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Consistent analysis of the 2+ excitation of the 12C Hoyle state populated in proton and α-particle inelastic scattering

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The ^{12}C excitation energy spectra populated in both proton and α-particle inelastic scattering measurements are examined. The data indicate the existence of a 2^+ state at $E_x = 9.75(0.15)$ MeV with a width of 750(150) keV. It is believed that this state corresponds to the 2^+ excitation of the 7.65 MeV, 0^+, Hoyle state, which acts as the main path by which carbon is synthesized in stars. A simultaneous R-matrix analysis of the two sets of data indicates that the 2^+ state possesses a very large α-reduced width, approaching the Wigner limit. This would indicate that the state is associated with a highly clustered structure. The potential geometric arrangements of the clusters is discussed.

I. INTRODUCTION

One of the longest standing challenges is to understand, in detail, the structure of light nuclei. Such systems have a large range of structural possibilities that are very sensitive to details of the nucleon-nucleon interaction. For example, both momentum and spatial correlations are extremely important. These manifest themselves as clusters, preformed inside the nucleus. The strong pairing interaction results in the α-particle being pre-eminent amongst cluster possibilities. Not only does it have a very large binding energy, but it has a first excited state close to 20 MeV which makes it a rather inert object. It is due to the very large α-particle binding energy that the nucleus ^{8}Be is unbound to α-decay even in its ground state. One of the great advances in the field in recent years has been the ability to calculate the structure of nuclei up to $A = 12$ from first principles, *ab initio* [1–4]. In this approach an in-medium nucleon-nucleon interaction is motivated from a starting point of the free nucleon-nucleon two-body interaction and then includes additional three-body contributions. These calculations reproduce the binding and excitation energies of

many light systems remarkably well. In the case of ^{8}Be they clearly reveal α-particle like correlations [1], highlighting the important role of an α-α cluster structure in the ground state, as already suggested by the experimentally measured decay width and rotational properties.

Historically, one of the pre-eminent tests of our understanding of the structure of light nuclei lies in the nature of the second excited state in ^{12}C. Since this system resides at the limits of the *ab initio* approach it is an important test. This state has character $J^\pi = 0^+$ and lies at $E_x = 7.65$ MeV. It is known as the Hoyle state as it was predicted by Fred Hoyle [5,6] as a solution to the discrepancy between the observed and predicted abundance of ^{12}C. ^{12}C is synthesized in the triple-α process, whereby the two α particles briefly fuse to make ^{8}Be and at sufficient densities there is a finite probability of capturing a third α particle to form ^{12}C. The 7.65 MeV state serves as a *doorway* resonance, substantially enhancing the reaction rate. Without this resonance, or even if its energy were slightly different, the abundance of carbon would be dramatically reduced as would that of carbon based lifeforms. A possible 2^+ excitation of this state is included in the NACRE compilations of astrophysical reaction rates at 9.1 MeV [7]. In scenarios in which stellar temperatures are significantly increased such a 2^+ resonance increases the reaction rate by a factor of 10. Thus, the existence of these states is significant [8].

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The question thus arises: Why should the 7.65 MeV state exist just above the ^{8}Be$+\alpha$ decay threshold? This is just at the right place for it to play a significant role in the helium-burning process. Is this a happy accident, or is there a deeper truth? Ikeda and co-workers in the late 1960s [9] postulated a rule that cluster states should lie very close to a cluster-decay threshold (see also [10]). The 7.65 MeV state lies just above the α-decay threshold (7.365 MeV). Calculations using a shell-model (no clustering) framework rather dramatically fail to reproduce the excitation energy of this state [11,12], whereas it is found at the right energy in cluster models [13]. On the face of it, this would confirm its cluster-like structure. Indeed, it is found experimentally to have a large α width (decay probability) which supports the claim for its 3α cluster nature. However, the arrangement of those clusters is unknown. Do they have a linear arrangement, a triangular structure or even some other form? One of the best tests would be to measure the moment of inertia of the system by determining the rotational excitations. The rotational energy of the first excited state, 2^+, is equal to $3\hbar^2/I$, where I is the moment of inertia. The larger the moment of inertia the lower the energy of the 2^+ state and the larger the corresponding nuclear deformation.

