Particle spectra in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$

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Particle production in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV is studied in the (3 + 1)-dimensional viscous hydrodynamic model. The shapes of the calculated transverse momentum spectra of π^+ , K^+ , protons, Ξ^- , and Ω ⁻ are in satisfactory agreement with preliminary data of the ALICE Collaboration, while the particle ratio p/π ⁺ is slightly overpredicted, and the strange baryon yields are underpredicted.

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I. INTRODUCTION

Particle production in Pb-Pb collisions at the highest available energy $\sqrt{s_{NN}} = 2.76$ TeV has been studied experimentally at the CERN Large Hadron Collider (LHC) [\[1–3\]](#page-5-0). The observation of elliptic and triangular flows indicates that a collectively expanding fireball of dense matter is formed, confirming results obtained in collisions at lower energies [\[4\]](#page-5-0). The hydrodynamic model of the dynamics provides a quantitative explanation for observables defined for particles emitted with soft momenta [\[5\]](#page-5-0). In particular, hydrodynamic models are applied to describe the anisotropic flow of charged particles produced in Pb-Pb collisions at the LHC [\[6–14\]](#page-5-0).

Statistical models of the particle production in heavy-ion collisions predict the production rates of specific hadrons assuming a chemically equilibrated system [\[15\]](#page-5-0). Recent results for Pb-Pb collisions at the LHC seem to be incompatible with this simple mechanism. The number of protons emitted is lower than expected from the rates of the emission of other particles, which raises a doubt in the assumption that the production of different particle species happens at a common chemical freeze-out temperature [\[16\]](#page-5-0). Particle abundances may undergo significant modifications in the nonequilibrium dynamics after hadronization, e.g., annihilation processes may reduce baryon multiplicities [\[17\]](#page-5-0).

Transverse momentum spectra of identified particles represent a more basic observable, as they involve the overall multiplicity as well as the momentum distribution for each particle species. The hydrodynamic evolution of a fireball of thermally equilibrated fluid until the freeze-out temperature T_f leads to a common chemical and kinetic freeze-out temperature. A similar idea constitutes the basic assumption of the single freeze-out model of particle emission [\[18\]](#page-5-0). Additional freedom is allowed in hydrodynamic models assuming that below the chemical freeze-out temperature the matter is in kinetic equilibrium but particle abundances remain frozen [\[19\]](#page-5-0). In such models particle ratios correspond to a fixed chemical freeze-out temperature *T*_{chem}, while the transverse momentum spectra are determined by the convolution of the collective velocity with the thermal emission at the kinetic freeze-out temperature T_{kin} . The genuinely nonequilibrium phase at the end of the evolution may be addressed using a hybrid model with hydrodynamics for the dense phase and a hadronic cascade afterburner for the latter evolution [\[20–24\]](#page-5-0).

The evolution in the hadronic cascade changes the chemical composition in the system and the momentum spectra of particles.

Deviations from local equilibrium in the hydrodynamic model of the dynamics are introduced as viscosity corrections to the energy-momentum tensor $[25-35]$. In particular, shear viscosity is important in quantitative predictions for the elliptic and triangular collective flows. The rapid expansion of the fireball introduces sizable corrections from bulk viscosity, if the equilibration processes are not fast enough to restore local equilibrium. Such deviations are twofold. First, the chemical composition remains effectively frozen at some stage, while the local energy density drops. Second, the local momentum distributions of particles in the fluid become softer. Bulk viscosity leads to both effects, depending on the local expansion rate [\[34,36–38\]](#page-5-0).

We present calculations for the transverse momentum spectra of identified particles in a $(3 + 1)$ -dimensional $[(3 + 1)$ -D] hydrodynamic model with bulk and shear viscosities for Pb-Pb collisions at LHC energies. The presence of bulk viscosity in the hadronic phase yields nonequilibrium effects for the chemical composition and for the transverse momentum spectra. We find a satisfactory agreement with preliminary data of the ALICE Collaboration [\[3,16\]](#page-5-0) for pion, kaon, proton spectra, and abundances. At the same time we constrain the freeze-out temperature using the interferometry data [\[2\]](#page-5-0). Reproducing the charged-particle density in pseudorapidity [\[3\]](#page-5-0) and the transverse momentum spectra allows for a direct prediction on the rapidity distributions of identified hadrons.

