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Application of the complex-scaling method to four-nucleon scattering above break-up threshold

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A formalism based on the complex-scaling method is developed and applied to solve the four-nucleon scattering problem above the break-up threshold. Converged calculations are presented for the isospin $T = 0$ and $T = 1$ channels in several energy regions above both the three- and the four-particle break-up thresholds.

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I. INTRODUCTION

The theoretical description of quantum-mechanical collisions turns out to be one of the most complex and challenging fundamental problems in theoretical physics. The calculations of the scattering process in configuration space require boundary conditions. Moreover, the complexity of these boundary conditions drastically increases with increasing number of open channels and, in particular, when the break-up channels are open. Momentum-space calculations do not involve complex boundary conditions; nevertheless, one has to deal with Green's functions, which carry singularities of increasing complexity. The description of the break-up process in the three-nucleon system became available already a while ago both in configuration space, by imposing complex boundary conditions, as well as in momentum space, by integrating moving singularities of the Green's functions [1]. Nevertheless, such direct approaches have not been extended beyond the three-body system, while their further development is stalled due to the rapid rise in complexity when the number of particles increases. Therefore, finding a method which would allow one to solve the scattering problem by avoiding difficulties due to the complex boundary conditions (singularities) is of great importance. One can acknowledge several recent efforts [2–7]. First of all, one should mention the Lorentz integral transform method [2], which allows one to calculate the integral cross section of the scattering process using bound-state-like techniques. However, this method becomes rather involved for the differential observables. Some time later a complex-energy method was presented [3]; this may provide full information about the scattering process. Lately, after some technical improvements, this method has even been applied to describe realistic four-nucleon break-up process [8]. Other recent developments include a momentum lattice technique [4] and a method based on a discretized continuum solution [7], which, respectively, must be tested in the four-body sector and for the break-up case.

In [6] we have presented the application of the complex-scaling method to describe three-nucleon scattering, including break-up reactions. This method allows one to treat a strong interaction of almost any complexity: realistic local and nonlocal potentials, optical potentials, and strong interactions in conjunction with a repulsive Coulomb force. Furthermore, both elastic and break-up amplitudes might be addressed by this method, thus representing the full description of three-particle collisions. There are no formal obstacles in extending this method to treat the collisions involving any number of particles, as long as one is able to handle a large-scale numerical problem. The logical evolution of the former work is to perform the first calculations in the four-body sector, treating nuclear reactions above the four-nucleon break-up threshold.

One should mention that an alternative variant of the complex-scaling method to calculate scattering observables above the break-up threshold has been proposed by Giraud *et al.* [9]. It relies on the spectral function formalism and requires diagonalization of the full N-body matrices to get converged results; the last fact makes it difficult to extend the method of Giraud *et al.* beyond the $N = 3$ case.

II. FORMALISM

In this study the four-body problem is treated using Faddeev-Yakubovski (FY) equations in configuration space. Using the FY formalism the wave function of the system is naturally decomposed into so-called FY components (FYCs) For the $A = 4$ system, two types of FYCs appear: type K ($K^l_{ij,k}$) and type H (H^{kl}_{ij}), where i, j, k, and l are particle indexes. By permuting particle indexes one may construct 12 independent components of type K as well as 6 independent components of type H. The asymptotes of the components $K^l_{ij,k}$ and H^{kl}_{ij} incorporate (3 + 1)- and (2 + 2)-particle channels, respectively (see Fig. 1).

