# Robustness of the excitation energy at scission as a novel probe of nuclear dissipation at high energy

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(Received 14 July 2012; published 17 September 2012)

In a recent paper [Phys. Rev. C **85**, 011601(R) (2012)] we proposed that excitation energy at scission ($E^*_{sc}$) measured at high initial excitation energy ($E^*$), which can be provided via proton induced spallation reaction approach, is a novel probe of nuclear dissipation ($\beta$). The present work is devoted to studying within the framework of the stochastic Langevin model the robustness of the conclusion with respect to the apparent changes in the spin of compound nuclei (CNs), in the magnitudes of the neutron-to-proton ratio of a CN and its fissility and size. In addition, the role of deformation effects as well as the influence resulting from different coefficients applied to deformation-dependent level-density parameters are investigated. All these factors have been found to have important effects on particle multiplicity, evaporation residue cross section, etc., which have widely been used as probes of nuclear dissipation. However, we find that the significant sensitivity of $E^*_{sc}$ to $\beta$ at high $E^*$ is little affected by these factors. The result demonstrates that a measurement of $E^*_{sc}$ in proton-nucleus reactions can place a robust constraint on the friction strength in the fission process of highly excited nuclei.

DOI: 10.1103/PhysRevC.86.034605 PACS number(s): 25.70.Jj, 25.70.Gh, 25.40.Sc, 24.60.Ky

## I. INTRODUCTION

In the past two decades, intensive efforts have been made to obtain information of nuclear dissipation, which is crucial for resolving the discrepancy between measured excitation functions of prescission particle multiplicity and evaporation residue cross sections and predictions using the standard Bohr-Wheeler statistical models, especially at high energy. But a diverse magnitude of the extracted dissipation strength ($\beta$) covering from $2 \times 10^{21}$ s$^{-1}$ to $20 \times 10^{21}$ s$^{-1}$ has been reported in the literature [1–21]. An essential reason for this is that for compound nuclei (CNs) with excitation energy around 150 MeV produced in heavy-ion fusions, a change in the spin of a formed CN, in its neutron-to-proton ratio and fissility, etc., appreciably modifies various particle emissions and fission (or evaporation-residue) cross sections, two primary quantities that are currently widely employed to probe nuclear dissipation and, as a result, these modifications significantly affect the sensitivity of the two observables to friction, as has been demonstrated in recent works [17,22]. Moreover, deformation effects [23–25] and different coefficients [26,27] applied to the deformation dependence of level-density parameters have also been noted to affect the particle emission [20].

In this context, in order to extract a precise $\beta$ value it becomes not only very urgent, but also very necessary to carry out a robust analysis of suggested various observables (including new ones) that are sensitive to $\beta$, though few have performed such analyses.

In a recent paper [28] we proposed that excitation energy at scission ($E^*_{sc}$) is a novel probe of $\beta$ in fission of highly excited CNs and, hence, energetic proton induced spallation reactions could provide a very useful experimental approach. The proposal was based on the Langevin model calculations for a fissioning $^{200}$Hg system and, in addition, under a condition of a low spin that is considered to be a typical feature of CNs formed in proton-nucleus reactions [29–32].

Since the above-mentioned factors have been shown to have important effects on prescission particles and further given that the number of emitted light particles has a decisive influence on the magnitude of $E^*_{sc}$, the present work is devoted to a robust study of the significant sensitivity of $E^*_{sc}$ to $\beta$ at high energy with respect to these factors.

## II. BASIS OF THEORETICAL MODEL

Stochastic equations like Langevin equations [17,18,21, 33–39] have demonstrated a successful description of the fission process of a highly excited CN, see Refs. [20,39,40] for a review. Moreover, the stochastic approach was recently applied to fission-yield calculations by Randrup *et al.* [41] and fission study of superheavy nuclei by Aritomo [42]. The model used here was developed by Fröbrich and Gontcher [20,40]. It combines both the Langevin equation with a statistical decay model (CDSM). The dynamic part of CDSM is described by entropy. The one-dimensional overdamped Langevin equation [20] is employed to perform the trajectory calculations:

$$\frac{dq}{dt} = \frac{T}{M\beta}\frac{dS}{dq} + \sqrt{\frac{T}{M\beta}}\Gamma(t). \quad (1)$$

Here $q$ is the dimensionless fission coordinate and is defined as half the distance between the center of mass of the future fission fragments divided by the radius of the compound nucleus, $M$ is the inertia parameter, and $\beta$ is the dissipation strength. The temperature in Eq. (1) is denoted by $T$ and $\Gamma(t)$ is a fluctuating force whose average and correlation function are $< \Gamma(t) > = 0$ and $< \Gamma(t)\Gamma(t') > = 2\delta(t - t')$, respectively. The driving force of the Langevin equation is calculated from the entropy:

$$S(q, E^*) = 2\sqrt{a(q)[E^* - V(q)]}, \quad (2)$$

where $E^*$ is the total internal energy of the system, and $a(q)$ is deformation-dependent level density parameter. Equation (2) is constructed from the Fermi-gas expression with a finite-range liquid-drop potential [43–45] $V(q)$ in the $\{c, h, \alpha\}$

 

parametrization [46]. The $q$-dependent surface, Coulomb, and rotation energy terms are included in the potential $V(q)$.

In the CDSM, light-particle evaporation is coupled to the fission mode by a Monte Carlo procedure. The emission width of a particle of kind $\nu$ is given by [47]

$$\Gamma_\nu = (2s_\nu + 1)\frac{m_\nu}{\pi^2\hbar^2\rho_c(E^*_{\rm intr})} \times \int_0^{E^*_{\rm intr}-B_\nu} d\varepsilon_\nu \rho_R(E^*_{\rm intr} - B_\nu - \varepsilon_\nu)\varepsilon_\nu\sigma_{\rm inv}(\varepsilon_\nu), \quad (3)$$

where $s_\nu$ is the spin of the emitted particle $\nu$, and $m_\nu$ its reduced mass with respect to the residual nucleus. The level densities of the compound and residual nuclei are denoted by $\rho_c(E^*_{\rm intr})$ and $\rho_R(E^*_{\rm intr} - B_\nu - \varepsilon_\nu)$. $E^*_{\rm intr}$ is the intrinsic excitation energy of the system, and $B_\nu$ are the liquid-drop binding energies. $\varepsilon$ is the kinetic energy of the emitted particle. The inverse cross sections are given by [47]

$$\sigma_{\rm inv}(\varepsilon_\nu) = \begin{cases} \pi R_\nu^2(1 - V_\nu/\varepsilon_\nu) & \text{for } \varepsilon_\nu > V_\nu, \\ 0 & \text{for } \varepsilon_\nu < V_\nu, \end{cases} \quad (4)$$

with

$$R_\nu = 1.21\left[(A - A_\nu)^{1/3} + A_\nu^{1/3}\right] + \frac{3.4}{\varepsilon_\nu^{1/2}}\delta_{\nu,n}, \quad (5)$$

where $A_\nu$ is the mass number of emitted particle $\nu = n, p, \alpha$.

