Neutron number dependence of the energies of the γ-vibrational states in nuclei with $Z \sim 100$ and the manifestation of pseudospin symmetry

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Background: The symmetry-guided methods have shown their power over many years of nuclear physics research. It was found that the concept of the pseudospin symmetry is very helpful in providing a simple explanation of many features of heavy nuclei.
Purpose: To show, based on the results of calculations of the energies of the γ-vibrational states in nuclei with $Z \sim 100$, that the experimental data on these energies will give us information indicating how well the pseudospin symmetry is realized in these exotic nuclei.
Method: The quasiparticle-phonon model is used to calculate the energies of the γ-vibrational states.
Results: It is shown that the energies of the γ-vibrational states in the Cm, Cf, Fm, No, and Rf isotopes considered as the functions of the number of neutrons N have a minimum at $N = 156$ when the neutron Fermi level lies just between the following neutron single particle levels: 3/2[622] and 1/2[620] which belong to the pseudospin doublet. It is shown that the corresponding two-quasiparticle component gives the main contribution to the structure of the γ phonon.
Conclusion: The experimental information on the energies of the γ-vibrational states in nuclei with $Z \sim 100$ can be used to determine a splitting of the $[\widetilde{521}]$ pseudospin doublet.

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I. INTRODUCTION

It is well known that in nuclear theory the mean field concept plays the role of the basic approach for several more specific advanced theories. These theories can be built upon introducing the single particle mean field basis. Therefore it is very important for the whole field of nuclear structure to examine and use the consequences of the underlying mean field symmetries even if these symmetries are approximate.

The pseudospin symmetry [1–3] is known as an approximate symmetry of the nuclear mean field. This symmetry is manifested in the nuclear excitation spectra by the presence of the quasidegenerate doublets. At the same time, the existence of this symmetry is strongly related to the strength of the spin-orbit interaction term of the nuclear mean field.

Dynamical symmetry implies the existence of a characteristic multiplet structure. These multiplets are characterized by a magnitude of the multiplet splitting. The characteristic magnitude of the splitting of the pseudospin doublets in spherical nuclei is of the order of $0.1\hbar\omega_0$, where $\hbar\omega_0$ is the frequence of the harmonic oscillator approximating nuclear mean field. However, this splitting demonstrates a dependence on the ratio between the numbers of protons and neutrons in the nucleus, and it is very small in some nuclei.

Single particle pseudospin doublets in the well-deformed nuclei are characterized by a projection of the pseudo-orbital momentum on the axial symmetry axis. The splitting of these doublets are several times smaller than in spherical nuclei. The calculations performed in [4] have shown that the goodness of the pseudospin symmetry improves if the nucleon binding energy decreases and the pseudo-orbital momentum decreases also. Therefore, weakly bound exotic nuclei are the most exciting ones to search for the pseudospin symmetry manifestation.

It is well known from many calculations of the structure of the γ-vibrational states in the well-deformed axially symmetric nuclei [5] that the structure of the γ phonons is mainly exhausted by a rather small number of the two-quasiparticle components. Due to this fact the energy of the γ-vibrational state can be strongly influenced by the presence of the low-energy two-quasiparticle state with $K^\pi = 2^+$. This happens if near the Fermi level are located two nearly lying single particle states having the same parities and the projections of the angular momentum on the axial symmetry axis whose sum or difference is equal to $K = 2$. Such closely lying single particle states can be the members of the pseudospin doublet. It is the aim of the present paper to calculate the energies of the γ-vibrational states for the sequences of the isotopes of the elements with $Z \sim 100$ and investigate the influence of the appearance of the pseudospin doublets near the Fermi level on the energies of the γ-vibrational states

In Sec. II we present information on the theoretical approach used in the calculations and on the single particle level scheme on which our calculations are based. In Sec. III the results of calculations of the energies and the structure of the γ-vibrational states in the even-even isotopes of Cm, Cf, Fm, No, and Rf are discussed.

II. QUASIPARTICLE-PHONON MODEL

The Hamiltonian of the quasiparticle-phonon model [5] contains the mean fields for protons and neutrons, monopole

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TABLE I. Parameters of the Woods-Saxon potential used in the calculations.

A	isospin	V_0, MeV	r_0, fm	a, fm	κ, fm^2
243	n	46.0	1.26	0.72	0.430
243	p	62.0	1.24	0.65	0.370

pairing, and the multipole-multipole interaction, both isoscalar and isovector, acting in the particle-hole and the particle-particle channels:

$$H = H_{sp} + V_{\text{pair}} + V_M^{ph} + V_M^{pp}. \quad (1)$$

Here H_{sp} is a one-body Hamiltonian, V_{pair} describes the monopole pairing, V_M^{ph} and V_M^{pp} are the particle-hole separable multipole interaction and particle–particle multipole pairing interaction, respectively.