We consider a four-nucleon system in the isospin formalism, where neutrons and protons are treated as isospin-degenerate states of the same particle nucleon. FY components which differ by the order of the particle indexing become related due to the symmetry of particle permutation. There remain only two independent FYCs, which we denote K and H by omitting their indexing. The FY equations for a case of four identical particles read [10,11]

$$(E - H_0 - V_{12})K = V_{12}(P^+ + P^-)[(1 + Q)K + H],$$
$$(E - H_0 - V_{12})H = V_{12}\tilde{P}[(1 + Q)K + H], \qquad (1)$$

where H_0 is a kinetic energy operator, whereas V_{ij} describes the interaction between the ith and jth nucleons. FYCs may be converted from one coordinate set to another by using particle permutation operators, which are summarized as follows: $P^+ = (P^-)^{-1} \equiv P_{23}P_{12}$, $Q \equiv -P_{34}$, and

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$\tilde{P} \equiv P_{13}P_{24} = P_{24}P_{13}$, where P_{ij} indicates operator permuting particles i and j.

In terms of the FYCs, the total wave function of an $A = 4$ system is given by

$$\Psi = [1 + (1 + P^+ + P^-)Q](1 + P^+ + P^-)K + (1 + P^+ + P^-)(1 + \tilde{P})H. \quad (2)$$

Each FY component $F = (K, H)$ is considered as a function, described in its proper set of Jacobi coordinates $(\vec{x}, \vec{y}, \vec{z})$ (see Fig. 1), and defined, respectively, by

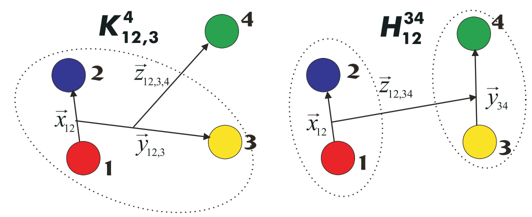

FIG. 1. (Color online) The FY components $K^4_{12,3}$ and H^{34}_{12} for a given particle ordering. As $z \to \infty$, the K components describe (3 + 1)-particle channels, while the H components contain asymptotic states of 2 + 2 channels.

$$\begin{aligned} \vec{x}_K &= \vec{r}_2 - \vec{r}_1, & \vec{x}_H &= \vec{r}_2 - \vec{r}_1, \\ \vec{y}_K &= \sqrt{\frac{4}{3}}\left(\vec{r}_3 - \frac{\vec{r}_1 + \vec{r}_2}{2}\right), & \vec{y}_H &= \vec{r}_4 - \vec{r}_3, \\ \vec{z}_K &= \sqrt{\frac{3}{2}}\left(\vec{r}_4 - \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}\right), & \vec{z}_H &= \sqrt{2}\left(\frac{\vec{r}_3 + \vec{r}_4}{2} - \frac{\vec{r}_1 + \vec{r}_2}{2}\right). \end{aligned} \quad (3)$$

Angular, spin, and isospin dependence of these components is described using tripolar harmonics $\mathcal{Y}_\alpha(\hat{x}, \hat{y}, \hat{z})$, i.e.,

$$\langle \vec{x}\vec{y}\vec{z}|F\rangle = \sum_\alpha \frac{F_\alpha(xyz)}{xyz}\, \mathcal{Y}_\alpha(\hat{x}, \hat{y}, \hat{z}). \quad (4)$$

The quantities $F_\alpha(xyz)$ are called regularized FY amplitudes, where the label α holds for a set of 10 intermediate quantum numbers describing a given four-nucleon quantum state (J^π, T, T_z). By using the LS-coupling scheme the tripolar harmonics are defined for components K and H, respectively, by

$$\mathcal{Y}_{\alpha_K} \equiv \{[(l_x l_y)_{l_{xy}} l_z]_L [((s_1 s_2)_{s_x} s_3)_{S3} s_4]_S\}_{J^\pi \mathcal{M}} \otimes [((t_1 t_2)_{t_x} t_3)_{T3} t_4]_{T\mathcal{T}_Z}, \quad (5)$$

$$\mathcal{Y}_{\alpha_H} \equiv \{[(l_x l_y)_{l_{xy}} l_z]_L [(s_1 s_2)_{s_x} (s_3 s_4)_{s_y}]_S\}_{J^\pi \mathcal{M}} \otimes [(t_1 t_2)_{t_x} (t_3 t_4)_{t_y}]_{T\mathcal{T}_Z}. \quad (6)$$

