Gauge-invariant formulation of $NN \to NN\gamma$

H. Haberzettl[1,*] and K. Nakayama[2,3,†]

[1]*Institute for Nuclear Studies and Department of Physics, The George Washington University, Washington, DC 20052, USA*

[2]*Department of Physics and Astronomy, University of Georgia, Athens, GA 30602, USA*

[3]*Institut für Kernphysik and Jülich Center for Hadron Physics, Forschungszentrum Jülich, 52425 Jülich, Germany*

(Received 8 November 2010; revised manuscript received 5 May 2012; published 4 June 2012)

A complete, rigorous relativistic field-theory formulation of the nucleon-nucleon (*NN*) bremsstrahlung reaction is presented. The resulting amplitude is analytic, Lorentz covariant and unitary as a matter of course and it is gauge invariant; i.e., it satisfies a generalized Ward-Takahashi identity. The novel feature of this approach is the consistent microscopic implementation of local gauge invariance across all interaction mechanisms of the hadronic systems, thus serving as a constraint for all subprocesses. The formalism is quite readily adapted to approximations and thus can be applied even in cases where the microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. We point out how the interaction currents resulting from the photon being attached to nucleon-nucleon-meson vertices can be treated by phenomenological four-point contact currents that preserve gauge invariance. In an advance application of the present formalism [Nakayama and Haberzettl, Phys. Rev. C **80**, 051001(R) (2009)], such interaction currents were found to contribute significantly in explaining experimental data. In addition, we provide a scheme that permits—through an introduction of phenomenological five-point contact currents—the approximate treatment of current contributions resulting from pieces of the *NN* interaction that cannot be incorporated exactly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude. We also discuss the necessary modifications when taking into account baryonic states other than the nucleon N; in detail, we consider the $\Delta(1232)$ resonance by incorporating the couplings of the *NN* to the $N\Delta$ and $\Delta\Delta$ systems and the $\gamma N \to \Delta$ transitions.

DOI: 10.1103/PhysRevC.85.064001 PACS number(s): 25.10.+s, 25.40.−h, 25.20.Lj, 13.75.Cs

I. INTRODUCTION

The two-nucleon system is one of the simplest strongly interacting systems. The study of the nucleon-nucleon (*NN*) bremsstrahlung reaction, therefore, offers one of the most fundamental and direct avenues for understanding how the electromagnetic field interacts with strongly interacting hadronic systems. In the past, the *NN* bremsstrahlung reaction had been applied extensively mainly to learn about off-shell properties of the *NN* interaction. It should be clear, however, that off-shell effects are model dependent and cannot be measured, and therefore they are meaningless quantities for the purpose of comparison.

Even though the original motivation for investigating the *NN* bremsstrahlung reaction has fallen away, understanding the dynamics of such a fundamental process, nevertheless, is of great importance from a general theoretical perspective. This is highlighted by the fact that none of the past models of *NN* bremsstrahlung could describe the high-precision proton-proton bremsstrahlung data from the Kernfysisch Versneller Instituut (KVI) [1,2] for coplanar geometries involving small proton scattering angles. This was generally considered all the more surprising since the irrelevance of off-shell effects was taken as implicit proof positive that the coupling of a photon to the interacting two-nucleon system was under control. The discrepancy between the KVI data and the existing theoretical models, therefore, was quite unexpected. This longstanding discrepancy of nearly a decade was resolved recently by the present authors [3], who put forward a novel approach to the *NN* bremsstrahlung reaction that takes into account details of the photon coupling to interacting systems that had previously been neglected. The study, in particular, revealed the importance of accounting for the corresponding interaction currents in a manner consistent with the gauge-invariance constraint.

Another recent bremsstrahlung experiment concerns the hard bremsstrahlung process $p + p \to pp(^1S_0) + \gamma$ measured for the first time by the COSY-ANKE Collaboration [4]. In the absence of free systems of bound diprotons, this process was considered as an alternative to the $\gamma + pp(^1S_0) \to p + p$ process, which complements the photodisintegration of the deuteron. Here, the hardness of the bremsstrahlung is due to the fact that the invariant mass of the two protons in the final state is constrained experimentally to be less than 3 MeV above its minimum value of twice the proton mass. In this kinematic regime, the two protons in the final state are practically confined to the $^1S_0$ state and most of the available energy is carried by the bremsstrahlung. Therefore, this kinematic regime is as far away from the soft-photon limit as possible. The proton-proton (pp) hard bremsstrahlung reaction has been also measured at CELSIUS-Uppsala [5]. In spite of extensive studies of the *NN* bremsstrahlung reaction in the past, no dedicated experiments of pp hard bremsstrahlung with the diproton in the final state had been available until these recent measurements [4,5]. Also, apart from the very recent study of Ref. [6], theoretical investigation of the $p + p \to pp(^1S_0) + \gamma$ reaction has been virtually nonexistent so far.

