# Predictions of the fusion-by-diffusion model for the synthesis cross sections of Z = 114-120elements based on macroscopic-microscopic fission barriers

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(Received 20 November 2011; revised manuscript received 22 May 2012; published 16 July 2012)

A complete set of existing data on hot fusion reactions leading to synthesis of superheavy nuclei of Z = 114-118, obtained in a series of experiments in Dubna and later in GSI Darmstadt and LBNL Berkeley, was analyzed in terms of an angular-momentum-dependent version of the fusion-by-diffusion (FBD) model with fission barriers and ground-state masses taken from the Warsaw macroscopic-microscopic model (involving nonaxial shapes) of Kowal *et al.* The only empirically adjustable parameter of the model, the injection-point distance  $(s_{inj})$ , has been determined individually for all the reactions. Very regular systematics of this parameter have been established. The regularity of the obtained  $s_{inj}$  systematics indirectly points at the internal consistency of the whole set of fission barriers used in the calculations. (In an attempt to fit the same set of data by using the alternative theoretical fission barriers of Möller *et al.* we did not obtain such a consistent result.) Having fitted all the experimental excitation functions for elements Z = 114-118, the FBD model was used to predict cross sections for synthesis of elements Z = 119 and 120. Regarding prospects to produce the new element Z = 119, our calculations prefer the  $^{252}\text{Es}(^{48}\text{Ca,xn})^{300-x}119$  reaction, for which the synthesis cross section of about 0.2 pb in 4n channel at  $E_{c.m.} \approx 220$  MeV is expected. The most favorable reaction to synthesize the element Z = 120 turns out to be  $^{249}\text{Cf}(^{50}\text{Ti,xn})^{299-x}120$ , but the predicted cross section for this reaction is only 6 fb (for 3n and 4n channels).

DOI: 10.1103/PhysRevC.86.014611

PACS number(s): 25.70.Jj, 25.70.Gh

### I. INTRODUCTION

Superheavy nuclei of  $Z \ge 104$  were synthesized either in cold fusion reactions on closed-shell <sup>208</sup>Pb and <sup>209</sup>Bi target nuclei bombarded by projectiles ranging from Ti to Zn or in hot fusion reactions, in which the heaviest available actinide targets were bombarded with the neutron rich <sup>48</sup>Ca projectiles (see review articles [1–3]). In the cold fusion reactions only one neutron is emitted from the compound nucleus to form the final compound-residue nucleus in its ground state. In hot fusion reactions more neutrons are emitted. At each step of the deexcitation cascade the neutron evaporation competes with the dominating process of fission. Therefore the synthesis cross section represents only a small part of the fusion cross section.

A characteristic feature of the fusion-evaporation reactions leading to the synthesis of superheavy nuclei is enormous hindrance of the fusion process itself. Consequently, the cross sections for the synthesis of heaviest elements are measured in picobarns or even femtobarns. It is believed that the hindrance is caused by the highly dissipative dynamics of the fusing system in its passage over the saddle point on the way through the multidimensional potential energy surface from the initial configuration of two touching nuclei into the configuration of the compound nucleus. Zagrebaev and Greiner developed a method of solving Langevin equations of motion to describe this stochastic stage of the fusion process [4]. In spite of very time consuming Langevin trajectory calculations, in which only one of say a million trajectories leads to formation of the compound nucleus, the model is used effectively to calculate synthesis cross sections for various reactions [5]. Another approach to the process of fusion of a "dinuclear system" (DNS) was proposed in Ref. [6]. It was assumed in this model that the dinuclear system stays in contact configuration

and undergoes successive transfer of all nucleons from the lighter nucleus to the heavier partner (in competition with the quasifission processes). Applications of this concept have been used in recent years by several groups. In still another approach, the "fusion-by-diffusion" model [7], the stochastic process of shape fluctuations that lead to the overcoming of the saddle point was described as the solution of the Smoluchowski diffusion equation in the deformation space along the fission valley.

The cold fusion reactions leading to the synthesis of nuclei of  $Z \leq 113$  were studied systematically with the DNS model in Ref. [8], with the fusion-by-diffusion (FBD) model [7,9] and with the Langevin dynamics model [5]. The hot fusion reactions leading to the synthesis of the heaviest nuclei of  $Z \geq 114$ have not been studied so systematically. In Ref. [5] excitation functions for some selected reactions were calculated although they were not confronted with experimental cross sections. Most of the publications on this topic concentrated on the predictions concerning possible ways of synthesis of the heaviest elements of Z = 119 and 120 [5,10–16]. Only very recently an extensive study of cold and hot fusion reactions in terms of a phenomenological approach based on the DNS model was reported [17].

