

**Bound state form factors from knockout in  $^{10}\text{Be}(d,t)$  neutron pickup**N. Keeley,<sup>1,\*</sup> K. W. Kemper,<sup>2</sup> and K. Rusek<sup>1,3</sup><sup>1</sup>*National Centre for Nuclear Research, ul. Andrzeja Sołtana 7, 05-400 Otwock, Poland*<sup>2</sup>*Department of Physics, The Florida State University, Tallahassee, Florida 32306, USA*<sup>3</sup>*Heavy Ion Laboratory, University of Warsaw, ul. Pasteura 5A, 02-093 Warsaw, Poland*

(Received 10 May 2012; published 27 July 2012)

Existing  $^{10}\text{Be}(d,t)^9\text{Be}$  neutron pickup data are analyzed using the same bound-state form factors for the  $\langle^{10}\text{Be}|^9\text{Be} + n\rangle$  overlap as in a recent analysis of single-neutron knockout [Grinyer *et al.*, *Phys. Rev. Lett.* **106**, 162502 (2011)]. While the knockout data were well described by a bound-state form factor calculated using the variational Monte Carlo (VMC) technique, including an appropriate neutron binding potential, the  $^{10}\text{Be}(d,t)^9\text{Be}$  pickup data are significantly overpredicted using this form factor. In addition, the no-core shell model (NCSM) and VMC form factors yield the same calculated pickup cross sections whereas the knockout results using these form factors differed by 20%. We explore possible sources of ambiguity in the pickup calculations that affect our ability to compare the absolute magnitudes in cross sections between these two very different reactions.

DOI: [10.1103/PhysRevC.86.014619](https://doi.org/10.1103/PhysRevC.86.014619)

PACS number(s): 25.60.Bx, 25.60.Je, 24.10.Eq, 25.70.Bc

**I. INTRODUCTION**

Low-energy—of the order of a few tens of MeV—single-nucleon pickup reactions induced by light ions such as  $p$ ,  $d$ , and  $^3\text{He}$  have been used as probes of single-particle structure in nuclei for many years, see, e.g., Ref. [1] for a general summary of a large body of work of this type or Refs. [2,3] for a recent systematic study of  $(d,p)$  and  $(p,d)$  reactions. In recent years, with the advent of fast radioactive beams single-nucleon knockout reactions at energies of the order of 100 MeV/nucleon have been employed to extract similar information on the structure of exotic nuclei, see, e.g., Ref. [4] for a convenient summary of this important body of work. Since spectroscopic information depends solely on the structure of the particular nucleus under investigation, different experimental probes should give identical results, within a reasonable level of uncertainty, provided that the analysis tools adequately model the dynamics of the processes employed.

Perhaps the largest single source of ambiguity in the analysis of low-energy direct nuclear reactions to extract spectroscopic information is the choice of bound-state form factor. A recent analysis of new single-neutron knockout data for  $^{10}\text{Be}$  [5] employed form factors derived from standard shell model (SM), variational Monte Carlo (VMC) and no-core shell model (NCSM) calculations. Existing data for the  $^{10}\text{Be}(d,t)^9\text{Be}$  single-neutron pickup reaction at an incident deuteron energy of 15 MeV [6] provide an excellent opportunity to compare the analysis of standard direct reaction data using the same bound-state form factors as for the knockout data. We report such an analysis in this work using standard direct reaction models and find that while the VMC form factor gives excellent agreement with the knockout data, all the form factors investigated significantly overpredict the low-energy pickup data.

**II. TEST CALCULATIONS**

It is important to use the same bound-state form factors when comparing spectroscopic information obtained from single-nucleon pickup and single-nucleon knockout reactions due to the sensitivity of the results to these quantities. However, there is the added complication for pickup reactions that *two* form factors must be defined, one for the target-like overlap (which may be fixed as that used in the knockout reaction) and one for the projectile-like overlap. In light ion reactions this latter is usually fixed using some theoretical or parametrized wave function, e.g., those of Reid [7] or Hulthén have often been employed for the  $\langle d|p + n\rangle$  overlap in  $(p,d)$  pickup analyses. In this work, we used the bound-state form factor of Eiró and Thompson [8] for the  $\langle t|d + n\rangle$  overlap, as in a previous study of the  $^{40}\text{Ca}(d,t)$  and  $(d,^3\text{He})$  reactions [9].