The barriers for the charged particles are [47,48]

$$V_\nu = \frac{(Z - Z_\nu)Z_\nu K_\nu}{R_\nu + 1.6}, \quad (6)$$

with $K_\nu = 1.32$ for $\alpha$, and 1.15 for proton.

Moreover, a formula suggested by Fröbrich and Gontchar [20] is used to evaluate the deformation dependence of the charged-particle emission barriers:

$$V_c(q) = V_\nu \times B_c(q). \quad (7)$$

Here $B_c(q)$ is given by Eq. (12).

For the emission of giant dipole resonance (GDR) $\gamma$ quanta we take the formula of Lynn [49],

$$\Gamma_\gamma = \frac{3}{\rho_c(E^*)}\int_0^{E^*} d\varepsilon\rho_c(E^* - \varepsilon)f(\varepsilon), \quad (8)$$

with

$$f(\varepsilon) = \frac{4}{3\pi}\frac{1+\kappa}{m_n c^2}\frac{e^2}{\hbar c}\frac{NZ}{A}\frac{\Gamma_G\varepsilon^4}{(\Gamma_G\varepsilon)^2 + \left(\varepsilon^2 - E_G^2\right)^2}, \quad (9)$$

with $\kappa = 0.75$. $E_G$ and $\Gamma_G$ are the position and width of the GDR, and their values are taken from Refs. [20,48].

The present Monte Carlo simulation allows for the discrete emission of light particles. The procedure is as follows: We calculate the decay widths for light particles at each Langevin time step $\tau$. Then the emission of particle is allowed by asking along the trajectory at each time step $\tau$ whether a random number $\zeta$ is less than the ratio of the Langevin time step $\tau$ to the decay time $\tau_{\rm dec} = \hbar/\Gamma_{\rm part}$: $\zeta < \tau/\tau_{\rm dec}$ $(0 \leqslant \zeta \leqslant 1)$, where $\Gamma_{\rm part}$ is the sum of light particle decay widths. If this is the case, a particle is emitted and we ask for the kind of particle $\nu$ $(\nu = n, p, \alpha, \gamma)$ by a Monte Carlo selection with the weights $\Gamma_\nu/\Gamma_{\rm part}$. This procedure simulates the law of radioactive decay for the different particles.

After each emission act of a particle of kind $\nu$ the energy of the emitted particle is calculated by a hit-and-miss Monte Carlo procedure, using the integrand of the formula for the corresponding decay width as weight function. Then the intrinsic energy, the entropy, and the temperature in the Langevin equation are recalculated and the dynamics is continued. The loss of angular momentum is taken into account by assuming that a neutron carries away $1\hbar$, a proton $1\hbar$, and an $\alpha$-particle $2\hbar$.

The excitation energy at scission ($E^*_{\rm sc}$) is determined by using energy conservation law

$$E^* = E^*_{\rm sc} + E_{\rm coll} + V(q) + E_{\rm evap}(t_{\rm sc}), \quad (10)$$

where $E_{\rm coll}$ is the kinetic energy of the collective degrees of freedom, and $E_{\rm evap}(t_{\rm sc})$ is the energy carried away by all evaporated particles by the scission time $t_{\rm sc}$. Equation (10) has been demonstrated [50] in the high energy fission case to describe excellently the experimental $E^*_{\rm sc}$ for a great number of fissioning systems which cover a wide range of fissilities and CN mass regions.

In CDSM calculation, the surface energy $B_s(q)$ and Coulomb energy $B_c(q)$ as a function of $q$ can be parametrized in the form [51]

$$B_s(q) = \begin{cases} 1 + 2.844(q - 0.375)^2, & \text{if } q < 0.452, \\ 0.983 + 0.439(q - 0.375), & \text{if } q \geqslant 0.452 \end{cases} \quad (11)$$

and

$$B_c(q) = \begin{cases} 1 + (1 - B_s(q) + B_f/E_{ssp})/2X & \text{for } q \geqslant 0.452, \\ 1 - 1.422(q - 0.375)^2 & \text{for } q < 0.452, \end{cases} \quad (12)$$

where $B_f$ and $E_{ssp}$ are the fission barrier and the surface energy of a spherical nucleus with fissility $X$, respectively [48]. For a sphere $B_s = B_c = 1$. Regarding the parametrization of rotation energy, see Ref. [48].

Because mass formula [45] contains the deformation-dependent surface energy term and the Coulomb energy term, the particle binding energy $B_i$ $(i = n, p, \alpha)$ is also a function of $q$ [23–25], and it can be written as [20]

$$B_i(q) = M_p(q) - M_d(q) + M_i, \quad (13)$$

where $M_i$ $(i = n, p, \alpha)$ is the mass of the emitted particles. $M_p(q)$ and $M_d(q)$ are the masses of the mother and daughter nuclei, respectively.

The CDSM describes the fission process as follows. At early times, the decay of the system is modelled by means of the Langevin equation. After the fission probability flow over the fission barrier attains its quasistationary value, the decay of the compound system is described by a statistical branch [52]. In the statistical branch we calculate the decay widths for particle emission and the fission width and use a standard Monte Carlo cascade procedure with the weights $\Gamma_i/\Gamma_{\rm tot}$ with $(i = \text{fission}, n, p, \alpha, \gamma)$ and $\Gamma_{\rm tot} = \sum_i \Gamma_i$. This procedure allows for multiple emissions of light particles and higher chance fission. In case fission is decided there, one switches again to the Langevin equation for computing the evolution from saddle to scission.

## III. RESULTS AND DISCUSSION

In this section we present the results of the Langevin model calculations for $E^*_{sc}$ at high energy, in particular for CNs with a marked difference in their spins, neutron-to-proton ratios, fissilities, and sizes. Additionally, the influences coming from deformation effects as well as from different descriptions of the coefficients in the deformation dependence of level-density parameters are discussed in detail.

