Processes $e^+e^- \to \pi\pi(\pi')$ in the extended Nambu–Jona-Lasinio model

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The process $e^+e^- \to \pi^+\pi^-$ is described in the framework of the extended NJL model in the mean-field approximation. Intermediate vector mesons $\rho^0(770)$, $\omega(782)$, and $\rho'(1450)$ are taken into account. Our results are in satisfactory agreement with experimental data. The prediction for the process $e^+e^- \to \pi\pi'(1300)$ is given. Here the main contribution is given by the diagram with intermediate $\rho'(1450)$ meson.

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I. INTRODUCTION

Recently, the processes $e^+e^- \to \pi^0\gamma,\ \pi'\gamma,\ \pi^0\omega,\ \pi^0\rho^0$ have been described in the framework of the extended NJL model in the mean-field approximation where we take into account only quark loops [1–3]. Intermediate vector mesons in the ground and excited states were taken into account. In all these works satisfactory agreement with experimental data was obtained without using any arbitrary parameters.

In this work the same method was used for description of the processes $e^+e^- \to \pi\pi(\pi')$ in the P state.[1] Now this process is thoroughly investigated from both experimental (see Ref. [6] and other references in this work) and theoretical [7–12] points of view. However, in all these works it is necessary to use the number of additional parameters. The extended NJL model allows us to describe these processes with using only one additional parameter.

Let us note that for the description of the processes $e^+e^- \to \pi^+\pi^-$ with intermediate photon and vector mesons in the ground state it is sufficient to use the standard NJL model [13–22]. On the other hand, for the description of the amplitude of this process with the intermediate $\rho'(1450)$ meson the extended NJL model [22–26] should be used.

II. LAGRANGIAN AND PROCESS AMPLITUDES

The amplitude of the process $e^+e^- \to \pi^+\pi^-$ is described by the diagrams given on Figs. 1 and 2.

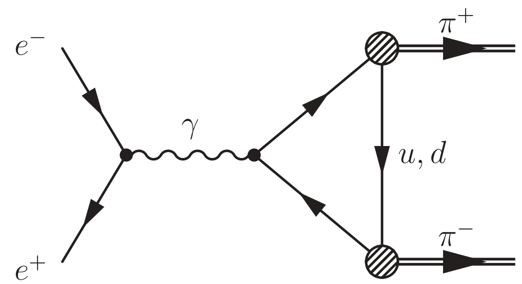

FIG. 1. Contact interaction of the photon with a pion pair through the photon propagator.

For the description of the first three diagrams with intermediate γ, ρ, and ω mesons in the ground state we need the part of the standard NJL Lagrangian which describes interactions of photons, pions, and the ground states of vector mesons with quarks

$$\Delta\mathcal{L}_1 = \bar{q}\left[i\hat{\partial} - m + \frac{e}{2}\left(\tau_3 + \frac{1}{3}I\right)\hat{A} + ig_\pi\gamma_5\tau_\pm\pi^\pm + \frac{g_\rho}{2}\gamma_\mu(I\hat{\omega} + \tau_3\hat{\rho}^0)\right]q, \quad (1)$$

where $\bar{q} = (\bar{u}, \bar{d})$ with u and d quark fields; $m = \mathrm{diag}(m_u, m_d)$, $m_u = 280$ MeV is the constituent quark mass, $m_d - m_u \approx 3.7$ MeV as will be shown below; e is the electron charge; $A, \pi^\pm, \omega$, and ρ^0 are the photon, pion, ω, and ρ meson fields, respectively; g_π is the pion coupling constant, $g_\pi = m_u/f_\pi$, where $f_\pi = 93$ MeV is the pion decay constant; g_ρ is the vector meson coupling constant, $g_\rho \approx 6.14$ corresponds to the standard relation $g_\rho^2/(4\pi) \approx 3$; $\tau_\pm = (\tau_1 \mp i\tau_2)/\sqrt{2}$, $I = \mathrm{diag}(1, 1)$ and $\tau_{1,2,3}$ are Pauli matrices. For the description of the radial excited mesons interactions we use the extended version of the NJL Lagrangian [2,24,25]