*helmut.haberzettl@gwu.edu

†nakayama@uga.edu

Apart from the intrinsic interest in the elementary NN bremsstrahlung process, the investigation of this reaction has also an immediate impact in the area of heavy-ion physics. Indeed, dilepton production in heavy-ion collisions is used intensively as a probe of hadron dynamics in the nuclear medium. Due to their weak interaction with hadrons, dileptons are well suited to probe the hadron dynamics in the dense region of heavy-ion collisions. The HADES Collaboration, in particular, is currently engaged actively in the study of dielectron production in heavy-ion as well as in elementary NN collisions in the 1–2 GeV/u energy domain [7]. In order to interpret the experimental data in heavy-ion collisions, it is imperative to understand the underlying basic elementary processes. Unfortunately, these basic elementary processes are not yet fully under control. In fact, there are a number of theoretical efforts to understand these basic reaction processes [8,9]. According to these studies, among the various competing mechanisms, the NN bremsstrahlung is one of the major mechanisms for producing dielectrons in these reactions.

On the theoretical side, the majority of the existing models of NN bremsstrahlung are potential models. They have been applied to the analyses of experimental data (mostly in coplanar geometries) obtained up until the early 2000s, before the more recent experiments mentioned above were performed. Among those models, the most recent and sophisticated ones that have been used in the analysis of the high-precision KVI data [1,2] are the microscopic meson-exchange models of Refs. [10–12]. There were also a number of other microscopic model calculations made throughout the 1990s [13–21], which are dynamically similar to those of Refs. [10–12], addressing a variety of issues in the NN bremsstrahlung process. All these models satisfy current conservation (at least in the soft-photon approximation),[1] but none of them obey the more general gauge-invariance condition in terms of the generalized Ward-Takahashi identity (WTI) employed in Ref. [3] (and explained in more detail in the present work). Quite recently, it was shown formally [22] how to maintain gauge invariance in a theory of undressed nonrelativistic nucleons if one introduces a finite cutoff in a reference theory that is presumed to be already gauge invariant. When applied to effective field theories, in particular, this implies that gauge invariance can be maintained in such theories order by order in the expansion.

The purpose of the present paper is to present the complete, rigorous covariant formulation of the NN bremsstrahlung reaction whose successful advance application was reported in Ref. [3]. The approach is based on a relativistic field theory in which the photon is coupled in all possible ways to the underlying two-nucleon T matrix obtained from the corresponding covariant Bethe-Salpeter-type NN scattering equation. This formulation follows the basic procedures of the field-theoretical approach of Haberzettl [23] developed for pion photoproduction off the nucleon. The resulting bremsstrahlung amplitude satisfies analyticity, unitarity, Lorentz covariance, and gauge invariance as a matter of course. Gauge invariance, in particular, is shown explicitly by deriving the corresponding generalized WTI.

We emphasize that the rigorous formalism presented here is complete in its description of the underlying hadronic dynamics and the resulting electromagnetic couplings, and its applications are only limited by the available computing power. However, it is quite readily adapted to approximations and thus can be applied even in cases where the microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. As a case in point, we mention that the success of the advance application [3] of the present formalism to the KVI data [1,2] was due to the incorporation of phenomenological four-point contact currents that preserve gauge invariance following the approach of Haberzettl, Nakayama, and Krewald [24] based on the original ideas of Refs. [23,25]. In addition, we provide a scheme that permits the approximate treatment of current contributions resulting from pieces of the NN interaction that cannot be incorporated explicitly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude. We also discuss the necessary modifications when taking into account baryonic states other than the nucleon N. In detail, we consider the $\Delta(1232)$ resonance by incorporating the couplings of the NN to the $N\Delta$ and $\Delta\Delta$ systems and the $\gamma N \to \Delta$ transitions. The resulting expressions are quite generic in their topological structure and thus may be used as a template for other baryonic states.