As the mean field term we have taken the Woods-Saxon potential

$$\begin{aligned} V_{sp}(\vec{r}) &= V_{WS}(\vec{r}) + V_{so}(\vec{r}), \\ V_{WS}(\vec{r}) &= -V_0\,(1+\exp[(r-R(\theta,\varphi))/a])^{-1}, \\ V_{so}(\vec{r}) &= -\kappa(\vec{p}\times\sigma)\nabla V_{WS}(\vec{r}) \end{aligned} \quad (2)$$

with the parameters taken from [6]. They are presented in Table I. These parameters are quite close to those used in the calculations for the rare earth nuclei [6,7].

In the last years many calculations of the self-consistent nuclear mean field have been performed. They are based on different choices of the energy density functional [8–11]. Our approach is not self-consistent. For this reason it is necessary to justify a possibility to use the Woods-Saxon shape of the single particle potential for nuclei with $Z \sim 100$. This was discussed in details in [12]. Here we repeat the main points of the discussion.

The experimentally investigated nuclei with $Z \sim 100$ are deformed [13–15]. Beautiful experimental confirmation of the quadrupole deformation of the heavy elements with $Z \sim 100$ comes from γ-ray spectroscopy around $Z = 102$ and $N = 152$. The deformation leads to a more equal distribution of the single particle states emerging from the high-j (for these states the maximum of the single particle wave function is shifted closer to the nuclear surface) and low-j spherical subshells. For this reason a density profile of a deformed nucleus is relatively flat inside a nucleus [16–18]. This resembles the use of the phenomenological Woods-Saxon potential for these nuclei.

A comparison of the results of the self-consistent calculations with the single particle spectra obtained with the Woods-Saxon potential demonstrates a very good agreement for neutrons [19]. The large gaps in the single particle spectra for neutrons at $N = 152$ and protons at $Z = 96$ and 100 are clearly seen. Our proton single particle scheme confirms an existence of the gaps at $Z = 96$ and $Z = 100$. In addition, the results obtained in [20] and [21] indicate on the existence of the proton $1/2^-[521]$ and $7/2^-[514]$ single particle states near the Fermi level at $Z = 103$ in agreement with our scheme.

The values of the diffusion parameter used in our calculations have been fixed in the investigations of the properties of the actinide nuclei [6]. They are close to the values used in [22] where $a = 0.70$ fm for neutrons and $a = 0.68$ fm for protons.

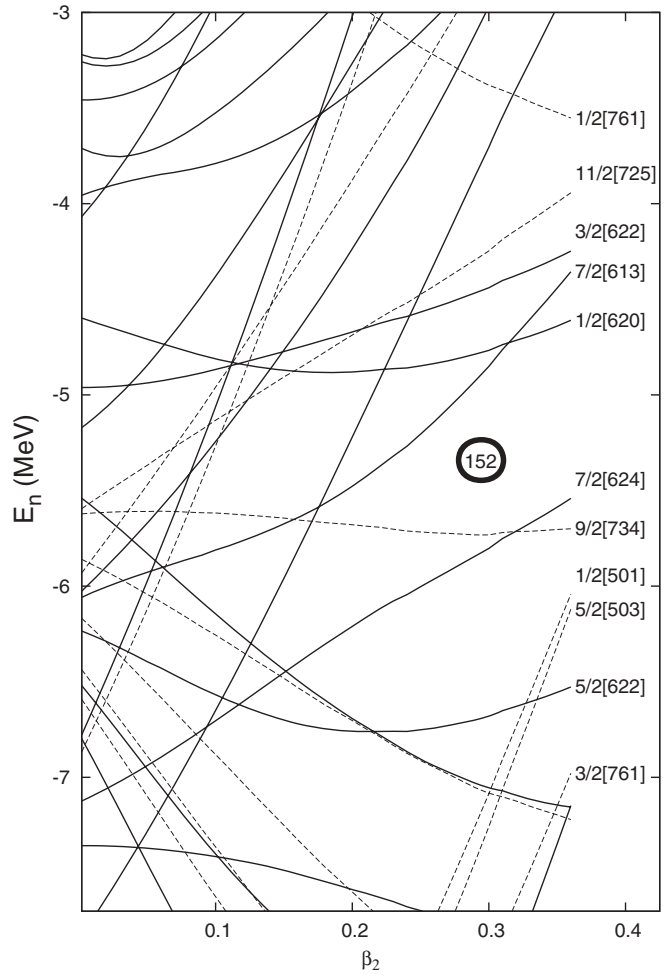

FIG. 1. The neutron single particle level scheme of the Woods-Saxon potential calculated with the parameters given in Table I.

