$ in a system of N unstable states satisfying the GOE statistics inside the system and interacting with the single open channel through Gaussian random amplitudes. The general reaction theory [23] is constructed in terms of the elements of the scattering matrix in the space of open channels a, b, \dots ,

$$S^{ba}(E) = \delta^{ba} - iT^{ba}(E). \quad (4)$$

Within the formalism of the effective non-Hermitian Hamiltonian $\mathcal{H} = H - (i/2)W$, the T matrix is defined as

$$T^{ba}(E) = \sum_{m,n=1}^N A_m^{b*} \left[\frac{1}{E - \mathcal{H}} \right]_{mn} A_n^a \quad (5)$$

in terms of the amplitudes A_n^a connecting an internal basis state n with an open channel a . Here we neglect the potential part of scattering that is not related to the internal dynamics of the compound nucleus. The anti-Hermitian term of the effective Hamiltonian is exactly represented by the sum over k open channels,

$$W_{mn} = \sum_{a=1}^k A_m^a A_n^{a*}, \quad (6)$$

where the amplitudes can be considered real in the case of time-reversal invariance. It is important that the factorized structure of the effective Hamiltonian guarantees the unitarity of the scattering matrix. The amplitudes A_n^a are uncorrelated Gaussian quantities with zero mean and variance defined as $\overline{A_n^a A_{n'}^b} = \delta^{ab} \delta_{nn'} \eta / N$. The trace of the anti-Hermitian part of the effective Hamiltonian, $\eta = \text{Tr} W$, i.e., the total sum of all N widths used in Eq. (2), is a quantity invariant under orthogonal transformation of the intrinsic basis. The detailed discussion of the whole approach, numerous applications, and relevant references can be found in the recent review article [21].

The simplest version of the R -matrix description uses instead of the amplitude T^{ba} its approximate form, where the denominator contains poles on the real energy axis corresponding to the eigenvalues of the Hermitian part H of the effective Hamiltonian. Then the continuum coupling occurs only at the entrance and exit points of the process while the influence of this coupling on the intrinsic dynamics of the compound nucleus is neglected (in general, H should also be renormalized by the off-shell contributions from the presence of the decay channels). In contrast to that, the full amplitude T^{ba} , Eq. (5), accounts for this coupling during the entire process including the virtual excursions to the continuum and back from intrinsic states. The poles are the eigenvalues of the full effective Hamiltonian in the lower half of the complex energy plane. The experimental treatment corresponds to this full picture. According to the original paper [8], the R -matrix code SAMMY [24] had been used in the experimental analysis where the relevant expression is given in the form

$$R_{cc'} = \sum_{\lambda} \frac{\gamma_{\lambda c} \gamma_{\lambda c'}}{E_{\lambda} - E - i\Gamma_{\lambda}/2} \delta_{JJ'}, \quad (7)$$

and the treatment included a careful segregation of s and p resonances, $J = J' = 1/2$ for an even target nucleus. In the notation of [24] λ represents a particular resonance and E_{λ} is the energy of the resonance. Here we can identify the intermediate states λ and their complex energies $E_{\lambda} - i\Gamma_{\lambda}/2$ with the eigenstates and complex eigenvalues of \mathcal{H} , while the numerator includes the amplitudes transformed to this new basis (time-reversal invariance preserves the symmetry of the scattering matrix). In terms of the reduced width $\gamma_{\lambda c}^2$ and the penetration factor P_c , the partial width is $\Gamma_{\lambda c} = 2P_c \gamma_{\lambda c}^2$.

Under the assumption of a single channel and universal energy dependence of penetration factors, the statistics of the total widths is the same as that of $\gamma_{\lambda c}^2$.