In Sec. II, we present the details of the full four-dimensional relativistic formulation, including a proof of the gauge invariance of the resulting bremsstrahlung amplitude. In Sec. III, we introduce the necessary modifications for a covariant three-dimensional reduction and discuss its implications for the description of the dynamics of the process. We then point out, in Sec. IV, that if one aims for a dynamically consistent microscopic description of all reaction mechanisms, one must implement gauge invariance in terms of generalized Ward-Takahashi identities for each subprocess—mere global current conservation is not sufficient. We also show how one can preserve the gauge invariance of the amplitude even if some interaction-current mechanisms—both for hadronic three- and four-point functions—cannot be incorporated exactly. In Sec. V, we discuss what needs to be done to add additional baryonic degrees of freedom, in particular, the coupling of NN, $N\Delta$, and $\Delta\Delta$ channels. A summarizing assessment of the present work is given in Sec. VI.

We emphasize that the present formalism is completely general and applies to proton-proton, proton-neutron, as well as neutron-neutron bremsstrahlung processes. Furthermore, the photon can be either real or virtual. The former corresponds to the usual NN bremsstrahlung process while the latter case is suited for applications in dilepton production in both elementary NN and heavy-ion collisions.

[1]The current conservation of earlier models usually comes about because, for pp bremsstrahlung, there are no exchange currents for (uncharged) mesons and the four-point contact current discussed in Ref. [3] is absent for phenomenological meson-nucleon-nucleon form factors that depend only on the momentum of the exchanged meson. For pn bremsstrahlung (see, e.g., Ref. [13]) the meson-exchange currents are taken into account via Siegert's theorem, which preserves current conservation in the soft-photon limit.

II. FULL FORMALISM

The bremsstrahlung current is obtained from the nucleon-nucleon T matrix by attaching an outgoing photon to all reaction mechanisms of T in all possible ways. To this end, we use here the gauge-derivative procedure developed in Ref. [23] in the context of pion photoproduction. This procedure is formally equivalent to employing minimal substitution for the connected part of the hadronic Green's function, and then taking the functional derivative with respect to the electromagnetic four-potential A^μ, in the limit of vanishing A^μ (for details, see [23]). The current is then obtained by removing the propagators of all external hadron legs from this derivative in a Lehmann-Szymanzik-Zimmermann (LSZ) reduction procedure [26,27].

A. Deriving the bremsstrahlung current

The nucleon-nucleon T matrix is determined by the corresponding four-dimensional Bethe-Salpeter scattering equations,

$$T = V + VG_0T \quad \text{or} \quad T = V + TG_0V, \tag{1}$$

where V is the *NN* interaction given by the set of all two-nucleon irreducible scattering mechanisms. G_0 describes the intermediate propagation of two noninteracting nucleons; i.e., schematically we have

$$G_0 = [t_1 \circ t_2], \tag{2}$$

where t_i denotes the propagator of the individual nucleon i and "$\circ$" stands for the convolution of the intermediate loop integration.

The basic bremsstrahlung current $\tilde{B}^\mu$ is obtained by evaluating the LSZ-type equation

$$\tilde{B}^\mu = -G_0^{-1}\{G_0TG_0\}^\mu G_0^{-1}, \tag{3}$$

where $-\{\cdots\}^\mu$ denotes the gauge derivative [23] taken here of the connected hadronic *NN* Green's function G_0TG_0, with μ being the Lorentz index of the current. Using then the product rule $\{YX\}^\mu = Y\{X\}^\mu + \{Y\}^\mu X$ for an (ordered) product YX of a two-step sequence of hadronic reaction mechanisms described by operators X (first step) and Y (second step), we employ Eq. (1) repeatedly to find

$$\tilde{B}^\mu = (1 + TG_0)(d^\mu + V^\mu)(G_0T + 1) - d^\mu, \tag{4}$$

where d^μ defined by

$$d^\mu = -G_0^{-1}\{G_0\}^\mu G_0^{-1} \tag{5}$$

subsumes the one-body current contributions from the individual nucleons and

$$V^\mu = -\{V\}^\mu \tag{6}$$

is the interaction current resulting from attaching the photon to any internal mechanisms of the *NN* interaction. Details of V^μ will be discussed below.

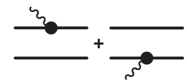
$G_0 d^\mu G_0 =$ 

FIG. 1. Graphical representation of Eq. (8). Solid lines depict nucleons and wavy lines indicate the outgoing bremsstrahlung photon.