There is some experimental information about the quadrupole deformation parameter β_2. Although the lifetimes have not been measured, the $B(E2)$ values of rotors are related to the energies of the first 2^+ levels by the empirical Grodzins relation [23]. Using this relation the value $\beta_2 = 0.27(2)$ has been deduced for ^{254}No [24]. The recent calculations [25] give the value $\beta_2 = 0.26$ for nuclei with $Z = 96$–104 and $N = 148$–156. The results of calculations presented in [26] demonstrate a stability of the quadrupole deformation for nuclei with $Z = 100$–104. For these reasons we have used the same value of $\beta_2 = 0.26$ for all considered nuclei. A comparison of the single particle energies of the levels lying around the Fermi surface and calculated for $\beta_2 = 0.25$ and $\beta_2 = 0.27$ with those obtained for $\beta_2 = 0.26$ show that in many cases deviations are less than 100 keV. Although for some states they take the values 150 keV or even 350 keV.

The neutron and proton single particle energies calculated with the Woods-Saxon potential as function of the deformation parameter β_2 are show in Figs. 1 and 2 taken

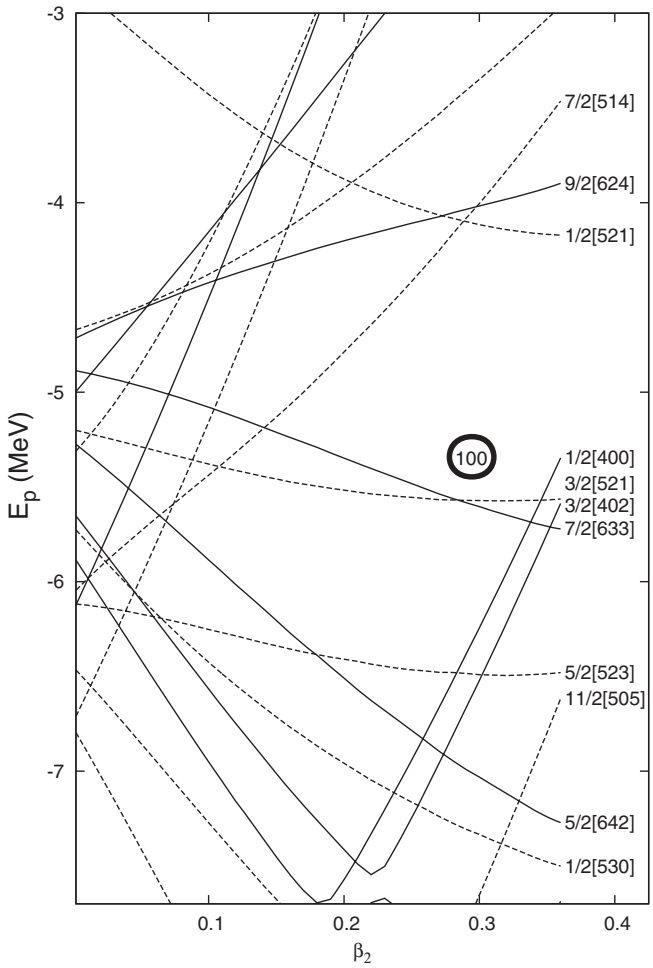

FIG. 2. The proton single particle level scheme of the Woods-Saxon potential calculated with the parameters given in Table I.

from [12]. Only fragments of the single particle schemes are presented.

In these figures the single particle levels are characterized by the asymptotic quantum numbers $K[Nn_z\Lambda]$. The parity of the single particle states π is given by $(-1)^N$.

If we compare the neutron single particle level scheme in Fig. 1 with the single particle energies extracted from the experimental data in [27] (Fig. 6 in [27]) we see in both cases just above $N = 152$ (only these neutron levels are shown in [27]) the same closely lying single particle levels, namely, $7/2^+[613]$, $1/2^+[620]$, $3/2^+[622]$, and $11/2^-[725]$. There is, however, a small difference in the ordering of the first two levels. We mention, however, that in [27] the energies of the neutron single particle levels have been extracted from the experimental data without taking into account the particle-vibration coupling.

The analysis of the proton single particle states [28] shows that the lowest states just above the gap at $Z = 96$ are $7/2^+[633]$ and $3/2^-[521]$ orbitals. The next three orbitals are $7/2^-[514]$, $1/2^-[521]$, and $9/2^+[624]$. This is just in a correspondence with the level scheme presented in Fig. 2 and calculated at $\beta_2 = 0.26$.

TABLE II. The experimental and the calculated energies of the $K^\pi = 2^+$ γ-vibrational states. The energies are given in keV. The quadrupole interaction constant κ_{22} is given in fm^2/MeV. This dimension of the interaction constant is determined by the use of the radial derivative of the Woods-Saxon potential as a form factor of the multipole-multipole interaction. The experimental data are taken from [31].