$$\Delta\mathcal{L}_2^{\mathrm{int}} = \bar{q}(k')\{A_\pi\tau_\pm\gamma_5\pi(p) - A_{\pi'}\gamma_5\tau_\pm\pi'(p) + A_{\omega,\rho}(\tau_3\hat{\rho}(p) + I\hat{\omega}(p)) - A_{\rho'}\tau_3\hat{\rho}'(p)\}q(k), \quad (2)$$

$$p = k - k',$$

$$\begin{aligned}
A_\pi &= g_{\pi_1}\frac{\sin(\alpha+\alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2}f(k^{\perp 2})\frac{\sin(\alpha-\alpha_0)}{\sin(2\alpha_0)},\\
A_{\pi'} &= g_{\pi_1}\frac{\cos(\alpha+\alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2}f(k^{\perp 2})\frac{\cos(\alpha-\alpha_0)}{\sin(2\alpha_0)}, \quad (3)\\
A_{\omega,\rho} &= g_{\rho_1}\frac{\sin(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2}f(k^{\perp 2})\frac{\sin(\beta-\beta_0)}{\sin(2\beta_0)},\\
A_{\rho'} &= g_{\rho_1}\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} + g_{\rho_2}f(k^{\perp 2})\frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)}.
\end{aligned}$$

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[1] In order to describe the pions pair in S or D states it is necessary to consider two virtual photos after annihilation e^+e^-. This contribution is suppressed by the factors $\alpha = 1/137$ and m_e^2/s [4]. Note that for consideration of a creation of pion pairs in the S or D states it is necessary to take into account not only quark but also meson loops (see, e.g., Ref. [5]).

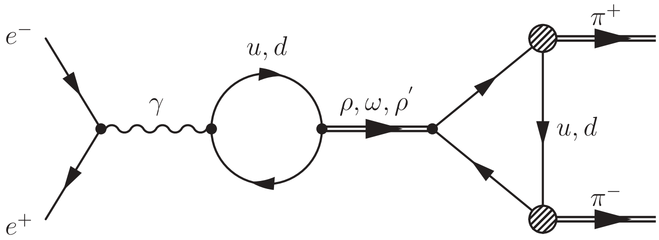

FIG. 2. Interaction with intermediate vector mesons.

The radially excited states were introduced in the NJL model with the help of the form factor in the quark-meson interaction:

$$f(k^{\perp 2}) = (1 - d|k^{\perp 2}|)\Theta(\Lambda^2 - |k^{\perp 2}|),$$
$$k^\perp = k - \frac{(kp)p}{p^2}, \qquad d = 1.788\ \mathrm{GeV}^{-2}, \tag{4}$$

where k and p are the quark and meson momenta, respectively. The cut-off parameter $\Lambda = 1.03$ GeV. Note that the NJL model itself and its extended version can only be used for sufficiently low energies. In this study we attempt to get qualitative results working at energies up to 2 GeV. The coupling constants $g_{\rho_1} = g_\rho$ and $g_{\pi_1} = g_\pi$ are the same as in the standard NJL version. The constants $g_{\rho_2} = 10.56$ and $g_{\pi_2} = g_{\rho_2}/\sqrt{6}$, and the mixing angles $\alpha_0 = 58.39^\circ$, $\alpha = 58.70^\circ$, $\beta_0 = 61.44^\circ$, and $\beta = 79.85^\circ$ were defined in Refs. [2,25].

Therefore, the amplitude of the process $e^+e^- \to \pi^+\pi^-$ has the form

$$T = \bar{e}\gamma_\mu e \frac{4\pi\alpha_e}{s}(B_{\gamma\rho} + B_\omega + B_{\rho'}) f_{a_1}(s)\left(p^\mu_{\pi^+} - p^\mu_{\pi^-}\right)\pi^+\pi^-, \tag{5}$$

where $\alpha_e = e^2/4\pi \approx 1/137$, $s = (p_{e^+} + p_{e^-})^2$, and

$$f_{a_1}(p^2) = Z + (1 - Z) + \left(\frac{p^2 - m_\pi^2}{(g_\rho F_\pi)^2}\right)\left(1 - \frac{1}{Z}\right)$$
$$= 1 + \left(\frac{p^2 - m_\pi^2}{(g_\rho F_\pi)^2}\right)\left(1 - \frac{1}{Z}\right), \tag{6}$$