There is one important aspect of all the models of the synthesis of superheavy nuclei that was not treated with proper attention so far. This is the question of the choice of theoretical fission barriers and ground-state masses, which have to be adopted for the description of the deexcitation of the compound nucleus. It is well known that calculations of the cross sections for synthesis of superheavy nuclei are extremely sensitive to the height of the fission barrier, especially in case of hot fusion reactions, in which three or four neutrons are emitted from the compound nucleus. When the barrier heights are not known precisely, an error in evaluation of the  $\Gamma_n / \Gamma_f$  ratio in each step of the (xn) deexcitation cascade accumulates x times leading to enormous errors in the calculation of the synthesis cross sections. (Here,  $\Gamma_n$  and  $\Gamma_f$  denote the neutron decay width and fission width, respectively.) Thus, precise knowledge of theoretical fission barriers and neutron binding energies (ground-state masses) is crucial for reasonable predictions of the synthesis cross sections.

In the last decade the mass tables of Möller *et al.* [18] have most frequently been used in the field of superheavy nuclei. Unfortunately, fission-barrier heights are not given in these tables. Therefore, in most of the above mentioned calculations of the synthesis cross sections the ground-state shell effect of the compound nucleus (which is listed in these tables) was used as the barrier height. In this simplification, both the macroscopic deformation energy and the shell effect at the saddle configuration are neglected. It seems, therefore, that these approximate values of the fission barrier are not sufficiently accurate to guarantee reliable predictions of the synthesis cross sections. (Absolute value of both these neglected effects may be of about 1-2 MeV each, while a 1-MeV shift of the barrier height may result in a change of the calculated cross section of 3n or 4n reaction by 2–3 orders of magnitude.)

Only in recent years have systematic compilations of theoretical fission barriers of superheavy nuclei (combined with the necessary information on the ground-state masses) become available in literature. Calculations in framework of the macroscopic-microscopic approach were reported by Muntian *et al.* [19] and later by Möller *et al.* [20]. The model [19] has been extended recently by Kowal *et al.* [21,22] by the inclusion of nonaxiality as an important new degree of freedom. Fission barriers of superheavy nuclei have been calculated also in a number of other papers within various models (see Ref. [23], Table IV for a review), however no sufficiently systematic information on the fission barriers and, simultaneously, ground-state masses has been provided.

In the present study we adopt the fusion-by-diffusion (FBD) model [7,9] for calculating the synthesis cross sections of the heaviest nuclei in hot fusion (xn) reactions by using the information on the fission-barrier heights [21,22] and other properties of the superheavy nuclei obtained within the Warsaw macroscopic-microscopic model [19].

The whole set of experimental data [3,24-34] on the synthesis of new superheavy elements of Z = 114-118 (obtained in Dubna by Oganessian and coworkers and later in a series of confirming experiments at GSI Darmstadt and LBNL Berkeley) was analyzed. Based on this test of the model predictions, the calculations were then performed for experimentally unexplored reactions aimed at the synthesis of new elements of Z = 119 and 120.

#### **II. REVIEW OF THE FUSION-BY-DIFFUSION MODEL**

The fusion-by-diffusion (FBD) model [7,9] serves to calculate cross sections for the synthesis of superheavy nuclei. Recently the model was modified in order to describe both cold

fusion (1*n*) and hot fusion (*xn*) reactions. In this extended version [9], for each angular momentum *l* the partial evaporation-residue cross section  $\sigma_{\text{ER}}(l)$  for production of a given final nucleus in its ground state is factorized as the product of the partial capture cross section  $\sigma_{\text{cap}}(l) = \pi \lambda^2 (2l+1)T(l)$ , the fusion probability  $P_{\text{fus}}(l)$ , and the survival probability  $P_{\text{surv}}(l)$ ,

$$\sigma_{\rm ER} = \pi \lambda^2 \sum_{l=0}^{\infty} (2l+1)T(l)P_{\rm fus}(l)P_{\rm surv}(l). \tag{1}$$

The capture transmission coefficients T(l) are calculated in a simple sharp cutoff approximation, where the upper limit  $l_{\text{max}}$  of full transmission, T(l) = 1, is determined by the capture cross section known from the systematics described in Refs. [9,35]. Here  $\lambda$  is the wavelength,  $\lambda^2 = \hbar^2/2\mu E_{\text{c.m.}}$ , and  $\mu$  is the reduced mass of the colliding system. The fusion probability  $P_{\text{fus}}(l)$  is the probability that the colliding system, after reaching the capture configuration (sticking), will eventually overcome the saddle point and fuse, thus avoiding reseparation. The other factor in Eq. (1), the survival probability  $P_{\text{surv}}(l)$ , is the probability for the compound nucleus to decay to the ground state of the final residual nucleus via evaporation of light particles and  $\gamma$  rays, thus avoiding fission.