Nucleus	$E(2^+_\gamma)_{\rm exp}$	$E(2^+_\gamma)_{\rm calc}$		
		$\kappa_{22} = 0.0174$	$\kappa_{22} = 0.0165$	$\kappa_{22} = 0.0150$
^{246}Cm	1124	1225	1432	1680
^{248}Cm	1049	997	1229	1492
^{248}Cf	–	1289	1478	1708
^{250}Cf	1032	1079	1282	1517
^{252}Cf	805	781	987	1207
^{254}Cf	–	553	758	957
^{256}Cf	–	612	834	1058
^{250}Fm	–	1181	1354	1543
^{252}Fm	–	1021	1225	1462
^{254}Fm	694	735	933	1148
^{256}Fm	682	510	713	909
^{258}Fm	–	586	812	1042
^{252}No	–	1261	1461	1703
^{254}No	–	1065	1274	1520
^{256}No	–	809	1018	1251
^{258}No	–	531	725	914
^{260}No	–	602	826	1052
^{254}Rf	–	1291	1478	1626
^{256}Rf	–	1077	1277	1497
^{258}Rf	–	833	1049	1286
^{260}Rf	–	539	731	917
^{262}Rf	–	606	832	1057

The strength of the monopole pairing interaction has been adjusted to reproduce the experimental values of the odd-even mass differences for all considered nuclei.

The quadrupole-quadrupole interaction in the particle-hole channel is used below as the residual forces. The factorized multipole-multipole forces used in this paper are rather schematic. However, because the universal nucleon-nucleon interaction in heavy nuclei is not yet derived microscopically, it is preferred to use the forces adjusted to selected experimental data. The factorized multipole-multipole interactions have been used in many calculations of the properties of the low-lying collective and two-quasiparticle states performed in the framework of the quasiparticle-phonon model [5,7,29]. Thus, we can say that these forces have been tested in the known regions of the nuclide chart. Good agreement with the experimental data obtained previously shows that the model can be used to predict the properties of nuclei in the new region. As the result of the previous calculations we know the values of the multipole-multipole interaction constants determined for the rare earth and actinide regions. At the same time we know from the self-consistent estimates of the interaction constants of the multipole-multipole forces that they are smooth functions of the mass number A [30]. Therefore we can extrapolate the interaction constants from the known regions of Z and A to nuclei with $Z \sim 100$.

TABLE III. The quasiparticle structure of the calculated $K^\pi = 2^+$ γ-vibrational states of ^{246,248}Cm, $^{248-256}$Cf, and $^{250-258}$Fm. The quantum numbers of the most important two-quasiparticle components and their contribution (in %) to the norm of the γ phonon are shown. The quadrupole interaction constant κ_{22} is given in fm^2/MeV.

