Rapidity dependence of particle densities in *pp* and *AA* collisions

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We use multiple scattering and energy conservation arguments to describe $dn/d\eta|_{N_AN_A}$ as a function of $dn/d\eta|_{pp}$ in the framework of string percolation. We discuss the pseudorapidity η and beam rapidity Y dependence of particle densities. We present our results for pp, Au-Au, and Pb-Pb collisions at RHIC and LHC.

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I. INTRODUCTION

As nuclei are made up of nucleons it is natural to look at nucleus-nucleus (AA) collisions as resulting from the superposition of nucleon-nucleon (pp) collisions, in the spirit of the Glauber model approach and generalizations of it. In the single scattering limit the average number of participating nucleons per nucleus N_A behave incoherently and

$$\left. \frac{dn}{dy} \right|_{N_A N_A} = \frac{dn}{dy} \right|_{pp} N_A. \tag{1}$$

Equation (1) corresponds to the wounded nucleon model [1-3]. This model is expected to dominate at very low energy. In general, data do not agree with Eq. (1).

At higher energy one has to take into account multiple scattering and one finds

$$\left. \frac{dn}{dy} \right|_{N_A N_A} = \left. \frac{dn}{dy} \right|_{pp} (N^{1+\alpha(s)} - N_A), \tag{2}$$

where $N_A^{1+\alpha(s)}$ is the estimated total number of nucleonnucleon collisions and single scattering was subtracted [4].

It should be noticed that energy momentum conservation constrains the combinatorial factors of the Glauber calculus at low energy. The problem is that the energy momentum of N_A valence strings has to be shared by $N_A^{4/3}$ (mostly) sea strings. There are proposals to cure this problem, for instance, by reduction of the height of the rapidity plateau for sea strings [5]. In the same spirit, but reducing the effective number of sea strings rather than reducing the sea plateau, we write (see [4])

. ...

with

$$N_A^{4/3} \to N_A^{(1+\alpha(s))},$$
 (3)

$$\alpha(s) = \frac{1}{3} \left(1 - \frac{1}{1 + \ln(\sqrt{s/s_0} + 1)} \right),\tag{4}$$

(1 . ())

such that for $\sqrt{s} \ll \sqrt{s_0}$, $\alpha(\sqrt{s}) \to 0$, we are back to the wounded nucleon model, and for $\sqrt{s} \gg \sqrt{s_0}$, $\alpha(\sqrt{s}) \to \frac{1}{3}$, and we have fully developed Glauber calculus. The need to take

multiple scattering contribution was experimentally shown at RHIC [6].

Here as in [2,3] our framework is the dual parton model with parton saturation, and we work with Schwinger strings, with fusion and percolation [7].

In *pp* and Au-Au collisions or in general $N_A N_A$ collisions the interactions occur with the formation of longitudinal strings in rapidity. The particle density dn/dy is expected to be proportional to the average number of strings (twice the number of elementary collisions) $N_{N_A}^s$ (see [4]),

$$\frac{dn}{dy}|_{N_A N_A} \sim \bar{N}_{N_A}^s. \tag{5}$$

The string percolation model describes the multiparticle production in terms of color strings stretched between the partons of the projectile and the target. In the impact parameter plane due to the confinement, the color of strings is confined to a small area in transverse space $S_1 = \pi r_0^2$ with $r_0 \sim 0.2-0.3$ fm, these strings decay into new ones by $q\bar{q} - q\bar{q}$ pair production and subsequently hadronize to produce the observed hadrons. In the impact parameter plane the strings appear as discs and as energy-density increases the discs overlap, fuse, and percolate, leading to the reduction of the overall color [8–10]. A cluster of *n* strings behaves as a single string with energy momentum corresponding to the sum of the individual ones. An essential quantity is the color reduction factor,

$$F(\eta_{N_{A}}^{t}) = \sqrt{\frac{1 - e^{-\eta_{N_{A}}^{t}}}{\eta_{N_{A}}^{t}}},$$
(6)

where $\eta_{N_A}^t$ is the string density in the impact parameter plane for $N_A N_A$ collisions given as (see [4])

$$\eta_{N_A}^t \equiv \frac{\pi r_0^2}{S_{N_A}} \bar{N}_{N_A}^s.$$
 (7)

 S_{N_A} is the area of the impact parameter projected overlap region of the interaction covered by N_A nucleons from nucleus A. Note that $N_{N_A}^s = N_p^s N_A^{1+\alpha}$ and instead of (5) we have now

$$\frac{dn}{dv}|_{N_A N_A} \sim F(\eta^t_{N_A})\bar{N}^s_{N_A}.$$
(8)

The color reduction factor $F(\eta_{N_A}^t)$ is a tool to slow down the increase of dn/dy with energy and number of participating