II. INITIAL CONDITIONS AND HYDRODYNAMIC EVOLUTION

The expansion of the fireball is described using second order viscous hydrodynamics. Hydrodynamic equations are solved in $(3 + 1)$ -D together with the Israel-Stewart equations for the stress corrections $\pi^{\mu\nu}$ and Π to the energy-momentum tensor (for details see Ref. $[37]$). We use a constant ratio of shear viscosity to entropy density $\eta/s = 0.08$. The bulk viscosity is nonzero only in the hadronic phase; we use $\zeta/s = 0.04$ and $\zeta/s = 0.08$. The equation of state is a combination of the lattice QCD [\[39\]](#page-5-0) and hadron gas equations of state, obtained in a thermodynamically consistent way [\[40\]](#page-5-0). The initial time for the hydrodynamic expansion is 0*.*6 fm/*c*. The relaxation times in the Israel-Stewart equations are $\tau_{\pi} = \tau_{\Pi} = \frac{3\eta}{Ts}$. For the chosen values of the relaxation times the stress corrections are close to the ones from the Navier-Stokes expression, at latter stages of the expansion. For a strong temperature dependence of the viscosity coefficients, the value of the relaxation time can influence the evolution [\[41\]](#page-5-0).

The initial entropy density $s(\eta_{\parallel}, x, y)$ for the $(3 + 1)$ -D hydrodynamic evolution in the space-time rapidity η_{\parallel} and the transverse coordinates x , y is

$$
s(\eta_{\parallel}, x, y) \propto \left(\frac{(y_b + \eta_{\parallel})N_+ + (y_b - \eta_{\parallel})N_-}{y_b(N_+ + N_-)}\right) \times \left[\frac{1 - \alpha}{2}\rho_{\text{part}} + \alpha \rho_{\text{bin}}\right] f(\eta_{\parallel}).
$$
 (1)

In the transverse plane the density is defined as a combination of the participant nucleon $\rho_{part} = N_+ + N_-$ and binary collision ρ_{bin} densities. The factor $(\frac{(y_b+\eta_1)N_+(y_b-\eta_1)N_-}{y_b(N_++N_-)})$ in Eq. (1) implements in the initial density the assumption that forward (backward) going nucleons *N*⁺ (*N*−) emit particles preferentially in the forward (backward) rapidity hemisphere [\[42\]](#page-5-0). The parameters of the longitudinal profile

$$
f(\eta_{\parallel}) \exp\left(-\frac{(\eta_{\parallel} - \eta_0)^2}{2\sigma_{\eta}^2} \theta(|\eta_{\parallel}| - \eta_0)\right) \tag{2}
$$

are adjusted to reproduce the charged-particle density in pseudorapidity; $\eta_0 = 2.3$, $\sigma_n = 1.4$, and y_b is the beam rapidity. The parameters of the Glauber model used to calculate the entropy profiles for Pb-Pb collisions ($A = 208$) are $R_A =$ 6.48 fm and $a = 0.535$ fm; the nucleon-nucleon cross section is $\sigma = 62$ mb.

At the freeze-out temperature particles are emitted from the freeze-out hypersurface according to the Cooper-Frye formula with viscosity corrections. The nonequilibrium modifications of the equilibrium momentum distribution f_0 from shear viscosity are quadratic in momentum [\[26\]](#page-5-0),

$$
\delta f_{\text{shear}} = f_0 \left(1 \pm f_0 \right) \frac{1}{2T^2(\epsilon + p)} p^{\mu} p^{\nu} \pi_{\mu \nu}.
$$
 (3)

The corrections from bulk viscosity are taken from the relaxation-time formula [\[34,43\]](#page-5-0)

$$
\delta f_{\text{bulk}} = C_{\text{bulk}} f_0 \left(1 \pm f_0 \right) \left(c_s^2 u^\mu p_\mu - \frac{(u^\mu p_\mu)^2 - m^2}{3u^\mu p_\mu} \right) \Pi, \tag{4}
$$

where c_s is the sound velocity and C_{bulk} is a normalization constant. At the freeze-out hypersurface, the flow velocity and the stress corrections from viscosity are exported to a Monte Carlo statistical emission code [\[44\]](#page-5-0) that is used to generate particle spectra. Bulk viscosity corrections are calculated with respect to the equilibrium distribution having the same energy density. Imposing constraints corresponding to conserved charges leads to deviations from the chemical equilibrium with respect to this equilibrium reference state. Effectively, it means that due to incomplete equilibration the chemical equilibrium temperature is shifted up and the effective kinetic temperature is shifted down (due to a redshifting of particle

