Fluctuations in hadronizing quark gluon plasma

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The dynamical development of the cooling and hadronizing quark gluon plasma (QGP) is studied in a simple model assuming critical fluctuations in the QGP to hadronic matter and a first-order transition in a small finite system. We consider an earlier determined free-energy density curve in the neighborhood of the critical point, with two local minima corresponding to the equilibrium hadronic and QGP configurations. In this approach the divergence at $e = 0$ eliminates fluctuations with negative or zero energy. The barrier between the equilibrium states is obtained from an estimated value of the surface tension between the two phases. We obtain a characteristic behavior for the skewness and the kurtosis of energy density fluctuations, which can be studied via a beam energy scan program.

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In central heavy-ion collisions fluctuations may occur due to critical phenomena arising from a phase transition in the equation of state (EoS). Fluctuations arising from the initial asymmetry are smaller in head-on reactions. In the neighborhood of the critical point the shear viscosity of quark gluon plasma (QGP) is becoming small [1], which facilitates the appearance of fluctuations.

Molecular dynamics and fluid dynamics simulations of heavy-ion collisions suggest that collective flow asymmetries can be measured [2], both if the global asymmetry or random flow arising from the initial state transverse momentum fluctuations or longitudinal center of mass rapidity, $y_{\rm CM}$, fluctuations are causing it. These alternative sources may also lead to specific statistical characteristics as discussed in Ref. [3]. These phenomena contribute to a spatial spread of the matter and energy density variations are present even if we do not have a phase transition in our EoS.

Here, on the other hand, we study random fluctuations of thermodynamical origin caused by the phase transition based on the considerations described in Ref. [4]. Recently the field is under intensive research, and several studies of the phase transition have addressed the non-Gaussian fluctuations [5]. We focus on the reactions where rapid hadronization and freeze-out (FO) happen simultaneously, as hadronization in chemical equilibrium in phase mixture would take too long time [6].

In high-energy heavy-ion reactions around 30 GeV/nucleon or above [7] when QGP is formed, we have low mass quarks (5–10 MeV) and massless gluons in large numbers and the conserved baryon charge has a minor effect. On the hadronic side mainly mesons and baryon-antibaryon pairs are formed. The observable signatures of the phase transition are formed when the plasma expands, hadronizes, and freezes out. At this stage the system may be in the vicinity of the critical point so critical fluctuations may appear. In a finite volume this would show up as energy density fluctuation, which would then lead to charged hadron number fluctuation (and much less in net baryon charge fluctuation).

In the present simple model we assume two coexisting phases in a finite system near a first-order phase transition. We study the abundance of the energy density distribution of the two phases in the mixed phase domain in terms of the volume ratio.

Following Ref. [4] and the ideas of the Landau-Ginsburg theory for critical phenomena we briefly sketch the way to describe critical fluctuations. In case of QGP to hadronic matter (HM) transition the essential difference is that in phase equilibrium the energy density of the HM phase is much lower than that of the QGP phase. As a consequence the usual fourth-order polynomial to describe the free energy of the system is not realistic as it would lead to considerable population of negative energy density states, which would be unphysical. Thus the fourth-order polynomial was replaced by a Laurent series.

Using the simple bag model EoS the equilibrium value of the energy density in the low-temperature, low-energy-density phase (HM) denoted by e_h, and the equilibrium value of the energy-density in the high-temperature, high-energy-density, QGP, phase denoted by e_q can be calculated as

$$e_h(T) = \frac{\pi^2 T^4}{10(\hbar c)^3}, \quad e_q(T) = \frac{\pi^2}{(\hbar c)^3}\left(\frac{37}{30}T^4 + \frac{34}{90}T_c^4\right). \quad (1)$$