Since the choice of form factor for the  $\langle t|d + n\rangle$  overlap could have a significant influence on the overall normalization of the  $(d,t)$  angular distribution we repeated the zero-range distorted wave Born approximation (DWBA) calculations of Ref. [6] in the full finite-range DWBA using the original optical potentials and  $\langle^{10}\text{Be}|^9\text{Be} + n\rangle$  form factor with the Eiró and Thompson  $\langle t|d + n\rangle$  form factor. The  $\langle^{10}\text{Be}|^9\text{Be} + n\rangle$  binding potential was of Woods-Saxon form with  $r_0 = 1.15$  fm,  $a = 0.57$  fm, and a spin-orbit component of the same geometry with depth  $V_{\text{SO}} = 6.0$  MeV, the depth of the central potential well being adjusted to give the correct neutron binding energy. The corresponding spectroscopic factor was 2.19. We confine our attention to the  $^{10}\text{Be}(d,t)^9\text{Be}_{\text{g.s.}}$  pickup since this corresponds directly to the knockout reaction of Ref. [5].

The calculations were performed using the code FRESKO [10] and prior-form DWBA with the full complex remnant term. The calculations of Ref. [6] were unable to describe satisfactorily the data without the use of a lower radial cutoff, a cutoff of 4 fm being found to give reasonable fits to the shape of the angular distributions while predicting substantially the same value for the peak cross section as calculations with no cutoff. We therefore performed finite-range calculations with and without a lower cutoff of 4 fm; both calculations are

\*keeley@fuw.edu.pl

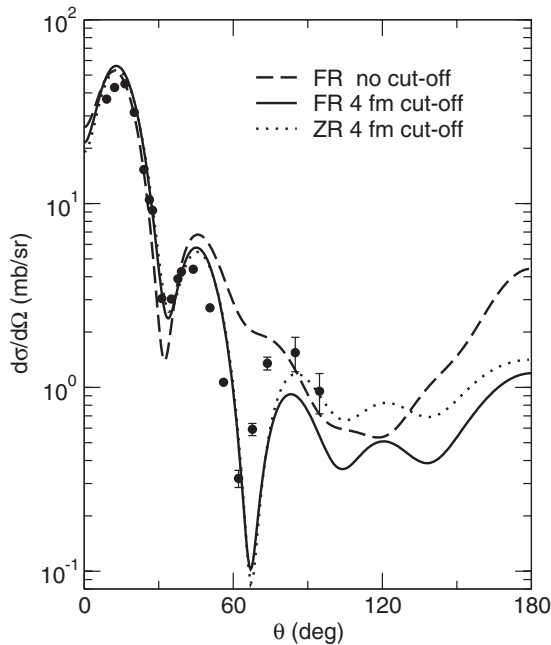


FIG. 1. Finite-range DWBA calculations for the  $^{10}\text{Be}(d,t)^9\text{Be}$  pickup to the  $3/2^-$  ground state of  $^9\text{Be}$  at  $E_d = 15$  MeV with (solid curve) and without (dashed curve) a lower radial cutoff of 4 fm. Optical potentials and  $\langle^{10}\text{Be}|^9\text{Be} + n\rangle$  form factors were taken from Ref. [6] and the  $\langle t|d + n\rangle$  form factor from Ref. [8]. The dotted curve denotes the result of a zero-range DWBA calculation using the same normalization factor as Ref. [6]. The data are from Ref. [6].

compared with the data in Fig. 1. Also included in Fig. 1 is the result of a zero-range DWBA calculation performed using the code DWUCK4 [11] and the same normalization as in Ref. [6] (plus the appropriate finite-range correction factor of 0.845) with a lower radial cutoff of 4 fm.

It will be noted from Fig. 1 that the use of a radial cutoff in the finite-range calculations considerably improves the agreement with the shape of the experimental angular distribution while the effect on the value of the peak cross section is small, as found in the original zero-range calculations of Auton [6]. More importantly in the context of the current work, we find that the use of the Eiró and Thompson  $\langle t|d + n\rangle$  form factor leads to a slight overprediction of the data, by factors of 15% and 10% for the calculations with and without cutoff, respectively. However, the zero-range DWUCK4 calculation agrees very well with the full finite-range calculation, so we may conclude that the Eiró and Thompson  $\langle t|d + n\rangle$  form factor is at least as good as one widely used choice of zero-range  $(d,t)$  normalization (there are a number of different normalizations in use in the literature). The apparent off-set in normalization between the present DWUCK4 calculations and the original JULIE calculations of Auton may be accounted for by the finite-range correction factor; if this is omitted from the DWUCK4 calculations the experimental peak cross section is well matched. In any case, the differences between the calculated and measured peak cross sections lie within the stated uncertainty of the spectroscopic factor given by Auton [6] for pickup to the  $3/2^-$  ground state of  $^9\text{Be}$ .

