Hyperfine structure anomaly and magnetic moments of neutron deficient Tl isomers with $I = 9/2$

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The hyperfine structure of 276.9-nm atomic transition has been studied by the resonant ionization spectroscopy method at mass-separator IRIS (Investigation of Radioactive Isotopes on Synchrocyclotron), Petersburg Nuclear Physics Institute (PNPI) for the odd Tl isomers with $I = 9/2$ and $A = 187-197$. A differential hyperfine structure anomaly for $6p^2P_{1/2}$ and $7s^2S_{1/2}$ atomic states in Tl isomers with $I = 9/2$ has been determined. It is described by the recently developed theoretical approach fairly well. This enables one to recalculate the magnetic moments of ¹⁸⁷−193Tl*^m* (*I* = 9*/*2) from previously measured hyperfine splittings for 7*s* ² *S*1*/*² states and to determine for the first time the magnetic moments for ¹⁹⁷Tl^{*m*} and ¹⁹⁵Tl^{*m*} (*I* = 9/2) from hyperfine splittings for $6p^2P_{1/2}$ states with properly taking into account the rather great hyperfine structure anomaly. Similar measurements with greater accuracy have been proposed for the other nuclear states in odd-odd Tl isotopes. These measurements could shed light on the nuclear magnetization distribution in these isotopes.

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I. INTRODUCTION

It is well known that the study of the isotope shift (IS) in the long isotopic chains using the laser spectroscopy technique leads to an understanding of the charge distribution change as a function of a neutron number. Atomic spectroscopy can give insight into the more subtle nuclear property also: It is the hyperfine structure anomaly (HFA) that enables one to estimate the isotopic change of the nuclear magnetization distribution.

The hyperfine anomaly ${}^{1}\Delta^{2}$ is defined as

$$
{}^{A_1}\Delta {}^{A_2} = \frac{a_1}{g_{I,1}} \cdot \frac{g_{I,2}}{a_2} - 1,\tag{1}
$$

where g_I is the nuclear *g* factor, *a* is the magnetic hyperfine constant, and indices 1 and 2 point to two different isotopes with atomic numbers A_1 and A_2 . HFA arises from the differences in charge and magnetization distribution within the nucleus, through the "Breit-Rosenthal" (BR) [\[1\]](#page-3-0) and "Bohr-Weisskopf" (BW) [\[2\]](#page-3-0) effects, respectively. If we denote a magnetic hyperfine constant for the point-like nucleus as a_{point} , then the observable magnetic hyperfine constant a can be presented as follows:

$$
a = a_{\text{point}}(1 + \varepsilon)(1 + \delta), \tag{2}
$$

where small quantities ε and δ are responsible for the BW and BR effects, respectively. Then HFA acquires the simple expression

$$
{}^{A_1}\Delta {}^{A_2} = {}^{A_1}\Delta {}^{A_2}_{BW} + {}^{A_1}\Delta {}^{A_2}_{BR} = (\varepsilon_1 - \varepsilon_2) + (\delta_1 - \delta_2). \tag{3}
$$

To determine HFA one should have independent values for magnetic moments μ and α constants for the pair of isotopes under study, measured with high accuracy. Independent magnetic moment values for ground and isomeric states of isotopes far from stability are hard to obtain. This is why the HFA data as a rule were restricted to stable and long-lived isotopes only. Usually the HFA value is within the range of 10^{-2} – 10^{-4} .

It was proposed [\[3\]](#page-3-0) to compare the ratios of *a* constants for different atomic states with different quantum numbers *n*, *l* to extract the differential HFA (DHFA) $^{n_1 l_1}_{A_1} \Delta^{n_2 l_2}_{A_2}$ rather than to rely upon the hardly accessible independently measured magnetic moments

$$
\rho_{n_1l_1,n_2l_2}^A = \frac{a_{n_1l_1}^A}{a_{n_2l_2}^A}, \quad\n\begin{array}{c}\nn_1l_1 \Delta_{l_2}^{n_2l_2} = \frac{\rho_{n_1l_1,n_2l_2}^{A_1}}{\rho_{n_1l_1,n_2l_2}^{A_2}} - 1 \\
= \frac{A_1 \Delta_{l_2}(n_1l_1) - A_1 \Delta_{l_2}(n_2l_2)}{\rho_{n_2l_2}^{A_2}}.\n\end{array} \tag{4}
$$

*A*¹

The ratio $\rho_{n_1l_1,n_2l_2}^A$ can have a different value for different isotopes because the atomic states with different n, l have a different sensitivity to the nuclear magnetization distribution. This approach was successfully applied to $7S_{1/2}$ and $7P_{1/2}$ states for several Fr isotopes [\[4\]](#page-3-0). The extension of this kind of investigation is highly desirable because it can shed light on the very important ground state characteristic, isotopic changes in nuclear magnetization distribution, for isotopes far from stability.