Nucleus	Structure (in %)		
	$\kappa_{22} = 0.0174$	$\kappa_{22} = 0.0165$	$\kappa_{22} = 0.0150$
^{246}Cm	*nn* 7/2[624] ⊗ 3/2[622] 23	*nn* 7/2[624] ⊗ 3/2[622] 27	*nn* 7/2[624] ⊗ 3/2[622] 35
	nn 5/2[622] ⊗ 1/2[620] 20	*nn* 5/2[622] ⊗ 1/2[620] 22	*nn* 5/2[622] ⊗ 1/2[620] 25
	pp 3/2[651] ⊗ 1/2[660] 7	*pp* 3/2[651] ⊗ 1/2[660] 7	*pp* 3/2[651] ⊗ 1/2[660] 6
^{246}Cm	*nn* 7/2[624] ⊗ 3/2[622] 30	*nn* 7/2[624] ⊗ 3/2[622] 36	*nn* 7/2[624] ⊗ 3/2[622] 51
	nn 3/2[622] ⊗ 1/2[620] 12	*nn* 3/2[622] ⊗ 1/2[620] 12	*nn* 3/2[622] ⊗ 1/2[620] 12
	nn 5/2[622] ⊗ 1/2[620] 12	*nn* 5/2[622] ⊗ 1/2[620] 12	*nn* 5/2[622] ⊗ 1/2[620] 10
	pp 3/2[651] ⊗ 1/2[660] 5		
^{248}Cf	*nn* 7/2[624] ⊗ 3/2[622] 24	*nn* 7/2[624] ⊗ 3/2[622] 29	*nn* 7/2[624] ⊗ 3/2[622] 37
	nn 5/2[622] ⊗ 1/2[620] 20	*nn* 5/2[622] ⊗ 1/2[620] 23	*nn* 5/2[622] ⊗ 1/2[620] 26
	nn 3/2[622] ⊗ 1/2[620] 6	*nn* 3/2[622] ⊗ 1/2[620] 6	*nn* 3/2[622] ⊗ 1/2[620] 5
	pp 3/2[521] ⊗ 1/2[521] 8	*pp* 3/2[521] ⊗ 1/2[521] 8	*pp* 3/2[521] ⊗ 1/2[521] 8
^{250}Cf	*nn* 7/2[624] ⊗ 3/2[622] 31	*nn* 7/2[624] ⊗ 3/2[622] 38	*nn* 7/2[624] ⊗ 3/2[622] 52
	nn 3/2[622] ⊗ 1/2[620] 12	*nn* 3/2[622] ⊗ 1/2[620] 13	*nn* 3/2[622] ⊗ 1/2[620] 11
	nn 5/2[622] ⊗ 1/2[620] 11	*nn* 5/2[622] ⊗ 1/2[620] 11	*nn* 5/2[622] ⊗ 1/2[620] 10
	pp 3/2[521] ⊗ 1/2[521] 7	*pp* 3/2[521] ⊗ 1/2[521] 7	*pp* 3/2[521] ⊗ 1/2[521] 6
^{252}Cf	*nn* 3/2[622] ⊗ 1/2[620] 39	*nn* 3/2[622] ⊗ 1/2[620] 48	*nn* 3/2[622] ⊗ 1/2[620] 63
	nn 7/2[624] ⊗ 3/2[622] 16	*nn* 7/2[624] ⊗ 3/2[622] 16	*nn* 7/2[624] ⊗ 3/2[622] 13
	nn 7/2[613] ⊗ 3/2[611] 6	*nn* 7/2[613] ⊗ 3/2[611] 5	
	nn 5/2[622] ⊗ 1/2[620] 6		
^{254}Cf	*nn* 3/2[622] ⊗ 1/2[620] 55	*nn* 3/2[622] ⊗ 1/2[620] 66	*nn* 3/2[622] ⊗ 1/2[620] 81
	nn 7/2[624] ⊗ 3/2[622] 8	*nn* 7/2[624] ⊗ 3/2[622] 6	
	nn 7/2[613] ⊗ 3/2[611] 6		
^{256}Cf	*nn* 3/2[622] ⊗ 1/2[620] 47	*nn* 3/2[622] ⊗ 1/2[620] 59	*nn* 3/2[622] ⊗ 1/2[620] 76
	nn 7/2[613] ⊗ 3/2[611] 7	*nn* 7/2[613] ⊗ 3/2[611] 6	
	nn 9/2[615] ⊗ 5/2[613] 7	*nn* 9/2[615] ⊗ 5/2[613] 6	
^{250}Fm	*nn* 7/2[624] ⊗ 3/2[622] 15	*nn* 7/2[624] ⊗ 3/2[622] 15	*nn* 7/2[624] ⊗ 3/2[622] 10
	nn 5/2[622] ⊗ 1/2[620] 14	*nn* 5/2[622] ⊗ 1/2[620] 14	*nn* 5/2[622] ⊗ 1/2[620] 10
	pp 3/2[521] ⊗ 1/2[521] 33	*pp* 3/2[521] ⊗ 1/2[521] 40	*pp* 3/2[521] ⊗ 1/2[521] 41
			pp 7/2[514] ⊗ 3/2[521] 23
^{252}Fm	*nn* 7/2[624] ⊗ 3/2[622] 23	*nn* 7/2[624] ⊗ 3/2[622] 26	nn 7/2[624] ⊗ 3/2[622] 31
	nn 3/2[622] ⊗ 1/2[620] 10	*nn* 3/2[622] ⊗ 1/2[620] 10	*nn* 3/2[622] ⊗ 1/2[620] 9
	nn 5/2[622] ⊗ 1/2[620] 10	*nn* 5/2[622] ⊗ 1/2[620] 10	*nn* 5/2[622] ⊗ 1/2[620] 8
	pp 3/2[521] ⊗ 1/2[521] 22	*pp* 3/2[521] ⊗ 1/2[521] 25	*pp* 3/2[521] ⊗ 1/2[521] 31
^{254}Fm	*nn* 3/2[622] ⊗ 1/2[620] 37	*nn* 3/2[622] ⊗ 1/2[620] 45	*nn* 3/2[622] ⊗ 1/2[620] 58
	nn 7/2[624] ⊗ 3/2[622] 15	*nn* 7/2[624] ⊗ 3/2[622] 15	*nn* 7/2[624] ⊗ 3/2[622] 12
	nn 7/2[613] ⊗ 3/2[611] 6		
	pp 3/2[521] ⊗ 1/2[521] 11	*pp* 3/2[521] ⊗ 1/2[521] 11	*pp* 3/2[521] ⊗ 1/2[521] 9
^{256}Fm	*nn* 3/2[622] ⊗ 1/2[620] 54	*nn* 3/2[622] ⊗ 1/2[620] 65	*nn* 3/2[622] ⊗ 1/2[620] 80
	nn 7/2[624] ⊗ 3/2[622] 8	*nn* 7/2[624] ⊗ 3/2[622] 6	
	nn 7/2[613] ⊗ 3/2[611] 6		
	pp 3/2[521] ⊗ 1/2[521] 6	*pp* 3/2[521] ⊗ 1/2[521] 5	
^{258}Fm	*nn* 3/2[622] ⊗ 1/2[620] 44	*nn* 3/2[622] ⊗ 1/2[620] 55	*nn* 3/2[622] ⊗ 1/2[620] 73
	nn 7/2[613] ⊗ 3/2[611] 7	*nn* 7/2[613] ⊗ 3/2[611] 6	
	nn 9/2[615] ⊗ 5/2[613] 7	*nn* 9/2[615] ⊗ 5/2[613] 6	
	pp 3/2[521] ⊗ 1/2[521] 8	*pp* 3/2[521] ⊗ 1/2[521] 7	*pp* 3/2[521] ⊗ 1/2[521] 5