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nucleons. Note that in Eq. (1) nucleons interact incoherently and S_{N_A} is in fact S_p , while in Eq. (2), due to coherence, S_{N_A} is the overall area of interaction. For details see [4]. We finally have at $\eta = 0$,

$$\frac{1}{N_A} \frac{dn}{dy} \Big|_{N_A N_A} = \kappa \frac{dn}{dy} \Big|_{pp} \left[1 + \frac{F(\eta_{N_A}^t)}{F(\eta_p^t)} \left(N_A^{\alpha(\sqrt{s})} - 1 \right) \right], \quad (9)$$

with κ being a normalization factor,

$$\eta_{N_A}^t = \eta_p^t N_A^{\alpha} \left(\frac{A}{N_A^{2/3}}\right),\tag{10}$$

and $F(\eta_{N_A,p}^t) \rightarrow \frac{1}{\sqrt{\eta_{N_A,p}^t}}$ and $\alpha(\sqrt{s}) \rightarrow \frac{1}{3}$, where N_p^s is the number of proton strings. At low energy N_p^s is around 2 growing with energy as $e^{2\lambda Y}$ (faster than $\frac{dn}{dy}|_{pp}$) so that we can approximately write

$$N_p^s = 2 + 4 \left(\frac{r_0}{R_p}\right)^2 e^{2\lambda Y}.$$
 (11)

We now generalize the results obtained in Ref. [4].

Based on the good description on data obtained by using the formula (9) for different atomic number and number of participants for different energies at midrapidity, we now apply the same formalism as used in pp to describe the rapidity evolution as suggested in Refs. [5,11,12] obtaining a general formula for pseudorapidity dependence of AA collisions:

$$\frac{1}{N_A} \frac{dn_{ch}^{N_A N_A}}{d\eta} \bigg|_{\eta} = \kappa' J F(\eta_p^t) N_p^s \frac{\left(1 + \frac{F(\eta_{N_A}^t)}{F(\eta_p^t)} \left(N_A^{\alpha(\sqrt{s})} - 1\right)\right)}{\exp\left(\frac{\eta - (1 - \alpha)Y}{\delta}\right) + 1},$$
(12)

where *J* is the usual Jacobean $J = \frac{\cosh \eta}{\sqrt{k_1 + \sinh^2 \eta}}$ and $\kappa' = \frac{\kappa}{J(\eta=0)} (\exp(\frac{-(1-\alpha)Y}{\delta}) + 1)$. We now apply the formula to describe the charge multiplic-

We now apply the formula to describe the charge multiplicity in *pp* collisions for different energies in pseudorapidity. From our general formula (12) by using $N_A = 1$ and A = 1, to consider *pp* collisions the expression is reduced to

$$\frac{dn_{\rm ch}^{pp}}{d\eta}\Big|_{\eta} = \kappa' F\left(\eta_p^t\right) N_p^s \frac{1}{\exp\left(\frac{\eta - (1-\alpha)Y}{\delta}\right) + 1}.$$
 (13)

We observe that the central pseudorapidity $\eta = 0$; the energy and number of participants behavior of formula (12) is mainly given by the second term of the bracket of (12), which reads

$$N_s^p F\left(\eta_{N_A}^t\right) \frac{\left(N_A^{\alpha(\sqrt{s})} - 1\right)}{\left(\exp\frac{(\alpha - 1)}{\delta}Y + 1\right)},\tag{14}$$

where the product $N_s^p N_A^{\alpha(\sqrt{s})}$, at high energies where $\alpha(\sqrt{s}) = 1/3$, is the number of collisions and $F(\eta_{N_A}^t)$ is the reduction factor of formula (6) based on the interaction of strings which means that (13), instead of being proportional to the number of collisions, is proportional to the number of participants. This reduction is due to the fact that the strength of the color

field inside a cluster of n strings instead of being n times the strength of the color field of a single string is \sqrt{n} due to the random direction of the individual color field in color space. This fact also is key in the color glass condensate, where the random color directions of the gluons produces the approximate saturation of the multiplicity with centrality. In color glass condensate, $\frac{dN}{d\eta}$ is proportional to $\frac{1}{\alpha_s}Q_s^2R_A^2$, where α_s is the strong quantum chromodynamics (QCD) coupling and Q_s the saturation momentum. As far as Q_s^2 is proportional to $N_A^{1/3}$, $Q_s^2 R_A^2$ is proportional to N_A and $\frac{1}{N_A} \frac{dN}{d\eta}$ becomes approximately independent of N_A . There is an additional dependence on N_A coming from $\frac{1}{\alpha_s}$, the number of gluons, which is proportional to log N_A . The formula (13) also has an additional N_A dependence coming from the factor $\sqrt{1 - e^{-\eta^t}}$ inside $F(\eta)$ [Eq. (6)]. Notice that $1 - e^{-\eta^t}$ is the fraction of the total collision area covered by strings and therefore it is natural that its relation to $1/\alpha_s$ is the number of gluons produced in the collision area. Although the functional dependencies of $1/\alpha_s$ and $\sqrt{1 - e^{-\eta^t}}$ on N_A and s are different, numerically they are not very different and in both cases they increase smoothly with N_A and s.