### A. Influence of angular momentum on $E^*_{sc}$ vs. $\beta$

The spin distribution of a CN, which can be expressed as $\frac{d\sigma_{fus}(\ell)}{d\ell} = \frac{2\pi}{k^2}\frac{2\ell+1}{1+\exp[(\ell-\ell_c)/\delta\ell]}$ where the parameters $\ell_c$ and $\delta\ell$ are the critical angular momentum for fusion and diffuseness, is an essential ingredient in statistical decay models formulated to give a successful description of de-excitation processes of an excited CN coming from heavy-ion fusions [53,54]. It has a critical influence on decay products of a CN, including the amplitude of fission probability and prescission particle number, two main observables that have been used to get information of nuclear dissipation based on heavy-ion-induced fission data.

In the previous publication [28], $E^*_{sc}$ as a function of $\beta$ at high energy was calculated for a $^{200}$Hg system at critical angular momentum $\ell_c = 10\hbar$ only. While the low spin is a characteristic of CNs formed in spallation reactions, like heavy-ion fusions the formed CN also has a spin distribution; that is, its spin has a fluctuation.

Besides in statistical type models, the $\sigma_{fus}(\ell)$ is also a key input parameter in dynamical model [17,18,34–37,39] calculation of a CN decay. It strongly affects the yields of fission cross sections and particle multiplicities as well as their sensitivity to $\beta$, as has been illustrated in calculation for CNs with excitation energy around 150 MeV (see, e.g., Ref. [20]).

Given the role played by $\sigma_{fus}(\ell)$ in the decay mechanism of a hot CN, in the present work Langevin calculations are performed at several $\ell_c$ in order to investigate the evolution of the curve of $E^*_{sc}$ vs. $\beta$ with $\ell_c$.

Shown in Fig. 1 are $E^*_{sc}$ as a function of $\beta$ calculated at $E^* = 450$ MeV for three $\ell_c = 10\hbar$, $30\hbar$, and $50\hbar$. As can be seen from the figure, angular momentum has a minor effect on the magnitude of $E^*_{sc}$, implying that the sensitivity of $E^*_{sc}$ to $\beta$ at high energy is robust with respect to the spin of CNs. The reason is that at high energy of 450 MeV, particle emission is dominated by excitation energy and the friction strength rather than by angular momentum. This is in contrast with the case of the lowly excited ($E^* \sim 150$ MeV) CNs produced in heavy-ion fusions where an apparent influence of angular momentum on particle emission was observed [10,20]. Thus, the present result demonstrates that it is obviously advantageous to choose spallation reactions induced by energetic protons as an avenue of probing nuclear dissipation in comparison with heavy-ion fusion reaction approach.

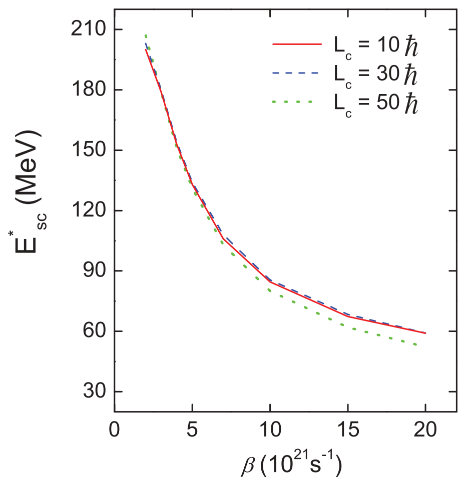


FIG. 1. (Color online) Excitation energy at scission $E^*_{sc}$ as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV for critical angular momenta of $^{200}$Hg compound systems $\ell_c = 10\hbar$ (red line, taken from Fig. 1 in Ref. [28]), $30\hbar$ (dashed blue line), and $50\hbar$ (dotted green line).

Apart from these known observables, $\ell$-related observables, i.e., the spin dependence of the evaporation residues $\sigma_{ER}(\ell)$ that can be measured by $\gamma$-ray multiplicity distributions in fusion reactions, has recently been suggested to be a sensitive probe of nuclear friction [55–58]. It has been found [17] that the magnitude of $\ell_c$ has a strong influence on the extracted $\beta$ value from measured $\sigma_{ER}(\ell)$ data. CNs populated via fusion reactions have a spin distribution $\sigma_{fus}(\ell)$, i.e., there exists a range of angular momentum (0–$\ell_c$) for the formed CN. So when the excited CN decays, its decay products, including evaporation residues, are of a spin distribution. This is the reason for the origin of the spin dependence of the evaporation residues $\sigma_{ER}(\ell)$. Moreover, in stochastic Langevin calculation, for each trajectory simulating the fission motion an angular momentum $L = \hbar\ell$ is sampled from the spin distribution of the CN, i.e., $d\sigma_{fus}(\ell)/d\ell$. The final calculated results [including for $\sigma_{ER}(\ell)$] are weighted over all relevant waves. From the origin of the $\sigma_{ER}(\ell)$ and its calculation procedure mentioned, it is clear that constraining the friction strength by comparing the calculated and measured $\sigma_{ER}(\ell)$ requires precise information of $\sigma_{fus}(\ell)$.

Thus an uncertainty in $\sigma_{fus}(\ell)$ (mainly in $\ell_c$) has a sizable contribution to the diverse friction value derived through a comparison between experiment and theory. In order to precisely derive the friction strength, it is ideal to reduce the dependence of fission observables on the CN spin distribution. Currently, information on $\sigma_{fus}(\ell)$ is from estimations by model calculation or by fitting experimental fusion cross sections. However, an accurate determination of it is not very easy.

Combining the result obtained in Ref. [28], where it has been shown that at high energy $E^*_{sc}$ is sensitively dependent on $\beta$, with the present results of calculations, i.e., the insensitivity of the $E^*_{sc}$ value to $\ell_c$, one can conclude that the $E^*_{sc}$ measured in spallation reactions could offer not only a sensitive but also a robust probe of nuclear dissipation. The latter feature of the $E^*_{sc}$ is favorable to a stringent constraint of $\beta$.

### B. Influence of neutron-to-proton ratio on $E^*_{sc}$ vs. $\beta$

In heavy-ion reaction experiments a situation that is frequently encountered is that particle multiplicity or evaporation residue cross section was measured for CNs with the same $Z$ but with a difference in their neutron-to-proton ratios ($N/Z$) resulting from various combinations of projectile and targets. These CNs having different $N/Z$ values were used to extract information of nuclear friction, for example, see Ref. [59]. A decreasing trend of fission cross sections with $N/Z$ of a CN formed at similar excitation energies and angular momenta has been observed experimentally in the excitation function of fission cross sections for isotopes of Pt, Po, etc. [53,60–63]. Additionally, a considerable change in the sensitivity of evaporation residue cross section [64] and its spin distribution [17] to $\beta$ arising from $N/Z$ was found in Langevin calculations.