IV. FROM ENSEMBLE DISTRIBUTION TO SINGLE WIDTH DISTRIBUTION

For a single-channel case, the joint distribution $P(\vec{E}; \vec{\Gamma})$ of all complex energy poles has been rigorously derived in [19] under assumptions of the GOE intrinsic dynamics in the closed system and Gaussian distributed random decay amplitudes. The result is given by

$$P(\vec{E}; \vec{\Gamma}) = C_N \prod_{m < n} \frac{(E_m - E_n)^2 + \frac{(\Gamma_m - \Gamma_n)^2}{4}}{\sqrt{(E_m - E_n)^2 + \frac{(\Gamma_m + \Gamma_n)^2}{4}}} \prod_n \frac{1}{\sqrt{\Gamma_n}} e^{-NF(\vec{E}; \vec{\Gamma})}, \quad (8)$$

where the “free energy” F contains interactions of $N \gg 1$ complex poles in the interval $2a = ND$ of energies,

$$F(\vec{E}; \vec{\Gamma}) = \frac{1}{a^2} \sum_n E_n^2 + \frac{1}{2a^2} \sum_{m < n} \Gamma_m \Gamma_n + \frac{1}{2\eta} \sum_n \Gamma_n. \quad (9)$$

For given N , this distribution contains two parameters, the semicircle radius a for the intrinsic dynamics and η characterizing the total trace of the imaginary part of the effective Hamiltonian.

Considering this free energy in the “mean-field” approximation, we see that the original mean value $\langle \Gamma \rangle_0 = \eta / N$ is substituted by $\langle \Gamma \rangle$, which is determined by the competition of two terms, $1/\langle \Gamma \rangle = 1/\langle \Gamma \rangle_0 + \langle \Gamma \rangle / 4D^2$. The first product in front of $\exp(-NF)$ in Eq. (8) substitutes the GOE-level repulsion by the repulsion in the complex plane and interaction of the poles with their negative- Γ “images” [19]. The structure of this result guarantees that all widths Γ are positive. The difficulty with the distribution of Eq. (8) is that it is not an analytic function of complex energies.

Our first step is to specify a single N th pole $(E_N, \Gamma_N) \equiv (E, \Gamma)$ and, using the fact that the distribution ensures $\Gamma_n \geq 0$, return to the absolute values of the amplitudes, $\sqrt{\Gamma_n} = \xi_n$, for other roots. In this form we can apply the steepest descent method owing to a large parameter $N \gg 1$ and a saddle point inside the integration interval that was absent in the initial expression. Integration of $\prod_{n=1}^{N-1} d\xi_n$ over all but one variable $\Gamma \equiv \Gamma_N = \xi_N^2$ leads to the following result for $P(\vec{E}; \Gamma)$ as a function of multiple energy variables \vec{E} and a single width variable Γ :

$$P(\vec{E}; \Gamma) = C_N \prod_{m < n} |E_m - E_n| \prod_n \sqrt{(E_n - E)^2 + \frac{\Gamma^2}{4}} \times \exp \left[-\frac{N}{a^2} \left(\sum_n E_n^2 + E^2 \right) \right] \times \frac{\exp \left[-\frac{N}{2\eta} \Gamma \right]}{\sqrt{\Gamma}} \left(\sqrt{\frac{2\pi}{N \left(\frac{\Gamma}{a^2} + \frac{1}{\eta} \right)}} \right)^{N-1}. \quad (10)$$

Introducing the new variables $2a/N = D$ and $\lambda = \eta N$, we shall examine the behavior of one of the N -dependent factors in Eq. (10) in the limit of $N \rightarrow \infty$:

$$\lim_{N \rightarrow \infty} \left[\frac{\exp\left[-\frac{N}{2\eta}\Gamma\right]}{\sqrt{\Gamma}} \left(1 + \frac{\lambda\Gamma}{a^2 N}\right)^{-\frac{N-1}{2}} \right] = \frac{\exp\left[-\frac{N}{2\eta}\Gamma\right]}{\sqrt{\Gamma}} \exp\left[-\frac{\lambda}{2a^2}\Gamma\right]. \quad (11)$$

The scaling properties of the parameters η , a^2 , and λ are as follows: $\eta \propto N$, $a^2 \propto N^2$, and $\lambda \propto N^2$. As a result, the product of two exponents produces a well-defined limit that brings in the desired dependence on the coupling strength κ .