The cross sections for the synthesis of superheavy nuclei are dramatically small because the fusion probability  $P_{\text{fus}}(l)$ is hindered (in some reactions even by several orders of magnitude) due to the fact that the saddle configuration of the heaviest compound nuclei is much more compact than the configuration of two colliding nuclei at sticking. It is assumed in the FBD model that after the contact of the two nuclei, a neck between them grows rapidly at an approximately fixed mass asymmetry and constant length of the system. This "neck zip" is expected to carry the system towards the bottom of the asymmetric fission valley. This is the "injection point," from where the system starts its climb uphill over the saddle in the process of thermal fluctuations in the shape degrees of freedom. Theoretical justification of the above picture of fast zipping the neck was given in Ref. [36], where the later stage of the stochastic climb uphill was described by solving the two-dimensional Langevin equation. Theoretical location of an effective injection point can be deduced from this model [36]. Also in a modified fusion-by-diffusion model [37] the location of the injection point was estimated theoretically. In our model we rely, however, on empirical determination of the injection point. Its location in the asymmetric fission valley,  $s_{inj}$ , is the only adjustable parameter of the FBD model.

By solving the Smoluchowski diffusion equation, it was shown in Ref. [38] that the probability of overcoming a parabolic barrier for the system injected on the outside of the saddle point at an energy H below the saddle is

$$P_{\rm fus} = \frac{1}{2}(1 - {\rm erf}\sqrt{H/T}),$$
 (2)

where T is the temperature of the fusing system. The energy threshold H opposing fusion in the diffusion process is thus the difference between the energy of the saddle point  $E_{\text{saddle}}$ and the energy of the combined system at the injection point  $E_{\text{inj}}$ , where  $E_{\text{inj}}$  is calculated using algebraic expressions given in Ref. [9], which approximate the potential energy surface along the fission valley. The energy of the saddle point is given by the adopted theoretical value of the fission barrier  $B_f$  and the ground-state energy of the compound nucleus. The corresponding values of the rotational energy at the injection point and at the symmetric saddle point are calculated assuming the rigid-body moments of inertia at these configurations [9].

As regards the survival probability  $P_{surv}$ , the standard statistical-model calculations were done by applying the Weisskopf formula for the particle (neutron) emission width  $\Gamma_n$ , and the conventional expression of the transition-state theory for the fission width  $\Gamma_f$ . The level density parameters  $a_n$  and  $a_f$  for neutron evaporation and fission channels were calculated as proposed by Reisdorf [39], with shell effects accounted for by the Ignatyuk formula [40]. All details regarding the calculations of the survival probability  $P_{surv}$  can be found in our recent paper [9]. In case of calculating multiple evaporation (xn) channels a simplified algorithm avoiding the necessity of using the Monte Carlo method was applied [41].

#### III. CALCULATIONS FOR Z = 114-120 ELEMENTS WITH THE MACROSCOPIC-MICROSCOPIC BARRIERS

As pointed out in the Introduction, calculations of the cross sections for synthesis of superheavy nuclei are extremely sensitive to the height of the fission barrier, especially in case of hot fusion reactions because at each step of deexcitation cascade the competition between neutron emission and fission strongly depends on the difference of energy thresholds for these two decay modes. Therefore, in attempts to reasonably calculate the synthesis cross sections, the choice of realistic and consistent theoretical information on the fission-barrier heights and the ground-state masses is essential. In our previous applications of the FBD model, devoted mostly to analysis of cold fusion reactions (of Z of the compound nucleus  $Z_{CN} \leq$ 113), fission barriers based on the Thomas-Fermi model [42] were used. In Ref. [43] it was observed, however, that for heavier nuclei of  $Z_{CN} \ge 114$  produced in hot fusion reactions the fission barriers based on the Thomas-Fermi model are evidently too high, while barriers based on the Warsaw macroscopic-microscopic model [19] lead to better agreement with experimental observations. Therefore results of the new macroscopic-microscopic calculations of the Warsaw group [21], involving an extended multidimensional deformation space, have been chosen as the saddle-point and ground-state input to the FBD model. The published [21] results for even-even nuclei have been supplemented with unpublished vet results for odd-Z and/or odd-N nuclei [22].

In the first stage of calculations a complete set of experimental data [3,24–34] on the synthesis of Z = 114-118 elements in reactions induced by <sup>48</sup>Ca projectiles on <sup>242,244</sup>Pu, <sup>243</sup>Am, <sup>245,248</sup>Cm, <sup>249</sup>Bk, and <sup>249</sup>Cf targets was analyzed with the aim to determine location of the injection point  $s_{inj}$ . Here  $s_{inj}$  is defined as the excess of the total length of the combined system over the length of the initial system (at the touching configuration) when the neck-zip process brings the system to the asymmetric fission valley.