Up until now, definitive evidence for a 2^+ excitation is not available. Within a reasonable energy reach from the 7.65 MeV state only one tentative 2^+ state is tabulated at 11.16 MeV, though this has only ever been seen once in the ^{11}B(^{3}He,d) reaction [14] and is likely to be spurious [15]. Precision β-decay measurements indicate the existence of a 2+ state close to this region (11 MeV) [16], though this is unlikely to be connected with the state which was proposed in the measurements of the ^{11}B(^{3}He,d) reaction. Two measurements clearly indicate a 2^+ state with a width of several hundred keV close to 9.6–9.8 MeV [17,18]. One significant problem is that this state is buried beneath a narrow, but dominant, 9.64 MeV 3^- state and a very broad 0^+ state at 10.3 MeV. From an individual measurement an unambiguous result cannot be achieved. Here we provide a consistent analysis of the two sets of data which shows a broad ^{12}C state close to 9.75 MeV which would be the collective excitation of the Hoyle state. This, in principle, permits the cluster configuration of the state discovered over 50 years ago, through which carbon-12 is formed, to be defined.

II. INELASTIC SCATTERING MEASUREMENTS AND ANALYSIS

All of the measurements described here were performed using magnetic spectrometers. The ^{12}C(p,p') study was performed at iThemba Labs in South Africa with a 66 MeV proton beam, with a ^{12}C excitation energy resolution of 24 keV. Full details are described in Ref. [18]. The ^{12}C(α,α') measurement was performed at RCNP Osaka with a 386 MeV ^{4}He beam and a spectrometer providing an excitation energy resolution of $\sim$150 keV, see Ref. [17].

A. ^{12}C(α,α')

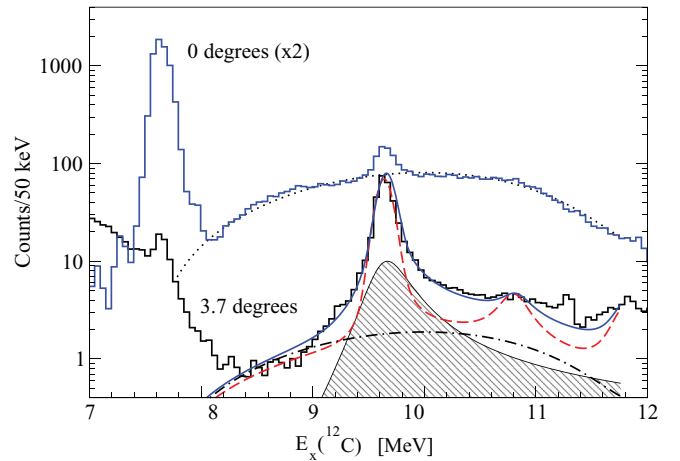Figure 1 shows the excitation energy spectrum for the ^{12}C(α,α') reaction for spectrometer angles of $\theta_{\rm lab} = 0°$ and

FIG. 1. (Color online) ^{12}C excitation energy spectra (with the ^{16}O background subtracted) from α-particle inelastic scattering at 386 MeV at $\theta_{\rm lab} = 0$ and 3.7°. The black-dotted curve characterizes the shape of the 10.3 MeV 0^+ state (and any other broad components). The data at $\theta_{\rm lab} = 3.7°$ have been fitted with two curves. The red dashed line corresponds to all known ^{12}C states. The solid blue curve includes an additional 2^+ contribution from an R-matrix calculation. The hatched region shows the asymmetric shape of the 2^+ state predicted by the R-matrix calculation [Eq. (1)], see text for details. Note there is a contaminant from the hydrogen in the target between 5 and 8 MeV (labeled H in Fig. 2.) The black dot-dashed line is the scaled broad 0^+ background used in the fit which includes the proposed 2^+ contribution.

3.7°. For the $0°$ spectrum it can be seen that the 0^+ states at 7.65 MeV and 10.3 MeV (broad) dominate. There is a small contribution from the 9.64 MeV 3^- state. Any further contributions are hidden. In order to reveal any more weakly populated states a technique for suppressing the dominant contributions has to be found. It is known that the angular distributions for the ^{12}C(α,α') reaction involved in the populating a 0^+ state are highly oscillatory and indeed reach a minimum close to $\theta_{\rm lab} = 3.7°$ (as is shown of Fig. 2 of Ref. [17]). By measuring the ^{12}C excitation energy spectrum at this angle, the 0^+ component should be reduced by nearly a factor of $\sim$100 compared with the $0°$ spectrum, whereas the 2^+ contribution would be slightly enhanced (a similar analysis is performed in Ref. [17]). The difference to the spectrum this makes, shown in Fig. 1, is marked. For example, the amplitude of the 7.65 MeV state is strongly suppressed compared with that of the 9.64 MeV 3^- state. It should be noted that the shape of broad resonances will be slightly modified by the variation of the inelastic scattering probability with excitation energy—though this effect is not believed to dominate.

