Pesudoscalar transition form factors within the light-front quark model

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We study the transition form factors of the pseudoscalar mesons (π, η, and η') as functions of the momentum transfer Q^2 within the light-front quark model. We compare our results with the recent experimental data by CELLO, CLEO, BaBar and Belle collaborations. By considering the possible uncertainties from the quark masses, we illustrate that our predicted form factors can fit with all the data, including those at the large Q^2 regions.

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Recently, the Belle Collaboration [1] published its data on the transition form factor ($F_{\pi\gamma}$) of $\pi^0 \to \gamma^*\gamma$, previously measured by BaBar [2], CLEO [3], and CELLO [4] collaborations. However, for the momentum transfer Q^2 above 10 GeV^2 the new data by Belle seem to be much lower than those by BaBar. As a result, the argument for the violation of the QCD asymptotic limit [5] is weakened despite the extensive theoretical studies in the literature [6–30].

In addition, following the pion data [2], the transition form factors $[F_{(\eta,\eta')\gamma}(Q^2)]$ of $\eta, \eta' \to \gamma^*\gamma$ have been reported by the BaBar Collaboration [31] for Q^2 up to about 35 GeV^2. Many theoretical works on the $\eta^{(\prime)}$ transition form factors have been also done [32–42] and the results are in agreement with the data by BaBar [31]. In particular, some of the studies have also tried to combined the analyses on the three pseudoscalar mesons of π^0, η, and $\eta^{(\prime)}$ to fit all data simultaneously.

Motivated by the Belle data, in this Brief Report we reexamine the transition form factor of the pion along with those of η and η' within the light-front quark model (LFQM) by including uncertainties of quark masses to check if we can accommodate all the data. Similar studies in other QCD models have been performed recently in Refs. [43–47].

We will use the phenomenological light-front (LF) meson wave function [48,49] to evaluate $Q^2|F_{\eta^{(\prime)}}(Q^2)|$ in all allowed kinematic regions. The LF wave function can be constructed by the simple structure of the meson constituent in terms of a quark-antiquark ($Q\bar{Q}$) pair [49]. The decay amplitude of $Q\bar{Q} \to \gamma^*\gamma^*$ with Lorentz structure is given by [50]

$$A(Q\bar{Q}(P) \to \gamma^*(q_1,\epsilon_1)\gamma^*(q_2,\epsilon_2)) = ie^2 F_{Q\bar{Q}}(q_1^2,q_2^2)\varepsilon_{\mu\nu\rho\sigma}\epsilon_1^\mu \epsilon_2^\nu q_1^\rho q_2^\sigma \,, \quad (1)$$

where $F_{Q\bar{Q}}(q_1^2,q_2^2)$ is a symmetric function under the interchange of q_1^2 and q_2^2, which can be found to be [48,49]

$$F_{Q\bar{Q}}(q_1^2,q_2^2) = -4\sqrt{\frac{N_c}{3}}\int \frac{dx\, d^2k_\perp}{2\,(2\pi)^3}\Phi_{Q\bar{Q}}(x,k_\perp^2) \times \frac{e_Q^2}{1-x}\frac{m_Q+(1-x)m_Q k_\perp^2\Theta}{x(1-x)q_2^2-m_Q^2-k_\perp^2} + (q_2 \leftrightarrow q_1), \quad (2)$$

with

$$\Theta = \frac{1}{\Phi_{Q\bar{Q}}(x,k_\perp^2)}\frac{d\Phi_{Q\bar{Q}}(x,k_\perp^2)}{dk_\perp^2}\,, \quad (3)$$

where $N_c = 3$ is the number of colors, $e_Q = 2/3(-1/3)$ for $Q = u(d,s)$, m_Q is the quark mass, and $\Phi_{Q\bar{Q}}(x,k_\perp^2)$ is the meson wave function, defined by

$$\Phi_{Q\bar{Q}}(x,k_\perp^2) = \sqrt{N_c\frac{x(1-x)}{2M_0^2}}\phi_{Q\bar{Q}}(x,k_\perp)\,, \quad (4)$$