In the present paper we have presented new results concerning DHFA in neutron deficient Tl isomers with spin $I = 9/2$. The good description of these results by the theoretical approach [\[5\]](#page-3-0) has enabled one to determine HFA for these isomers more reliably than was done previously and therefore to improve the values of their magnetic moments.

II. EXPERIMENTAL SETUP

Resonant ionization spectroscopy (RIS) has been used for hyperfine structure (hfs) and IS measurements for neutron deficient Tl isotopes. The detailed description of the experimental setup is presented in Ref. [\[6\]](#page-3-0).

The nuclides under study are produced in the uranium carbide target of a high density [\[7\]](#page-3-0) of the mass separator by 1 GeV protons of the Petersburg Nuclear Physics Institute synchrocyclotron. The atoms are thermally released from the target to the ion source cavity (tungsten tube with a length of 40 mm and 1.5 mm in diameter).

For the excitation and ionization of Tl atoms the radiation of the narrow-band dye laser (NBL; fundamental wavelength

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bandwidth is of 1 GHz), pumped by the copper vapor lasers (CVL), and the CVL radiation were used. The two-step resonance scheme was used for Tl isotopes ionization

$$
6p^2P_{1/2} \overset{276.9 \text{ nm}}{\rightarrow} 6d^2D_{3/2} \overset{511 \text{ nm}+578 \text{ nm}}{\rightarrow} \text{continuum}.
$$

Beams from the lasers are introduced into the ion source cavity through the quartz window in the front-end back side to provide the two-step resonance ionization of Tl atoms. The wavelength of the NBL is scanned across the chosen transition. The photoion current at the collector of the mass separator increases at the resonance. Thus, the experimental spectra represent the dependence of the ion current on the scanned laser frequency. The detection of the ion current was provided by *γ* counting. The corresponding detector is installed at the moving tape station.

III. EXPERIMENTAL RESULTS

In Fig. 1 some experimental hfs spectra of Tl isotopes are shown. IS and hfs in a 276.9-nm transition for 18 Tl isotopes and isomers were measured during our experiments. In this paper we will discuss the hfs data for odd Tl isomers with $I = 9/2$ only. Other results will be presented elsewhere.

In Table I the *a* constants for the Tl ground state $6p^2P_{1/2}$ are presented. In Refs. [\[8,9\]](#page-3-0) the *a* constants for the excited $7s²S_{1/2}$ state of Tl isomers with $A = 187, 189, 191,$ and 193 were measured. They are presented in column 3 of the

FIG. 1. (Color online) Hfs spectra of some Tl isotopes. In the inserts the atomic number, spin, and energies of the γ lines for ion current monitoring for the isotope in question are presented. Full lines are the results of the fitting with the Voigt profile. Vertical dashed lines mark the center of gravity of the corresponding hfs.

TABLE I. Hfs constants for ground and excited states of Tl isomers with $I = 9/2$ and corresponding $\rho_{6P_{1/2},7S_{1/2}}^A$ and DHFA values.

\boldsymbol{A}	$a(6p^2P_{1/2})^a$ $(MHz)^a$	$a(7s^2S_{1/2})$ (MHz)	$\rho^A_{6P_{1/2},7S_{1/2}}$	$^{6P_{1/2}}_{205}\Delta^{7S_{1/2}}_{A}$
187	5374(50)	$3163(5)^{b}$	1.699(16)	$2.02(0.97) \times 10^{-2}$
189	5474(56)	$3204.5(4.8)^c$	1.708(18)	$1.47(1.05) \times 10^{-2}$
191	5506(32)	$3225.9(3.4)^c$	1.707(10)	$1.56(60) \times 10^{-2}$
193	5583(34)	$3263.0(2.4)$ ^c	1.711(10)	$1.31(62) \times 10^{-2}$
195	5634(55)			
197	5871(82)			
2 _D	п.			

a Present work.