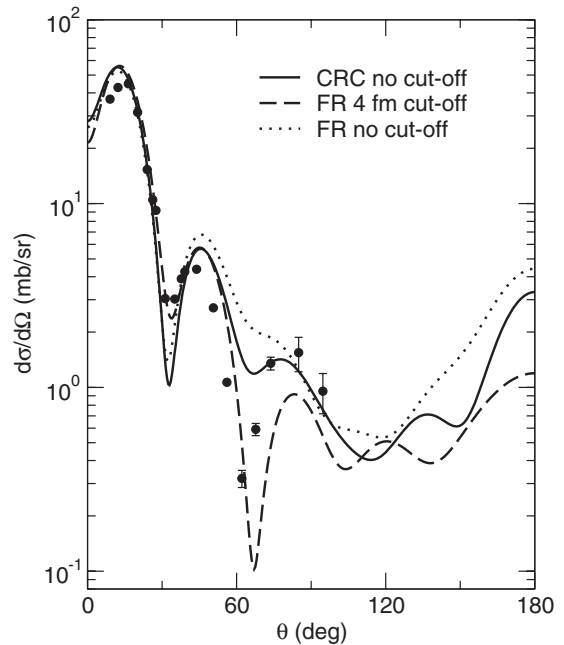


FIG. 2. CRC calculation (solid curve) for the  $^{10}\text{Be}(d,t)^9\text{Be}$  pickup to the  $3/2^-$  ground state of  $^9\text{Be}$  at  $E_d = 15$  MeV where deuteron breakup has been modeled using CDCC. No radial cutoff was employed. The dashed curve denotes the result of a finite-range DWBA calculation including a lower radial cutoff of 4 fm while the dotted curve denotes the result of a finite-range DWBA calculation with no radial cutoff. The data are from Ref. [6].

It is often assumed that the need for a lower radial cut-off in DWBA calculations involving deuterons in entrance or exit channels simulates the influence of deuteron breakup. To test this we performed a coupled reaction channels (CRC) calculation where the entrance channel optical potential was replaced by a continuum discretized coupled channels (CDCC) calculation similar to that described in Ref. [12]. The necessary diagonal and transition potentials were calculated using Watanabe-type folding based on the global nucleon optical potential of Ref. [13] and the deuteron internal wave function of Ref. [7]. The  $n + p$  continuum was discretized into bins in momentum ( $k$ ) space of width  $\Delta k = 0.125 \text{ fm}^{-1}$  up to a maximum value  $k_{\text{max}} = 0.625 \text{ fm}^{-1}$ , corresponding to a deuteron “excitation energy” of 18.6 MeV. The transfer step was modeled using CRC and thus included the non-orthogonality correction but was otherwise unchanged.

The elastic scattering data of Auton [6] were well described by this calculation and the calculated angular distribution for the  $^{10}\text{Be}(d,t)^9\text{Be}$  pickup to the  $3/2^-$  ground state of  $^9\text{Be}$  is compared to the relevant data in Fig. 2 as well as the finite-range DWBA calculations with and without the 4 fm cutoff. Figure 2 shows that the majority of the improvement in the description of the shape of the  $(d,t)$  angular distribution produced by the radial cutoff in the DWBA calculation does indeed appear to be due to the effect of deuteron breakup. In particular, the magnitude of the second peak is much better described compared to the no cut-off DWBA calculation and the second minimum now appears unambiguously, although

it is still not as deep as that in the DWBA calculation with cutoff or the data. We have thus established that CRC/CDCC calculations with no radial cutoff and using the Eiró and Thompson  $\langle t|d+n\rangle$  form factor provide a solid basis for testing the three  $\langle {}^{10}\text{Be}|{}^9\text{Be}+n\rangle$  form factors used in the analysis of the knockout data [5].

### III. CALCULATIONS WITH FORM FACTORS FROM KNOCKOUT

We now proceed to compare the results of calculations using the three bound-state form factors from the knockout study of Grinyer *et al.* [5] with the  ${}^{10}\text{Be}(d,t){}^9\text{Be}_{g.s.}$  pickup data of Auton [6]. We performed a series of CDCC/CRC calculations using each of the three form factors of Grinyer *et al.* [5]—SM, NCSM, and VMC—and different choices for the exit channel  $t+{}^9\text{Be}$  optical potential. These were: the potential used by Auton [6] in the original DWBA analysis (parameter set H'), the global triton potential of Becchetti and Greenlees [14], the global mass-3 potential of Pang *et al.* [15], and finally the four empirical  $t+{}^9\text{Be}$  optical potentials of Schmelzbach *et al.* [16], the latter being at almost precisely the required energy. All calculations employed the Koning and Delaroche global nucleon potential [13] as the basis for the CDCC part.

