Predictions for the first two positive-parity states of ^{13}F

H. T. Fortune
Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania, 19104, USA

R. Sherr
Department of Physics, Princeton University, Princeton, New Jersey 08544, USA
(Received 31 May 2012; revised manuscript received 23 July 2012; published 4 September 2012)

We have used a potential model, together with information from ^{13}Be, to compute expected energies and widths for the first two positive-parity states of ^{13}F. Results are (all in MeV) $E_p = 2.30$ and 4.94 (or 5.26), width ∼0.6 and 0.3 (or 0.4), for $1/2^+$ and $5/2^+$, respectively.

DOI: 10.1103/PhysRevC.86.034301 PACS number(s): 21.10.Dr, 21.10.Sf, 23.50.+z, 27.20.+n

I. INTRODUCTION

New experimental techniques have allowed observation of an increasing number of light nuclei beyond the proton drip line. Even several years ago, some of them (^{11}N, ^{15}F, ^{19}Na) had been populated as final states in light- [1–3] (^{11}N); [4,5] (^{15}F); [6,7] (^{19}Na) and heavy-ion induced reactions [8–10] (^{11}N); [11] (^{11}N and ^{15}F) and as elastic-scattering resonances in experiments [12–14] (^{11}N); [15–17] (^{15}F); [18,19] (^{19}Na) with radioactive beams incident on a hydrogen target. Proton decays of ^{15}F and ^{19}Na were observed after forming them in neutron removal and/or fragmentation reactions [20,21]. More recently, others have been reported: ^{14}F [22], ^{18}Na [23], and ^{19}Mg [24,25]. A simple potential model worked reasonably well for ^{14}F [26]. Here we report our predictions for ^{13}F.

II. MODEL

The model assumes the same nuclear interaction in a neutron-rich nucleus and its proton-rich mirror, with only a Coulomb term added for the latter. Then, if energies are known for the neutron-rich nucleus, they can be computed for the proton-rich mirror. This procedure has worked reasonably well for several light nuclei. Of special note is the case of ^{19}Mg, for which predictions [27,28] and experiments [29] had proven to be difficult for many years. The potential-model prediction [30] of $E_{2p} = 0.87(7)$ MeV was followed by an experimental finding of 0.75(5) MeV [24,25].

We use a Woods-Saxon potential well and vary its depth to reproduce the neutron energy in the core + neutron system. To this potential is then added the Coulomb potential of a uniformly charged sphere to compute the proton energy in the core′ + p system, where "core′" is the mirror of "core." The calculations are especially sensitive to competition between the $2s_{1/2}$ and $1d_{5/2}$ orbitals in these nuclei, because the $s_{1/2}$ energy is significantly lower (relative to $d_{5/2}$) in the proton-rich member of a mirror pair. This effect (called the Thomas-Ehrman shift) is well known and is well reproduced in potential model calculations. We have applied that model to the case of ^{13}Be-^{13}F.

III. APPLICATION TO ^{13}F

In ^{12}Be, the ground state (g.s.) [31–34] contains about 68% of the configuration ^{10}Be$_{1p}$ x $(sd)^2$, and only about 32% of the normal p-shell ^{12}Be$_{1p}$ configuration. (Here, we use subscript $1p$ to denote a pure p-shell nucleus.) Use of this wave function, and the orthogonal one for the excited 0^+ state, in predictions for various observables in ^{12}Be has produced consistency for many quite different features. The latest is the B(GT) strength from ^{12}B(g.s.) to the two 0^+ states [35]. This was the first direct measurement of the $1p$-shell intensity in the excited 0^+ state. A recent summary of the use of these wave functions in ^{12}Be can be found in Ref. [36]. Adding $s_{1/2}$ and $d_{5/2}$ nucleons to this ground state could cause difficulty with the Pauli principle, especially for $s_{1/2}$. Thus, we adopt a slightly different approach, which we now outline.

The lowest positive-parity states expected in ^{13}Be are $2s_{1/2}$ and $1d_{5/2}$ coupled to a p-shell ^{12}Be ground state, resulting in only two states, with $J^\pi = 1/2^+$ and $5/2^+$. Next in importance are states with three sd-shell neutrons coupled to a p-shell ^{10}Be. The three lowest states of this configuration are expected to be $1/2^+$, $3/2^+$, and $5/2^+$, but they should lie considerably higher in excitation. Also possible are $s_{1/2}$ and $d_{5/2}$ coupled to the 2^+ p-shell state of ^{12}Be. This configuration is undoubtedly present at some level, but we ignore it for now. We return to it later.