The red-dashed curve in Fig. 1 shows a line-shape which includes contributions from a scaled 0^+ state (with the shape being extracted from the $0°$ data, with any other broad contributions—black-dotted line) the 9.64 MeV 3^-, 10.84 MeV 1^-, and 11.83 MeV 2^- states. Here we assume a single broad 0^+ state at 10.3 MeV, but there may be contributions from additional broad resonances which are unaccounted for. Where the width of the states is less than
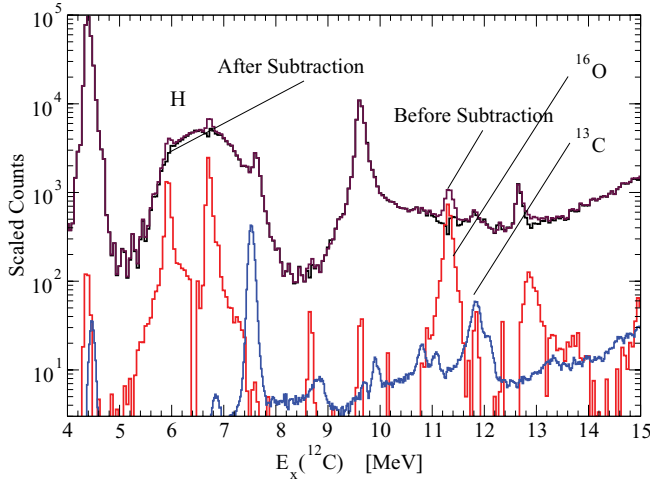
FIG. 2. (Color online) Analysis of background contributions to the ^{12}C excitation energy spectrum populated in α-inelastic scattering at $\theta_{\rm lab} = 3.7°$. The maroon histogram (labeled "Before Subtraction") is the data before background subtraction. The hydrogen contaminant is labeled H. The red histogram (labeled "^{16}O") shows the ^{16}O contaminant spectrum, which when subtracted gives the black histogram (labeled "After Subtraction"). The blue histogram (labeled "^{13}C") shows a measurement of ^{13}C(α,α') measured at $\theta_{\rm lab} = 3.7°$ [19]. The broad feature in the ^{12}C excitation energy spectrum between 9 and 10 MeV cannot be interpreted in terms of target contaminants.

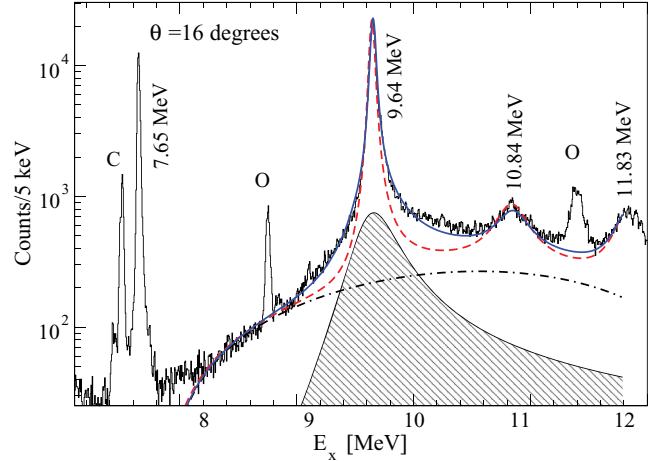

FIG. 3. (Color online) ^{12}C excitation energy spectrum from a measurement of proton inelastic scattering for a 66 MeV proton beam and a scattering angle of $\theta_{\rm lab} = 16°$. Contaminants from ^{13}C and ^{16}O target contaminants are labeled C and O, respectively. Two fits are shown. The red dashed line corresponds to all known ^{12}C states. The solid blue curve includes an additional 2$^+$ contribution from an R-matrix calculation. The hatched region shows the asymmetric shape of the 2$^+$ state predicted by the R-matrix calculation [Eq. (1)], see text for details. The black dot-dashed line is the scaled broad 0$^+$ background used in the fit which includes the proposed 2$^+$ contribution.

the experimental resolution the line shape corresponds to a convolution of the natural line shape with a Gaussian function reflecting the experimental resolution. For the 3$^-$ state the line shape is the result of a convolution of that from an R-matrix calculation (see later for details) with the experimental resolution. The broad 0$^+$ strength has been normalized to the data at $E_x = 8.5$ MeV. It is clear that these contributions alone cannot reproduce the measurement. Hence, there appears to be an additional component.