The next step is to separate the incoming plane wave of two colliding clusters from K (or H) partial components:

$$K(\vec{x}, \vec{y}, \vec{z}) = K^{\text{out}}(\vec{x}, \vec{y}, \vec{z}) + K^{\text{in}}(\vec{x}, \vec{y}, \vec{z}), \quad (7)$$

$$H(\vec{x}, \vec{y}, \vec{z}) = H^{\text{out}}(\vec{x}, \vec{y}, \vec{z}) + H^{\text{in}}(\vec{x}, \vec{y}, \vec{z}). \quad (8)$$

The expansion of the incoming plane wave in tripolar harmonics provides

$$F^{\text{in}}_{\alpha_K}(x, y, z) = \delta_{3+1}\kappa^{(3)}_{\alpha_K}(x, y) \cdot \hat{j}_{l_z}(q_3 z)/q_3, \quad (9)$$

$$F^{\text{in}}_{\alpha_H}(x, y, z) = \delta_{2+2}\kappa^{(22)}_{\alpha_H}(x, y) \cdot \hat{j}_{l_z}(q_{22} z)/q_{22}. \quad (10)$$

Here $\delta_{3+1} = 1$ and $\delta_{2+2} = 0$ if one considers the incoming state of one particle projected on the bound cluster of three particles (like $n + {}^3$H). Alternatively, $\delta_{3+1} = 0$ and $\delta_{2+2} = 1$ if one considers the incoming state of (2 + 2)-particle clusters (like $d + d$). The functions $\kappa^{(3)}_{\alpha_K}(x, y)$ and $\kappa^{(22)}_{\alpha_H}(x, y)$ represent regularized Faddeev amplitudes of the corresponding bound-state wave functions containing three- and (2 + 2)-particle clusters, respectively. The terms $q_3^2 = \frac{m}{\hbar^2}(E - \epsilon_3)$ and $q_{22}^2 = \frac{m}{\hbar^2}(E - \epsilon_2 - \epsilon_2)$ are the momenta of the relative motion of the free clusters. Here we suppose that the system possesses only one three-particle bound state and only one two-particle bound state where the binding energies equal ϵ_3 and ϵ_2, respectively. By inserting Eq. (7) into Eq. (1) one may rewrite FY equations in their driven form:

$$\begin{aligned} (E - H_0 - V_{12})K^{\text{out}} - V_{12}(P^+ + P^-)[(1 + Q)K^{\text{out}} + H^{\text{out}}] &= V_{12}(P^+ + P^-)[(1 + Q)H^{\text{in}} + QK^{\text{in}}], \\ (E - H_0 - V_{12})H^{\text{out}} - V_{12}\tilde{P}[(1 + Q)K^{\text{out}} + H^{\text{out}}] &= V_{12}\tilde{P}[(1 + Q)K^{\text{in}}]. \end{aligned} \quad (11)$$

One may note that the K^{out} and H^{out} components in the asymptote contain only various combinations of the outgoing waves. The FY components of both types retain parts of the outgoing wave of the break-up into three and four clusters. In addition, the components K^{out} retain an outgoing wave in the (3 + 1)-particle channel, whereas the components H^{out} retain an outgoing wave in the (2 + 2)-particle channel. In the asymptote, where at least one particle recedes from the others, they take the following forms:

$$\begin{aligned} K^{\rm out}(\vec{x},\vec{y},\vec{z}) &= A_{31}(\hat{z})\psi^{(3)}(\vec{x},\vec{y})\frac{\exp(iq_3z)}{|z|} + A^K_{211}(\hat{y},\hat{z})\psi^{(2)}(\vec{x})\frac{\exp(iq_2X)}{|X|^{5/2}} + A^K_{1111}(\hat{x},\hat{y},\hat{z})\frac{\exp(iq_1R)}{|R|^4},\\ H^{\rm out}(\vec{x},\vec{y},\vec{z}) &= A_{22}(\hat{z})\psi^{(22)}(\vec{x},\vec{y})\frac{\exp(iq_3z)}{|z|} + A^H_{211}(\hat{y},\hat{z})\psi^{(2)}(\vec{x})\frac{\exp(iq_2X)}{|X|^{5/2}} + A^H_{121}(\hat{x},\hat{z})\psi^{(2)}(\vec{y})\frac{\exp(iq_2Y)}{|Y|^{5/2}}\\ &\quad + A^H_{1111}(\hat{x},\hat{y},\hat{z})\frac{\exp(iq_1R)}{|R|^4}, \end{aligned} \tag{12}$$

where terms A represent various types of amplitudes of scattering in two, three, and four clusters. Wave functions $\psi^{(3)}$, $\psi^{(22)}$, and $\psi^{(2)}$ represent various cluster bound states and thus are exponentially bound.

One may introduce the complex-scaling operator (CSO)

$$\widehat{S} = e^{i\theta r\frac{\partial}{\partial r}} = e^{i\theta(x\frac{\partial}{\partial x}+y\frac{\partial}{\partial y}+z\frac{\partial}{\partial z})}, \tag{13}$$

where $r^2 = x^2 + y^2 + z^2$, with the complex-scaling angle θ of free choice. The action of the complex-scaling operator on an outgoing wave gives

$$\widehat{S}\exp(ikr) = \exp(-kr\sin\theta)\exp(ikr\cos\theta). \tag{14}$$

Therefore, if one acts on $K^{\rm out}$ (or $H^{\rm out}$) with the complex-scaling operator and chooses the complex-scaling angle $0 < \theta < \pi/2$, one must get exponentially decreasing functions $\widetilde{K^{\rm out}} \equiv \widehat{S}K^{\rm out}$ and $\widetilde{H^{\rm out}} \equiv \widehat{S}H^{\rm out}$. By acting on Eq. (11) with the complex-scaling operator $\widehat{S}$ one gets

$$\begin{aligned} \widehat{S}(E-H_0-V_{12})\widehat{S}^{-1}\widetilde{K^{\rm out}} - \widehat{S}V_{12}\widehat{S}^{-1}(P^++P^-)[(1+Q)\widetilde{K^{\rm out}}+\widetilde{H^{\rm out}}] &= \widehat{S}V_{12}\widehat{S}^{-1}(P^++P^-)[(1+Q)\widetilde{H^{\rm in}}+Q\widetilde{K^{\rm in}}],\\ \widehat{S}(E-H_0-V_{12})\widehat{S}^{-1}\widetilde{H^{\rm out}} - \widehat{S}V_{12}\widehat{S}^{-1}\tilde{P}[(1+Q)\widetilde{K^{\rm out}}+\widetilde{H^{\rm out}}] &= \widehat{S}V_{12}\widehat{S}^{-1}\tilde{P}[(1+Q)\widetilde{K^{\rm in}}], \end{aligned} \tag{15}$$

based on the fact that permutation operators, being independent of r, commute with the complex-scaling operator. Since the components $\widetilde{K^{\rm out}}$ and $\widetilde{H^{\rm out}}$ are exponentially bound, one may use standard bound-state techniques to represent them and eventually solve Eqs. (11). Nevertheless, in order to obtain stable numerical results—either using discretization techniques on finite grids or square-integrable bases to represent $\widetilde{K^{\rm out}}$ ($\widetilde{H^{\rm out}}$) components—one must ensure that the inhomogeneous term on the right-hand side of Eq. (11) is also exponentially bound. It is easy to realize that the short-range potential term nullifies the expression on the right-hand side everywhere except for $x < r_V$, where r_V is the range of the interaction. Therefore, in order to see the convergence of the inhomogeneous term one should concentrate on its behavior in the $x \ll (y^2 + z^2)^{1/2}$ region of space. In a similar manner, as has been demonstrated for a three-body case [6], convergence of the inhomogeneous term implies some limiting conditions for the complex-scaling angle to be used. For a system of four identical particles, these conditions turn out to be