FIG. 1. (Color online) Systematics of the injection-point distance  $s_{inj}$  as a function of the kinetic energy excess  $E_{c.m.} - B_0$  above the Coulomb barrier  $B_0$ . Values of  $s_{inj}$  have been determined for each reaction and each particular xn channel by fitting the theoretical cross section at the maximum of a given xn excitation function to the data. The calculations have been done for the fission-barrier heights and ground-state masses of Kowal *et al.* [21,22]. Complete list of the analyzed reactions with references is given in the text. Identical symbols for a given Z and a given experiment refer to data for consecutive xn channels.

In order to determine systematics of  $s_{inj}$  for the set of hot fusion reactions [3,24–34], the individual values of  $s_{inj}$  were deduced for each reaction and each particular xn channel by adjusting the assumed  $s_{inj}$  value to the experimental synthesis cross section at the maximum of a given xn excitation function. The compilation of so-deduced  $s_{inj}$  values is displayed in Fig. 1 as a function of the kinetic energy excess  $E_{c.m.} - B_0$  above the Coulomb barrier  $B_0$ . (For the definition of  $B_0$  see Ref. [9].)

It should be commented here that values of  $s_{inj}$  are inferred from the synthesis cross sections in a model-dependent way, assuming particular ground-state masses and fission barriers. Therefore the result of this procedure obviously depends to some extent on these theoretical input data used in the calculations. Consequently, the systematics of  $s_{inj}$  obtained in calculations employing different sources of the theoretical input data may appear different (cf. the  $s_{inj}$  systematics obtained in recent calculations of cold fusion reactions [9] analyzed assuming masses and fission barriers based on the Thomas-Fermi model [42]).

It is clearly seen from Fig. 1 that the injection distance  $s_{inj}$  increases with the decreasing energy  $E_{c.m.} - B_0$ , in agreement with expectations based on the dynamics of nucleus-nucleus collisions, for example the classical trajectory calculations [44]. Very good correlation between the  $s_{inj}$  values and the corresponding energies  $E_{c.m.} - B_0$  can be viewed as an argument in favor of the fission barriers of Kowal *et al.* [21,22] because such a striking correlation would be very unlikely if the theoretical barrier heights were inconsistent with experimental values.

A linear fit to the dependence of  $s_{inj}$  on  $E_{c.m.} - B_0$  in Fig. 1,

$$s_{\rm inj} \approx 4.09 \text{ fm} - 0.192(E_{\rm c.m.} - B_0) \text{ fm/MeV},$$
 (3)

represents the only empirical input to our model and once this systematics of the injection-point distance is determined



FIG. 2. (Color online) Energy dependence of the cross section for synthesis of superheavy nuclei in hot fusion reactions. Full circles represent data for 3n, 4n, and 5n reaction channels obtained in Dubna experiments for elements Z = 114-118 [3,24–27,29,34]; open circles represent data obtained at GSI Darmstadt for Z = 114 and 116 [30,32]. Data are compared with excitation functions for separate xn channels, calculated with the FBD model assuming fission barriers and ground-state masses of Kowal *et al.* [21,22] and the systematics of the injection-point distance [Eq. (3)].

in form of Eq. (3), one can use the FBD model to calculate excitation functions of fusion-evaporation reactions without any adjustable parameters.

In Fig. 2 we present a comparison of our FBD model predictions of excitation functions for different *xn* channels with experimental synthesis cross sections (assigned to the corresponding *xn* channels) in the following hot fusion reactions:  ${}^{244}$ Pu( ${}^{48}$ Ca,*xn*) ${}^{292-x}$ 114 [3,25,26,30,33],  ${}^{243}$ Am( ${}^{48}$ Ca,*xn*) ${}^{291-x}$ 115 [3,24,34],  ${}^{245}$ Cm( ${}^{48}$ Ca,*xn*) ${}^{293-x}$ 116 [3,25],  ${}^{248}$ Cm( ${}^{48}$ Ca,*xn*) ${}^{296-x}$ 116 [3,26,32],  ${}^{249}$ Bk( ${}^{48}$ Ca,*xn*) ${}^{297-x}$ 117 [29], and  ${}^{249}$ Cf( ${}^{48}$ Ca,*xn*) ${}^{297-x}$ 118 [3,27]. The largest

deviations of our general fit to the data approach a factor of 10 that corresponds effectively to a difference of about 0.5 MeV in the assumed height of the theoretical fission barrier. Given this high sensitivity of the model predictions to the assumed fission-barrier heights, the overall agreement between the FBD predictions and measured cross sections is quite satisfactory. (It is rather unlikely that the accuracy of the theoretical predictions of individual fission barriers might be much better than  $\pm 0.5$  MeV.)