In contrast to the knockout analysis, where the VMC form factor gave excellent agreement with the measured cross section, all three form factors considerably overpredict the pickup data regardless of which exit channel  $t+{}^9\text{Be}$  optical potential is employed. The closest agreement with the magnitude of the peak cross section was obtained with the global potential of Pang *et al.* [15], while potential BA17 of Schmelzbach *et al.* [16] gave a slightly better description of the shape of the angular distribution. In Fig. 3 we compare the results of calculations with the SM, NCSM, and VMC form factors and the Pang *et al.* and BA17 potentials with the data of Auton [6].

The first thing to note from Fig. 3 is that the NCSM and VMC form factors predict identical angular distributions for the  ${}^{10}\text{Be}(d,t){}^9\text{Be}$  pickup whereas in the knockout analysis the NCSM cross section is  $\sim 20\%$  larger than the VMC one. Why this should be so is unclear. The reduction factors required to fit the experimental peak cross sections in each case are given in Table I, together with the corresponding factors from the knockout analysis of Ref. [5]. The differences cannot be accounted for by the slight ( $\sim 15\%$ ) overprediction of the  $(d,t)$  cross section in the test calculations due to the use of the Eiró and Thompson [8]  $\langle t|d+n\rangle$  form factor: the ratio of the pickup to knockout reduction factors is not constant. Nor can they be accounted for by the experimental uncertainty in the measured knockout cross section. This suggests that the discrepancy could be due to a too simplistic picture of the reaction mechanism in the pickup analysis. Two sets of couplings not included in the calculations presented in Fig. 3 suggest themselves as possible sources of the apparent discrepancy between the pickup and knockout analyses: pickup proceeding via the strongly coupled  $3.37\text{ MeV } 2^+$  first excited state of  ${}^{10}\text{Be}$  in the entrance partition and coupling to the breakup degree of freedom of  ${}^9\text{Be}$  in the exit partition.

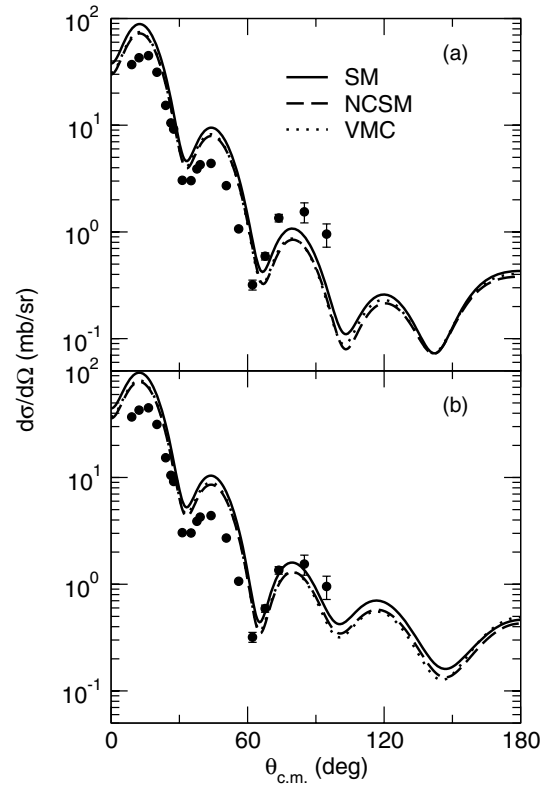


FIG. 3. (a) CRC calculations for the  ${}^{10}\text{Be}(d,t){}^9\text{Be}$  pickup with the SM, NCSM, and VMC form factors of Ref. [5] and the Pang *et al.* [15] global triton potential in the exit channel. (b) As for (a), but with potential BA17 from Ref. [16]. The data are from Auton [6].

We first investigated the effect of including pickup via the  $3.37\text{ MeV } 2^+$  state of  ${}^{10}\text{Be}$  and performed a series of calculations based on the SM form factor and with the  $t+{}^9\text{Be}$  optical potential from the global systematics of Pang *et al.* [15]. The tests were carried out with the SM form factor in order that a consistent set of spectroscopic factors for the  $\langle {}^{10}\text{Be}(2^+)|{}^9\text{Be}+n\rangle$  overlap could be used. Spectroscopic factors for the  $1p_{3/2}$  and  $1p_{1/2}$  neutrons were taken from Cohen and Kurath [17], the same source as for the  $\langle {}^{10}\text{Be}(0^+)|{}^9\text{Be}+n\rangle$  overlap SM spectroscopic factor of Grinyer *et al.* [5], and increased by the same  $A/(A-1)$  center-of-mass motion correction. The same parameters were used for the binding potential well as for the  $\langle {}^{10}\text{Be}(0^+)|{}^9\text{Be}+n\rangle$  overlap [5];

TABLE I. Reduction factors for the CRC calculations shown in Fig. 3 required to fit the experimental peak cross sections, plus the corresponding factors for the knockout analysis of Ref. [5]. The final two columns give the ratios of the reduction factors for the  $(d,t)$  analyses to those from the knockout analysis.