To study the fluctuations we find the free energy density, $f(e)$, for arbitrary values of e, and not just for the energy densities e_q and e_h. For these equilibrium points

$$f[e_q(T)] = -p_q(T) = -\frac{\pi^2}{90(\hbar c)^3}\left(37T^4 - 34T_c^4\right), \quad (2)$$

$$f[e_h(T)] = -p_h(T) = -\frac{\pi^2}{30(\hbar c)^3}T^4. \quad (3)$$

Following the Landau theory we approximate now the free energy density as a polynomial in the neighborhood of an e_0 energy density ($e_0 \in [e_h, e_q]$), where it has a local maximum. In order to obtain the required divergence at $e = 0$, a slightly modified functional form is assumed:

$$f(e) = f_1 + \frac{K_1}{e} + K_2(e-e_0) + K_3(e-e_0)^2 + K_4(e-e_0)^3. \quad (4)$$

The constants, f_1, K_1, K_2, K_3, K_4 can be determined from thermodynamic considerations as done in Ref. [4]:

$$K_1 = \frac{\sigma}{\xi_0}\frac{1}{A_0 A_1}, \quad K_2 = \frac{K_1}{e_0^2}, \tag{5}$$

$$K_3 = -\frac{K_1}{2}\frac{e_h^2(e_q+e_0)+e_q^2(e_0+e_h)-e_0^2(e_q+e_h)}{e_q^2 e_h^2 e_0^2}, \tag{6}$$

$$K_4 = \frac{K_3}{3}\frac{e_h e_q + e_q e_0 + e_0 e_h}{e_q^2 e_h^2 e_0^2}, \tag{7}$$

where

$$A_0 = \frac{(e_q-e_0)(e_0-e_h)}{e_0^3 e_h^2 e_q^2}(2e_q e_h + e_0 e_q + e_h e_0),$$

$$A_1 = \frac{(e_q-e_h)^2}{3\sqrt{2}} + \frac{[e_0-(e_q+e_h)/2]^2\sqrt{\pi}}{2}.$$

In Eq. (5) σ represents the surface tension of a hadronic bubble and ξ_0 is the characteristic size of a hadronic droplet. Once the K_i values are expressed as function of $e, e_0, e_q, e_h, \xi_0, \sigma$, the unknown f_1 and e_0 parameters are obtained at any temperature from Eqs. (2) and (3). Following our previous work [4], we have used the $T_c = 169$ MeV, $\xi_0 = 3$ fm and $\sigma = 0.05$ GeV/fm^2 values in all our calculations. For temperatures in the vicinity of the critical temperature the free-energy-density curve as a function of the energy density shows two local minima at the energy densities, $e_h(T)$ and $e_q(T)$. For $T = T_c$ the two minima have the same free energy density, while for $T < T_c$ the hadronic, $f(e_h)$, and for $T > T_c$ the QGP, $f(e_q)$, free energy is lower.

Once the free energy curve is known, one can estimate the probability density of finding the system in a state with energy density e: $P(e) \propto \exp[-\beta F(e)]$, where $F(e) = \Omega f(e)$, with Ω the volume of the created QGP. In Fig. 1 we plot at the critical temperature the characteristic $P(e)/P(e_q)$ curves, considering different volumes for the QGP ($\Omega = 10$ fm^3, 50 fm^3, and 500 fm^3 values). Also, in Fig. 2 we show the $P(e)/P(e_q)$ curves for $\Omega = 50$ fm^3 and different temperatures.

It is also important to mention that different thermodynamical parameters (especially intensives and extensive ones) do not have to show the same critical fluctuation properties, so we have to study the fluctuations of several parameters.

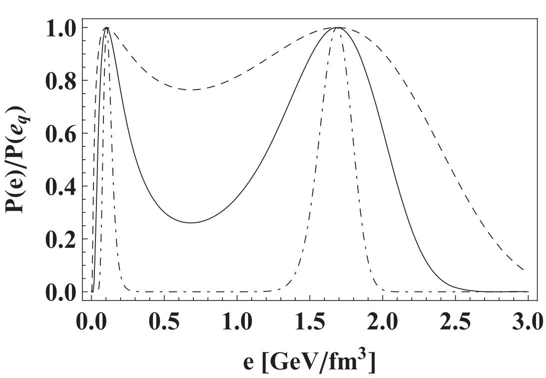

FIG. 1. The relative probability $P(e)/P(e_q)$ of finding a state of a given energy density e for $T = T_c$, in a system of volume $\Omega = 10$ fm^3 (dashed), $\Omega = 50$ fm^3 (full line), and $\Omega = 500$ fm^3 (dot-dashed).