It is instructive to compare results of calculations presented in Figs. 1 and 2 with predictions for an alternative set of



FIG. 3. (Color online) Dependence of the injection-point distance  $s_{inj}$  on the kinetic energy excess  $E_{c.m.} - B_0$  above the Coulomb barrier  $B_0$ , deduced from analysis of experimental data [3,24–34] the same way as in Fig. 1, but assuming the fission-barrier heights [20] and ground-state masses [18] of Möller *et al.* (see text).

theoretical fission barriers. In Fig. 3 we present individual values of the injection distance  $s_{inj}$  deduced for the same set of data on hot fusion reactions [3,24–34], but obtained assuming fission barriers of Möller et al. [20], the only alternative, complete set of necessary information available in literature. The barriers of Möller et al. are considerably higher than barriers of Kowal et al. [21,22], thus resulting in larger values of the calculated survival probability  $P_{surv}$ . Consequently, the procedure of calibrating the individual  $s_{ini}$  values by fitting the predictions to experimental cross sections resulted in larger values of the determined injection distance  $s_{ini}$ . Contrary to the consistent systematics of  $s_{inj}$  values shown in Fig. 1, Fig. 3 demonstrates the evident inconsistency of the set of  $s_{ini}$  values obtained for the barriers of Ref. [20]. It is seen from Fig. 3 that the  $s_{inj}$  values range from 5.5 fm to 8.5 fm and are too large to have a reasonable physical meaning. (In most cases, they correspond to the injection distance that exceeds the distance of the scission configuration.) Most importantly, the individual points in Fig. 3 seam to be almost randomly scattered and do not show any correlation with energy.

There is one more inconsistency that can be noticed when the fission barriers of Ref. [20] and the ground-state masses [18] are used. Namely, for these high fission barriers and corresponding Q values, the predicted positions of the maxima of the xn excitation functions are shifted by some 5–7 MeV toward lower energies as compared with the data (and also with respect to the predictions for barriers of Refs. [21,22]). This effect is illustrated in Fig. 4, where the data for 3nand 4n channels in the  ${}^{243}$ Am( ${}^{48}$ Ca,xn) ${}^{291-x}$ 115 reaction are compared with the excitation functions calculated for these two reaction channels. This considerable energy shift, seen also for other reactions, stems from the fact that for the Möller's barriers [20] and the corresponding ground-state masses [18], the fission barrier  $B_f$  is larger than the neutron binding energy  $B_n$  for all the compound nuclei formed in the studied reactions. Consequently, the  $\Gamma_n / \Gamma_f$  ratio rises very fast at low excitation energies thus influencing the position and shape of the xn excitation functions.





FIG. 4. (Color online) Excitation functions for the 3*n* and 4*n* channels of the <sup>243</sup>Am(<sup>48</sup>Ca,*xn*)<sup>291-*x*</sup>115 reaction calculated with the FBD model assuming the fission barriers [20] and ground-state masses [18] of Möller *et al.* (dashed lines) compared with the experimental cross sections [3,24,34] and the predictions for the fission barriers and ground-state masses of Kowal *et al.* [21,22] (solid lines). In the absence of clear correlation between  $s_{inj}$  and  $E_{c.m.} - B_0$  for the barriers of Möller *et al.* (see Fig. 3), the dashed lines were calculated for a fixed value  $s_{inj} = 7.2$  fm (the mean value).

From Figs. 1 and 2 it is seen that contrary to the generally higher fission barriers of Ref. [20], the input data of Kowal *et al.* [21,22] give a reasonable agreement of the calculated and measured cross sections as well as the very clear correlation between  $s_{inj}$  and  $E_{c.m.} - B_0$  that calibrates the injection distance  $s_{inj}$ . This entitles us to believe that the set of theoretical fission-barrier heights and ground-state masses [21,22] is quite adequate for a wide range of the heaviest nuclei considered in this study. Therefore we are going to use them for predictions of cross sections of yet unexplored reactions aimed at the synthesis of new elements Z = 119 and 120.