	Pang	BA17	knockout [5]	Pang/knockout	BA17/knockout
SM	0.52	0.49	0.76	0.68	0.64
NCSM	0.64	0.59	0.84	0.76	0.70
VMC	0.62	0.57	1.00	0.62	0.57

while in principle they could be somewhat different for the  $\langle {}^{10}\text{Be}(2^+)|{}^9\text{Be} + n \rangle$  overlap this choice should be sufficient for the purposes of our test calculations. The  $B(E2; 0^+ \rightarrow 2^+)$  value was taken from Raman *et al.* [18] while the nuclear deformation length was taken from Table 5 of Auton [6]. (Note: the  $B(E2; 0^+ \rightarrow 2^+)$  given in Auton [6] is approximately a factor of two smaller than the recommended value of Raman *et al.* [18]. However, the  $B(E2)$  of Auton gives the same result as that of Raman *et al.*; as might be expected the inelastic excitation of  ${}^{10}\text{Be}$  by deuterons is insensitive to the Coulomb part.)

Inclusion of this transfer path had no influence on the pickup cross section, provided that the entrance channel potentials were readjusted to yield the same elastic scattering as in the calculations without coupling to the  ${}^{10}\text{Be}$   $2^+$  state (if this was not done the peak cross section for pickup was slightly *larger* than for the case where no  $2^+$  coupling was included). We therefore conclude that this transfer path may be safely ignored and its omission is not responsible for the apparent discrepancy between the pickup and knockout results.

Assessing the influence of  ${}^9\text{Be}$  breakup in the exit partition is more problematical. To model this process accurately would require the use of four-body continuum discretized coupled channels (CDCC) theory, which has to date not been attempted for  ${}^9\text{Be}$  breakup and is beyond the scope of the present work, representing a formidable theoretical challenge. Nevertheless, the  $3/2^-$  ground, 2.43 MeV  $5/2^-$ , and 6.38 MeV  $7/2^-$  states of  ${}^9\text{Be}$  are reasonably well approximated as a  $K = 3/2$  rotational band. We therefore simulated the effect of  ${}^9\text{Be}$  breakup coupling using the standard coupled channels method, deriving the necessary Coulomb coupling strengths from the measured ground state quadrupole moment [19] and taking the nuclear deformation lengths from Ref. [20]. The exit channel  $t + {}^9\text{Be}$  optical potential was based on parameter set BA17 of Ref. [16], retuned using SFRESCO, the searching version of FRESCO [10], to recover a reasonable fit to the 17 MeV elastic scattering data of Schmelzbach *et al.* [16] with the inelastic couplings included.

The pickup calculation employed our standard CRC/CDCC methodology and included transfer couplings to the  $3/2^-$  ground state using the VMC form factor of Grinyer *et al.* [5] and the 2.43 MeV  $5/2^-$  state with the form factor of Auton [6], the spectroscopic factor for the latter transition being reduced by a factor of 15% so that the calculated peak cross section matched the measured one. We did not include pickup to the 6.38 MeV  $7/2^-$  state of  ${}^9\text{Be}$  due to the lack of a value for the spectroscopic factor for this overlap (this state was not observed in the experiment of Auton [6] so is presumably weakly populated). The influence of coupling to the 2.43 MeV  $5/2^-$  and 6.38 MeV  $7/2^-$  states of  ${}^9\text{Be}$  had only a small effect on the cross section for pickup to the  $3/2^-$  ground state, leading to a slight *increase* its magnitude and thus worsening the agreement with the data. We therefore conclude that coupling to the breakup of  ${}^9\text{Be}$  is also probably not responsible for the apparent discrepancy between pickup and knockout reactions. This is not a definitive conclusion due to the necessarily approximate way in which the breakup coupling has been modeled and it is certainly possible that

a more realistic calculation of the breakup effects could modify it.