where

$$\phi_{Q\bar{Q}}(x,k_\perp) = N\sqrt{\frac{dk_z}{dx}}\exp\left(-\frac{\vec{k}^2}{2\omega_{Q\bar{Q}}^2}\right)\,, \quad (5)$$

$$M_0^2 = \frac{m_Q^2+k_\perp^2}{x} + \frac{m_Q^2+k_\perp^2}{1-x}\,, \quad (6)$$

with $N = 4(\pi/\omega_{Q\bar{Q}}^2)^{\frac{3}{4}}$, $\vec{k} = (k_\perp, k_z)$, $k_z = (x+1/2)M_0$, and $\omega_{Q\bar{Q}}$ the parameter related to the physical size of the pseudoscalar meson ($P = \sum Q\bar{Q}$) in the wave function. If q_1 or q_2 is on mass shell, the form factor of $Q\bar{Q} \to \gamma^*\gamma$ can be written as

$$F_{Q\bar{Q}\to\gamma^*\gamma}(Q^2,0) = -4\sqrt{\frac{N_c}{3}}\int \frac{dx\, d^2k_\perp}{2(2\pi)^3}\frac{e_Q^2\Phi_{Q\bar{Q}}(x,k_\perp^2)}{1-x} \times \left\{\frac{m_Q+(1-x)m_Q k_\perp^2\Theta}{x(1-x)Q^2-m_Q^2-k_\perp^2} - \frac{m_Q+(1-x)m_Q k_\perp^2\Theta}{m_Q^2+k_\perp^2}\right\}, \quad (7)$$

where $Q^2 = q_1^2$ or q_2^2 is the momentum transfer. From Eq. (7), by summing up the relevant Fock states we obtain the transition

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for the transition form factors of $P \to \gamma^*\gamma$ to be

$$F_{P\to\gamma^*\gamma}(Q^2) \equiv F_{P\gamma}(Q^2) = \sum F_{Q\bar{Q}\to\gamma^*\gamma}(Q^2,0). \quad (8)$$

For the π^0 meson, we use $|\pi^0\rangle = |u\bar{u} - d\bar{d}\rangle/\sqrt{2}$ and $m_u = m_d = m_q$. The states of η and η' can be expressed in terms of the two orthogonal states of $|\eta_q\rangle$ and $|\eta_s\rangle$, parametrized as [51–54]

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \quad (9)$$

where $|\eta_q\rangle = |u\bar{u} + d\bar{d}\rangle/\sqrt{2}$ and $|\eta_s\rangle = |s\bar{s}\rangle$. The mixing angle has been studied in various decay processes and constrained to be $\phi \simeq 37^\circ \sim 42^\circ$ [54]. Under this scheme, the valence states of $\eta^{(\prime)}$ can be written as

$$\begin{aligned} |\eta\rangle &= \cos\phi \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} - \sin\phi |s\bar{s}\rangle, \\ |\eta'\rangle &= \sin\phi \frac{|u\bar{u} + d\bar{d}\rangle}{\sqrt{2}} + \cos\phi |s\bar{s}\rangle. \end{aligned} \quad (10)$$

Consequently, the transition form factors of $\eta^{(\prime)} \to \gamma^*\gamma$ have the forms

$$\begin{aligned} F_{\eta\gamma} &= \cos\phi F_{\eta_q} - \sin\phi F_{\eta_s}, \\ F_{\eta'\gamma} &= \sin\phi F_{\eta_q} + \cos\phi F_{\eta_s}. \end{aligned} \quad (11)$$