Regarding possibilities to produce the element Z = 119 we consider, first of all, the most preferred reactions induced by the favorable beam of <sup>48</sup>Ca on two isotopes of einsteinium, <sup>252</sup>Es and <sup>254</sup>Es. These extremely difficult to produce targets could be available in the near future. Therefore we present in Figs. 5(a) and 5(b) the predicted energy dependence of the xn cross sections in reactions on these two isotopes. The largest cross section, which turns out to be at the edge of experimental possibilities (about 0.2 pb in 4n channel at  $E_{\rm c.m.} \approx$ 220 MeV), is predicted for the  ${}^{252}$ Es( ${}^{48}$ Ca,xn) ${}^{300-x}$ 119 reaction. Surprisingly, the cross section in the reaction on a more neutron-rich target,  ${}^{254}\text{Es}({}^{48}\text{Ca},xn){}^{302-x}119$ , is by one order of magnitude lower (only about 15 fb). This is a consequence of lower fission barriers [21,22] in the chain of subsequent neutron-emitting nuclei,  $B_f = 4.87$  MeV, 4.98 MeV, 5.77 MeV in  $^{302}119$ ,  $^{301}119$ , and  $^{300}119$ , while in a chain of neutron decays starting from the 300119 nucleus, the predicted fission barriers are 5.77 MeV, 5.55 MeV, and 6.03 MeV, respectively. Very recently Zagrebaev et al. [15] have reported a prediction for the same reaction,  ${}^{254}\text{Es}({}^{48}\text{Ca},xn){}^{302-x}119$  (about 0.3 pb for 3*n* channel). No prediction for the  ${}^{252}$ Es( ${}^{48}$ Ca,*xn*) ${}^{300-x}$ 119 reaction was given.

In case of inaccessibility of Es targets, the most promising target-projectile combination to synthesize the element



FIG. 5. (Color online) Synthesis cross sections of yet undiscovered superheavy nuclei of Z = 119 and 120 predicted by using the fusion-by-diffusion (FBD) model with fission barriers and ground-state masses of Kowal *et al.* [21,22] and the systematics of the injection-point distance [Eq. (3)] (see text).

Z = 119 is the <sup>249</sup>Bk(<sup>50</sup>Ti,*xn*)<sup>299-x</sup>119 reaction. Predictions for this reaction are shown in Fig. 5(c). Both 3*n* and 4*n* channels are expected to have comparable cross sections of about 30 fb (at maximum) at  $E_{c.m.} \approx 225$  and 232 MeV, respectively. Almost an equally small cross section for the <sup>249</sup>Bk(<sup>50</sup>Ti,*xn*)<sup>299-x</sup>119 reaction (about 60 fb) was predicted in Ref. [5], and somewhat larger value (about 110 fb) in Ref. [16]. Unfortunately, such small cross sections seem to be beyond the reach of present-state experiments. More optimistic predictions for the same reaction appeared recently in Ref. [45], however a relatively large cross section (about 0.6 pb) was obtained for probably overestimated values of the fission barrier taken as the pure ground-state shell effect from tables of Ref. [18]. Prospects for the synthesis of element Z = 120 are considerably worse than those for Z = 119. First of all, there is no chance to use the favorable beam of <sup>48</sup>Ca because the complementary <sup>257</sup>Fm target cannot be produced. We consider therefore reactions with <sup>50</sup>Ti beam on two available isotopes of californium, <sup>249</sup>Cf(<sup>50</sup>Ti,*xn*)<sup>299-*x*</sup>120 and <sup>251</sup>Cf(<sup>50</sup>Ti,*xn*)<sup>301-*x*</sup>120, which seem to be best choice. Excitation functions for these two reactions are shown in Figs. 5(d) and 5(e). The largest cross section is expected in the former reaction (about 6 fb at maximum in both 3*n* and 4*n* channels), in the latter reaction the maximum cross section is about 3 fb for 4*n* channel. Again, similarly as in case of reactions on two isotopes of einsteinium discussed above, a smaller cross section for more neutron rich compound nucleus

is associated with respectively lower fission barriers predicted in Refs. [21,22].

In Fig. 5(f) we present results of calculations for the <sup>248</sup>Cm(<sup>54</sup>Cr,*xn*)<sup>302-*x*</sup>120 reaction that is a more symmetric combination of even-*Z* target and projectile, next to Ti + Cf. The obtained cross sections of the order of 1 fb for 3*n* and 4*n* reaction channels clearly demonstrate that fusion processes are too strongly hindered in more symmetric systems. For completeness, we calculated also cross sections in two reactions of much more symmetric systems, <sup>238</sup>U(<sup>64</sup>Ni,*xn*)<sup>302-*x*</sup>120 and <sup>244</sup>Pu(<sup>58</sup>Fe,*xn*)<sup>302-*x*</sup>120 (not shown in figures), for which attempts to produce the element Z = 120 were done [46,47]. The calculated 3*n* and 4*n* cross sections in these two reactions are dramatically small, about 0.3 fb and 0.1 fb, respectively. Note that experimental upper limits for these two reactions had been established at 90 fb [46] and 400 fb [47], respectively.