#### IV. SUMMARY AND CONCLUSIONS

Using the same  $\langle {}^{10}\text{Be}|{}^9\text{Be} + n \rangle$  bound-state form factors as a recent knockout study [5] in a reanalysis of existing  ${}^{10}\text{Be}(d,t){}^9\text{Be}$  pickup data [6], by removing the largest single source of ambiguity, enabled an objective test of whether the two experimental methods could yield the same spectroscopic factors for the  $\langle {}^{10}\text{Be}_{g.s.}|{}^9\text{Be}_{g.s.} + n \rangle$  overlap. Despite the fact that the VMC form factor gave excellent agreement with the knockout cross section in the analysis of Grinyer *et al.* [5], none of the three form factors used in that work were able to describe the pickup data—they all significantly overpredicted the peak cross section. An additional puzzling result was that the VMC and NCSM form factors gave identical results in the pickup analysis whereas they yielded significantly different cross sections in the analysis of the knockout data.

In principle, the SM, NCSM, and VMC calculations represent increasing levels of sophistication. The SM calculations [17] assume an inert core plus a number of active nucleon(s) interacting via an effective force tuned to a specific shell. The NCSM calculations [21] dispense with the inert core, all nucleons being active and interacting via the CD-Bonn 2000 two-nucleon ( $NN$ ) force [22]. Finally, the VMC calculations [23] employ a Hamiltonian constructed using the Argonne  $v_{18}$  [24] and Urbana IX [25] two- and three-body forces, respectively and include interactions with the continuum. The VMC calculations should thus, in principle, give the most realistic result for the bound-state form factor. The knockout analysis [5] seems to bear this out, but our  $(d,t)$  analysis is unable to differentiate between the VMC and NCSM results.

Test calculations established that the discrepancy between the pickup and knockout results could not be accounted for by either transfer paths proceeding via the 3.37 MeV  $2^+$  first excited state of  ${}^{10}\text{Be}$  or the influence of coupling to the breakup of the weakly bound  ${}^9\text{Be}$  in the exit partition. This latter conclusion is, however, somewhat tentative due to the necessarily approximate modeling of the breakup process. As a subsidiary result, our calculations appear to confirm the supposition that the need for radial cutoffs in DWBA calculations involving deuterons simulates to some extent the influence of deuteron breakup.

Taken at their face value, consideration of these results forces us to conclude that the knockout and pickup data cannot be described using the same bound-state form factors with existing reaction theories. However, there are three caveats to this statement (in addition to that concerning the effect of  ${}^9\text{Be}$  breakup couplings): (i) the  $\langle t|d + n \rangle$  form factor used in our calculations [8] gives pickup cross sections some 15–20% larger than the data of Auton [6] when using the same  $\langle {}^{10}\text{Be}|{}^9\text{Be} + n \rangle$  bound-state form factor as the original analysis (ii) the pickup data [6] were taken in direct kinematics with a  ${}^{10}\text{Be}$  target manufactured in a reactor and the data normalization is consequently rather more uncertain than usual

(iii) corrections due to the Perey effect [26] have not been taken into account. The first two issues are linked, in that they both have a direct bearing on the normalization of the calculations relative to the data, while the effect of the third may be checked directly in zero-range DWBA calculations using the code DWUCK4.

The first point is difficult to resolve completely, as without analyzing a large data set of  $(d,t)$  reactions involving different targets it is impossible to decide objectively whether a given  $\langle t|d+n\rangle$  form factor or normalization (for zero-range calculations) is the most realistic. Our choice agrees well with zero-range DWUCK4 [11] calculations using the same normalization as Auton [6] and applying an appropriate finite-range correction factor, although we note that significantly smaller normalization factors for  $(d,t)$  reactions exist in the literature, e.g., that used by Cossairt *et al.* [27]. The normalization used by Auton is that due to Bassel [28], calculated assuming the Hulthén wave function for the deuteron and the Irving-Gunn wave function for the triton. Cossairt *et al.* employed the normalization due to Hering *et al.* [29], calculated using a Hulthén  $n+d$  potential and giving a normalization some 24% lower than that of Bassel. However, it is possible to state unambiguously that by itself the larger normalization of the pickup cross section due to our use of the Eiró and Thompson overlap [8] cannot account for the apparent discrepancy between pickup and knockout results. Regarding the second point, Auton [6] states that the absolute cross sections are accurate to within 30%, including both statistical and systematic errors. It is also stated (in a note to Table 1 of Ref. [6]) that the amount of  $^{10}\text{Be}$  in the target was determined to within  $\pm 10\%$  by normalizing the elastic scattering data to various optical model calculations. Based on this, it seems reasonable to assume a value of at most about  $\pm 20\%$  as the systematic uncertainty in the data normalization. However, we note that the  $^{10}\text{Be}+d$  elastic scattering at  $E_d = 15.0$  MeV was recently remeasured in inverse kinematics as part of a  $^{10}\text{Be}(d,p)$  study [30] and that the data of Auton [6] had to be renormalized by a factor greater than 1.0 to match the new data.