Then, the Hamiltonian (1) is expressed in terms of the quasiparticle creation and annihilation operators obtained from the corresponding particle operators through the u-v Bogoliubov transformation. The quasiparticle Hamiltonian is then adopted to solve the RPA eigenvalue equations and generates the RPA phonon operators.

III. GAMMA-VIBRATIONAL STATES

The γ-vibrational states can be considered as the most collective vibrational excitations in many well-deformed axially symmetric nuclei. They have been observed in many nuclei and are well-understood theoretically. However, the experimental information on these excitations in nuclei with

TABLE IV. The quasiparticle structure of the calculated $K^\pi = 2^+$ γ-vibrational states of $^{252-260}$No and $^{254-262}$Rf. The quantum numbers of the most important two-quasiparticle components and their contribution (in %) to the norm of the γ phonon are shown. The quadrupole interaction constant κ_{22} is given in fm^2/MeV.

Nucleus	Structure (in %)		
	$\kappa_{22} = 0.0174$	$\kappa_{22} = 0.0165$	$\kappa_{22} = 0.0150$
^{252}No	*nn* 5/2[622] ⊗ 1/2[620] 17	*nn* 5/2[622] ⊗ 1/2[620] 18	*nn* 5/2[622] ⊗ 1/2[620] 19
	nn 7/2[624] ⊗ 3/2[622] 17	*nn* 7/2[624] ⊗ 3/2[622] 18	*nn* 7/2[624] ⊗ 3/2[622] 19
	nn 3/2[622] ⊗ 1/2[620] 5	*nn* 3/2[622] ⊗ 1/2[620] 5	
	pp 3/2[521] ⊗ 1/2[521] 21	*pp* 3/2[521] ⊗ 1/2[521] 25	*pp* 3/2[521] ⊗ 1/2[521] 33
^{254}No	*nn* 7/2[624] ⊗ 3/2[622] 25	*nn* 7/2[624] ⊗ 3/2[622] 29	*nn* 7/2[624] ⊗ 3/2[622] 37
	nn 3/2[622] ⊗ 1/2[620] 11	*nn* 3/2[622] ⊗ 1/2[620] 11	*nn* 3/2[622] ⊗ 1/2[620] 11
	nn 5/2[622] ⊗ 1/2[620] 11	*nn* 5/2[622] ⊗ 1/2[620] 10	*nn* 5/2[622] ⊗ 1/2[620] 9
	pp 3/2[521] ⊗ 1/2[521] 16	*pp* 3/2[521] ⊗ 1/2[521] 18	*pp* 3/2[521] ⊗ 1/2[521] 21
^{256}No	*nn* 3/2[622] ⊗ 1/2[620] 34	*nn* 3/2[622] ⊗ 1/2[620] 41	*nn* 3/2[622] ⊗ 1/2[620] 55
	nn 7/2[624] ⊗ 3/2[622] 16	*nn* 7/2[624] ⊗ 3/2[622] 16	*nn* 7/2[624] ⊗ 3/2[622] 14
	nn 7/2[613] ⊗ 3/2[611] 6	*nn* 7/2[613] ⊗ 3/2[611] 5	
	nn 5/2[622] ⊗ 1/2[620] 6	*nn* 5/2[622] ⊗ 1/2[620] 5	
	pp 3/2[521] ⊗ 1/2[521] 10	*pp* 3/2[521] ⊗ 1/2[521] 9	*pp* 3/2[521] ⊗ 1/2[521] 8
^{258}No	*nn* 3/2[622] ⊗ 1/2[620] 56	*nn* 3/2[622] ⊗ 1/2[620] 66	*nn* 3/2[622] ⊗ 1/2[620] 81
	nn 7/2[624] ⊗ 3/2[622] 7	*nn* 7/2[624] ⊗ 3/2[622] 6	
	nn 7/2[613] ⊗ 3/2[611] 6		
	pp 3/2[521] ⊗ 1/2[521] 5		
^{260}No	*nn* 3/2[622] ⊗ 1/2[620] 46	*nn* 3/2[622] ⊗ 1/2[620] 57	*nn* 3/2[622] ⊗ 1/2[620] 75