Our calculations show that if the fission barriers of Refs. [21,22] were correct, there is no chance to synthesize the element Z = 120, even in the most favorable reaction  $^{249}Cf(^{50}Ti,xn)^{299-x}120$ , for which the predicted cross section is only 6 fb. Note that other model calculations for the  $^{249}Cf(^{50}Ti,xn)^{299-x}120$  reaction, published previously [5,10, 12-14,16], predicted considerably larger cross sections though also too small to be measurable (typically of the order of 50 fb). The dispersion of these different theoretical results has to be linked, first of all, to different fission barriers and ground-state masses used in these calculations.

We would like to emphasize that our predictions concerning the synthesis of Z = 119 and 120 nuclei are based on the *consistency* of the FBD model calculations with the adopted

- [1] S. Hofmann, Rep. Prog. Phys. 61, 639 (1998).
- [2] S. Hofmann and G. Münzenberg, Rev. Mod. Phys. 72, 733 (2000).
- [3] Yu. Oganessian, J. Phys. G: Nucl. Part. Phys. 34, R165 (2007).
- [4] V. Zagrebaev and W. Greiner, J. Phys. G: Nucl. Part. Phys. 31, 825 (2005); 34, 1 (2007).
- [5] V. Zagrebaev and W. Greiner, Phys. Rev. C 78, 034610 (2008).
- [6] G. G. Adamian, N. V. Antonenko, W. Scheid, and V. V. Volkov, Nucl. Phys. A 627, 361 (1997).
- [7] W. J. Światecki, K. Siwek-Wilczyńska, and J. Wilczyński, Phys. Rev. C 71, 014602 (2005).
- [8] E. A. Cherepanov, *Fusion Dynamics at the Extremes*, edited by Yu. Ts. Oganessian and V. I. Zagrebaev (World Scientific, Singapore, 2001), p. 186.
- [9] T. Cap, K. Siwek-Wilczyńska, and J. Wilczyński, Phys. Rev. C 83, 054602 (2011).
- [10] Z. H. Liu and Jing-Dong Bao, Phys. Rev. C 80, 054608 (2009).
- [11] G. G. Adamian, N. V. Antonenko, and W. Scheid, Eur. Phys. J. A 41, 235 (2009).
- [12] K. Siwek-Wilczyńska, T. Cap, and J. Wilczyński, Int. J. Mod. Phys. E 19, 500 (2010).
- [13] A. Nasirov, G. Fazio, G. Giardina, G. Mandaglio, M. Manganaro, and A. Muminov, Int. J. Mod. Phys. E 20, 406 (2011).
- [14] A. K. Nasirov, G. Mandaglio, G. Giardina, A. Sobiczewski, and A. I. Muminov, Phys. Rev. C 84, 044612 (2011).

ground-state masses and fission barriers of Refs. [21,22] and with *all* the existing experimental data on the synthesis of superheavy nuclei in hot fusion reactions [3,24–34]. Therefore the accuracy of these predictions is expected to be comparable with the accuracy of our overall fit to the data for the synthesis of Z = 114-118 nuclei, shown in Fig. 2.

In summary, we analyzed a complete set of existing data on hot fusion reactions leading to the synthesis of superheavy nuclei of Z = 114-118 [3,24-34] in terms of an *l*-dependent version of the FBD model with fission barriers and ground-state masses taken from the macroscopic-microscopic model of Kowal et al. [21,22]. By calibrating the assumed injection-point distances (sinj) to the measured cross sections, perfect systematics of  $s_{inj}$  values have been established for a wide range of hot fusion reactions enabling, hopefully, reliable predictions of the synthesis cross sections for yet unexplored reactions. Regarding prospects to produce the new element Z = 119, our calculations prefer the <sup>252</sup>Es(<sup>48</sup>Ca,*xn*)<sup>300-*x*</sup>119 reaction, for which the synthesis cross section of about 0.2 pb in 4n channel at  $E_{\rm c.m.} \approx 220$  MeV is expected. According to the microscopic-macroscopic model predictions [21,22], fission barriers for heavier isotopes of the element Z = 119are significantly lower leading to a considerably smaller cross section in the alternative  ${}^{254}\text{Es}({}^{48}\text{Ca},xn){}^{302-x}119$  reaction. Also the reaction  ${}^{249}\text{Bk}({}^{50}\text{Ti},xn){}^{299-x}119$  gives little chances for a measurable cross section (the predicted cross section is about 30 fb for both 3n and 4n channels). The most favorable reaction to synthesize the element Z = 120 is the  ${}^{249}Cf({}^{50}Ti,xn){}^{299-x}120$  reaction, but the predicted cross section is only 6 fb (for 3n and 4n channels).