The question then arises as to if this may be attributed to a target contaminant. Figure 2 shows an analysis of various background contributions to the excitation energy spectrum. It should be noted that the spectra in Fig. 1 has already had a contribution from a ^{16}O background measurement subtracted. The slight mismatch between the ^{12}C and background spectrum resulting in the bipolar feature close to 11.3 MeV. The ^{16}O spectrum was deduced from the difference in measurements with a silicon oxide and a silicon target and is shown as the red histogram in Fig. 2. The maroon and black histograms show the ^{12}C excitation energy spectra before and after the ^{16}O subtraction (labeled "Before Subtraction" and "After Subtraction", respectively). It can be seen that between 9–11 MeV that the ^{16}O contribution is negligible. The broad bump between 5 and 8 MeV corresponds to the hydrogen contaminant in the target. The blue histogram represents a measurement of the ^{13}C(α,α') reaction at an angle of 3.7° and a beam energy of 388 MeV [19]. There appear to be very few features in the spectrum from the ^{12}C target which indicate a significant ^{13}C contribution. We have therefore adjusted the ^{13}C strength such that it reproduces the small peak close to 11.8 MeV (which is in fact most likely the

11.83 MeV state in ^{12}C)—this then provides an upper limit on the contribution to the spectrum. Once again, there are no broad features in the ^{13}C spectrum which can account for the discrepancy observed in Fig. 1. Aside from these components, there may also be a contribution from the ^{12}C($\alpha,\alpha'\alpha$) knock-on reaction. To resolve this, a study with an alternative projectile is required.

B. ^{12}C(p,p')

Figure 3 shows the high resolution measurement of the ^{12}C(p,p') reaction. These data also correspond to the ^{12}C excitation energy spectrum measured in a minimum in the inelastic scattering angular distribution (see Fig. 1, Ref. [17]). A similar analysis to that shown in Fig. 1 is performed. The red-dashed curve shows the closest reproduction of the data if a broad 0$^+$ and narrow 3$^-$, 1$^-$, and 2$^-$ contributions are normalized to the data—again the 0$^+$ strength function being normalized to $E_x = 8.5$ MeV. In this instance the 0$^+$ strength was parametrized based on the line shape from Ref. [8]. As shown in Fig. 1 an additional component is again required, which was also demonstrated in Ref. [18] not to be associated with ^{16}O and ^{13}C target components. A further measurement of the ^{12}C(p,p') reaction using a 25 MeV proton beam also finds evidence for an excess yield in the same region [20]. The nature of the broad component associated with the 0$^+$ state is slightly different with the proton and α-projectiles. This may be in part due to the differing influence of the 11.83 MeV 2$^-$ state which is more strongly excited in the proton measurements and that the α-inelastic scattering will also contain components from the ^{12}C($\alpha,\alpha'\alpha$) reaction.

III. THE 2^+ LINE SHAPE

The hatched region shown in both Figs. 1 and 3 corresponds to a broad 2^+ resonance predicted by an R-matrix calculation [21]. In this analysis the 2^+ resonance was generated for the $^8\text{Be}+\alpha$ channel, where a channel radius of $R = 1.34(8^{1/3} + 4^{1/3})$ fm was used.

This is a single-channel R-matrix calculation where the parameters are the resonance energy, channel radius, and α-particle partial decay width. It was assumed that the resonance shape is independent of the excitation process (inelastic scattering), i.e., the excitation probability does not change substantially across the resonance. This latter assumption is partially justified by the very large range of excitation energies observed to be populated in the reaction. The amplitude of the resonance line shape, $A(E)$, was calculated from the form

$$A(E) = N \frac{\Gamma_\alpha}{(E_{\text{res}} - E - \Delta)^2 + (\Gamma_\alpha/2)^2}, \qquad (1)$$