Explicitly, the photon contributions from the two-nucleon propagator are found as

$$d^\mu = \Gamma_1^\mu\left(\delta_2 t_2^{-1}\right) + \left(\delta_1 t_1^{-1}\right)\Gamma_2^\mu, \tag{7}$$

where Γ_i^μ is the electromagnetic current operator of nucleon i; δ_i denotes an implied δ function that makes the incoming and outgoing momenta for the intermediate spectator nucleon i the same. We thus have

$$G_0 d^\mu G_0 = \left[t_1\Gamma_1^\mu t_1 \circ t_2\right] + \left[t_1 \circ t_2\Gamma_2^\mu t_2\right], \tag{8}$$

which is represented graphically in Fig. 1.

Note that Eq. (4)—apart from the subtraction by d^μ—possesses the structure of a distorted-wave Born approximation (DWBA), with the factors $(G_0T + 1)$ and $(1 + TG_0)$ supplying the Møller operators producing the initial and final scattering states, respectively, distorted by the *NN* interaction. The d^μ contribution by itself—without any initial-state or final-state *NN* interactions—is disconnected, as one sees clearly from Fig. 1. The overall subtraction of d^μ in Eq. (4), therefore, is necessary to remove this (unphysical) disconnected structure from $\tilde{B}^\mu$ and retain only connected physical contributions.

It is possible—and indeed desirable for the following—to equivalently rewrite Eq. (4) to provide a full DWBA structure for $\tilde{B}^\mu$ of the form

$$\tilde{B}^\mu = (1 + TG_0)\tilde{J}^\mu(G_0T + 1), \tag{9}$$

where

$$\tilde{J}^\mu = d^\mu G_0 V + VG_0 d^\mu + V^\mu - VG_0 d^\mu G_0 V \tag{10}$$

is the completely connected current that describes the bremsstrahlung reaction in the absence of any hadronic initial-state or final-state interactions (with the exception of the subtraction $VG_0d^\mu G_0V$; see below); we shall refer to this tree-level-type current as the basic production current. The equivalence of this form of $\tilde{B}^\mu$ to Eq. (4) is easily seen by repeated applications of Eq. (1). The *NN* T matrices appearing to the right or left of the basic production current $\tilde{J}^\mu$ in Eq. (9) thus provide the initial-state interaction (ISI) or final-state interaction (FSI), respectively, of the two nucleons external to the basic production current $\tilde{J}^\mu$.

The subtraction term $VG_0d^\mu G_0V$ in $\tilde{J}^\mu$ removes here the double counting of contributions $TG_0d^\mu G_0T$ to the full current $\tilde{B}^\mu$ that arise from the two d^μ contributions in $\tilde{J}^\mu$. As such, therefore, it is not a dynamically independent contribution to $\tilde{B}^\mu$ and appears here only because, for formal reasons, we wish to retain the DWBA form of $\tilde{B}^\mu$ in Eq. (9). We shall see below, in Sec. III, where we treat the covariant three-dimensional reduction of Eq. (9), that special considerations are needed for this subtraction term if one wants to maintain gauge invariance for the reduced amplitude.

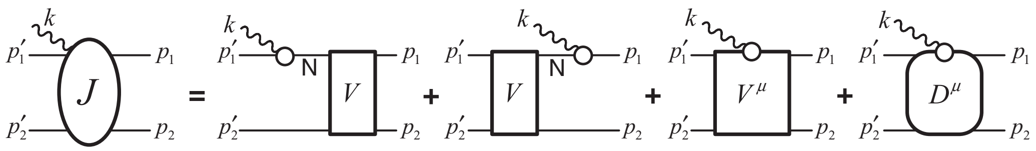


FIG. 2. Basic production current J^μ of Eq. (11) for $NN \to NN\gamma$. Time proceeds from right to left. External legs are labeled by the four-momenta of the respective particles. Boxes labeled V subsume all two-nucleon irreducible contributions to the NN interaction that drive the Bethe-Salpeter equation (1). The interaction current V^μ contains all mechanisms where the photon emerges from *within* the interaction V (i.e., any current mechanism not associated with an external leg of the interaction V). The last diagram, D^μ, as given in Eq. (12), subsumes all possible completely transverse contributions [cf. Eq. (15)], in addition to the subtraction that corrects the double counting arising from the first two contributions (see text). Diagrams where the photon emerges from the lower nucleon line are suppressed; the antisymmetrization of nucleons is implied.