$$\tan\theta < \sqrt{\frac{2B_3}{E_{\rm c.m.}}}, \tag{16}$$

if the incoming plane wave of $1 + 3$ type is considered with a binding energy of a three-particle cluster equal to B_3, and

$$\tan\theta < \sqrt{\frac{2B_2}{E_{\rm c.m.}}}, \tag{17}$$

when the incoming plane wave of $2 + 2$ type is considered with a binding energy of a two-particle cluster equal to B_2. $E_{\rm c.m.}$ is the scattering energy of two clusters in the center-of-mass frame. In the last equation one should take a smaller value of the two-body binding energies if the $2 + 2$ incoming plane wave combines two-body clusters in different energy states.

Expressions (16) and (17) are compatible with rather large rotation angle region at low energies. For example, at the four-particle break-up threshold this limit gives $\theta < 54.7^\circ$. The choice of a large angle at low energy is moreover restricted by the ability to use analytically continued short-range potentials $\widehat{S}V_{12}\widehat{S}^{-1}$, which may become divergent and strongly oscillating when using large θ values [12,13]. Well above the four-particle break-up threshold the aforementioned limit may become important, as demonstrated in [6] for a three-body case. However, at these higher energies the solution converges at smaller θ values, since exponents in the outgoing wave components $\widetilde{K^{\rm out}}$ and $\widetilde{H^{\rm out}}$ decrease faster because the values of the momenta q increase with energy [see Eqs. (12) and (14)]. The most complicated regions in which to obtain the converged solution are situated around the thresholds where one of the momentum q value is small and thus one of the outgoing waves is dominated by avery slow exponent.

The scattering amplitudes may be obtained by using Green's theorem

$$\begin{aligned} A(\vec{k}) = \frac{m}{\hbar^2}[\langle F^{\rm in}(\vec{k})|H_0 - E|\Psi(\vec{k})\rangle\\ - \langle\Psi(\vec{k})|H_0 - E|F^{\rm in}(\vec{k})\rangle], \end{aligned} \tag{18}$$

where $F^{\rm in}(\vec{k})$ is a part of the wave function describing the incoming plane wave, whereas $\Psi(\vec{k})$ is the total wave function.

For the transition amplitudes to (3 + 1)-particle channels, using Eq. (11) and employing symmetry properties of FY components, one gets

$$A_{31}(\vec{k}) = \frac{m}{\hbar^2}\iiint_V d\mathcal{V}_{xyz}\{6\mathcal{K}_K^\dagger(H_0 - E)(K^{\text{out}} + K^{\text{in}}) + 3\mathcal{K}_H^\dagger(H_0 - E)(H^{\text{out}} + H^{\text{in}})\}, \tag{19}$$

with $\mathcal{K}_K = \frac{1}{2}(P^+ + P^-)Q(1 + P^+ + P^-)K^{\text{in}}$ and $\mathcal{K}_H = \tilde{P}(1 + P^+ + P^-)K^{\text{in}}$. The scattering amplitudes are scalars and do not depend on r; therefore they are not affected by the complex-scaling operator $\hat{S}A(\vec{k}) = A(\vec{k})$. This allows us to extract the scattering amplitudes utilizing the complex-scaled wave functions, obtained by solving Eq. (15):

$$A_{31}(\vec{k}) = \frac{m}{\hbar^2}\iiint_V d\widetilde{\mathcal{V}_{xyz}}\{6\widetilde{\mathcal{K}_K^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}(\widetilde{K^{\text{out}}} + \widetilde{K^{\text{in}}}) + 3\widetilde{\mathcal{K}_H^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}(\widetilde{H^{\text{out}}} + \widetilde{H^{\text{in}}})\}. \tag{20}$$