To numerically calculate the transition form factors of $P \to \gamma^*\gamma$, we need to specify the parameters appearing in $\phi_{Q\bar{Q}}(x, k_\perp)$. To constrain the quark masses of $m_{u,d,s}$ and the meson scale parameters of $\omega_{Q\bar{Q}}$ in Eq. (5), we use the branching ratios of $P \to 2\gamma$ and the decay constants of the $Q\bar{Q}$ states, defined by

$$\mathcal{B}(P \to 2\gamma) = \frac{(4\pi\alpha)^2}{64\pi\Gamma_P} m_P^3 |F(0,0)_{P\to 2\gamma}|^2, \quad (12)$$

and

$$f_{Q\bar{Q}} = 4\frac{\sqrt{N_c}}{\sqrt{2}} \int \frac{dx\, d^2k_\perp}{2(2\pi)^3} \phi_{Q\bar{Q}}(x, k_\perp) \frac{m_Q}{\sqrt{m_Q^2 + k_\perp^2}}, \quad (13)$$

respectively, where $Q = q$ or s denotes the quark in the Fock state. Explicitly, we use [55]

$$\begin{aligned} &\mathcal{B}(P \to 2\gamma) \\ &\quad = (98.832 \pm 0.034),\ (39.30 \pm 0.20),\ (2.12 \pm 0.14)\%, \end{aligned} \quad (14)$$

which lead to $|F(0,0)_{P\to 2\gamma}| \equiv |F_{P\gamma}(0)| = 0.274$, 0.260, and 0.341 in GeV^{-1} for $P = \pi^0$, η, and η', respectively. For the decay constants, we take [36]

$$f_\pi = 132, \quad f_{q\bar{q}} = 140, \quad f_{s\bar{s}} = 168 \text{ MeV}. \quad (15)$$

To illustrate the pion transition form factor, we have to specify the up and down quark masses. In our previous study in Ref. [49], we fixed $m_q = 0.24$ GeV ($q = u, d$). To fit all the experimental data including the new data from Belle, we would like to include the uncertainty from the quark masses.

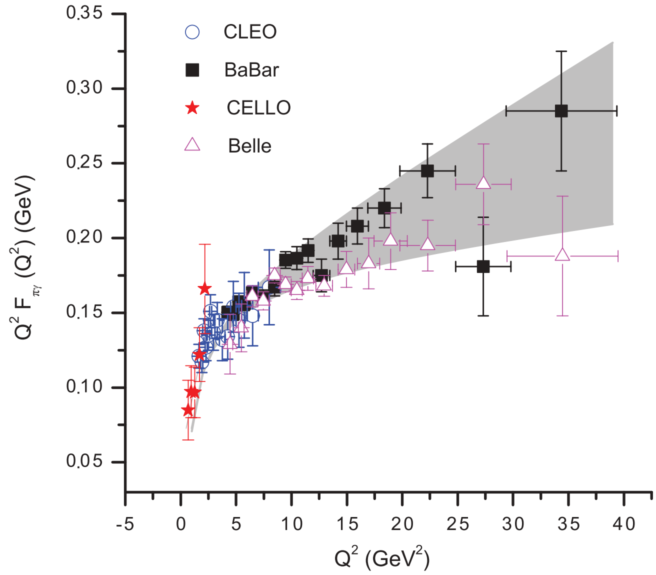

FIG. 1. (Color online) $Q^2F_{\pi\gamma}(Q^2)$ as a function of Q^2 in the LFQM with $m_q = 0.22 \sim 0.30$ GeV.

Explicitly, we revise our input with a possible range of m_q, i.e., $m_q = 0.22 \sim 0.30$ GeV. As a result, we can derive various meson scale parameters of ω_π from the pion decay constant and $F_{\pi\gamma}(0)$ from the decay rate of $\pi^0 \to \gamma\gamma$. In Fig. 1, we show the Q^2 dependence of the π^0 transition form factor $Q^2F_{\pi\gamma}(Q^2)$ in the LFQM (gray band) with $m_q = 0.22 \sim 0.30$ GeV, where we have also plotted the experimental data of BaBar [2], Belle [1], CELLO [4], and CLEO [3] collaborations. Note that the upper and lower edges of the gray band in Fig. 1 correspond to $m_q = 0.30$ and 0.22 GeV, respectively. From the figure, we see that either the experimental data by CELLO, CLEO, and BaBar or those by CELLO, CLEO, and Belle can be simultaneously fitted well in the LFQM.