If these two factors are combined in the same sense, i.e., if we reduce the spectroscopic factors for the  $\langle t|d+n\rangle$  overlap by 15% and increase the data normalization by 20%, together they could just account for the apparent discrepancy, as Fig. 4 shows. It should be recalled, however, that the data normalization could equally well be 20% lower than its nominal value. There is also the question of why the NCSM and VMC form factors give identical angular distributions in the  $^{10}\text{Be}(d,t)$  analysis but significantly different cross sections in the knockout analysis, which cannot be accounted for by a simple normalization problem. Nevertheless, as may be seen in Table I, the reduction factors extracted from the  $(d,t)$  and knockout analyses do agree to within an uncertainty of 30–40%, of the same order as the total uncertainty in the direct reaction model ingredients and the overall data normalization.

Corrections for the Perey effect, due to the use of wave functions generated by the local equivalents of intrinsically nonlocal optical potentials, can be important for nucleons but usually quickly diminish in importance with mass. We applied the standard corrections to the entrance and exit channel distorting waves in DWUCK4 calculations, using nonlocality

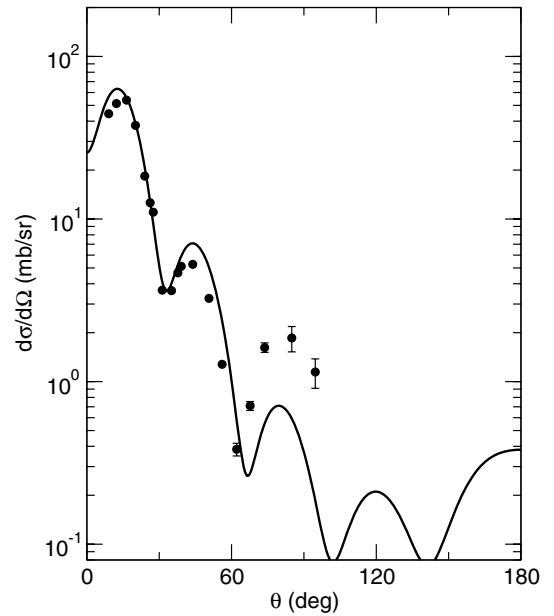


FIG. 4. CRC calculation for the  $^{10}\text{Be}(d,t)^9\text{Be}$  pickup to the  $3/2^-$  ground state of  $^9\text{Be}$  at  $E_d = 15$  MeV (solid curve) compared with the data of Ref. [6] (filled circles) renormalized by a factor of 1.2. The calculation employed the  $\langle t|d+n\rangle$  form factor of Ref. [8] with the spectroscopic factors multiplied by 0.85 and the VMC  $\langle ^{10}\text{Be}|^9\text{Be}+n\rangle$  form factor from Ref. [5]. The exit channel  $t+^9\text{Be}$  potential was calculated using the global systematics of Pang *et al.* [15].

parameters  $\beta_d = 0.54$  and  $\beta_t = 0.25$  [31], and found that the results were unchanged.

Returning to the question of why the NCSM and VMC form factors give identical results for the calculated  $^{10}\text{Be}(d,t)$  cross section while the knockout cross sections calculated with these form factors differ by  $\sim 20\%$ , it may simply be a “kinematic” effect, in that the high-energy knockout and low-energy pickup reactions probe different regions of the bound-state radial wave function. The plausibility of this explanation may be demonstrated by performing DWBA calculations at a much higher incident deuteron energy—200 MeV—and comparing the resulting pickup cross sections. When this is done it is found that, except at extreme forward scattering angles ( $\theta_{c.m.} \leq 5^\circ$ ) where the two calculations agree in magnitude, the cross section calculated with the NCSM form factor is uniformly  $\sim 30\%$  larger than that calculated with the VMC form factor.

In summary, it seems that the overall conclusion is that while the spectroscopic information extracted from these two very different reactions cannot be completely reconciled using existing direct reaction models, the level of agreement is such ( $\sim 30\text{--}40\%$ ) that it almost falls within the total uncertainty of the absolute data normalization and the ingredients of the pickup calculations.

#### ACKNOWLEDGMENTS

K.W.K. acknowledges partial support from the Florida State University Robert O. Lawton Fund.