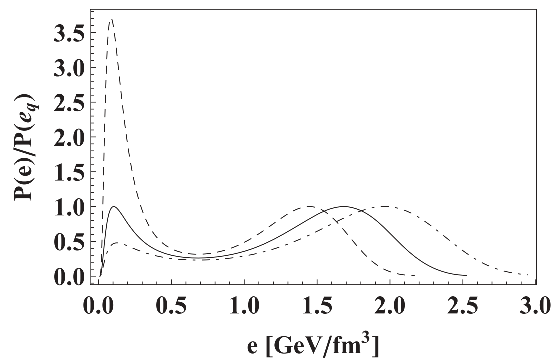

FIG. 2. The relative probability of finding a state of a given energy density e for the $T = 0.95T_c$ (dashed), $T = T_c$ (continuous), and $T = 1.05T_c$ (dot-dashed) temperatures. The volume of QGP is $\Omega = 50$ fm^3.

Furthermore, the statistical physics estimates assume a single thermal source at or near the critical point, while in heavy-ion reactions we have the problem of spatial fluctuations, which arise from a dynamically expanding fluid flow even in the least fluctuating configuration and without any phase transition in the EoS [3]. Assuming central collisions only, to avoid the effects from azimuthal flow asymmetries, we can have additional sources of random fluctuations from particle emission from projectile and target residues (spectator evaporation) [2].

The dynamically developing flow pattern leads to a spatial distribution of all thermodynamical quantities, while the system expands rapidly. Finally the supercooled QGP can hadronize rapidly and almost simultaneously it freezes out. This final stage of the reaction is best described by a nonequilibrium model. To complement the fluctuations arising exclusively from the flow dynamics [3], in this Brief Report we study only the fluctuations arising from the phase transition.

In a theoretical approach we can assume a spatial distribution of a thermodynamical quantity, x, usually an order parameter, like the specific energy density. For the variable x the averages and various order moments distributions can be written as $\langle x^n\rangle = \int x^n P(x)dx$, and

$$M^{(n)} = \langle (x-\langle x\rangle)^n\rangle = \int (x-\langle x\rangle)^n P(x)dx, \tag{8}$$

where $P(x)$ is the spatial distribution weighted [e.g., by the baryon charge density in the center of mass frame (CF)]. The spatial variance, the skewness, and the kurtosis can be obtained from these moments:

$$\Delta x = \langle (x-\langle x\rangle)^2\rangle = M^{(2)}, \tag{9}$$

$$S = \frac{\langle (x-\langle x\rangle)^3\rangle}{(\Delta x)^{3/2}} = \frac{M^{(3)}}{(M^{(2)})^{3/2}}, \tag{10}$$

$$K = \frac{\langle (x-\langle x\rangle)^4\rangle}{(\Delta x)^2} - 3 = \frac{M^{(4)}}{(M^{(2)})^2} - 3. \tag{11}$$

By using these averages, first we can calculate specific extensives, which are governed by strict conservation laws. The total baryon charge, energy, and momentum conservations are governed by the continuity equation and by the relativistic

Euler equation.

$$N^{\mu}{}_{,\mu} = 0, \quad T^{\mu\nu}{}_{,\nu} = 0, \tag{12}$$

and as a consequence, the total momentum in the CF should remain zero during the development, while the average specific energy per net nucleon number

$$\langle \varepsilon^{\mathrm{CF}} \rangle \equiv T^{00}/N^{0} = \text{const.} \tag{13}$$

should remain constant in CF.

The total net baryon number, N_{tot}, is exactly conserved in the reaction. At the same time the average baryon charge density is decreasing and it may have significant spatial and/or event by event (EbE) fluctuation. This leads to similar fluctuation in the particle number densities or in the EbE charged particle numbers. This last quantity is the most frequently studied observable.