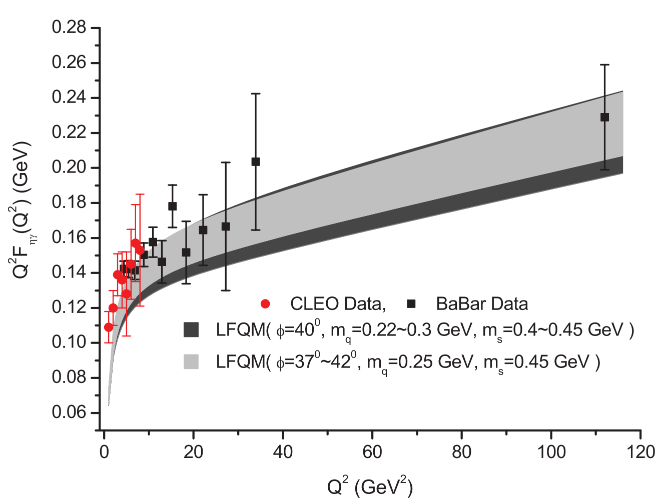

FIG. 2. (Color online) $Q^2F_{\eta\gamma}(Q^2)$ as a function of Q^2 in the LFQM, where the dark-gray band represents the inputs of $m_q = 0.22 \sim 0.30$, $m_s = 0.40 \sim 0.45$ in GeV and $\phi = 40^\circ$, while the light-gray one stands for those of $m_q = 0.25$, $m_s = 0.45$ GeV and $\phi = 37 \sim 42^\circ$.

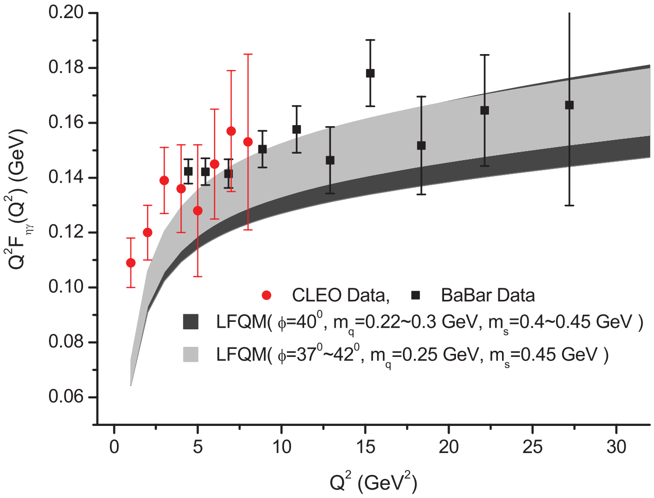

FIG. 3. (Color online) Same as Fig. 2, but for the lower Q^2 region of $Q^2 < 30\ \text{GeV}^2$.

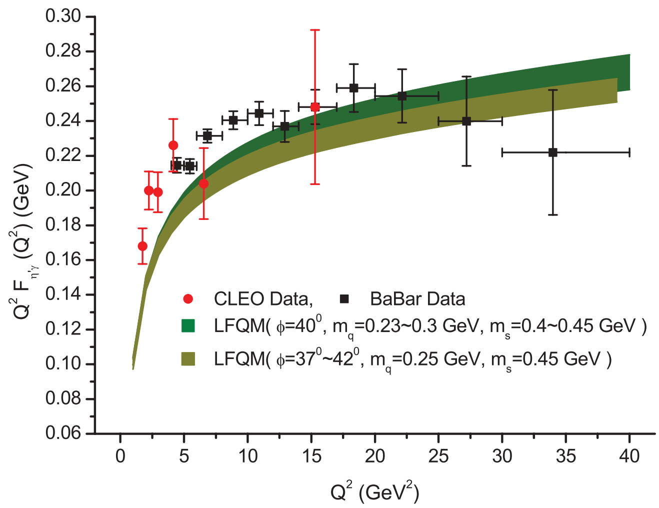

FIG. 4. (Color online) Same as Fig. 2, but for η'.