where $Z = (1 - 6m_u^2/m_{a_1}^2)^{-1}$ is the additional renormalizing factor pion fields that appeared after the inclusion of a_1–π transitions. This function describes the creation of pions at the ends of the triangle quark diagram with the possibility of creation of these pions through the intermediate axial-vector $a_1(1260)$ meson (see Fig. 3). The first term of this amplitude corresponds to the triangle diagram without a_1–π transitions, the second term corresponds to diagram with a_1–π transition on the one of the pion lines and the third term corresponds to the diagram with transitions on both pion lines.[2]

The transition γ–ρ takes the form (see Ref. [15])

$$\frac{e}{g_\rho}(g^{\nu\nu'}q^2 - q^\nu q^{\nu'}). \tag{7}$$

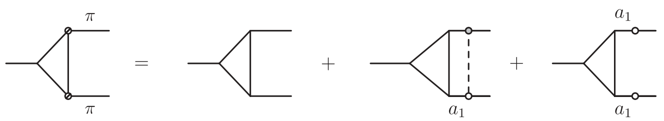

FIG. 3. Triangle diagrams with a_1–π transitions.

Thus, one can write $B_{\gamma\rho}$ contribution in the form

$$B_{\gamma\rho} = 1 + \frac{s}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} = \frac{1 - i\sqrt{s}\Gamma_\rho(s)/m_\rho^2}{m_\rho^2 - s - i\sqrt{s}\Gamma_\rho(s)} m_\rho^2. \tag{8}$$

Let us note that this expression is close to vector meson dominance model (see, e.g., Ref [15]).

The term describing the transition γ–ω is equal to the term γ–ρ multiplied by the factor 1/3 [2,15]. The $\omega \to \pi\pi$ process was described in Ref. [15]

$$C\left(m_\rho^2\right)\omega_\mu\left(p^\mu_{\pi^+} - p^\mu_{\pi^-}\right), \tag{9}$$

where $C(s) = C_1(s) + C_2(s)$. C_1 describes the amplitude of transition $\omega \to \rho$ due to the difference of two quark loops. The first of them contains only u quark and second contains only d quark. Using the last experimental data for the decay $\omega \to \pi\pi$ [27] we obtain $m_d - m_u \approx 3.7$ MeV. This difference allows us to describe not only the decay $\omega \to \pi\pi$, but the mass difference of charge and neutral pion and kaon (see [15]) and obtain the interference ω–ρ in process $e^+e^- \to \pi^+\pi^-$ in good agreement with experimental data

$$C_1(s) = \frac{8(\pi\alpha_\rho)^{3/2} m_\omega^2}{3\left(m_\omega^2 - s - i\sqrt{s}\Gamma_\rho(s)\right)} \frac{3}{(4\pi)^2} \log\left(\frac{m_d}{m_u}\right)^2, \tag{10}$$

and C_2 describes the amplitude $\omega \to \gamma \to \rho$

$$C_2(s) = -\sqrt{\frac{\pi}{\alpha_\rho}} \frac{2\alpha s}{3\left(m_\omega^2 - s - i\sqrt{s}\Gamma_\rho(s)\right)}. \tag{11}$$

Then for the part of the amplitude with intermediate ω meson we get

$$B_\omega = \frac{C(s)}{3g_\rho} \frac{s}{m_\omega^2 - s - i\sqrt{s}\Gamma_\omega(s)}. \tag{12}$$

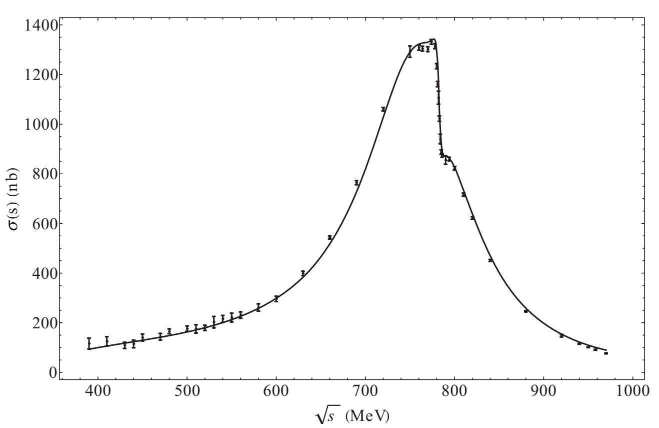

FIG. 4. Comparison of experimental results for $e^+e^- \to \pi^+\pi^-$ with the NJL prediction.