We focus now on the skewness and kurtosis of the specific energy density and consequently the charged particle densities according to Eqs. (10) and (11). We determine these quantities as a function of the system's temperature and also as a function of the volume abundance (r_h) of the hadronic matter. We assume that in a rapid transition, where critical fluctuations dominate and the two phases are not separated these two phases are in thermal equilibrium. The temperature decreases rapidly starting from QGP where $T > T_c$ until the hadronization completes at $T < T_c$. As the phases are not separated the simplest estimate for any temperature T, is that the volume abundance of the hadronic matter is

$$r_h = \frac{P(e_h)}{P(e_q) + P(e_h)}, \tag{14}$$

where e_h and e_q are the energy densities of the pure phases defined in Eq. (1), and the probability densities, $P(e)$, are defined previously. This relation makes a one to one correspondence between the volume abundance and the equilibrium temperature in a rapidly expanding and cooling system during the process of a phase transition.

The skewness (Fig. 3) is first negative (indicating a longer tail on the lower-energy side), then at 80% HM volume abundance ($r_h = 0.8$) it turns into positive (indicating a longer tail on the high-energy side). The hadronization can be parametrized both as a function of the volume abundance of the growing hadronic phase or the decreasing equilibrium temperature of the system showing critical fluctuations.

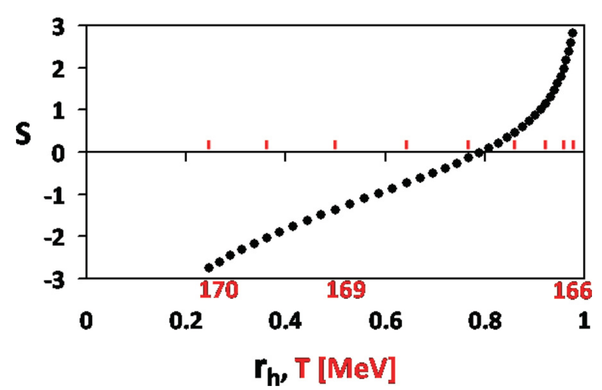

FIG. 3. (Color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity, additional tick marks (red ones) represent increments of 0.5 MeV in T. Results for $\Omega = 500$ fm^3.

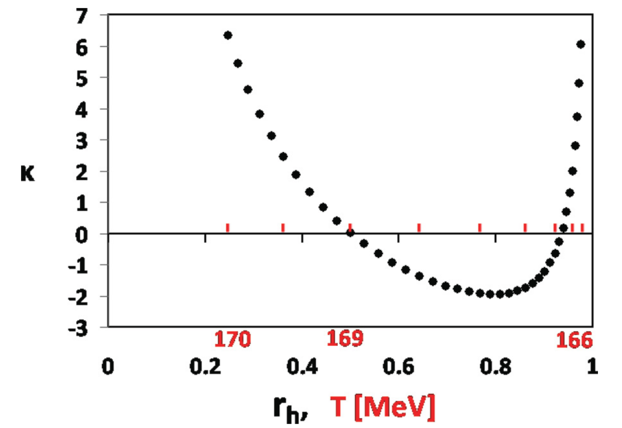

FIG. 4. (Color online) Kurtosis as a function of the volume abundance of the hadronic matter (denoted as r_h, where 1 represents complete hadronization). The temperature scale is also indicated for clarity, additional tick marks (red ones) represent increments of 0.5 MeV in T. Results for $\Omega = 500$ fm^3.

In Fig. 4, we can see that the kurtosis is positive at first, then turns to be negative (the distribution becomes wider) in the phase transition domain, while it becomes positive again as the phase transition completes. The minimum of kurtosis is at 80% HM volume abundance. Notice that the kurtosis is increasing much sharper on the hadronic side, which is a clear consequence of the energy difference between the two phases. This asymmetry appears as a result of the Laurent series expansion, and it would not show up with the usual fourth-order polynomial approximation.