where $\Gamma_\alpha = 2P_l(E)\gamma_\alpha^2$, E_{res} being the resonance energy, E the energy in the center-of-mass, γ_α is the reduced α width, and $P_l(E)$ is the barrier penetrability factor for the given orbital angular momentum l; $l = 2$ in the present case. N is a normalization constant, which is connected with the strength of the population of the state in the inelastic scattering process. In the present case γ_α was set equal to the Wigner limit ($\sqrt{3\hbar^2/2\mu R^2}$). The energy shift is given by $\Delta = \gamma_\alpha^2(S(E) - B)$, where $S(E)$ is the shift function and B is the boundary condition defined as the value of $S(E_{\text{res}})$, where

$$S(E) = \frac{\rho(FF' + GG')}{F^2 + G^2}, \qquad (2)$$

where $\rho = kR$ and F, G, F', and G' are regular and irregular Coulomb wave functions and their derivatives, respectively.

The asymmetric shape of the resonance shown in Figs. 1 and 3 is due to the fact that the state lies below the top of the Coulomb and centrifugal barriers. The calculation is for a state of width 750 keV centered at 9.75 MeV. It can be seen that the line shape developed provides a good description of the deficit in the α and proton inelastic scattering spectra.

A. Fitting procedure

The fitting procedure used in the analysis of the two sets of data used a consistent 2^+ line shape calculated using the method described above. The width and centroid of the 2^+ line shape were adjusted to provide the best fit to α-inelastic scattering data and then optimized to the proton-inelastic scattering data. This procedure was repeated until the best fit to both sets of data was achieved—which was not necessarily the optimal fit to the two sets of data independently. The energies and widths of the known 3^-, 1^-, and 2^- resonances were not free parameters, but convolved with the known experimental resolution—only their amplitudes were allowed to vary.

In the case of the α and proton inelastic scattering the improvement in the χ^2/d.o.f. following the inclusion of the R-matrix line shape is a factor of 10 for the α-particle scattering and 4.5 for the protons (corresponding to the region $E_x = 8.5$ to 11.0 MeV interval). The values of χ^2/d.o.f.

are 24 and 45, respectively, indicating that the analysis does not completely account for the shape of the broad 0^+ line shape, other similar components and contributions from the α-particle knock-on reaction. The uncertainty in the fitting process indicates an excitation energy of 9.75(0.15) MeV and a width of 750(150) keV for both sets of inelastic scattering data. This should be compared with the values of $E_x = 9.6(1)$ MeV with a width of 600(100) keV deduced in Ref. [18] and $E_x = 9.84(6)$ MeV with a width of 1010(150) keV [17]. In the latter instance the width was deduced from a gaussian fit rather than R-matrix analysis. Nevertheless, the values are consistent.

One of the biggest uncertainties in this analysis is the behavior of the very broad 0^+ contribution together with any background contribution to the spectra. These components are hard to quantify. In the α-inelastic scattering a broad component associated with the 0^+ strength associated with the zero degree measurement was employed. In reality, this contains both the 0^+ strength function and any background contributions. For the proton inelastic scattering an alternate approach was used as the 0^+ strength function from the α-inelastic scattering contains the population energy dependence and contributions from α-knockout. Here the 0^+ contribution was parameterized from the β-decay measurements [8]. In practice, over the region being fitted, these are both smoothly varying functions and in the region close to 9.7 MeV the variation with energy is not significant. An analysis in which the 0^+ contributions were exchanged between the proton and α-inelastic scattering data resulted in relatively small differences in the quality of the fit. There are other broad states in this region. However, given the >1 MeV separation to the 1^- state at 10.84 MeV, uncertainties in its energy and width have relatively small impact on the quality of the fit and the extracted width and energy of the 2^+ component. The factor that is hardest to quantify is the energy dependence of the excitation probability. Since this should also be a smoothly varying function, its effect is, like that of the 0^+ strength function, expected to be relatively minor. Nevertheless, the uncertainties on the 2^+ centroid and width quoted above are chosen to be conservative to account for these uncertainties.