In the last equation the terms combining two incoming plane waves are the slowest to converge. However, since these terms are easy to calculate without complex scaling they may also be evaluated by using unscaled expressions, thus finally giving

$$A_{31}(\vec{k}) = \frac{m}{\hbar^2}\iiint_V d\widetilde{\mathcal{V}_{xyz}}\{6\widetilde{\mathcal{K}_K^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}\widetilde{K^{\text{out}}} + 3\widetilde{\mathcal{K}_H^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}\widetilde{H^{\text{out}}}\} + \frac{m}{\hbar^2}\iiint_V d\mathcal{V}_{xyz}\{6\mathcal{K}_K^\dagger(H_0 - E)K^{\text{in}} + 3\mathcal{K}_H^\dagger(H_0 - E)H^{\text{in}}\}. \tag{21}$$

In a similar manner one may derive the transition amplitudes into (2 + 2)-particle channels:

$$A_{22}(\vec{k}) = \frac{m}{\hbar^2}\iiint_V d\widetilde{\mathcal{V}_{xyz}}\{8\widetilde{\mathcal{H}_K^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}\widetilde{K^{\text{out}}} + 4\widetilde{\mathcal{H}_H^\dagger}\widehat{S}(H_0 - E)\widehat{S}^{-1}\widetilde{H^{\text{out}}}\} + \frac{m}{\hbar^2}\iiint_V d\mathcal{V}_{xyz}\{8\mathcal{H}_K^\dagger(H_0 - E)K^{\text{in}} + 4\mathcal{H}_H^\dagger(H_0 - E)H^{\text{in}}\}, \tag{22}$$

with $\mathcal{H}_K = \mathcal{H}_H = \frac{1}{2}(P^+ + P^-)(1 + \tilde{P})H^{\text{in}}$.

Four- and three-particle break-up amplitudes may equally be calculated. However, we do not report on them in this paper.

III. RESULTS

To test the applicability of our approach we consider a system of four identical nucleons with masses $\frac{\hbar^2}{m} = 41.4711$ MeV fm^2, where the nucleon-nucleon (NN) interaction is described by the spin-dependent S-wave MT I-III potential, defined as

$$V_S(r) = -A_S\frac{\exp(-1.55r)}{r} + 1438.72\frac{\exp(-3.11r)}{r}, \tag{23}$$

where $V_S(r)$ is in MeV and r is in femtometers. The attractive Yukawa strengths are given by $A_{s=0} = -513.968$ MeV fm and $A_{s=1} = -626.885$ MeV fm for the two-nucleon interaction in the spin singlet and triplet states, respectively. Coulomb repulsion between the protons is neglected. Within this model the nuclear interaction turns out to be isospin independent and thus nucleonic systems conserve the total isospin (T). In addition, due to the S-wave limitation of the model, nucleonic systems separately conserve the total spin and the orbital angular momentum. The MT I-III model is fitted to reproduce the correct binding energies of the deuteron (^{2}H) and the triton (^{3}H), at −2.230 and −8.535 MeV, respectively. However, the absence of the Coulomb interaction relocates the ^{3}He ground state to the same energy as the ^{3}H ground state. Two-cluster collisions are available in $T = 1$ and $T = 0$ channels, which will be discussed further on.

The calculations are performed by employing the numerical method described in [6,10,11] and using 50 discretization points in each direction (x, y, z). The complex-scaling angle is fixed at $\theta = 9^\circ$. Vanishing boundary conditions for FY partial amplitudes were imposed at the borders of the discretized grid, which was varied from 35 to 50 fm. The results have been tested to be stable when modifying the scaling angle and the grid parameters. Basically, the extracted amplitudes turn out to be accurate to three digits, which guarantees three-digit accuracy for the extracted phase shifts. Nevertheless, this method is slightly less accurate for the inelasticity parameter, especially once its value is very close to 1. Due to the S-wave limitation of the interaction model, partial amplitudes with $l_x \neq 0$ do not contribute in solving the FY equation (15); however, one must include these amplitudes in evaluating the integrals of

TABLE I. Neutron-triton scattering phase shifts (in degrees) and inelasticity parameters. The accuracy for calculated phase shifts is about 0.1° whereas the inelasticity parameter has an accuracy of around 0.005.