- [1] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon, Oxford, 1983).
- [2] M. B. Tsang, J. Lee, S. C. Su, J. Y. Dai, M. Horoi, H. Liu, W. G. Lynch, and S. Warren, *Phys. Rev. Lett.* **102**, 062501 (2009).
- [3] J. Lee, M. B. Tsang, and W. G. Lynch, *Phys. Rev. C* **75**, 064320 (2007).
- [4] A. Gade and J. Tostevin, *Nuclear Physics News* **20**, 11 (2010).
- [5] G. F. Grinyer, D. Bazin, A. Gade, J. A. Tostevin, P. Adrich, M. D. Bowen, B. A. Brown, C. M. Campbell, J. M. Cook, T. Glasmacher, S. McDaniel, P. Navrátil, A. Obertelli, S. Quaglioni, K. Siwek, J. R. Terry, D. Weisshaar, and R. B. Wiringa, *Phys. Rev. Lett.* **106**, 162502 (2011).
- [6] D. L. Auton, *Nucl. Phys. A* **157**, 305 (1970).
- [7] R. V. Reid Jr., *Ann. Phys. (NY)* **50**, 411 (1968).
- [8] A. M. Eiró and I. J. Thompson, *Phys. Rev. C* **59**, 2670 (1999).
- [9] N. Keeley and R. S. Mackintosh, *Phys. Rev. C* **77**, 054603 (2008).
- [10] I. J. Thompson, *Comput. Phys. Rep.* **7**, 167 (1988).
- [11] P. D. Kunz, [<http://spot.colorado.edu/kunz/DWBA.html>].
- [12] N. Keeley, N. Alamanos, and V. Lapoux, *Phys. Rev. C* **69**, 064604 (2004).
- [13] A. J. Koning and J. P. Delaroche, *Nucl. Phys. A* **713**, 231 (2003).
- [14] F. D. Becchetti, Jr., and G. W. Greenlees, *Polarization Phenomena in Nuclear Reactions* (The University of Wisconsin Press, Madison, 1971).
- [15] D. Y. Pang, P. Roussel-Chomaz, H. Savajols, R. L. Varner, and R. Wolski, *Phys. Rev. C* **79**, 024615 (2009).
- [16] P. A. Schmelzbach, R. A. Hardekopf, R. F. Haglund Jr., and G. G. Ohlsen, *Phys. Rev. C* **17**, 16 (1978).
- [17] S. Cohen and D. Kurath, *Nucl. Phys. A* **101**, 1 (1967).
- [18] S. Raman, C. W. Nestor Jr., and P. Tikkanen, *At. Data Nucl. Data Tables* **78**, 1 (2001).
- [19] D. R. Tilley, J. H. Kelley, J. L. Godwin, D. J. Millener, J. Purcell, C. G. Sheu, and H. R. Weller, *Nucl. Phys. A* **745**, 155 (2004).
- [20] V. Hnizdo, K. W. Kemper, and J. Szymakowski, *Phys. Rev. Lett.* **46**, 590 (1981).
- [21] P. Navrátil, S. Quaglioni, I. Stetcu, and B. R. Barrett, *J. Phys. G* **36**, 083101 (2009).
- [22] R. Machleidt, *Phys. Rev. C* **63**, 024001 (2001).
- [23] S. C. Pieper and R. B. Wiringa, *Annu. Rev. Nucl. Part. Sci.* **51**, 53 (2001).
- [24] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
- [25] B. S. Pudliner, V. R. Pandharipande, J. Carlson, and R. B. Wiringa, *Phys. Rev. Lett.* **74**, 4396 (1995).
- [26] F. G. Perey, in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clemental and C. Villi (Gordon and Breach, New York, 1963), p. 125.
- [27] J. D. Cossairt, S. B. Talley, D. P. May, R. E. Tribble, and R. L. Spross, *Phys. Rev. C* **18**, 23 (1978).
- [28] R. H. Bassel, *Phys. Rev.* **149**, 791 (1966).
- [29] W. R. Hering, H. Becker, C. A. Wiedner, and W. J. Thompson, *Nucl. Phys. A* **151**, 33 (1970).
- [30] K. T. Schmitt, K. L. Jones, A. Bey, S. H. Ahn, D. W. Bardayan, J. C. Blackmon, S. M. Brown, K. Y. Chae, K. A. Chipps, J. A. Cizewski, K. I. Hahn, J. J. Kolata, R. L. Kozub, J. F. Liang, C. Matei, M. Matos, D. Matyas, B. Moazen, C. Nesaraja, F. M. Nunes, P. D. O'Malley, S. D. Pain, W. A. Peters, S. T. Pittman, A. Roberts, D. Shapira, J. F. Shriner Jr., M. S. Smith, I. Spassova, D. W. Stracener, A. N. Villano, and G. L. Wilson, *Phys. Rev. Lett.* **108**, 192701 (2012).
- [31] J. J. Kolata and J. V. Maher, *Phys. Rev. C* **8**, 285 (1973).