In particular, a recent work [22] has indicated that the magnitude of $N/Z$ of a decaying CN can affect light particle emissions; that is, with increasing $N/Z$ more neurons and less light charged particles (LCPs), including protons and $\alpha$ particles, are emitted. The change in the number of emitted neutrons and LCPs may have a direct effect on the amplitude of $E^*_{sc}$.

Aside from the spin distribution of CNs, a crucial input parameter for decay-type model calculations mentioned previously, we will explore here under high-energy conditions the influence of $N/Z$ of a CN on $E^*_{sc}$. Three Hg isotopes, i.e., $^{194}$Hg, $^{200}$Hg, and $^{206}$Hg whose $N/Z$ is 1.425, 1.5, and 1.575 are chosen for this purpose.

As seen from Fig. 2, where the multiplicities of various light particles as functions of $N/Z$ and $\beta$ are shown, $M_n$ is an increasing function of the $N/Z$ of the three Hg nuclei, $M_p$ and $M_\alpha$ are a decreasing function of the $N/Z$. This is due to the systematics of the neutron number of fissioning sources, and these types of behavior can be explained in terms of the change of the particle separating energy for these fissioning sources with different $N/Z$.

A lower $E^*_{sc}$ for $^{206}$Hg relative to the case of $^{200}$Hg and $^{194}$Hg, as shown in Fig. 3, is mainly because the former evaporates more neutrons.

While a greater $M_n$ for $^{206}$Hg carries away more excitation energy from the decaying system compared to the $^{194}$Hg case, a slightly stronger LCPs emission [see Figs. 2(b) and 2(c)] for the latter also causes an extra cost of excitation energy. As a consequence of both neutron and LCPs emissions, the excitation energy taken away from both $^{206}$Hg and $^{194}$Hg decaying systems does not differ very much, i.e., $E^*_{sc}$ displays a similar sensitivity to $\beta$ for these Hg CNs with different $N/Z$. The expectation is confirmed in Fig. 3.

### C. Influence of fissility on $E^*_{sc}$ vs. $\beta$

Even though in the presence of friction, fission lifetimes may become short with increasing the fissility, because fission barriers are a decreasing function of fissility. In Fig. 4, fission barriers of $^{200}$Hg ($Z = 80$) and $^{200}$Rn ($Z = 86$) are displayed. For the two nuclei, their fissilities (defined as $Z^2/A$) are 32 and 37, respectively, which have a marked difference.

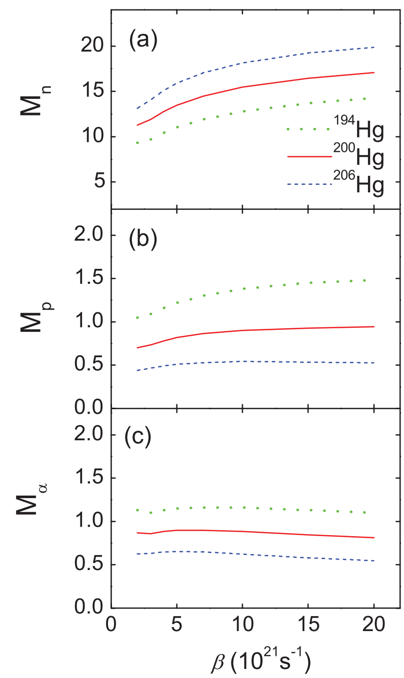


FIG. 2. (Color online) Comparison of evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for three compound systems $^{194}$Hg (dotted green lines), $^{200}$Hg (red lines), and $^{206}$Hg (dashed blue lines).

Our calculation shows that the magnitudes of the $E^*_{sc}$ calculated at $E^* = 350$ MeV for both $^{200}$Hg and $^{200}$Rn systems are comparable (Fig. 5). The physical understanding for the result is as follows. A high fission barrier (for low-fissility $^{200}$Hg) decreases the fission decay width and protects the system from disintegrating quickly, which could enhance particle emission because the number of emitted light particles

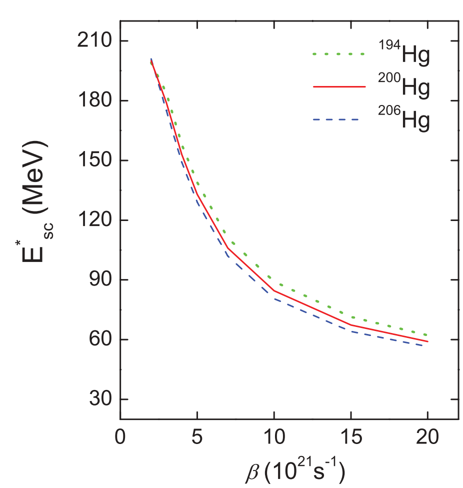


FIG. 3. (Color online) Comparison of excitation energy at scission $E^*_{sc}$ as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound nuclei $^{194}$Hg (dotted green line), $^{200}$Hg (red line, taken from Fig. 1 in Ref. [28]), and $^{206}$Hg (dashed blue line).

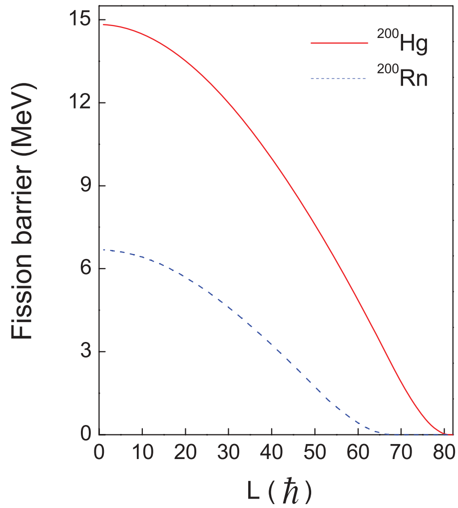


FIG. 4. (Color online) Fission barriers of $^{200}$Hg (red line) and $^{200}$Rn (dashed blue line) as a function of angular momentum.

is closely connected to the fission time length. But on the other side, there also exists a strong competition between neutron and LCP decay channels. It means that the enhancement of neutron emission leads to a weak LCP emission. Consequently, although $^{200}$Hg has a lower fissility, favoring neutron emission, as seen in Fig. 6(a), it evaporates less LCPs compared to the high-fissility $^{200}$Rn case, as also seen in Figs. 6(b) and 6(c).