^bReference [\[9\]](#page-3-0).

c Reference [\[8\]](#page-3-0).

Table I. The values of $\rho_{6P_{1/2},7S_{1/2}}^A$ and $_{205}^{6P_{1/2}}\Delta_A^{7S_{1/2}}$ calculated according to Eq. (4) are also shown in Table I. For the stable isotope ²⁰⁵Tl the following values, needed in the ${}^{6P_{1/2}}_{205} \Delta_A^{7S_{1/2}}$
calculation, were used: *a*(6*p*²*P*_{1/2}) = 21310.835(5) MHz $[10]$, $a(7s^2S_{1/2}) = 12294.5(1.5)$ MHz $[11]$, and $\rho_{6P_{1/2},7S_{1/2}}^{205} =$ 1.73336(21).

It is clearly seen from Table I that the DHFA for the $6p^2P_{1/2}$ and $7s^2S_{1/2}$ states in Tl isomers with $I = 9/2$ is not negligible. Moreover, it is more than two orders of magnitude greater than the well-known HFA value for the two stable Tl isotopes with $I = 1/2$: ${}^{205}\Delta_{6P_{1/2}}^{203} = 1.050(15) \cdot 10^{-4}$ [\[10\]](#page-3-0). Isotopic dependence of DHFA cannot be traced in view of our relatively large experimental uncertainties. Thus, for the comparison with the theory we used the mean weighted value of DHFA for Tl isomers with $I = 9/2$, $A = 187-193$: ${}^{6P_{1/2}}\Delta^{7S_{1/2}}_{A(I=9/2)} = 1.53(37) \cdot 10^{-2}.$

IV. CALCULATION OF DHFA

In Ref. [\[5\]](#page-3-0) the atomic many-body technique was applied to the calculation of HFA. The stable Tl isotopes were considered. We used this approach for DHFA calculation in neutron deficient Tl isotopes.

According to A.-M. Mårtensson-Pendrill (Ref. [[5\]](#page-3-0)), the BR part of HFA can be presented as follows:

$$
{}^{A_1}\Delta_{\rm BR}^{A_2} = f(nl) \cdot \lambda_{A_1, A_2},\tag{5}
$$

where λ_{A_1,A_2} is the well-known charge radius parameter extracted from the IS measurements, $\lambda_{A_1,A_2} = k_c(Z) \cdot \delta \langle r^2 \rangle_c^{A_1,A_2}$, where $\langle r^2 \rangle_c^{A_1, A_2}$ is the change in the mean square nuclear charge radius. For Tl atoms $k_c(81) = 0.94$ [\[12\]](#page-3-0). The Dirac-Fock calculation gives $[5] f(6p_{1/2}) = -4.14 \cdot 10^{-4} \text{fm}^{-2}, f(7s) =$ $[5] f(6p_{1/2}) = -4.14 \cdot 10^{-4} \text{fm}^{-2}, f(7s) =$ $-15.8 \cdot 10^{-4}$ fm⁻².

For the BW part of HFA the analogous factorization was proposed (see Ref. [\[5\]](#page-3-0) for a detailed description)

$$
\varepsilon = b_{2s} \cdot \lambda_m \cdot d_2,
$$

\n
$$
\lambda_m = \langle r^2 \rangle_m \cdot \left(1 + \frac{b_{4s} \cdot d_4}{b_{2s} \cdot d_2} \cdot \frac{\langle r^4 \rangle}{\langle r^2 \rangle} + \cdots \right) = k_m \cdot \langle r^2 \rangle_m, \quad (6)
$$

\n
$$
d_{2n} = C_s \cdot \left(1 + \frac{2n}{2n+3} \cdot \zeta \right) + \frac{3}{2n+3} \cdot (1 - C_s),
$$

where $\langle r^2 \rangle_m$ denotes a mean squared nuclear magnetization radius, b_{ns} are coefficients determined by the atomic state only. For the fractional spin contribution C_s and spin-asymmetry term ζ the simplest approximation was used [\[13\]](#page-3-0)

$$
\zeta = \frac{2I + 3}{4I}, \quad C_s = \frac{g_s}{g_I} \cdot \frac{g_I - g_L}{g_s - g_L}, \tag{7}
$$

where *gs* and *gL* are the spin and orbital effective *g* factors, respectively. We used $g_L = 1.16$ (according to the prescription from Ref. [\[4\]](#page-3-0)) and have chosen *gs* factors to match the known magnetic moments of the Tl isotopes in question. The result $g_s = (0.80 \div 0.84) \cdot g_{s,\text{free}}$ -fairly corresponds to the option used in Ref. [\[4\]](#page-3-0) $(g_s = 0.85 \cdot g_{s,\text{free}})$.