FIG. 1. (Color online) Charged-particle distributions in pseudrapidity in Pb-Pb collisions at $\sqrt{s}_{NN} = 2.76$ TeV, for centralities (from top to bottom) 0%–5%, 5%–10%, 10%–20%, ..., 50%–60%. Results from viscous hydrodynamics (lines) are compared to preliminary data of the ALICE Collaboration (symbols) [\[3\]](#page-5-0).

thermal momenta). An expanded discussion of this approach can be found in Refs. [\[37,38\]](#page-5-0).

III. RESULTS

The parameters of the initial entropy density for the hydrodynamic evolution are constrained by the charge particle density as function of pseudorapidity measured for different centrality classes (Fig. 1). We find that the optimal mixing parameter for the admixture of binary collisions $\alpha = 0.15$; the same value as used in $(2 + 1)$ -D viscous model simulations for heavy-ion collisions at the LHC [\[10\]](#page-5-0). The half-width of

FIG. 2. (Color online) Transverse momentum spectra of π^+ , K^+ , and p (from top to bottom) in Pb-Pb collisions with centralities 0% – 5%, obtained in the viscous hydrodynamic model with $\zeta/s = 0.04$ and $T_f = 150$ MeV (dashed lines), $\zeta/s = 0.04$ and $T_f = 140$ MeV (dotted lines), and $\zeta/s = 0.08$ and $T_f = 140$ MeV (solid lines). Symbols represent preliminary data of the ALICE Collaboration [\[3\]](#page-5-0). The dashed-dotted line represents the result of the viscous hydrodynamic calculation with $\zeta/s = 0.08$ and $T_f = 140$ MeV but without bulk viscosity corrections at freeze-out.

FIG. 3. (Color online) Interferometry radii in Pb-Pb collisions with centralities $0\% - 5\%$ as functions of the pion pair momentum, obtained in the viscous hydrodynamic model with $\zeta/s = 0.04$ and $T_f = 150$ MeV (dashed lines), $\zeta/s = 0.04$ and $T_f = 140$ MeV (dotted line), and $\zeta/s = 0.08$ and $T_f = 140$ MeV (solid lines). Symbols represent preliminary data of the ALICE Collaboration [\[2\]](#page-5-0).

the plateau in the distribution $\eta_0 = 2.3$ is larger as compared to $\eta_0 = 1.5$ that has been used for Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ [\[37\]](#page-5-0).

Transverse momentum spectra for p^+ , K^+ and protons in central collisions are presented in Fig. [2.](#page-1-0) The spectra get harder when the freeze-out temperature is lowered from $T_f = 150$ MeV (dashed lines) to 140 MeV (dotted lines), and using $\zeta/s = 0.04$. Especially, the pion spectra get too flat. The size of the fireball at freeze-out can be estimated from the in-terferometry radii [\[45\]](#page-5-0). The radius R_{side} is best described using a freeze-out temperature of 140 MeV (Fig. 3). Also the value of the ratio $R_{\text{out}}/R_{\text{side}}$ is closer to the experimental data if the evolution is longer. The proton yield is too large for the freezeout at 150 MeV, while the average transverse momentum of protons is too low. The proton spectra and yields are better reproduced for $T_f = 140$ MeV. To describe at the same time the pion and proton spectra a freeze-out temperature 140 MeV and a bulk viscosity coefficient $\zeta/s = 0.08$ are used (solid lines in Figs. [2](#page-1-0) and 3). Increasing the bulk viscosity makes the local momentum distributions of light particles (pions) softer, which results in the softening of their final transverse momentum spectra. At the same time, nonequilibrium corrections make the proton yield increase slightly as compared to the case of $\zeta/s = 0.04$, without changing the proton average momentum. In the following we set $T_f = 140$ MeV and $\zeta/s = 0.08$ as the optimal parameters for the freeze-out conditions that reproduce

FIG. 4. (Color online) Transverse momentum spectra of π^+ , K^+ , and p (from to to bottom) in Pb-Pb collisions with centralities 10% – 20% (a) and 30%–40% (b), obtained in the viscous hydrodynamic model (solid lines). Symbols represent preliminary data of the ALICE Collaboration [\[3\]](#page-5-0).

particle spectra and interferometry radii in central collisions. We note that the number of pions and kaons with very soft momenta is underestimated, while the proton yield is slightly overestimated.