[2]Let us note that $f_{a_1}(p^2)$ has the value which is close to the factor introduced in Ref. [12].

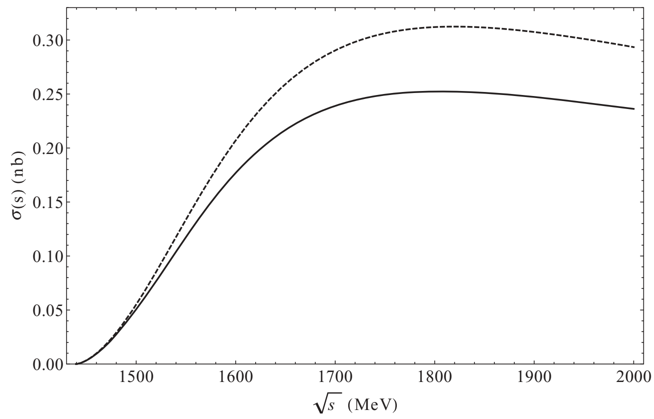

FIG. 5. NJL prediction for $e^+e^- \to \pi\pi'$. The solid line is the total cross section, the dashed line is the $\rho'(1450)$ contribution only.

The last part of the amplitude contains intermediate $\rho'(1450)$ meson. The transition γ–ρ' has the form (see Ref. [2])

$$C_{\gamma\rho'}\frac{e}{g_\rho}(g^{\nu\nu'}q^2 - q^\nu q^{\nu'}), \quad (13)$$

$$C_{\gamma\rho'} = -\left(\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)} + \Gamma\frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)}\right), \quad (14)$$

$$\Gamma = \frac{I_2^f}{\sqrt{I_2 I_2^{ff}}} = 0.54. \quad (15)$$

The vertex $\rho'\pi\pi$ is proportional to (see Ref. [25])

$$C_{\rho'\pi\pi} = -\left(\frac{\cos(\beta+\beta_0)}{\sin(2\beta_0)}g_{\rho_1} + \frac{\cos(\beta-\beta_0)}{\sin(2\beta_0)}\frac{I_2^f}{I_2}g_{\rho_2}\right) = 1.68. \quad (16)$$

Unfortunately, our model cannot describe a relative phase between $\rho(770)$ and $\rho'(1450)$ in $e^+e^- \to \pi\pi(\pi')$. Thus, we get the phase factor from e^+e^- annihilation and τ decays experiments: $B_{\rho'} \to e^{i\pi}B_{\rho'}$.

Thus, the ρ' meson contribution reads

$$B_{\rho'} = e^{i\pi}\frac{C_{\gamma\rho'}C_{\rho'\pi\pi}}{g_\rho}\frac{s}{m_{\rho'}^2 - s - i\sqrt{s}\Gamma_{\rho'}(s)}, \quad (17)$$

where the running width $\Gamma_{\rho'}$ reads [2]

$$\Gamma_{\rho'}(s) = \Theta(2m_\pi - \sqrt{s})\Gamma_{\rho'\to2\pi} + \Theta(\sqrt{s}-2m_\pi)\left(\Gamma_{\rho'\to2\pi} + \Gamma_{\rho'\to\omega\pi}\frac{\sqrt{s}-2m_\pi}{m_\omega - m_\pi}\right)\Theta(m_\omega + m_\pi - \sqrt{s}) + \Theta(m_{\rho'} - \sqrt{s})$$
$$\Theta(\sqrt{s} - m_\omega - m_\pi)\left(\Gamma_{\rho'\to2\pi} + \Gamma_{\rho'\to\omega\pi} + (\Gamma_{\rho'} - \Gamma_{\rho'\to2\pi} - \Gamma_{\rho'\to\omega\pi})\frac{\sqrt{s}-m_\omega - m_\pi}{m_{\rho'} - m_\omega - m_\pi}\right) + \Theta(\sqrt{s} - m_{\rho'})\Gamma_{\rho'}. \quad (18)$$

The values $\Gamma(\rho' \to 2\pi) = 22$ MeV and $\Gamma(\rho' \to \omega\pi^0) = 75$ MeV were calculated in Ref. [25]. The $\Gamma_{\rho'} = 340$ MeV was taken the value of lower boundary from the Particle Data Group [27].