It was shown [\[5\]](#page-3-0) that the ratios $\frac{b_{4s}}{b_{2s}}$, $\frac{b_{6s}}{b_{2s}}$, and so on, are small and to a good approximation independent on the atomic state $\frac{b_{4s}}{b_{2s}} = -3.2 \cdot 10^{-3}$ fm⁻², $\frac{b_{6s}}{b_{2s}} = 8.8 \cdot 10^{-6}$ fm⁻⁴. Using the liquid drop model for the estimation of the ratios $\frac{\langle r^{2n} \rangle}{\langle r^2 \rangle}$ (*n* > 1), $\langle r^2 \rangle_c^{1/2}$ (*A* = 205) = 5.475fm from Ref. [\[14\]](#page-3-0) and setting $\langle r^2 \rangle_m^{1/2} (A = 205) = \langle r^2 \rangle_c^{1/2} (A = 205)$ (cf. the same assump-tion in Ref. [\[4\]](#page-3-0) for odd Fr isotopes) we obtained $k_m(I = 9/2,$ TI) = 1.01.

The values of $b_{2s} (6p^2P_{1/2}) = -2.48 \cdot 10^{-4} \text{fm}^{-2}$ and $b_{2s}(7s^2S_{1/2}) = -7.62 \cdot 10^{-4} \text{fm}^{-2}$ were calculated by A.-M. Mårtensson-Pendrill (Ref. [[5\]](#page-3-0)) using the relativistic "coupledcluster" approach.

With these input parameters one can easily calculate the mean DHFA value for Tl isomers with $A = 187-193$ and $I = 9/2$

$$
{}_{205}^{6P_{1/2}}\Delta_{A(I=9/2)}^{7S_{1/2}}(\text{theor}) = 1.2 \cdot 10^{-2}
$$

in fair agreement with the experimental data. It is worth noting that the isotopic dependence of the calculated DHFA is approximately equal to $1 \cdot 10^{-4}$ per neutron (without taking into account the possible change of C_s and ζ factors due to the configuration mixing), which is obviously much smaller than the present experimental accuracy.

V. MAGNETIC MOMENTS FOR Tl ISOMERS WITH *I* **= 9***/***2**

Magnetic moments can be extracted from the hfs constants *a* by the standard relation

$$
\mu_A = \mu_{nl} \cdot (1 + {}^{205}\Delta_{nl}^A)
$$

\n
$$
\equiv \mu_{205} \cdot \frac{I_A}{I_{205}} \cdot \frac{a_A(nl)}{a_{205}(nl)} \cdot (1 + {}^{205}\Delta_{nl}^A). \tag{8}
$$

In Fig. 2 the comparison of the values of $\mu_{7S_{1/2}}$ and $\mu_{6P_{1/2}}(i)$. e., magnetic moments without taking into account HFA) are presented. The values of $\mu_{7S_{1/2}}$ are systematically higher than the values of $\mu_{6P_{1/2}}$. According to Eqs. [\(1\)](#page-0-0) and [\(4\),](#page-0-0) this discrepancy stems from the DHFA contribution

$$
\mu_{6P_{1/2}}(A) = \mu_{7S_{1/2}}(A) \cdot \left(1 - \frac{6P_{1/2}}{205} \Delta_A^{7S_{1/2}}\right). \tag{9}
$$

The experimental values of $\mu_{6P_{1/2}}$ agree fairly well with the values of $\mu_{6P_{1/2}}$ calculated by Eq. (9) with experimental values of $\mu_{7S_{1/2}}$ and DHFA (see Fig. 2). The theoretical ratio $\eta = (\varepsilon (7s^2S_{1/2}) + \delta (7s^2S_{1/2})) / (\varepsilon (6p^2P_{1/2}) + \delta (6p^2P_{1/2}))$

FIG. 2. (Color online) Magnetic moments for Tl isomers with $I = 9/2$. Circles: $\mu_{7S_{1/2}}$ [see Eq. (8)], squares: $\mu_{6P_{1/2}}$, triangles: $\mu_{6P_{1/2}}$ calculated by Eq. (9) by taking into account the DHFA contribution.