In the CDSM calculation, the loss of excitation energy of the decaying system due to neutron emission contains its separation energy and its kinetic energy. However, for a LCP emission, besides particle separation energy and kinetic energy, an extra energy is needed to help the LCP to overcome its Coulomb emission barrier. Evaporating a LCP thus costs more excitation energy of the decaying nucleus. So, a greater neutron multiplicity for low-fissility $^{200}$Hg as compared to that of high-fissility $^{200}$Rn does not mean more excitation energy

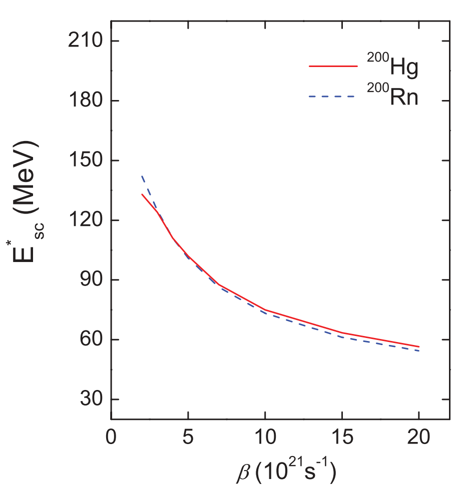


FIG. 5. (Color online) Comparison of excitation energy at scission $E^*_{sc}$ as a function of friction strength $\beta$ at initial excitation energy $E^* = 350$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound systems $^{200}$Hg (red line, taken from Fig. 1 in Ref. [28]) and $^{200}$Rn (dashed blue line).

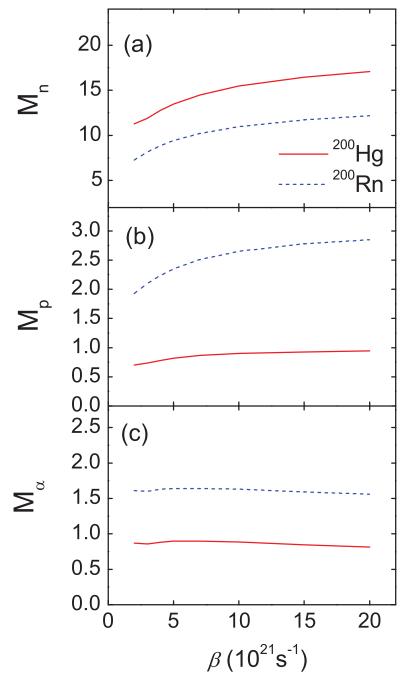


FIG. 6. (Color online) Comparison of evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) as a function of friction strength $\beta$ at initial excitation energy $E^* = 350$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound systems $^{200}$Hg (red lines, taken from Fig. 2 in Ref. [28]) and $^{200}$Rn (dashed blue lines).

is taken away from the former system. Furthermore, given that the $E^*_{sc}$ is determined by all particles evaporated throughout the fission process, the loss of the excitation energy originating from contributions of both neutron and LCP emissions for the two decaying systems with different fissilities is very close. This yields an analogous $E^*_{sc}$ (Fig. 5).

More importantly, we observe from Fig. 5 that the evolution of $E^*_{sc}$ with $\beta$ is alike for both $^{200}$Hg and $^{200}$Rn. The observation demonstrates that the sensitivity of $E^*_{sc}$ to $\beta$ at high energy almost does not depend upon the fissility of CNs involved.

### D. Influence of the size of CNs on $E^*_{sc}$ vs. $\beta$

To date, numerous measurements have been reported for particle emission from a broad range of a CN whose size $A$ spans from 150 to 250.

Prescission particles vary with the size of a fissioning system [34]. The heavier the decaying nucleus, the longer the saddle to scission path, which enhances postsaddle particles. It is therefore interesting to examine the possible influence of the size of a CN on the sensitivity of $E^*_{sc}$ vs. $\beta$. We use a heavy nucleus $^{240}$Cf as illustration.

In Fig. 7, one can notice a lower $E^*_{sc}$ for $^{240}$Cf than for $^{200}$Hg. This is because under the condition of high energy, even though a number of particles are emitted at an early stage of fission process, but due to a high initial excitation energy a considerable fraction of excitation energy is still left in the decay system that is available for further evaporating particles. Compared to the light $^{200}$Hg, a longer descent from saddle to

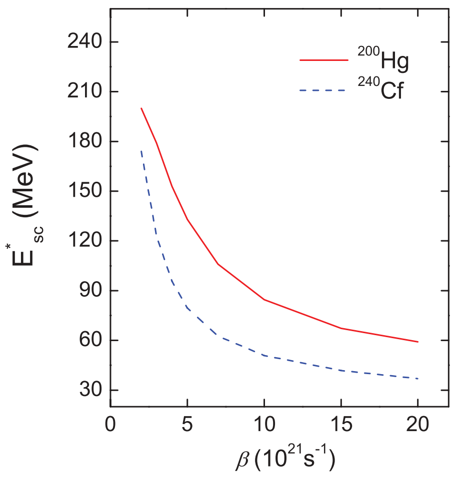


FIG. 7. (Color online) Comparison of excitation energy at scission $E^*_{sc}$ as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for systems $^{200}$Hg (red line, taken from Fig. 1 in Ref. [28]) and $^{240}$Cf (dashed blue line).

scission for heavy nuclei $^{240}$Cf significantly increases postsaddle particles, because the fission process of the latter is retarded more longer, providing more time for particle evaporation. Apart from increasing prescission light particles, a larger saddle-to-scission emission at high $E^*$ also leads to a larger cooling of the decaying system, i.e., a lower $E^*_{sc}$ at high $E^*$.

Although a rise in the size of CNs modifies the emitted particle number (Fig. 8), the significant sensitivity of $E^*_{sc}$ vs. $\beta$

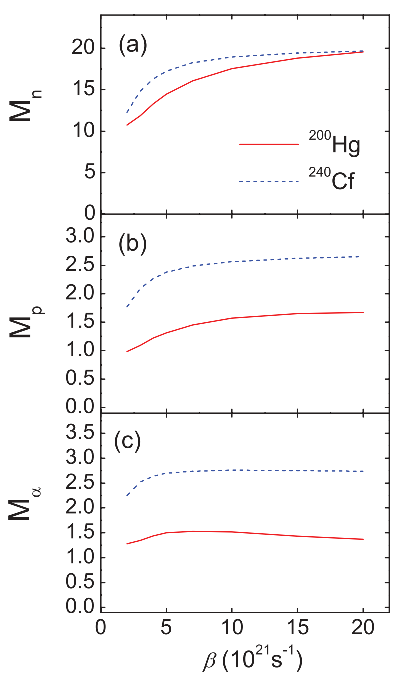


FIG. 8. (Color online) Comparison of evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound systems $^{200}$Hg (red lines) and $^{240}$Cf (dashed blue lines).