Our procedure is applicable only if the energies are known in the neutron-rich nucleus (^{13}Be in this case). Only two positive-parity states have been reported in ^{13}Be, and their energies are somewhat uncertain. Three separate experiments [37–39] have reported a $1/2^+$ state just at threshold ($E_n \sim 0$). A d-wave resonance (almost certainly $5/2^+$) exists near 2.0 to 2.3 MeV [40]. These are energies relative to decay into the physical ^{12}Be(g.s.) + neutron. It would be extremely difficult to treat an s-wave neutron state at threshold. But it occurred to us that such a state is *bound* with respect to a p-shell ^{12}Be + n,

TABLE I. Energies (MeV) of 0^+ states in ^{12}Be and ^{12}O.

Nucl.	State	E_x	Nucl.	State	E_x
^{12}Be	Physical ground state	0	^{12}O	Physical ground state	0
	Physical 0^{+}'	2.24		Physical 0^{+}'	∼1.8
	^{10}Be$_{1p}$ x $(sd)^2$	0.72		^{10}C$_{1p}$ x $(sd)^2$	0.58
	^{12}Be(g.s.)$_{1p}$	1.52		^{12}O(g.s.)$_{1p}$	1.22

TABLE II. Energies (MeV) in ^{13}Be and ^{13}F.

J^π	$E_x(^{13}\text{Be})$	E_n		E_p	
		Rel. to ^{12}Be(g.s.)	Rel. to $^{12}\text{Be}_{1p}$	Rel. to $^{12}\text{O}_{1p}$	Rel. to ^{12}O(g.s.)
$1/2^+$	∼0	0	− 1.52	1.18	2.40
$5/2^+$	∼2.3	2.0, 2.3	+ 0.48, 0.78	3.72, 4.04	4.94, 5.26

and that is the space we have chosen to work in. With our wave functions for the 0^+ states in ^{12}Be, the state $^{12}\text{Be}_{1p}$(g.s.) x $2s_{1/2}$ has spectroscopic factors of 0.32 and 0.68 to the ground state and excited 0^+ state, respectively. After a simple rotation of basis to pure $1p$-shell and pure $(sd)^2$ 0^+ states, the S's are 1.0 and 0.0, respectively. In a two-state model for the ground state and 2.24-MeV 0^+ states of ^{12}Be, and using the wave-function intensities mentioned earlier, it is a simple matter to compute the energy of the $^{12}\text{Be}_{1p}$ ground state (see Table I). The result is $E_x = 1.52$ MeV above the physical ^{12}Be(g.s.). Thus, in our space, the $1/2^+$ state is bound by 1.52 MeV. With our potential, we then obtain $E_p = 1.18$ MeV for $^{12}\text{O}_{1p} + p$. Our model always uses the same wave-function intensities in a nucleus and its mirror. Thus, if the state recently observed [41] in ^{12}O at 1.8 MeV is the excited 0^+ state, then the p-shell ground state of ^{12}O is at $E_x = 1.22$ MeV above the physical ^{12}O(g.s.). Then we arrive at $E_p = 2.40$ MeV for E_p relative to the physical $^{12}\text{O(g.s.)} + p$.

We now repeat the procedure for the $5/2^+$ state. For definiteness, we have performed the calculation for both 2.0 and 2.3 MeV for its excitation energy in ^{13}Be. The results are listed in Table II. If a better number becomes available for this energy, it is a simple matter to adjust our predictions.

Both these resonances are broad enough in ^{13}F that their single-particle widths are difficult to compute. For the $1/2^+$, we estimate $\Gamma_{sp} = 1.85(35)$ MeV, and for the $5/2^+$ $\Gamma_{sp} = 1.4(2)$ or 1.7(3). These are for decay to the physical ^{12}O (g.s.). The $1/2^+$ state is likely to be nearly single particle relative to $^{12}\text{O}_{1p}$, but thus (with our wave functions) only about 32% sp to the physical ^{12}O(g.s.), so $S \sim 0.32$, giving $\Gamma_{\text{pred}} \sim 0.32\Gamma_{sp}$. For the $5/2^+$, its spectroscopic factor could be significantly less than for the $1/2^+$. In Sec. IV below, we estimate $S(5/2^+) \sim 0.7$ to the ^{12}O p-shell ground state, and, hence, $S(5/2) \sim 0.22$ to the physical ground state, implying $\Gamma_{\text{pred}}\ (5/2) \sim 0.3$ to 0.4 MeV. These widths are listed in Table III.