The spin-2 strength function $[S_2(E_x)]$ extracted from the multipole decomposition analysis [17] is compared with a variety of R-matrix line shapes associated with different channel radii and excitation energies in Fig. 4. These give an impression of the range of parameters for the resonance width and centroid which are possible. The R-matrix curves correspond to (green solid line) a channel radius of $R = 1.4(8^{1/3} + 4^{1/3})$ fm and an excitation energy of 9.77 MeV, with a reduced width equal to the Wigner limit. The resulting width of the state is found to be $\Gamma = 840$ keV. Similar analysis with the same channel radius and reduced α width and lower excitation energy of 9.72 MeV (blue dot-dashed line), produces a width of $\Gamma = 790$ keV. The best fit (red dashed line) with a channel radius of $R = 1.35(8^{1/3} + 4^{1/3})$ fm corresponds to a reduced width equal to the Wigner limit, a total width of 690 keV and an excitation energy of 9.67 MeV. The χ^2/d.o.f. $= 2.9$ for this fit and indicate uncertainties on the width and centroid of 120 and 30 keV, respectively. This analysis is again consistent with an excitation energy of
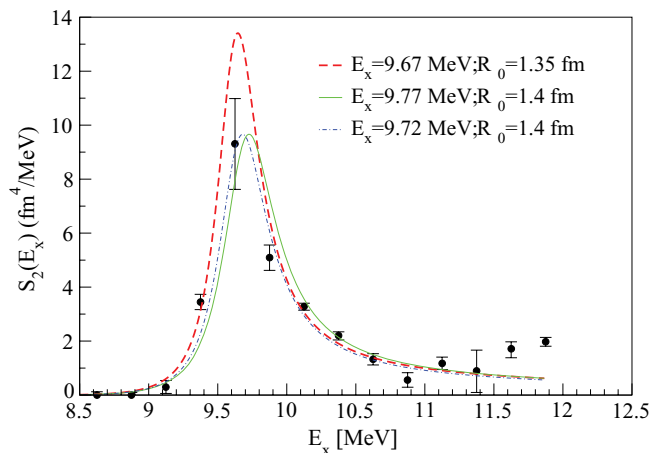
FIG. 4. (Color online) The spin-2 strength function $[S_2(E_x)]$ extracted from the multipole decomposition analysis [17] (data points) compared with the R-matrix line shapes [Eq. (1)] associated with different channel radii and excitation energies.

9.75(0.15) MeV and a width of 750(150) keV, with a reduced width close to the Wigner limit.

IV. DISCUSSION

The fact that an identical component is required for both proton and α inelastic scattering data strongly suggests the existence of a new state in ^{12}C. The high resolution measurements demonstrate that the feature cannot be attributed to tails on the peaks arising from features of the response function of the spectrometer focal plane detectors nor other similar phenomena. It is likely that this new state is the missing 2^+ excitation of the Hoyle state. The width of the structure is consistent with a 2^+ "alpha particle" state. Higher and lower spin states would have a much narrower and broader widths, respectively. Moreover, universally cluster model calculations only show a 2^+ state to lie in this region, and even shell-model calculations offer no other alternatives.

The separation between this new state and the Hoyle state is 2.10(0.15) MeV. This can provide an estimate of the moment of inertia of the structure and thus the arrangement of the α particles. The charge radius of ^{4}He has been determined to be $R_\alpha = 1.673(1)$ fm [22]. For two α particles separated by their charge radii $3\hbar^2/I = 4$ MeV. The first excited state of ^{8}Be (2^+) lies at 2.9 MeV and thus would indicate a separation of the two clusters of $1.2(2R_\alpha)$. If three α particles separated by this latter distance were arranged in a linear fashion then $3\hbar^2/I = 1.0$ MeV (microscopic cluster models indicate 0.8 MeV [24]), and in a triangle 2.75 MeV (rotated around an axis of symmetry passing through one α particle and bisecting the other two).

The ground state of ^{12}C has an oblate structure which has been ascribed a 3α D_{3h} point symmetry [23]. The energy of the corresponding 2^+ state is 4.4 MeV, which would indicate that the separation of the α particles is $0.8(2R_\alpha)$.

The experimental 0^+–2^+ separation of 2.10(0.15) MeV excludes a linear arrangement and is less than 2.75 MeV. If the triangular arrangement is relaxed a little such that the separation of one pair of particles is increased then a structure between a chain and triangle is reached (a bent chain). In this instance the bending angle of the chain would be $\sim$75° (i.e., closer to the triangle limit). Indeed an isosceles shape was suggested in three-body calculations of Ref. [25]. Other cluster models predict energy separations of 1.6 to 2.8 MeV [13].

In a recent calculation [26] the structure of the Hoyle state appears as a loose assembly of α particles, something like a Bose gas. This model indicates a 2^+ excitation would lie 2.3 MeV above the Hoyle state—very close to the present observation. A similar view of the state emerges in the Bose-Einstein condensate calculations in Ref. [27].