For the total cross section we get

$$\sigma(s) = \frac{\alpha^2\pi}{12s}f_{a_1}^2(s)\left(1 - 4m_\pi^2/s\right)^{3/2}|B_{\rho\gamma} + B_\omega + B_{\rho'}|^2. \quad (19)$$

The total cross section is defined by ρ and ω mesons, the ρ' meson contributes only to the differential cross section. Figure 4 shows that our results for the total cross section are in satisfactory agreement with experiments.

Let us note that for description of the $e^+e^- \to \pi'\pi$ not only the intermediate state with $\rho'(1450)$ but the intermediate state $\rho''(1700)$ can play the important role. However, here we take into account only ρ' meson contribution. Therefore, we can only expect to obtain a qualitative description of this process. The corresponding total cross section takes the form

$$\sigma(s) = \frac{\alpha^2\pi}{12s^2}\Lambda^{3/2}\left(s, m_{\pi'}^2, m_\pi^2\right)\left|B_{\rho\gamma}^{\pi\pi'} + B_{\rho'}^{\pi\pi'}\right|^2, \quad (20)$$

$$B_{\rho\gamma}^{\pi\pi'} = \frac{C_{\rho\pi\pi'}}{g_\rho}\left(1 + \frac{s}{m_\rho^2 - s - im_\rho\Gamma_\rho}\right)$$
$$= \frac{C_{\rho\pi\pi'}}{g_\rho}\frac{1 - i\Gamma_\rho/m_\rho}{m_\rho^2 - s - im_\rho\Gamma_\rho}m_\rho^2, \quad (21)$$

$$B_{\rho'}^{\pi\pi'} = e^{i\pi}\frac{C_{\gamma\rho'}C_{\rho'\pi\pi'}}{g_\rho}\frac{s}{m_{\rho'}^2 - s - im_{\rho'}\Gamma_{\rho'}}, \quad (22)$$

where $\Lambda(s, m_{\pi'}^2, m_\pi^2) = (s - m_{\pi'}^2 - m_{\pi'}^2)^2 - 4m_{\pi'}^2 m_\pi^2$, $m_{\pi'} = 1300$ MeV is mass of π' meson [27], $C_{\rho\pi\pi'}$ and $C_{\rho'\pi\pi'}$ is defined in a similar way as $C_{\rho'\pi\pi}$ with the use of the Lagrangian in Ref. [2]. Corresponding results are shown in Fig. 5.

Now for the processes $e^+e^- \to 4\pi$ we have the following experimental data [28]: $\sigma(e^+e^- \to \pi^+\pi^-\pi^0\pi^0) \approx 10$ nb at the energy from 1.5 to 2.0 GeV; $\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-) \approx 30$ nb at energy between 1.5 and 2.0 GeV. Therefore, we can see that our result does not contradict these data and can give a noticeable contribution to these processes.

III. CONCLUSIONS

Let us note than our version of the NJL model allows us to describe not only meson production in the e^+e^- processes but branching of the decay of tau lepton into mesons. Indeed, in the works [29–33] decays τ into $3\pi\nu$, $\pi\gamma\nu$, $\pi\pi\nu$, $\pi\omega\nu$, and $\eta\pi\nu$ were described in satisfactory agreement with experimental data. The calculation of the last process $\tau \to \pi\pi\nu$ is very close to process $e^+e^- \to \pi^+\pi^-$ which was considered here. In [31] we obtained satisfactory agreement of both branching and differential width with experimental data. With the help of the method used here we can obtain also a qualitative prediction for branching of the process $\tau \to \pi\pi'(1300)\nu$. This value equals approximately to 0.2%, which does not contradict modern experimental data referring to the decays $\tau \to 4\pi\nu$. This prediction can be useful for future experimental measurement.

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