	nn 7/2[613] ⊗ 3/2[611] 7	*nn* 7/2[613] ⊗ 3/2[611] 6	
	nn 9/2[615] ⊗ 5/2[613] 7	*nn* 9/2[615] ⊗ 5/2[613] 6	
	pp 3/2[521] ⊗ 1/2[521] 5		
^{254}Rf	*nn* 7/2[624] ⊗ 3/2[622] 21	*nn* 7/2[624] ⊗ 3/2[622] 21	*nn* 7/2[624] ⊗ 3/2[622] 82
	nn 5/2[622] ⊗ 1/2[620] 19	*nn* 5/2[622] ⊗ 1/2[620] 19	
	nn 3/2[622] ⊗ 1/2[620] 6		
	pp 3/2[521] ⊗ 1/2[521] 8	*pp* 3/2[521] ⊗ 1/2[521] 17	
	pp 5/2[512] ⊗ 1/2[521] 8	*pp* 5/2[52] ⊗ 1/2[521] 8	
^{256}Rf	*nn* 7/2[624] ⊗ 3/2[622] 31	*nn* 7/2[624] ⊗ 3/2[622] 36	*nn* 7/2[624] ⊗ 3/2[622] 39
	nn 3/2[622] ⊗ 1/2[620] 12	*nn* 3/2[622] ⊗ 1/2[620] 12	*nn* 3/2[622] ⊗ 1/2[620] 9
	nn 5/2[622] ⊗ 1/2[620] 12	*nn* 5/2[622] ⊗ 1/2[620] 11	*nn* 5/2[622] ⊗ 1/2[620] 8
	pp 3/2[521] ⊗ 1/2[521] 6	*pp* 3/2[521] ⊗ 1/2[521] 6	*pp* 3/2[521] ⊗ 1/2[521] 24
^{258}Rf	*nn* 3/2[622] ⊗ 1/2[620] 35	*nn* 3/2[622] ⊗ 1/2[620] 43	*nn* 3/2[622] ⊗ 1/2[620] 57
	nn 7/2[624] ⊗ 3/2[622] 16	*nn* 7/2[624] ⊗ 3/2[622] 17	*nn* 7/2[624] ⊗ 3/2[622] 15
	nn 7/2[613] ⊗ 3/2[611] 6	*nn* 7/2[613] ⊗ 3/2[611] 5	
	nn 5/2[622] ⊗ 1/2[620] 6	*nn* 5/2[622] ⊗ 1/2[620] 5	
^{260}Rf	*nn* 3/2[622] ⊗ 1/2[620] 57	*nn* 3/2[622] ⊗ 1/2[620] 68	*nn* 3/2[622] ⊗ 1/2[620] 82
	nn 7/2[624] ⊗ 3/2[622] 8	*nn* 7/2[624] ⊗ 3/2[622] 6	
	nn 7/2[613] ⊗ 3/2[611] 6		
^{262}Rf	*nn* 3/2[622] ⊗ 1/2[620] 47	*nn* 3/2[622] ⊗ 1/2[620] 59	*nn* 3/2[622] ⊗ 1/2[620] 77
	nn 7/2[613] ⊗ 3/2[611] 7	*nn* 7/2[613] ⊗ 3/2[611] 6	
	nn 9/2[615] ⊗ 5/2[613] 7	*nn* 9/2[615] ⊗ 5/2[613] 6	
	nn 7/2[624] ⊗ 3/2[622] 6		

$Z \sim 100$ is rather scarce. The γ-vibrational excitations have been observed only in ^{246,248}Cm, ^{250,252}Cf, and ^{254,256}Fm.

The results of our calculations of the energies and the two-quasiparticle structure of the γ-vibrational states are presented in Tables II, III, and IV for three values of the interaction constant of the quadrupole-quadrupole forces in the particle-hole channel. The value of $\kappa_{22} = 0.0174$ was fixed previously in the calculations for U isotopes. As it is seen from Table II with this value of κ_{22} we obtain a good description of the known experimental data. The results for the other two values of κ_{22} are shown in order to get an idea of sensitivity of the energies of γ phonons to variations of κ_{22}. The results presented in Tables II–IV are obtained without taking into account a mixing of the one-phonon and two-phonon states.