The pion, kaon, and proton spectra without the bulk viscosity corrections at freeze-out $[Eq. (4)]$ $[Eq. (4)]$ are shown by the dashed-dotted lines in Fig. [2.](#page-1-0) Bulk viscosity corrections make the spectra softer, especially for light particles, and introduce a correction in particle abundances, increasing the proton number. The corrections are important for pions with high momenta ($p_\perp > 1.5$ GeV). They make the proton number increase by 35%. If the corrections are large, formally a better ansatz for the distribution function with bulk viscosity corrections would be an exponential function [\[37\]](#page-5-0), but the final spectra are very similar as when using Eq. [\(4\).](#page-1-0) The shift in the effective chemical equilibrium temperature is approximately the same for all the particles, depending only on the local expansion rate. A more elaborate ansatz is possible, with different bulk viscosity corrections for mesons, baryons, or strange particles [\[38\]](#page-5-0); using additional parameters the measured particle ratios could be better reproduced.

FIG. 5. (Color online) Transverse momentum spectra of Λ^0 (dashed line), E^- (dotted line and stars), and Ω^- (solid line and diamonds) in Pb-Pb collisions with centralities 0%–20% (a) and 20%–40% (b). The lines and the symbols represent the results of the viscous hydrodynamic model and the preliminary data of the ALICE Collaboration [\[16\]](#page-5-0) respectively.

The spectra of identified particles in semi central collisions are well described by the hydrodynamic model (Fig. [4\)](#page-2-0). For momenta $p_T > 1.5$ GeV the pion spectra are underestimated by the hydrodynamic model with statistical particle emission. The discrepancy increases with centrality, and is visible for kaons as well for centralities 30%–40%. This effect indicates that a nonthermal component in the particle emission is present, e.g., jet fragmentation. A similar underestimation of the experimental particle yields at high momenta by the hydrodynamic model is seen in peripheral Au-Au collisions at lower energies [\[37\]](#page-5-0). The proton multiplicity and spectra are well described by the model for different centrality classes.

The production rate of strange baryons with higher masses Ξ and Ω is very sensitive to the chemical freeze-out temperature. In Fig. 5 are shown the transverse momentum spectra of Λ^0 , Ξ^- , and Ω^- particles for two centrality classes. The results of the viscous hydrodynamic model are in qualitative agreement with the preliminary data of the ALICE Collaboration. Nonequilibrium corrections in the expanding fireball increase the effective chemical freeze-out temperature. However, the effect is not strong enough to reproduce to

FIG. 6. (Color online) Ratios of particle yields in Pb-Pb collisions with centralities 0%–20%; preliminary data of the ALICE Collaboration (squares) [\[16\]](#page-5-0) are compared to results of viscous hydrodynamic calculations (lines).

observed yields of heavy baryons. It is instructive to look at the ratios of p_T integrated particle yields. The hydrodynamic model reproduces qualitatively the observed particles ratios (Fig. 6), although the nonequilibrium effects are described with only one parameter, the bulk viscosity coefficient. Deviations from chemical equilibrium at the freeze-out temperature $T_f = 140$ MeV shift the ratios of heavy particle yields to pion yields upward. The ratio K/π is very well reproduced, the ratio p/π is overpredicted by 17%, and the ratios Ξ/π and Ω/π are underpredicted by 30%–40%. The ratios obtained from the hydrodynamic simulation with $T_f = 140$ MeV correspond aproximately to a chemical equilibrium temperature of 150 MeV (Fig. 6). The deviations of the calculated particles ratios from the observations indicate that the data cannot be described using a single chemical freeze-out temperature; as mentioned above, the agreement could be improved using different equilibration rates for different particle species.