The real energy distribution does not change much in an open system with a single decay channel being still, at finite but large N , close to a semicircle. We are working in the central region of the spectrum where the level density is approximately constant and the energy spectrum is close to equidistant. (The maximum of the level spacing distribution is always at $s = \delta E/D \approx 1$ although the distribution in an open system changes at small spacings, $s < 1$, as we will comment later.) With $E_n = E - nD$, we are able to perform an exact calculation of the product:

$$C_N \prod_n \sqrt{(E_n - E)^2 + \frac{\Gamma^2}{4}} = \widehat{C}_N \left(\prod_{n=1}^N \left[1 + \frac{\Gamma^2/4}{(nD)^2} \right] \right)^{1/2}_{N \rightarrow \infty} = \widehat{C}_N \left(\frac{\sinh\left[\frac{\pi}{2} \frac{\Gamma}{D}\right]}{\frac{\pi}{2} \frac{\Gamma}{D}} \right)^{1/2}, \quad (12)$$

where we have used the famous Euler formula

$$\frac{\sinh x}{x} = \prod_{k=1}^{\infty} \left[1 + \frac{x^2}{k^2 \pi^2} \right]. \quad (13)$$

The width-independent factors will enter the normalization constant. Of course, the whole reasoning is valid in the limit $N \gg 1$. Finally, the width distribution for $\Gamma \ll \eta$ is represented by

$$P(\Gamma) = C \left(\frac{\sinh\left[\frac{\pi}{2} \frac{\Gamma}{D}\right]}{\frac{\pi}{2} \frac{\Gamma}{D}} \right)^{1/2} \frac{\exp\left[-\frac{N}{2\eta}\Gamma\right]}{\sqrt{\Gamma}} \exp\left[-\frac{\lambda}{2a^2}\Gamma\right]. \quad (14)$$

V. DOORWAY APPROACH

As an alternative derivation, we will apply the *doorway approach* [2,25,26]. Here we use the eigenbasis of the imaginary part W of the effective non-Hermitian Hamiltonian. Due to the factorized nature of W dictated by unitarity [19], the number of its nonzero eigenvalues is equal to the number of open channels. In our case we have only one eigenvalue, the *doorway* $\varepsilon_0 - i\eta/2$, which has a nonzero width equal to the imaginary part η of the trace of the Hamiltonian. The remaining basis states are stable, being driven by the Hermitian intrinsic Hamiltonian; its diagonalization produces their real energies ε_n . These states acquire the widths through the interaction with

the doorway state; the corresponding matrix elements will be denoted h_n . In this basis, the Hamiltonian is represented as

$$\begin{pmatrix} \varepsilon_0 - \frac{i}{2}\eta & h_1 & h_2 & \cdots & h_N \\ h_1^* & \varepsilon_1 & 0 & \cdots & 0 \\ h_2^* & 0 & \varepsilon_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_N^* & 0 & 0 & \cdots & \varepsilon_N \end{pmatrix}. \quad (15)$$

The complex eigenvalues $\mathcal{E} = E - i\Gamma/2$ are the roots of the secular equation

$$\mathcal{E} = \varepsilon_0 - \frac{i}{2}\eta + \sum_{n=1}^N \frac{|h_n|^2}{\mathcal{E} - \varepsilon_n}, \quad (16)$$

which is equivalent to the set of coupled equations for real and imaginary parts,

$$E = \varepsilon_0 + \sum_{n=1}^N |h_n|^2 \frac{E - \varepsilon_n}{(E - \varepsilon_n)^2 + \Gamma^2/4}, \quad (17)$$

$$\Gamma = \frac{\eta}{1 + \sum_{n=1}^N \frac{|h_n|^2}{(E - \varepsilon_n)^2 + \Gamma^2/4}} \equiv f(\Gamma, E). \quad (18)$$