E_{lab}	$L = 0$		$L = 1$		$L = 2$	
(MeV)	$S = 0$	$S = 1$	$S = 0$	$S = 1$	$S = 0$	$S = 1$
14.4	72.7	81.2	40.0	57.4	−3.92	−2.45
	0.993	0.988	0.988	1.00	0.999	0.988
18.0	65.5	74.4	38.8	55.4	−3.24	−1.98
	0.990	0.984	0.968	0.983	0.995	0.973
22.1	58.4	67.4	37.1	53.0	−2.40	−1.21
	0.988	0.983	0.944	0.952	0.988	0.955

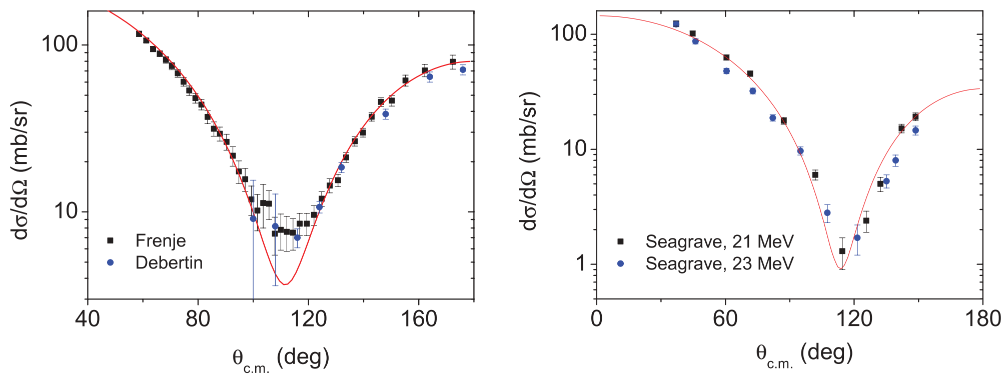

FIG. 2. (Color online) Calculated n-^{3}H elastic differential cross sections for neutrons of laboratory energies of 14.4 MeV (left panel) and 22.1 MeV (right panel) compared with the experimental results of Frenje *et al.* [14], Debretin *et al.* [15], and Seagrave *et al.* [16].

Eqs. (19)–(22) The expansion into tripolar harmonics was limited by the $\max(l_x, l_y, l_z) \leqslant 3$ condition. The results are converged to four significative digits with respect to the partial angular momentum basis.

First of all we present the results for the $T = 1$ case, which well reflects the reality of the n-^{3}H collisions. The values of the calculated phase shifts and the inelasticity parameters are summarized in Table I. The phase shifts are obtained with very high accuracy, with a variation observed only in the third digit. The variation of the inelasticity parameter is of the order 0.005, which looks to be a rather accurate result. Nevertheless, since the values of the inelasticity parameter are very close to unity such accuracy might be critical in determining the small value of the total break-up cross section.

In Table II the calculated total elastic cross sections are compared with the experimental values. One may notice a rather good agreement. These calculations have been performed for total orbital momentum states $L \leqslant 3$ and seem to be converged in this respect. In Fig. 2 we present the comparison of the differential elastic cross sections, calculated for an incident neutron at laboratory energies of 14.4 MeV (left panel) and 22.1 MeV (right panel), with the experimental values. One may notice that a rather good agreement is also obtained in this case. Only at the minimum region, for the 14.4-MeV neutrons, do the theoretical results underestimate the experimental values. Nevertheless, the overall agreement remains very good and is far beyond expectations for such a simplistic interaction model as MT I-III. This proves that the n-^{3}H cross sections at higher energy, beyond the resonance region, are rather insensitive to the details of the nucleon-nucleon interaction. As has been shown recently [8] the realistic interaction models further improve the description of n-^{3}H elastic cross sections, providing almost perfect agreement with the data also in the minimum region.