The $(3 + 1)$ -D hydrodynamic model does not assume boost invariance and gives predictions on the rapidity dependence of particle spectra. In practice, the initial density in the longitudinal direction is adjusted to reproduce the final pseudorapidity distribution (Fig. [1\)](#page-1-0). Once the transverse momentum spectra and the pseudorapidity distributions are

FIG. 7. Rapidity distributions of π^+ (solid line), K^+ (dashed line), and *p* (dotted line) emitted in Pb-Pb collisions with centralities 0%–5%, calculated in the $(3 + 1)$ -D viscous hydrodynamic model.

FIG. 8. (Color online) Elliptic flow coefficient of charged particles as function of transverse momentum for three centrality classes calculated in the viscous hydrodynamic model (lines); the symbols represent the experimental results of the ALICE Collaboration [\[46\]](#page-5-0).

found to be in agreement with the experimental observations, the rapidity distributions of identified particles can be reliably estimated. The rapidity distributions for pions, kaons, and protons are shown in Fig. [7.](#page-3-0) We find that the distributions are not boost invariant. However, in the range $|y| < 1$ the dependence of the particle densities on rapidity is very weak. This indicates that $(2 + 1)$ -D hydrodynamic models represent a good approximation for the dynamics and the mechanism of particle production in Pb-Pb collisions at the LHC at midrapidity.

Using the hydrodynamic model with optical Glauber initial conditions, one can calculate the elliptic flow coefficient. The elliptic flow coefficient for charged particles at different centralities is presented in Fig. 8. The calculation describes well the data for soft momenta. The results are consistent with

FIG. 9. (Color online) Elliptic flow coefficient of identified particles as function of transverse momentum calculated in the viscous hydrodynamic model (lines); the symbols represent the preliminary results of the ALICE Collaboration [\[47\]](#page-5-0), centrality 40%–50%.

predictions of other viscous hydrodynamic codes [\[6,8–14\]](#page-5-0). Calculations of the elliptic flow for central collisions or of the triangular flow require event-by-event simulations including fluctuations in the initial state, and are outside the scope of this paper. An observable related to the freeze-out conditions for specific particles is the elliptic flow of identified particles. As can be seen in Fig. 9, the hydrodynamic model with bulk viscosity corrections and freeze-out at 140 MeV gives a slightly too small splitting between the elliptic flow of pions, kaons, and protons.

IV. DISCUSSION

We present $(3 + 1)$ -D viscous hydrodynamic calculations of the spectra of particles emitted in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV. We find that the transverse momentum spectra are sensitive to bulk viscosity effects. Bulk viscosity in an exploding fireball leads to deviations from local equilibrium in the fluid elements. With the bulk viscosity coefficient $\zeta/s = 0.08$ and a freeze-out temperature $T_f = 140$ MeV we find a satisfactory agreement with preliminary experimental data for the transverse momentum spectra of pions, kaons, and protons. At the same time, the size of the fireball and the amount of the collective transverse flow of the fluid accumulated in the hydrodynamic phase are compatible with the experimental measurements of the momentum dependence of the interferometry radii R_{out} , R_{side} , and R_{long} .

Transverse momentum spectra of identified particles in semicentral collisions are reproduced as well. In particular, we find a strong transverse push visible in the p_T spectra of protons. On the other hand, bulk viscosity effects make the pion spectra softer, in agreement with the experiment. Using the same freeze-out conditions we calculate the Ξ^- and $\Omega^$ spectra and compare to the preliminary data of the ALICE Collaboration for two different centrality classes. Nonequilibrium corrections make the effective chemical freeze-out temperature to be 150 MeV and not 140 MeV as given by the energy density at freeze-out. The ratios of particle yields K/π are very well reproduced, while the ratio p/π is slightly overpredicted, and the strange baryon abundances are underpredicted. This shows that an effective single chemical freeze-out temperature (generated dynamically in viscous hydrodynamics) cannot describe all the measured particle ratios. The elliptic flow of identified particles from the hydrodynamic model shows a splitting according to the particle mass, but slightly smaller than observed experimentally. Distributions of identified particles in rapidity show that the system is not boost invariant. However, in the limited interval of central rapidities $|y| < 1$ an approximate plateau is seen in the rapidity distributions.

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