For a Gaussian distribution of coupling matrix elements with $\langle |h|^2 \rangle = 2\sigma^2/N$ (a scaling that was derived in [25]), we obtain

$$P(\Gamma) = \int_{-\infty}^{+\infty} \delta(\Gamma - f(\Gamma, E)) \exp\left[-\frac{N}{\sigma^2} \sum_{n=1}^N h_n^2\right] \prod_{n=1}^N dh_n. \quad (19)$$

The integration in (19) via the steepest descent method leads to Eq. (2). In order to get this result we use the possibility of finding the highest root $E = \varepsilon_N$, relative to which the energies ε_n can be counted as $E = \varepsilon_N$, $\varepsilon_n = \varepsilon_N - nD$. An important intermediate step is the evaluation of the infinite product of the Lorentzian peaks, which can be simplified as

$$\begin{aligned} & \left(\prod_{n=1}^{N-1} \left[1 - \frac{\frac{\Gamma^2}{4} + (E - \varepsilon_N)^2}{\frac{\Gamma^2}{4} + (E - \varepsilon_n)^2} \right] \right)^{-1/2} \\ &= \left(\prod_{n=1}^{N-1} \left[1 - \frac{\frac{\Gamma^2}{4}}{\frac{\Gamma^2}{4} + (nD)^2} \right] \right)^{-1/2} \\ &= \left(\frac{\sinh\left[\frac{\pi}{2} \frac{\Gamma}{D}\right]}{\frac{\pi}{2} \frac{\Gamma}{D}} \right)^{+1/2}. \end{aligned} \quad (20)$$

In a similar way one can analyze the resonance spacing distribution $P(s)$ along the real energy axis; spacings $s = \delta E/D$ are measured in units of their mean value D . As predicted in [19] and observed numerically in [22], the short-range repulsion disappears and the Wigner surmise with the standard linear preexponential factor s is substituted by the square root,

$$P(s) \propto \sqrt{s^2 + 4 \frac{\langle \Gamma^2 \rangle}{D^2}} \exp[-\text{const.} \cdot s^2]. \quad (21)$$

At spacing $s \ll 1$, the probability behaves as $a + bs^2$ with the quadratic dependence on s that, similar to the GUE, mimics

the violation of time-reversal invariance due to the open decay channel. The absence of short-range repulsion, $a \neq 0$, reflecting the energy uncertainty of unstable states, is the result of the anti-Hermitian interaction through the continuum which repels widths and attracts real energies [27], opposite to a normal Hermitian perturbation.

We demonstrated that two complementary approaches which reflect different physical aspects of the situation lead essentially to the equivalent (after identification of corresponding parameters) results which we prefer to write in the form (2). We expect that for other canonical ensembles the width distribution far from the superradiance can be expressed by a similar formula with the function $(\sinh \kappa / \kappa)^{\beta/2}$, where the standard index of the ensemble is $\beta = 1$ for the GOE, $\beta = 2$ for the GUE, and $\beta = 4$ for the Gaussian symplectic ensemble. In the same way we expect the square root in Eq. (21) to be substituted by the same power $\beta/2$.

The doorway approach naturally indicates the limits of the variable, $0 \leq \Gamma \leq \eta$. It also has an advantage in that one can possibly generalize the answer by taking into account explicitly the rigidity of the internal energy spectrum with fluctuations of level spacings around their mean value D [while, in our approximation, only this average value enters Eq. (2)]. Another direction of generalization includes the possible influence of a single-particle resonance depending on the position of its centroid with respect to the considered interval of the resonance spectrum. In particular, that centroid may be located under the threshold of our decay channel. In this case even the standard energy dependence of the widths can change, as was mentioned long ago [25] (see also [14]). The doorway state may or may not coincide with such a resonance, so that the effective Hamiltonian (15) may contain two special states coupled with the “chaotic” background, one by intrinsic interactions and another one through the continuum.

VI. ADDING γ CHANNELS

The goal of this section is to estimate in the same spirit the influence of γ channels on the resonance width distribution. Only a single open elastic neutron channel was taken into account in the analysis of data [8,9]. The presence of even weak additional open channels changes the unitarity conditions. Examples of the mutual influence of neutron and γ channels are well known in the literature from long ago (see, for example, [28]).