This phase transition dynamics as a function of the HM volume abundance is not directly observable. Only the final so-called FO stage is observable where the particle numbers and their momentum distributions do not change any more. To determine this time or HM volume abundance one would need a complex dynamical model, where the different stages of the whole reaction are all described accurately and are matched [8] to each other. It is estimated that this FO stage may be in the domain of the phase transition [9].

In Ref. [10] a mixed particle method is introduced, which could separate the fluctuations arising from local critical fluctuations. The mixed events are actually eliminating two-particle correlations, and only the single-particle distributions remains. The comparison of STAR data with model predictions analyzed with this method indicate a sign change of skewness in the RHIC Beam Energy Scan program.

As a conclusion we state that in this Brief Report we reiterated earlier results on critical fluctuations [4] in the phase transition between quark gluon plasma and hadronic matter. This model is specific because of the large difference of the energy density of the two phases, where $e_q \gg e_h$. Here this work is extended to the evaluation of frequently used statistical parameters like the skewness and kurtosis of typical parameters like charged particle multiplicities.

We study the dynamical change of these typical parameters during the hadronization process, where the development is very characteristic and informative for the phase transition we study. There are similar studies in a microscopic quark molecular dynamics model [11] and in phenomenological hadronization model with supercooling [12], following the idea of rapid simultaneous hadronization and FO. In such processes it is not excluded that fluctuations from the initial stages of the transition dominate.

In experiments one can measure these parameters (together with all other measurable quantities) at the FO space-time domain (or at the FO hypersurface). The recent RHIC Beam Energy Scan program scans the same statistical parameters at a series of different beam energies. If the FO would happen at all energies in the pure hadronic matter the signs of critical fluctuations would not be observable. However, in rapid transition the FO may happen in the critical fluctuation domain (with negative kurtosis) and in the beam energy domain the fluctuations may be dominated by the prehadronic QGP phase. This would be indicated by a change in the sign of skewness. Reference [10] suggests that this may be seen in the measurements.

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[1] L. P. Csernai, J. I. Kapusta, and L. D. McLerran, Phys. Rev. Lett. **97**, 152303 (2006).
[2] N. S. Amelin, E. F. Staubo, L. P. Csernai, V. D. Toneev, and K. K. Gudima, Phys. Rev. C **44**, 1541 (1991).
[3] D. J. Wang *et al*., arXiv:1205.4934v1 [nucl-th].
[4] L. P. Csernai and Z. Neda, Phys. Lett. B **337**, 25 (1994).
[5] M. A. Stephanov, Phys. Rev. Lett. **102**, 032301 (2009); R. V. Gavai and S. Gupta, Phys. Rev. D **78**, 114503 (2008); Tapan K. Nayaka (STAR collaboration), Nucl. Phys. A **830**, 555c (2009); Y. Zhou, S. S. Shi, K. Xiao, K. J. Wu, and F. Liu, Phys. Rev. C **82**, 014905 (2010); M. Nahrgang, T. Schuster, M. Mitrovski, R. Stock, and M. Bleicher, J. Phys. G **38**, 124150 (2011).
[6] L. P. Csernai and J. I. Kapusta, Phys. Rev. Lett. **69**, 737 (1992); Phys. Rev. D **46**, 1379 (1992).
[7] C. Alt *et al*., Phys. Rev. C **77**, 024903 (2008).
[8] Yun Cheng, L. P. Csernai, V. K. Magas, B. R. Schlei, and D. Strottman, Phys. Rev. C **81**, 064910 (2010).
[9] L. P. Csernai and I. N. Mishustin, Phys. Rev. Lett. **74**, 5005 (1995).
[10] Dai-Mei Zhou, Ayut Limphirat, Yu-liang Yan, Yun Cheng, Yu-peng Yan, Xu Cai, Laszlo P. Csernai, and Ben-Hao Sa, Phys. Rev. C **85**, 064916 (2012).
[11] S. Haussler, S. Scherer, and M. Bleicher, Phys. Lett. B **660**, 197 (2008).
[12] S. Zschocke *et al*., Phys. Rev. C **83**, 044903 (2011).