TABLE III. Energies and widths (MeV) calculated for ^{13}F.

J^π	E_p	Γ_{sp}	S(g.s.)[a]	Γ_{pred}
$1/2^+$	2.40	1.85(35)	∼0.32	∼0.60(11)
$5/2^+$	4.94	1.4(2)	∼0.22	∼0.31(5)
	or 5.26	1.7(3)	∼0.22	∼0.38(7)

[a]See text.

IV. POSSIBLE COMPLICATING FACTORS

Above, we mentioned two other sets of positive-parity states. The configuration $^{10}\text{Be}_{1p}$ x $(sd)^3$ is expected to possess three relatively close-lying states, with $J^\pi = 1/2, 3/2,$ and $5/2$, as in ^{17}C [42,43]. In ^{15}C, there is no sign of such ^{12}C x $(sd)^3$ states in the known spectrum, so they are not likely to present a problem in ^{13}Be. Because the three would be close together, if any mixing occurs between them and the sp states, such mixing would affect the $5/2^+$ sp state to a greater extent, because of its much higher energy. The other set of positive-parity states mentioned above are states with $s_{1/2}$ and $d_{5/2}$ sp coupled to the p-shell 2^+ state. Within the p-shell basis, the 2^+ state is calculated [44] to be 4.4 MeV above the ground state in ^{12}Be. So the possible presence of these additional couplings should have a minimal effect on the first two sp states. But, again, just because of energies, any complication would be expected to be much larger for the $5/2^+$ state than for $1/2^+$. (The 2 x $d_{5/2}$ $1/2^+$ state would lie many MeV above the $1/2^+$ ground state, but the 2 x $s_{1/2}$ $5/2^+$ state would be considerably closer to the $5/2^+$ sp state.) With reasonable energies and matrix elements, we estimate the $5/2^+$ mixing to be of order 30%. It is for these reasons that we assumed that $S(1/2^+) \sim 1$ but $S(5/2^+) \sim 0.7$ (relative to $^{12}\text{Be}_{1p}$) for the width estimates above.

Mixing of the 0 x d and 2 x s $5/2^+$ states, if present, would cause a decrease in energy of the lowest $5/2^+$ state from the value in Table II for ^{13}F because of the Thomas-Ehrman effect. But, we do not expect a major shift. (We have used the experimental energy in ^{13}Be, so any shift due to mixing there is already accounted for.)

A p-wave state (probably $1/2^-$) has been reported [40] at 0.51 MeV in ^{13}Be. Its configuration is probably primarily $^{11}\text{Be}(1/2^-)$ x $(sd)^2$. We have not computed its energy in ^{13}F, but we expect the $1/2^-$-$5/2^+$ splitting to be about the same in ^{13}Be and ^{13}F.

V. SUMMARY

We have used a potential model, together with information from ^{12,13}Be and ^{12}O to compute energies and widths of the lowest $1/2^+$ and $5/2^+$ states in ^{13}F, assumed to be mirrors of ^{13}Be. Results are summarized in Tables II and III. If experimental energies in ^{13}Be should change, our numbers will need to be adjusted.

[1] W. Benenson, E. Kashy, D. H. Kong-A-Siou, A. Moalem, and H. Nann, Phys. Rev. C **9**, 2130 (1974).
[2] V. Guimarães *et al.*, Phys. Rev. C **67**, 064601 (2003).
[3] V. Guimarães *et al.*, Nucl. Phys. A **588**, c161 (1995).

[4] W. Benenson, E. Kashy, A. G. Ledebuhr, R. C. Pardo, R. G. H. Robertson, and L. W. Robinson, Phys. Rev. C **17**, 1939 (1978).

[5] G. J. KeKelis, M. S. Zisman, D. K. Scott, R. Jahn, D. J. Vieira, J. Cerny, and F. Ajzenberg-Selove, Phys. Rev. C **17**, 1929 (1978).

[6] Joseph Cerny, R. A. Mendelson, Jr., G. J. Wozniak, John E. Esterl, and J. C. Hardy, Phys. Rev. Lett. **22**, 612 (1969).