Next we consider the total isospin $T = 0$ case. This isospin channel is a very rich one, combining the $d + d$, n-^{3}H, and p-^{3}He binary scattering modes in addition to three- and four-particle break-up ones. Due to the absence of the Coulomb interaction, the n-^{3}H and p-^{3}He thresholds coincide in our calculations. The soundness of these calculations is further shrouded by the fact that we neglect the Coulomb interaction in the asymptotes of the open channels. Therefore, there is not much sense in comparing the obtained results compiled in Table III with the experiment. One may notice (see Table IV) that our obtained values are also rather different from the ones calculated for the $J^\pi = 0^+$ case by Uzu *et al.* [3], who have used the same assumptions as in the present paper but employed a separable Yamaguchi interaction. The last fact indicates the strong sensitivity of the $T = 0$ channel to the details of the nucleon-nucleon interaction. However, this sensitivity is not surprising, as the $T = 0$ channel is strongly attractive and contains a series of resonances also above the four-particle break-up threshold. It is also confirmed by rather large inelastic cross sections (inelasticity parameters).

TABLE II. Neutron-triton elastic (σ_e), inelastic (σ_b), and total (σ_t) scattering cross sections (in units of millibarns) for the selected neutron laboratory energies (in units of MeV) compared with the experimental data. Calculations have been limited to the maximal total orbital angular momentum states $L \leqslant 3$.

E_{lab}	MT I-III			Experimental	
(MeV)	σ_e	σ_b	σ_t	σ_t	Ref.
14.4	922	11	933	978 ± 70	[17]
18.0	690	25	715	750 ± 40	[17]
22.1	512	38	550	620 ± 24	[18]

TABLE III. Nucleon-trinucleon scattering phase shifts (in degrees) and inelasticity parameters calculated for center-of-mass energies of 20.5 and 30 MeV and nucleon laboratory energies of 27.3 and 40 MeV, respectively.

		$E_{c.m.} = 20.5$ MeV		$E_{c.m.} = 30$ MeV	
		δ (deg)	η	δ (deg)	η
$L = 0$	$S = 0$	−56.6	0.650	−81.0	0.618
	$S = 1$	68.8	0.947	56.9	0.882
$L = 1$	$S = 0$	−85.3	0.945	78.9	0.918
	$S = 1$	64.9	0.886	52.8	0.843
$L = 2$	$S = 0$	47.1	0.678	44.7	0.720
	$S = 1$	1.09	0.896	4.49	0.851

TABLE IV. Nucleon-trinucleon scattering phase shifts (in degrees) and inelasticity parameters for the $J^\pi = 0^+$ case and at the chosen center-of-mass projectile energies (in units of MeV). The results of this manuscript using the MT I-III interaction are compared with the ones of Ref. [3], in which the Yamaguchi potential was employed.

$E_{\text{c.m.}}$	MT I-III (this work)		Yamaguchi [3]	
	δ (deg)	η	δ (deg)	η
7.3	−4.46	0.988	−5.51	0.899
20.5	−56.6	0.650	−61.7	0.746

IV. CONCLUSION

The complex-scaling method has been applied in this manuscript to study four-nucleon scattering above the three- and four-particle break-up thresholds. The restrictive condition has been derived for the complex-scaling angle to be used in the four-body calculations. When considering scattering at high energies, according to Eqs. (16) and (17), the scaling angle must be strongly restricted from above. However, this limitation should not hamper the method at high energies, since after the complex scaling the fast vanishing of the outgoing wave is ensured by the large-wave-number values.

The method turns out to be very reliable and provides a very accurate description of the scattering phases, with better than three-digit accuracy. The inelasticity parameters are obtained with two-digit accuracy. The complex-scaling method might be used with almost any configuration-space bound-state technique to solve scattering problems. It is straightforward to extend this method beyond the four-body case.

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