proves to be independent of the mass number [due to the factorization of BW and BR corrections, see Eqs. (5) and (6)] and is equal to 3.1. The fair agreement of the theoretical and experimental results for DHFA data gives confidence to this value calculated in the framework of the same theoretical approach. Using this ratio and the experimentally determined ${}^{6P_{1/2}}_{205} \Delta_{A(I=9/2)}^{7S_{1/2}}$ one can obtain experimental values of HFA

$$
\begin{aligned}\n\frac{6P_{1/2}}{205} \Delta_{A(I=9/2)}^{7S_{1/2}} &= \frac{205}{\Delta_{6P_{1/2}}^A} - \frac{205}{\Delta_{7S_{1/2}}^A} \\
&= (1 - \eta) \cdot \frac{205}{\Delta_{6P_{1/2}}^A}, \\
\frac{205}{\Delta_{6P_{1/2}}^A (I=9/2)} &= -0.007(2), \\
\frac{205}{\Delta_{7S_{1/2}}^A (I=9/2)} &= -0.023(5).\n\end{aligned}
$$
(10)

In the third column of Table II the magnetic moments for Tl isomers with $I = 9/2$, corrected by the HFA contribution, are presented. For $A = 187-193$ more accurate data from Refs. [\[8,9\]](#page-3-0) for the $7s^2S_{1/2}$ state were used. For $A = 195, 197$ the values of $a(6p^2P_{1/2})$, determined in the present work for the first time, were used.

TABLE II. Magnetic moments for Tl isomers with $I = 9/2$.

	μ (μ_N) (literature data)	μ (μ _N) with the new HFA correction
187	$3.7932(65)^{a}$	3.707(22)
189	$3.8776(63)^b$	3.756(22)
191	$3.9034(48)^{b}$	3.781(22)
193	$3.9482(39)^{b}$	3.824(22)
195		3.869(39)
197		4.032(57)

a Reference [\[9\]](#page-3-0), without HFA correction, errors due to HFA uncertainty are not included.

 b Reference [\[8\]](#page-3-0) with Moskowitz-Lombardi HFA correction [\[15\]](#page-3-0); errors due to HFA uncertainty are not included.

In the second column of Table [II](#page-2-0) old values of the magnetic moment determined by the authors of Refs. [8,9] are shown. In Ref. [8] HFA was estimated through the empirical Moskowitz-Lombardi rule [15]. This approach was successfully applied earlier to odd-neutron Hg isotopes [15], odd-proton Ir and Au isotopes $[16,17]$, and doubly odd Au isotopes $[18]$. It was found that for the *s* atomic states in the odd-proton nuclei a good description of the experimental data for HFA can be obtained with the same scaling constant *α*

$$
\varepsilon = \frac{\mp \alpha}{\mu}, \quad I = l \pm \frac{1}{2}, \quad \alpha = 1.2 \cdot 10^{-2} \mu_N.
$$
 (11)

In the framework of this parametrization the agreement with the experimental value of ${}^{205}\Delta_{7S_{1/2}}^A(I=9/2)$ for Tl isomers may be achieved with a different scaling parameter only: α = $2.6(7) \cdot 10^{-2} \mu_N$.

VI. CONCLUSION

In the present paper it is shown that the experimental determination of DHFA by the measurement of hfs constants

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in different atomic transitions gives the possibility to fix HFA correction and therefore to obtain more reliable magnetic moments values. The crucial point is the availability of the trustworthy atomic calculations [5]. Regrettably, due to the Doppler restricted resolution it is impossible in the framework of the RIS method to enable sufficient accuracy in the analogous measurements of *a* constants in odd-odd Tl isotopes and isomers with $I = 2$ and 7. It would be of importance to make measurements of $a(6p^2P_{1/2}, I = 2, 7)$ with the aid of collinear spectroscopy for a further check of the theoretical predictions of DHFA and a determination of the HFA corrections for corresponding magnetic moments. It should be stressed that for the investigated Tl isomers with $I = 9/2$ the nuclear state is quite a pure $h_{9/2}$ state. That is why the simplest one-configuration approximation for the fractional spin contribution *Cs* and spin-asymmetry term *ζ* proves to be quite sufficient. In the cases of more complex states with $I = 2$ and $I = 7$, as well as for DHFA isotopic dependence description, more refined nuclear models will be necessary. This can advance us in the understanding of the nuclear magnetization distribution.

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