[7] W. Benenson, A. Guichard, E. Kashy, D. Mueller, H. Nann, and L. W. Robinson, Phys. Lett. B **58**, 46 (1975).

[8] A. Azhari, T. Baumann, J. A. Brown, M. Hellström, J. H. Kelley, R. A. Kryger, D. J. Millener, H. Madani, E. Ramakrishnan, D. E. Russ, T. Suomijarvi, M. Thoennessen, and S. Yokoyama, Phys. Rev. C **57**, 628 (1998).

[9] A. Lépine-Szily *et al.*, Phys. Rev. Lett. **80**, 1601 (1998).

[10] J. M. Oliveira, Jr. *et al.*, Phys. Rev. Lett. **84**, 4056 (2000).

[11] A. Lépine-Szily *et al.*, Nucl. Phys. A **722**, c512 (2003).

[12] L. Axelsson *et al.*, Phys. Rev. C **54**, R1511 (1996).

[13] K. Markenroth *et al.*, Phys. Rev. C **62**, 034308 (2000).

[14] E. Casarejos *et al.*, Phys. Rev. C **73**, 014319 (2006).

[15] W. A. Peters *et al.*, Phys. Rev. C **68**, 034607 (2003).

[16] V. Z. Goldberg, G. G. Chubarian, G. Tabacaru, L. Trache, R. E. Tribble, A. Aprahamian, G. V. Rogachev, B. B. Skorodumov, and X. D. Tang, Phys. Rev. C **69**, 031302 (2004).

[17] F. Q. Guo *et al.*, Phys. Rev. C **72**, 034312 (2005).

[18] C. Angulo *et al.*, Phys. Rev. C **67**, 014308 (2003).

[19] M. G. Pellegriti *et al.*, Phys. Lett. B **659**, 864 (2008).

[20] I. Mukha *et al.*, Phys. Rev. C **82**, 054315 (2010).

[21] I. Mukha *et al.*, Phys. Rev. C **79**, 061301 (2009).

[22] V. Z. Goldberg *et al.*, Phys. Lett. B **692**, 307 (2010).

[23] T. Zerguerras *et al.*, Eur. Phys. J. A **20**, 389 (2004).

[24] I. Mukha *et al.*, Phys. Rev. C **77**, 061303 (2008).

[25] I. Mukha *et al.*, Phys. Rev. Lett. **99**, 182501 (2007).

[26] R. Sherr and H. T. Fortune, Phys. Lett. B **699**, 281 (2011).

[27] L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Phys. Rev. Lett. **85**, 22 (2000).

[28] L. V. Grigorenko and M. V. Zhukov, Phys. Rev. C **68**, 054005 (2003).

[29] N. Frank, T. Baumann, D. Bazin, R. R. C. Clement, M. W. Cooper, P. Heckman, W. A. Peters, A. Stolz, M. Thoennessen, and M. S. Wallace, Phys. Rev. C **68**, 054309 (2003).

[30] H. T. Fortune and R. Sherr, Phys. Rev. C **76**, 014313 (2007).

[31] H. T. Fortune, G.-B. Liu, and D. E. Alburger, Phys. Rev. C **50**, 1355 (1994).

[32] T. Suzuki and T. Otsuka, Phys. Rev. C **56**, 847 (1997).

[33] R. Sherr and H. T. Fortune, Phys. Rev. C **60**, 064323 (1999).

[34] A. Navin *et al.*, Phys. Rev. Lett. **85**, 266 (2000).

[35] R. Meharchand *et al.*, Phys. Rev. Lett. **108**, 122501 (2012).

[36] H. T. Fortune and R. Sherr, Phys. Rev. C **85**, 051303 (2012).

[37] M. Thoennessen, S. Yokoyama, and P. G. Hansen, Phys. Rev. C **63**, 014308 (2000).

[38] G. Christian *et al.*, Nucl. Phys. A **801**, 101 (2008).

[39] H. Simon *et al.*, Nucl. Phys. A **791**, 267 (2007).

[40] Y. Kondo *et al.*, Phys. Lett. B **690**, 245 (2010).

[41] D. Suzuki *et al.*, Phys. Rev. Lett. **103**, 152503 (2009).

[42] Z. Elekes *et al.*, Phys. Lett. B **614**, 174 (2005).

[43] Y. Kondo *et al.*, Phys. Rev. C **79**, 014602 (2009).

[44] S. Cohen and D. Kurath, Nucl. Phys. A **141**, 145 (1970).