is still robust with respect to the size of the fissioning nuclei, as clearly seen in Fig. 7. The result has an implication for one to choose different types observables to get knowledge of $\beta$ in fission. We explain it in the following way. When employing observables, for instance, fission probability [2,53,54], evaporation residue cross section [15], and excitation energy at saddle [11] to constrain the friction strength, theoretical calculations of particle number emitted before saddle and from saddle to scission are important, because the calculated presaddle particles directly affect the amplitudes of these observables. And a simulation of postsaddle particle emission is also crucial for constraining the friction strength through analyzing experimental data of kinetic-energy and mass distributions of fission fragments [13,18]. In other words, a precise extraction of $\beta$ by comparing calculations with above-mentioned observable data demands an accurate evaluation of pre- and postsaddle particle multiplicity.

In contrast, the $E^*_{sc}$ value is controlled by total prescission particles, not by a pre- or a postsaddle emission alone. Thus, using $E^*_{sc}$ to determine $\beta$ does not require information of pre- or postsaddle particles. Furthermore, experimentally the multiplicities of prescission particles can be extracted by fitting particle energy spectra in coincidence with fission fragments, but the multiplicities coming from inside the barrier or from saddle to scission points cannot. In addition, by measuring the particle energy spectra, the particle multiplicities from a prescission CN stage and from a postscission stage can be obtained simultaneously.

As indicated above, constraining the friction strength with fission probability, evaporation-residue cross section, etc. relies on the information of presaddle particle number that can only be available from theoretical simulations, because presaddle emission cannot be disentangled in experiment data. However, combining both $E^*_{sc}$ and precsission particles offers an opportunity that could put a more stringent constraint on the friction strength. This is not only because such a combination can incorporate more experimental information, i.e., particle emission from both the CN and two fission fragments, but also because prescission particles and $E^*_{sc}$ determined from measured postscission particles [2] are two kinds of independent and, thereby, complementary information sources.

### E. Influence of level density parameters on $E^*_{sc}$ vs. $\beta$

As pointed out in Refs. [40,65,66], the driving force of a hot system is not simply the negative gradient of the conservative potential but should contain a thermodynamical correction. So the crucial quantity adopted in dynamical model calculations is not bare potential $V(q)$ but entropy $S(q, E^*)$, which is constructed starting from the deformation-dependent level density parameter and determines the force in the Langevin equation [Eq. (1)]. Therefore, the level density parameter is a key parameter in dynamical calculations. It is expressed as

$$a(q) = a_1 A + a_2 A^{2/3} B_s(q), \quad (14)$$

where $A$ is the mass number, and $B_s(q)$ is defined by Eq. (11). Two sets of the coefficients $a_1$ and $a_2$ that are frequently used in calculation are considered here. One set

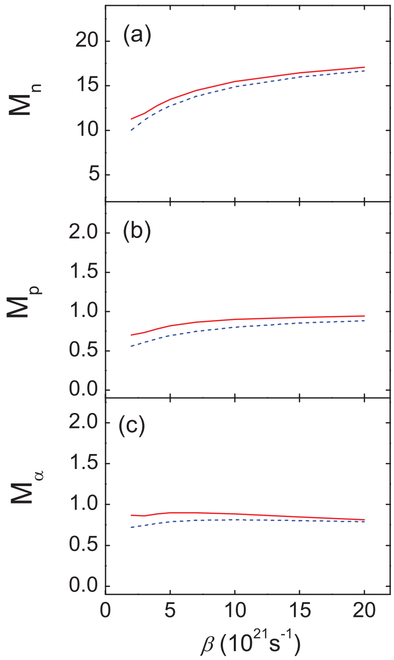


FIG. 9. (Color online) Comparison of evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) of $^{200}$Hg compound systems as a function of friction strength $\beta$ at initial excitation energy $E^* = 350$ MeV and critical angular momentum $\ell_c = 10\hbar$ for level density parameter of Ignatyuk *et al.* (red lines, taken from Fig. 2 in Ref. [28]) and of Tõke *et al.* (dashed blue lines).

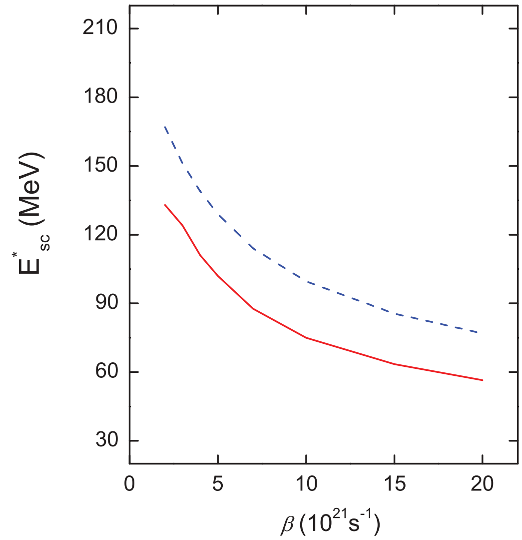


FIG. 10. (Color online) Comparison of excitation energy at scission $E^*_{\rm sc}$ of $^{200}$Hg compound systems as a function of friction strength $\beta$ at initial excitation energy $E^* = 350$ MeV and critical angular momentum $\ell_c = 10\hbar$ for level density parameter of Ignatyuk *et al.* (red line, taken from Fig. 1 in Ref. [28]) and of Tõke *et al.* (dashed blue line).

is from Ignatyuk *et al.*'s (Ign) prescription [26] in which $a_1 = 0.073$ MeV$^{-1}$ and $a_1 = 0.095$ MeV$^{-1}$. Another is taken from predictions by Tõke and Światecki [27] (TS), who suggested that $a_1 = 0.0685$ MeV$^{-1}$ and $a_2 = 0.274$ MeV$^{-1}$. The previous systematic calculations with stochastic models [20] for CNs with excitation energy ($\sim$150 MeV) formed in heavy-ion-induced fission indicated that Ign coefficients give a smaller fission probability but a larger particle multiplicity as compared to TS ones.

Since a difference in particle multiplicity was noted in the calculations with Ign and TS coefficients, it is necessary to check whether the difference can affect the sensitivity of $E^*_{\rm sc}$ to $\beta$ found in Ref. [28] where Ign level density parameter was adopted. The calculation results at high energy of $E^* =$ 350 MeV with TS $a(q)$ and a comparison with those obtained with Ign $a(q)$ are plotted in Fig. 9. As expected, the use of TS coefficients gives a slightly lower particle multiplicity, which is in line with the calculation made earlier at a lower excitation energy [20].