Generalizing the doorway description, we allow now each intrinsic state to decay by γ emission, which is always possible independently of the position of the neutron threshold. In the simplest approximation, the effective non-Hermitian Hamiltonian is now represented by

$$\begin{pmatrix} \varepsilon_0 - \frac{i}{2}\eta & h_1 & h_2 & \cdots & h_N \\ h_1^* & \varepsilon_1 - \frac{i}{2}\gamma_1 & 0 & \cdots & 0 \\ h_2^* & 0 & \varepsilon_2 - \frac{i}{2}\gamma_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ h_N^* & 0 & 0 & \cdots & \varepsilon_N - \frac{i}{2}\gamma_N \end{pmatrix}, \quad (22)$$

where we assumed that the intrinsic part of the matrix is pre-diagonalized and introduced γ_n as the widths for γ channels.

Analogously to Eq. (16), the complex energy eigenvalues $\mathcal{E} = E - \frac{i}{2}\Gamma$ are the roots of the secular equation

$$\mathcal{E} = \varepsilon_0 - \frac{i}{2}\eta + \sum_{n=1}^N \frac{|h_n|^2}{\mathcal{E} - (\varepsilon_n - \frac{i}{2}\gamma_n)}, \quad (23)$$

which is equivalent to a set of coupled equations,

$$E = \varepsilon_0 + \sum_{n=1}^N |h_n|^2 \frac{E - \varepsilon_n}{(E - \varepsilon_n)^2 + (\Gamma - \gamma_n)^2/4}, \quad (24)$$

$$\Gamma = \frac{\eta + \sum_{n=1}^N |h_n|^2 \frac{\gamma_n}{(E - \varepsilon_n)^2 + (\Gamma - \gamma_n)^2/4}}{1 + \sum_{n=1}^N |h_n|^2 \frac{1}{(E - \varepsilon_n)^2 + (\Gamma - \gamma_n)^2/4}} \equiv g(\Gamma, E, \gamma). \quad (25)$$

The resonance width distribution for an open quantum system with γ channels included is given by

$$P(\Gamma, \gamma) = \int_{-\infty}^{+\infty} \delta(\Gamma - g(\Gamma, E, \gamma)) \times \exp \left[-\frac{N}{\sigma^2} \sum_{n=1}^N h_n^2 \right] \prod_{n=1}^N dh_n. \quad (26)$$

Estimating the widths γ_n by their average value, γ , and acting in the same manner as in the case of a single open channel we come to the final expression for the resonance width distribution:

$$P(\Gamma, \gamma) = C \frac{(\eta - \gamma)}{\sqrt{\Gamma - \gamma} \sqrt{\eta - \Gamma}} \exp \left[-\frac{N}{2\sigma^2} (\Gamma - \gamma)(\eta - \Gamma) \right] \times \left(\frac{\sinh \left[\frac{\pi(\Gamma - \gamma)(\eta - \Gamma)}{2D} \right]}{\frac{\pi(\Gamma - \gamma)(\eta - \Gamma)}{2D}} \right)^{1/2} \quad (27)$$

which is shifted by $\Gamma \rightarrow \Gamma - \gamma$ compared to the previous result. The symmetry mentioned earlier between the ends of the distribution, $\Gamma = 0$ and $\Gamma = \eta$, would be substituted here by $\Gamma \rightarrow (\eta + \gamma) - \Gamma$. Thus, the effective influence of γ -channels on the resonance width distribution is reduced here to a shift