Of course, direct measurements confirming the 2^+ nature are important. Studies of the emission pattern of the α-decay products could, in principle, provide such evidence. An analysis of the correlations between the scattered α particle and the α particle from the ^{12}C $\rightarrow$ ^{8}Be+α decay have proved inconclusive [28] and higher resolution measurements are required. On the other hand, studies of the ^{12}C(γ,3α) reaction do indicate significant 2^+ strength in the current region [29]. It is clear that there is a need for further direct measurements.

V. SUMMARY

A combined analysis of proton and α-particle inelastic scattering, both performed at a scattering angle which coincides with a minimum in the dominant background contribution, demonstrates the existence of a new state in ^{12}C at 9.75(15) MeV [$\Gamma = 750(150)$ keV]. It is believed that this state is the missing 2^+ excitation of the 7.65 MeV Hoyle state. The present measurements indicate the state has a well-developed α-cluster structure and that the α particles are arranged in either an open triangular structure or a loose assembly of α particles. This analysis would appear to exclude a linear arrangement associated with the 3α chain.

[1] R. B. Wiringa, Steven C. Pieper, J. Carlson, and V. R. Pandharipande, Phys. Rev. C **62**, 014001 (2000).

[2] S. C. Pieper, K. Varga, and R. B. Wiringa, Phys. Rev. C **66**, 044310 (2002).

[3] R. B. Wiringa and S. C. Pieper, Phys. Rev. Lett. **89**, 182501 (2002).

[4] E. Epelbaum, H. Krebs, D. Lee, Ulf-G. Meißner, Phys. Rev. Lett. **106**, 192501 (2011).

[5] F. Hoyle, Astrophys. J **1**, 12 (1954).

[6] C. W. Cook *et al.*, Phys. Rev. **107**, 508 (1957).

[7] C. Angulo *et al.*, Nucl. Phys. A **656**, 3 (1999).

[8] H. O. U. Fynbo *et al.*, Nature **433**, 136 (2005).

[9] K. Ikeda *et al.*, Prog. Theor. Phys. Suppl. **Extra Number**, 464 (1968).

[10] M. Freer, Nature **487**, 309 (2012).

[11] S. Karataglidis, P. J. Dortmans, K. Amos, and R. de Swiniarski, Phys. Rev. C **52**, 861 (1995).

[12] P. Navrátil, J. P. Vary, and B. R. Barrett, Phys. Rev. Lett. **84**, 5728 (2000).

[13] P. Descouvemont and D. Baye, Phys. Rev. C **36**, 54 (1987).

[14] G. M. Reynolds, D. E. Rundquist, and R. M. Poichar, Phys. Rev. C **3**, 442 (1971).

[15] R. Smit *et al.*, Phys. Rev. C (to be published).

[16] S. Hyldegaard *et al.*, Phys. Rev. C **81**, 024303 (2010).

[17] M. Itoh *et al.*, Nucl. Phys. A **738**, 268 (2004); Phys. Rev. C **84**, 054308 (2011).

[18] M. Freer *et al.*, Phys. Rev. C **80**, 041303(R) (2009).

[19] Y. Sasamoto *et al.*, Mod. Phys. Lett. A **21**, 2393 (2006).

[20] W. R. Zimmerman, N. E. Destefano, M. Freer, M. Gai, and F. D. Smit, Phys. Rev. C **84**, 027304 (2011).

[21] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[22] E. Borie and G. A. Rinker, Phys. Rev. A **18**, 324 (1978).

[23] R. Bijker and F. Iachello, Phys. Rev. C **61**, 067305 (2000).

[24] A. C. Merchant and W. D. M. Rae, Nucl. Phys. A **549**, 431 (1992).

[25] D. V. Fedorov and A. S. Jensen, Phys. Lett. B **389**, 631 (1996).

[26] M. Chernykh, H. Feldmeier, T. Neff, P. von Neumann-Cosel, and A. Richter, Phys. Rev. Lett. **98**, 032501 (2007).

[27] A. Tohsaki, H. Horiuchi, P. Schuck, and G. Röpke, Phys. Rev. Lett. **87**, 192501 (2001).

[28] M. Itoh *et al.*, Mod. Phys. Lett. A **21**, 2359 (2006).

[29] M. Gai *et al.*, Acta Phys. Pol. B **42**, 775 (2011).