- [15] V. I. Zagrebaev, A. V. Karpov, and Walter Greiner, Phys. Rev. C 85, 014608 (2012).
- [16] Nan Wang, En-Guang Zhao, Werner Scheid, and Shan-Gui Zhou, Phys. Rev. C 85, 041601(R) (2012).
- [17] Ning Wang, Junlong Tian, and Werner Scheid, Phys. Rev. C 84, 061601(R) (2011).
- [18] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, At. Data Nucl. Data Tables 59, 185 (1995).
- [19] I. Muntian, Z. Patyk, and A. Sobiczewski, Acta Phys. Pol. B 32, 691 (2001); 34, 2141 (2003).
- [20] Peter Möller, Arnold J. Sierk, Takatoshi Ichikawa, Akira Iwamoto, Ragnar Bengtsson, Henrik Uhrenholt, and Sven Åberg, Phys. Rev. C 79, 064304 (2009).
- [21] M. Kowal, P. Jachimowicz, and A. Sobiczewski, Phys. Rev. C 82, 014303 (2010).
- [22] M. Kowal and A. Sobiczewski (unpublished); M. Kowal, P. Jachimowicz, J. Skalski, arXiv:1203.5013.
- [23] H. Abusara, A. V. Afanasjev, and P. Ring, Phys. Rev. C 85, 024314 (2012).
- [24] Yu. Ts. Oganessian et al., Phys. Rev. C 69, 021601(R) (2004).
- [25] Yu. Ts. Oganessian et al., Phys. Rev. C 69, 054607 (2004).
- [26] Yu. Ts. Oganessian et al., Phys. Rev. C 70, 064609 (2004).
- [27] Yu. Ts. Oganessian et al., Phys. Rev. C 74, 044602 (2006).
- [28] L. Stavsetra, K. E. Gregorich, J. Dvorak, P. A. Ellison, I. Dragojević, M. A. Garcia, and H. Nitsche, Phys. Rev. Lett. 103, 132502 (2009).
- [29] Yu. Ts. Oganessian et al., Phys. Rev. Lett. 104, 142502 (2010).

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- [30] Ch. E. Düllmann et al., Phys. Rev. Lett. 104, 252701 (2010).
- [31] P. A. Ellison *et al.*, Phys. Rev. Lett. **105**, 182701 (2010).
- [32] S. Hofmann *et al.*, GSI Scientific Report 2010 (GSI Report 2011-1), p. 197 (unpublished).
- [33] J. M. Gates *et al.*, Phys. Rev. C 83, 054618 (2011).
- [34] Yu. Ts. Oganessian et al., Phys. Rev. Lett. 108, 02252 (2012).
- [35] K. Siwek-Wilczyńska and J. Wilczyński, Phys. Rev. C 69, 024611 (2004).
- [36] David Boilley, Hongliang Lü, Cawan Shen, Yasuhisa Abe, and Bernard G. Giraud, Phys. Rev. C 84, 054608 (2011).
- [37] Zu-Hua Liu and Jing-Dong Bao, Phys. Rev. C 81, 044606 (2010).
- [38] W. J. Światecki, K. Siwek-Wilczyńska, and J. Wilczyński, Acta Phys. Pol. B 34, 2049 (2003).
- [39] W. Reisdorf, Z. Phys. A 300, 227 (1981).

- [40] A. V. Ignatyuk, G. N. Smirenkin, and A. S. Tishin, Yad. Fiz. 21, 485 (1975) [Sov. J. Nucl. Phys. 21, 255 (1975)].
- [41] T. Cap, K. Siwek-Wilczyńska, I. Skwira-Chalot, and J. Wilczyński, Acta Phys. Pol. B 43, 297 (2012).
- [42] W. D. Myers and W. J. Swiatecki, Nucl. Phys. A 601, 141 (1966); Lawrence Berkeley Laboratory Report LBL-36803, December 1994 (unpublished), available on [http://ie.lbl.gov/txt/ms.txt].
- [43] K. Siwek-Wilczyńska, A. Borowiec, and J. Wilczyński, Int. J. Mod. Phys. E 18, 1079 (2009).
- [44] H. Feldmeier, Rep. Prog. Phys. 50, 915 (1987).
- [45] Zu-Hua Liu and Jing-Dong Bao, Phys. Rev. C 84, 031602(R) (2011).
- [46] S. Hofmann *et al.*, GSI Scientific Report 2008 (GSI Report 2009-1), p. 131 (unpublished).
- [47] Yu. Ts. Oganessian et al. Phys. Rev. C 79, 024603 (2009).