Concerning the pseudorapidity dependence, it is described by the same factor, $\frac{1}{(\exp(\frac{1-(1-\alpha)Y}{\delta})+1)}$, in *pp* and *AA* collisions. This dependence was obtained in our previous work [5,11, 12] giving rise to an increase with energy smaller at central pseudorapidity $\eta = 0$ than at large pseudorapidity $\eta = Y$.

II. COMPARISON WITH EXPERIMENTAL DATA (RHIC, LHC)

In Fig. 1 is shown the comparison of the formula (13) applied to different energies at different pseudorapidities with data from different experiments and energies, showing a good agreement in the evolution in pseudorapidity and an increase in the plateau region as increasing with energy.

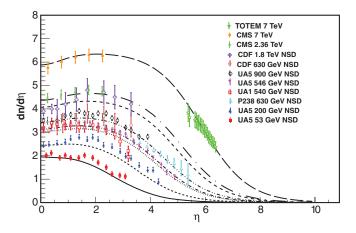


FIG. 1. (Color online) Comparison of the results from the evolution of the $dn_{ch}/d\eta$ with dependence in pseudorapidity from Eq. (13) for *pp* collisions at different energies (lines); data are taken from Refs. [13–15].

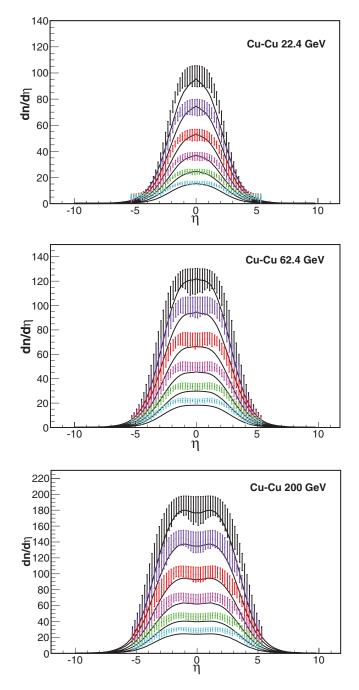


FIG. 2. (Color online) Comparison of the results from the evolution of the $dn_{ch}/d\eta$ with pseudorapidity from Eq. (12) for Cu-Cu collisions at 22.4-, 62.4-, and 200-GeV energies; data are taken from Ref. [16]. Error bars in color blue, green, pink, red, purple, and black are used for the corresponding centralities 45%–55%, 35%–45%, 25%–35%, 15%–25%, 6%–15%, 0%–6%, respectively; lines in black show our results.

In Figs. 2–5 is shown the comparison between our results from formula (12) for Cu-Cu, Au-Au, and Pb-Pb collisions at different energies, in agreement with data.

In Fig. 6 we show some predictions for 3.2, 3.9, and 5.5 TeV energies at centrality 0%-5%, for Pb-Pb collisions.

In the above computations we have used the following values of the parameters: $\kappa = 0.63 \pm 0.01$, $\lambda = 0.201 \pm 0.003$,

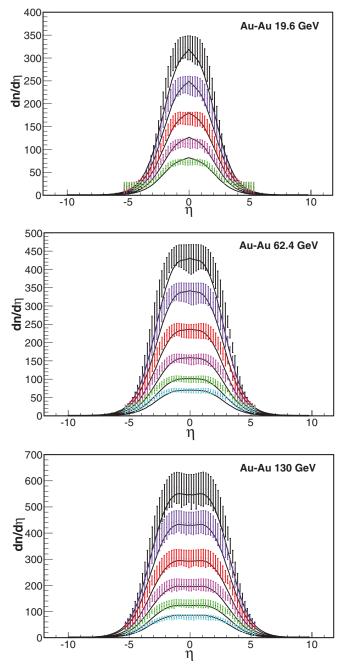


FIG. 3. (Color online) Comparison of the results from the evolution of the $dn_{ch}/d\eta$ with pseudorapidity from Eq. (12) for Au-Au collisions at 19.6-, 62.4-, and 130-GeV energies; data are taken from [17]. Error bars in color blue, green, pink, red, purple, and black are used for the corresponding centralities 45%–55%, 35%–45%, 25%–35%, 15%–25%, 6%–15%, 0%–6%, respectively; lines in black are the model results.

and $\sqrt{s_0} = 245 \pm 29$ GeV, the same as obtained in [4], to describe the particle density $\frac{dn}{d\eta}|_{N_A N_A}$ in the same power law as $\frac{dn}{d\eta}|_{pp}$. We had made here an extension to these descriptions to add the pseudorapidity evolution with the same aim as in Ref. [5].