As a consequence of a larger particle multiplicity obtained with Ign level density parameter, more excitation energy is taken away from the decay system, leaving a colder excitation energy at scission, i.e., a smaller $E^*_{\rm sc}$ relative to the TS level density parameter case, as observed in Fig. 10.

While a difference in the coefficients of $a(q)$ influences the magnitude of $E^*_{\rm sc}$, a prominent feature seen from Fig. 10 is that the slope of $E^*_{\rm sc}$ with $\beta$ predicted with two sets of the constants $a_1$ and $a_2$ is alike, meaning the sensitivity of $E^*_{\rm sc}$ to $\beta$ is little affected by the different description of the coefficients in the deformation-dependent level density parameter $a(q)$.

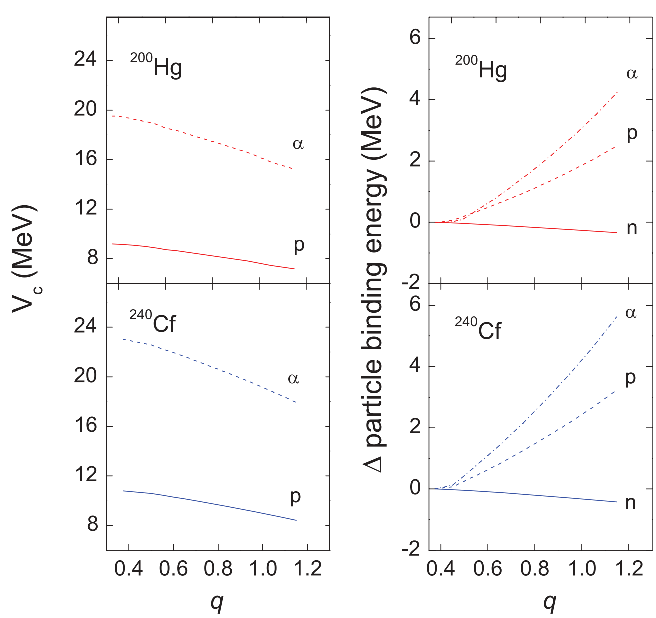


FIG. 11. (Color online) Left column: Emission barrier ($V_c$) of protons and $\alpha$ particles of the fissioning systems $^{200}$Hg (upper panel) and $^{240}$Cf (lower panel) as a function of deformation coordinate $q$. Right column: Change in neutron ($\Delta B_n$), proton ($\Delta B_p$), and $\alpha$-particle ($\Delta B_\alpha$) binding energies as a function of deformation coordinate $q$ relative to the spherical binding energies for compound systems $^{200}$Hg (upper panel) and $^{240}$Cf (lower panel).

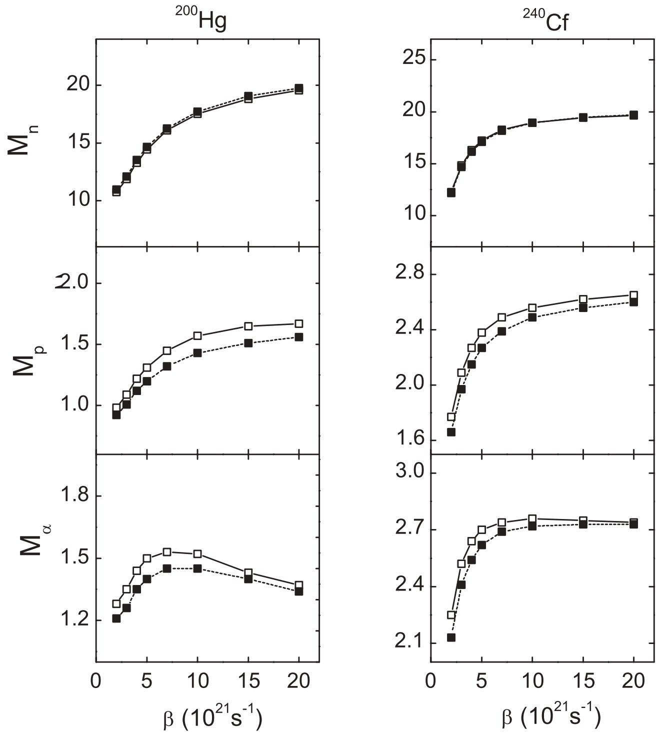


FIG. 12. Evaporated multiplicities of prescission neutrons (a), protons (b), and $\alpha$ particles (c) as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound systems $^{200}$Hg (left column) and $^{240}$Cf (right column) without deformation effects (open symbols connected by solid lines) and with deformation effects (full symbols connected by dashed lines).

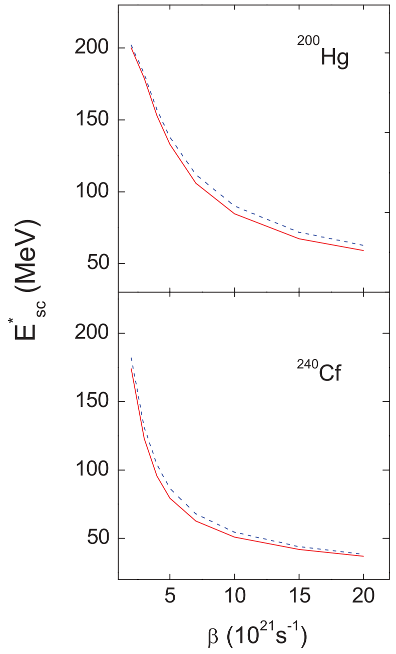


FIG. 13. (Color online) Excitation energy at scission $E^*_{\rm sc}$ as a function of friction strength $\beta$ at initial excitation energy $E^* = 450$ MeV and critical angular momentum $\ell_c = 10\hbar$ for compound systems $^{200}$Hg (upper panel, in which the red line is taken from Fig. 1 in Ref. [28]) and $^{240}$Cf (lower panel) without (red lines) and with (dashed blue lines) deformation effects.

### F. Influence of deformation on $E^*_{\rm sc}$ vs. $\beta$

As a CN evolves from ground state to scission, it experiences a deformation. The deformation effects in fission dynamics were noted and their importance in interpreting and determining the $\beta$ value from particle-multiplicity data has been stressed by many authors [23–25,67–70]. Here we study the possible influence of deformation on $E^*_{\rm sc}$ as a sensitive probe of $\beta$ at high energy.