We emphasize that to reproduce the BaBar (Belle) high-Q^2 tail, the higher (lower) quark mass is required.

In our numerical calculations of η and η', the first term in Eq. (7) dominates for the lower region of Q^2 and thus, it can be used to describe the experimental data of CLEO [56] and BaBar [31] with $Q^2 \leqslant 10\ \text{GeV}^2$. The second one in Eq. (7), related to the nonvalence quark contributions, is quite small for a small Q^2. In general, this term can be neglected in the low Q^2 region, but it may enhance the form factors of $Q^2 F_{\eta^{(\prime)}\gamma}$ at the high values of Q^2. Hence, we will take into account this term in our calculations. Similar to the pion case, we will also consider the uncertainties from the quark masses. Explicitly, we use $m_q = 0.22 \sim 0.30$ and $m_s = 0.40 \sim 0.45$ in GeV. Moreover, we will examine a possible range of $\phi = 37 \sim 42^\circ$ for the mixing angle in Eq. (11). In Fig. 2, we show our results for the Q^2 dependence of the η transition form factor in terms of $Q^2 F_{\eta\gamma}(Q^2)$, where the dark-gray band represents the inputs of $m_q = 0.22 \sim 0.30$, $m_s = 0.40 \sim 0.45$ in GeV and $\phi = 40^\circ$, while the light-gray one stands for those of $m_q = 0.25$, $m_s = 0.45$ GeV and $\phi = 37 \sim 42^\circ$. An enlarged view of Fig. 2 for the lower Q^2 region of $Q^2 < 30\ \text{GeV}^2$ is given in Fig. 3. In Fig. 4, we draw $Q^2 F_{\eta'\gamma}(Q^2)$ as a function of Q^2, where the dark-gray and light-gray bands represent the inputs of $m_q = 0.22 \sim 0.30$, $m_s = 0.40 \sim 0.45$ GeV and $\phi = 40^\circ$ and $m_q = 0.25$, $m_s = 0.45$ GeV and $\phi = 37^\circ$–42°, respectively.

As shown in Figs. 2–4, our results for $Q^2 F_{\eta^{(\prime)}\gamma}(Q^2)$ are in good agreement with the experimental data. Note that the upper (lower) edges of the dark-gray bands in Figs. 2–4 correspond to $m_q = 0.30\,(0.25)$ and $m_s = 0.45\,(0.4)$ GeV, while those of the yellow bands $\phi = 37^\circ\,(42^\circ)$. We remark that the form factors $Q^2 F_{\eta^{(\prime)}\gamma}$ increase (decrease) with quark masses m_q (the mixing angle ϕ), whereas the effect from the uncertainty from m_s is small due to the small quark charge. It is interesting to point out that the form factors can be better fitted for a larger m_q with a fixed ϕ or $\phi = 40^\circ$ with a fixed m_q in the lower Q^2 region.

In summary, motivated by the recent experimental measurements, we have shown the transition form factors of π^0, η, and $\eta' \to \gamma^*\gamma$ as functions of the momentum transfer Q^2 within the LFQM. We have recalculated $F_{\pi\gamma}(Q^2)$ by considering the allowed possible range of $m_q = 0.22 \sim 0.30$ GeV. We have illustrated that our result of the pion transition form factor in the LFQM can fit either the experimental data by CELLO, CLEO, and BaBar collaboratons or those by CELLO, CLEO, and Belle for a fixed quark mass. In particular, we have found that to reproduce the BaBar and the recent Belle high-Q^2 tails, the higher and lower quark masses are needed, respectively. With the same set of model parameters as the pion, we have also studied the form factors of $F_{\eta^{(\prime)}\gamma}$ by considering the possible ranges of the quark masses: $m_q = 0.22 \sim 0.30$ and $m_s = 0.40 \sim 0.45$ in GeV and the $\eta - \eta'$ mixing angle: $\phi = 37^\circ \sim 42^\circ$, and we have found that our results agree well with the CLEO and BaBar data in the η and η' cases.

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