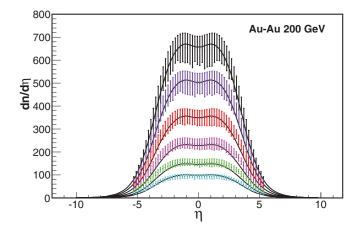


FIG. 4. (Color online) Comparison of the results from the evolution of the $dn_{ch}/d\eta$ in pseudorapidity from Eq. (12) for Au-Au collisions at 200 GeV; data are taken from [17]. Error bars in color blue, green, pink, red, purple, and black are used for the corresponding centralities 45%–55%, 35%–45%, 25%–35%, 15%–25%, 6%–15%, 0%–6%, respectively; lines in black are the model results.

The new parameters values $\alpha \simeq 0.34$, $\delta \simeq 0.84$, $k_1 = 1.2$ had been set to adjust Eq. (13) with data [13–15]; these values are close to those used previously. These results can be extended to describe proton-nucleus collisions.

The recent data from TOTEM gives a measurement in a high rapidity range unexplored before, allowing one to constrain more the parameters of our model to the behavior at high energies for pp collisions; once these parameters are set for $dn/d\eta_{pp}$ collisions data we use the same parameters to describe $dn/d\eta_{AA}$, obtaining a good description of the data.

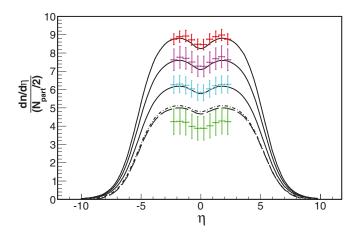


FIG. 5. (Color online) Comparison of the results from the evolution of the $\frac{dn_{ch}}{d\eta} \frac{1}{(N_{part}/2)}$ with the pseudorapidity from Eq. (12) for Pb-Pb collisions at 2.76 TeV; data are taken from [18]. Error bars in color green, blue, pink, and red are used for the corresponding centralities 85%–95%, 50%–55%, 0%–90%, and 0%–5%, respectively; lines in black are the corresponding results from the model to the respective centrality. For the smaller centrality we use the number of participants corresponding to 85%–95% shown in the dot- dashed line; dashed line is the minimum number of participants equal to 2.

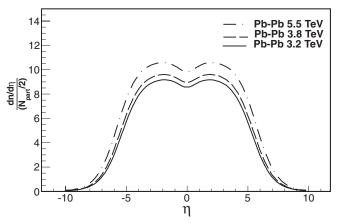


FIG. 6. Predictions on the evolution of the $\frac{dn_{\rm ch}}{d\eta} \frac{1}{(N_{\rm part}/2)}$ with pseudorapidity from Eq. (12) for Pb-Pb collisions at 3.2-, 3.9-, and 5.5-TeV energies at 0%–5% centrality.

III. CONCLUSIONS

We have discussed in a general way the physics of particle densities in pp and AA collisions. Our model gives a nonlinear dependence of $\frac{1}{N_A} dn/d\eta|_{AA}$ on $dn/d\eta|_{pp}$. Particle densities, as a function of η and Y, give a good description of Pb-Pb data at LHC in a wide region in η . The same is observed for pp in a wide range of rapidity Y.

Notice that recent data from TOTEM experiment measurements in the charged particle pseudorapidity density $dN_{\rm ch}/d\eta$ in *pp* collisions at $\sqrt{s} = 7$ TeV for 5.3 < $|\eta| < 6.4$ have been compared to several MC generators and none of them was found to fully describe the measurement, but our model is able to reproduce it.

We are able to describe the rise with energy in central AA and *pp* collisions with the same powerlike exponent, due to the energy conservation effects which give rise to an additional energy dependence vanishing at extremely high energies. In color glass condensate or gluon saturation models different explanations have been proposed, as additional entropy production in the pre-equilibrium phase [19], or enhanced parton showers in AA collisions due to the larger p_T of the initially produced minijets compared to pp collisions [20], or the interplay between the DGLAP evolution equation and the nucleus geometry that makes that the saturation momentum Q_s grow faster in AA central collisions than in pp collisions [21]. There is another model, the nonequilibrium statistical relativistic diffusion model [22], which gives reasonable description of AA and pp collisions, although it is not able to reproduce the TOTEM data at high η . In this model there are three sources, one at central rapidity and the two others in the fragmentation regions. In our approach, we have also three different regions because in the fragmentation regions we have only strings involving valence quarks and in the central region there are additional short rapidity strings between quarks and antiquarks. In this model we use a diffusion evolution equation in pseudorapidity which we also use to obtain the pseudorapidity dependence.

The predictions of our approach at higher energy will be confronted with data soon.

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