The role of deformation in particle evaporation is through modifying particle binding energies and Coulomb barriers of LCPs. Take $^{200}$Hg as an example. First, deformation decreases the neutron binding energy, because Fig. 11 exhibits that $\Delta B_n$ drops with increasing $q$ that only causes a tiny rise of $M_n$ for $^{200}$Hg calculated at $E^* = 450$ MeV (Fig. 12), see below for reasons.

Second, the emission barrier ($V_c$) of protons ($\alpha$ particles) decreases, as dynamic evolution of the decaying system proceeds from ground-state configuration at which $q = 0.375$ to scission-point configuration at which $q = 1.15$. Contrary to the behavior of $\Delta B_n$, $\Delta B_p$ and $\Delta B_\alpha$ are an increasing function of $q$, as depicted in the right upper panel of Fig. 11; that is, deformation increases the binding energies of LCPs. While a fall in $V_c$ favors LCP emission, a rise in $B_p$ ($B_\alpha$) hinders LCP emission. Thus, the opposite variations of $V_c$ and $B_p$ ($B_\alpha$) with $q$ have the counterbalanced effects on $M_p$ ($M_\alpha$). Moreover, given a strong competition among various decay channels during the shape evolution of a CN from equilibrium ground state to scission, an enhanced neutron decay will suppress LCP decay channels. Consequently, the influence of nuclear dissipation on LCPs illustrates a sensitivity to deformation (Fig. 12).

Because accounting for deformation effects in calculation has an opposite influence on the emissions of neutrons and LCPs, this could modify the $E^*_{\rm sc}$ value. The prediction is confirmed in Fig. 13. However, the effects of deformation on the significant sensitivity of $E^*_{\rm sc}$ to $\beta$ are minor, as can also be clearly seen in the upper panel of Fig. 13.

To further confirm the conclusion drawn from the $^{200}$Hg nucleus, we make a calculation for a heavy system $^{240}$Cf. This is because with increasing the size of a decaying CN, it will experience a more larger deformation when it fissions, which yields a greater modification to particle binding energy and to the emission barrier of LCPs (the lower panel of Fig. 11).

However, calculations indicate that the multiplicities of neutrons and LCPs of the heavy system Cf with and without deformation effects do not vary very much (the right column in Fig. 12). This is possibly because under high-energy conditions the role of deformation in modifying particle emission is not very prominent. As a consequence, the effects of deformation on $E^*_{\rm sc}$ and its sensitivity to $\beta$ are minor, as illustrated in the lower panel of Fig. 13. The result further demonstrates that the significant sensitivity of $E^*_{\rm sc}$ to $\beta$ at high energy remains unaltered even though taking account of the deformation effects in calculation.

## IV. SUMMARY AND CONCLUSIONS

Based on the Langevin model that incorporates the statistical description of particle evaporation along the fission path, we examine the robustness of the significant sensitivity of the excitation energy at scission ($E^*_{sc}$) to friction ($\beta$) against those important factors that are expected to strongly affect the multiplicity of light particles emitted in fission. They include: (i) two critical input parameters for solving the stochastic equations; that is, the spin of formed CNs and different coefficients applied to describe the deformation dependence of level-density parameters; (ii) important effects, such as neutron-to-proton ratio, fissility and the size of a CN that have been found in previous studies to have strong effects on the particle emission, and (iii) deformation effects in fission processes.

On the whole, our calculation results, namely the robustness of the significant sensitivity of $E^*_{sc}$ to $\beta$, underline the fundamental roles played by excitation energy and the friction strength on the production of the particle number emitted from CNs formed at high energy. In other words, under a high-energy condition, extracting $\beta$ with $E^*_{sc}$ can largely lower the dependence of $E^*_{sc}$ and hence $\beta$ on the previously mentioned various factors. The present result thus not only corroborates the conclusion reached in our recent work [28], but also it further demonstrates that $E^*_{sc}$ provided with spallation reaction approach is a robust probe of $\beta$. Compared to particle multiplicity and evaporation residue cross section, measured in fusion reactions and widely used to constrain the friction strength, the robust feature of the $E^*_{sc}$ established here could provide a more reliable and stringent constraint of $\beta$.

Besides these studied factors that affect a CN decay, the distribution of CN shapes produced in heavy-ion fusions has been found by Charity [71] to have a significant effect on evaporation. It indicates that it is important to consider the shape distribution in calculation, especially when confronting theoretical calculations with experimental observables involving heavy-ion fusion-fission and fusion-evaporation processes. Different from heavy-ion collisions, CNs formed in proton-nucleus collisions have a nearly spherical shape due to low spin involved [29,31,32,72].

Even for the calculation of the initial excitation energy of CNs coming from heavy-ion fusion reactions, it is complicated than originally expected. It is usually obtained by a simple formula $E^* = E_{\mathrm{c.m.}} + Q$, where $E_{\mathrm{c.m.}} = E_{\mathrm{lab}} A_T/(A_T + A_P)$ is the center-of-mass energy of the fusion heavy-ion system and $Q$ is the fusion $Q$ value. But Umar *et al.* [73] recently indicated that accurately evaluating the excitation energy of the systems formed during heavy-ion collisions needs an elaborate microscopic calculation. Also, although particle emission in proton-nucleus collision processes leads to a distribution of initial excitation energy for formed CNs, in heavy-ion reactions the phenomenon of preequilibrium emission also complicates the determination of the initial excitation energy of a formed CN [74–76].

While uncertainties and difficulties have been pointed out for a precise determination of the friction strength on the basis of decay properties of hot CNs produced in heavy-ion reactions, the use of spallation reaction approach has several issues that need to be treated properly. For instance, the formed CN via spallation reactions has a fluctuation in its initial excitation energy, as indicated in Ref. [28]. So, on the experimental side, for experimentalists who endeavor to acquire an accurate knowledge of the friction strength by means of the proton-nucleus reaction approach, efforts are required to find a more efficient way to select events with high initial excitation energy and have a method to evaluate it. This may require a detection of both fission fragments and various light particles. In addition, to better guide experimental exploration, a more complete calculation for energetic proton-induced fission processes is necessary. In this aspect, intranuclear cascade models developed by Cugnon *et al.* [72,77], quantum molecular dynamics models [78], etc., have been proposed to deal with collision processes between protons and nuclei in spallation reactions.

In light of the present study, $E^*_{sc}$ has been demonstrated to be a robust probe of $\beta$, but more experimental and theoretical researches are still needed to better employ the probe to tightly constrain the dissipation strength in nuclear fission.


## ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China under Grant No. 11075034.



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