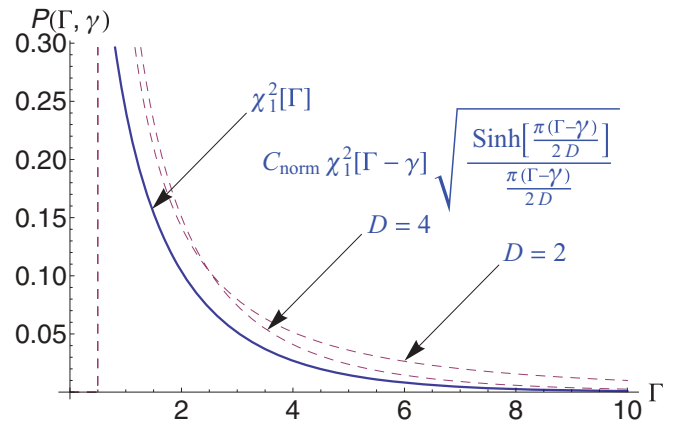


FIG. 2. (Color online) The proposed resonance width distribution according to Eq. (27) with a single neutron channel and N γ -channels in the practically important case $\eta \gg \Gamma$. The neutron width Γ , radiation width γ , and mean level spacing D are measured in units of the mean value $\langle \Gamma \rangle$.

of the whole distribution by a mean radiation width γ as seen in Fig. 2.

In the practical region far away from the super-radiance, $\Gamma \ll \eta$, we obtain

$$P(\Gamma, \gamma) = \chi_1^2[\Gamma - \gamma] \left(\frac{\sinh \left[\frac{\pi(\Gamma - \gamma)}{2D} \right]}{\frac{\pi(\Gamma - \gamma)}{2D}} \right)^{1/2}. \quad (28)$$

In order to extract the neutron width from the total resonance width, the treatment of the data has to be modified making in a sense an inverse shift. Of course, a more precise consideration should use a statistical distribution of the γ widths.

VII. CONCLUSION

In this article we propose a standard resonance width distribution for an open quantum system based on chaotic intrinsic dynamics and coupling of states with the same quantum numbers to the common decay channel. Two approximate methods lead to an equivalent analytical expression for the width distribution that does not belong to the class of chi-square distributions with the only parameter ν traditionally used in the analysis of data. In the limit of vanishing openness and return to a closed system we recover the standard PTD. The new result depends on the ratio (1) of the width to the mean level spacing, $\kappa \sim \Gamma/D$, that regulates the strength of the continuum coupling. The deviations from the PTD grow with κ up to the critical strength $\kappa \sim 1$, when the broad “superradiant” state becomes essentially part of the background, while the remaining “trapped” states return to the weak-coupling regime. This physics was repeatedly discussed previously, especially in relation to quantum signal transmission through mesoscopic devices [21,29], but it is outside of our interest here.

In the practical region of low-energy neutron resonances, the effects predicted here are relatively small. Although at small κ the derived neutron width distribution supports an experimental trend, the final judgment can be made only after the presence of γ channels was accounted for. We have to attract the attention of experimentalists to the fact that the data should be analyzed with the aid of a distribution that does not belong to the routinely used chi-square class; γ channels should be included in this consideration. We can also mention that the result agrees with numerical simulations [15] for the full many-resonance distribution function (8). Using the suggested distribution as a new reference point, one can ascribe the remaining deviations to the specific features of individual systems (level densities, single-particle structure in a given energy region, shape transformations, energy dependence of the widths, etc.). Unfortunately, we still do not have experimental tests for the full distribution (8). Although in nuclear physics it is hard to make such a detailed analysis for higher energies and greater degree of resonance overlap, the systems with tunable chaos, such as microwave cavities, acoustic blocks, or even elastomechanical devices [30], seem to provide appropriate tools for such studies.

ACKNOWLEDGMENTS

The authors thank P. E. Koehler and V. V. Sokolov for useful discussions, G. L. Celardo for numerical checks, D. V. Savin for communication, and M. Shapiro and S. Naboko for helpful remarks. GS is grateful to the Guidance Committee members represented by N. O. Birge, B. A. Brown, W. W. Repko, and R. Zegers for critical discussions. The authors are thankful to the referee for a number of helpful suggestions and remarks. Partial support from NSF Grant No. PHY-1068217 is gratefully acknowledged.

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