The results given in Table II show that for all considered elements the energy of the γ-vibrational state takes its minimum in nuclei with the number of neutrons equal to $N = 156$. To understand this fact let us analyze the quasiparticle structure of the γ phonon. It is seen from Tables III and IV that if the number of neutrons approaches the value $N = 156$ the contribution of the two-quasiparticle component 3/2[622] ⊗ 1/2[620] to the norm of the γ-vibrational one phonon state

becomes the largest one. It is seen from Fig. 1 (left part) that at $N = 156$ and $\beta_2 = 0.26$ the neutron Fermi level is located between the single particle states $3/2^+[622]$ and $1/2^+[620]$ and therefore the two-quasiparticle state consisting of these quasiparticles has the smallest energy compared to the other two-quasiparticle states. This energy is equal to 1.251 MeV. This explains why the energy of the γ-vibrational one phonon state has a minimum when the number of neutrons is equal to $N = 156$. The other important neutron two-quasiparticle component $7/2[613]\otimes 3/2[622]$ has at $N = 156$ the energy 1.343 MeV. However, our calculations have shown that the energy of this two-quasiparticle component has its minimum not at $N = 156$ but at $N = 154$. The energies of the others neutron two-quasiparticle components are larger than 2 MeV if $N = 156$.

To verify further the effect of the neutron two-quasiparticle component $3/2[622]\otimes 1/2[620]$ on the neutron number dependence of the energies of the γ-vibrational states we have shifted artificially the energy of the neutron single particle state $3/2[622]$ up, i.e., decreased its binding, by 0.6 MeV and 1.2 MeV. As the result there appear two and three others neutron single particle states, respectively, between the $1/2[620]$ and $3/2[622]$ neutron single particle states at $\beta_2 = 0.26$. This shift of the energy of the single particle state changes the neutron number dependence of the energy of the γ-vibrational state in such a way that the minimum at $N = 156$ disappears and $E(2^+_\gamma)$ decreases continuously if N increases from $N = 150$ to $N = 160$. The absolute value of $E(2^+_\gamma)$ is increased because of this shift of the energy of the single particle neutron state $3/2^+[622]$ for all considered isotopes. This fact indicate on the strong neutron number dependence of the $E(2^+_\gamma)$ and its absolute value on a relative position of the neutron single particle states $1/2[620]$ and $3/2[622]$.

The single particle neutron states $3/2^+[622]$ and $1/2^+[620]$ are the members of the pseudospin doublet with the quantum numbers $[\tilde{N}\tilde{n}_3\tilde{\Lambda}] = [521]$. The connection of the pseudospin quantum numbers to the Nilsson asymptotic quantum numbers $[Nn_z\Lambda_{1,2}]$ is the following: $\tilde{N} = N - 1, \tilde{n}_3 = n_3, \tilde{\Lambda} = \frac{1}{2}(\Lambda_1 + \Lambda_2)$. Our discussion above have shown that the difference between the single particle energies of these states influences on the neutron number dependence and the absolute energy value of the γ-vibrational state. Small splitting of the $1/2[620]$ and $3/2[622]$ single particle state will mean that the pseudospin symmetry is approximately preserved. Thus, the experimental observation (or nonobservation) of the minimum of the energy of the γ-vibrational one phonon state when the number of neutrons is equal to $N = 156$ is important for studying manifestation of the pseudospin symmetry in very heavy exotic nuclei. The value of $E(2^+_\gamma)$ at $N = 156$ gives an information on the splitting of the pseudospin doublet $[\tilde{N}\tilde{n}_3\tilde{\Lambda}] = [521]$.

To get the feeling of the effect of the mixing of the one-phonon and the two-phonon states we have performed the calculations for the Cf isotopes with and without the mixing. The results of the calculations have shown that the mixing of the one-phonon and the two-phonon states decreases the energies of the $E(2^+_\gamma)$ states approximately by 50 keV. However, this mixing does not influence on the neutron number dependence of the $E(2^+_\gamma)$ and keeps the minimum at $N = 156$. Approximately 98% of the norm of the state vectors corresponding to the γ-vibrational states obtained in the calculations including the mixing effect are provided with the one-phonon component.

IV. SUMMARY

Based on the quasiparticle-phonon model we have calculated the energies and the two-quasiparticle structure of the γ-vibrational states. The results of calculations show that in the isotopes of Cm, Cf, Fm, No, and Rf the energies of the one-phonon γ-vibrational states have a minimum if the number of neutrons is equal to $N = 156$. In addition, a contribution of the two-quasiparticle component $3/2^+[622]\otimes 1/2^+[620]$ to the norm of the γ-vibrational one-phonon state becomes the largest one at $N = 156$. The single particle states $3/2^+[622]$ and $1/2^+[620]$ are the members of the pseudospin doublet. Thus, the experimental information on the energies of the γ-vibrational states in nuclei with $Z \sim 100$ can be used to determine a splitting of the $[\widetilde{521}]$ pseudospin doublet.

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