Equations (9) and (10) are not the complete solution of the bremsstrahlung problem yet, because the gauge-derivative procedure—indeed any procedure based on minimal substitution—cannot produce current contributions that are completely transverse. Such contributions must be added to the mechanisms obtained above for $\tilde{B}^\mu$. Without lack of generality, we may do so by modifying the basic production current $\tilde{J}^\mu$ according to

$$\tilde{J}^\mu \to J^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + D^\mu, \tag{11}$$

where

$$D^\mu = T^\mu - V G_0 d^\mu G_0 V \tag{12}$$

contains the sum of all explicitly transverse five-point currents denoted by T^μ, in addition to the subtraction current $V G_0 d^\mu G_0 V$. In other words,

$$k_\mu T^\mu = 0 \tag{13}$$

is true irrespective of whether or not the external nucleons are on-shell. The complete bremsstrahlung current B^μ then is given by

$$\tilde{B}^\mu \to B^\mu = (1 + T G_0) J^\mu (G_0 T + 1). \tag{14}$$

We emphasize that this equation and Eq. (11) provide an exact generic description of the bremsstrahlung process off the NN system. The structure of the basic production current J^μ is depicted in Fig. 2.

The detailed nature of the transverse contribution T^μ in Eq. (12) must be specified by the underlying interaction Lagrangians. Examples are meson-transition currents J_M^μ as depicted in Fig. 3 and $\gamma N\Delta$-transition currents J_Δ^μ, i.e.,

$$T^\mu = J_M^\mu + J_\Delta^\mu + \cdots. \tag{15}$$

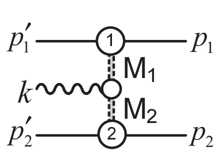


FIG. 3. Generic example for meson transition currents J_M^μ containing a transverse $\gamma M_1 M_2$ vertex, with $M_1 \neq M_2$, occurring in D^μ of Eq. (12) via its transverse contribution T^μ. Examples are $\gamma\rho\pi$ and $\gamma\omega\pi$ transition currents.

The latter will be discussed below, in Sec. V, when we consider $\Delta(1232)$ contributions in detail.

The specific details of any particular application, of course, depend on the mechanisms taken into account in the NN interaction V that drives the scattering process in the Bethe-Salpeter equation (1). For driving interactions based on single-meson exchanges, the complete structure of J^μ is discussed below, in Sec. IV A. In Sec. V, as mentioned already, we also consider the structures arising from Δ contributions that go beyond single-meson exchanges.

B. Proof of gauge invariance

It should be clear that the procedure used for deriving the bremsstrahlung current B^μ does produce a current that is gauge invariant as a matter of course. Nevertheless, we will now explicitly prove gauge invariance because this will guide us later, in Sec. III, in how to implement gauge invariance when we calculate B^μ in a covariant three-dimensional reduction.

To prove the gauge invariance of the current (14), we first note that, for an outgoing photon with four-momentum k, the four-divergence of d^μ may be schematically written as

$$k_\mu d^\mu = \hat{Q} G_0^{-1} - G_0^{-1} \hat{Q}, \tag{16}$$

where $\hat{Q}$ is short for

$$\hat{Q} = \hat{Q}_1 + \hat{Q}_2. \tag{17}$$

The operator $\hat{Q}_i$ describes the charge Q_i of nucleon i and it removes the four-momentum k carried away by the photon from any subsequent interaction of nucleon i (appearing on the left of $\hat{Q}_i$). This notation allows one to keep track of the kinematics without explicit four-momentum arguments; i.e., by specifying initial momenta for the two nucleons, the placement of the $\hat{Q}_i$ immediately allows one to find the momenta along each nucleon line. Equation (16) is an immediate consequence of the Ward-Takahashi identity for the nucleon current operator Γ_i^μ [27,28], i.e.,

$$k_\mu \Gamma_i^\mu = \hat{Q}_i t_i^{-1} - t_i^{-1} \hat{Q}_i, \tag{18}$$

applied to Eq. (7). Explicitly, with G_0 specified as in Eq. (2), we have

$$\begin{aligned}\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q} &= \left[\hat{Q}_1 t_1^{-1} - t_1^{-1}\hat{Q}_1\right] \circ t_2^{-1} + t_1^{-1} \circ \left[\hat{Q}_2 t_2^{-1} - t_2^{-1}\hat{Q}_2\right] \\ &= \left[Q_1 t_1^{-1}(p_1) - t_1^{-1}(p_1 - k)Q_1\right] \circ t_2^{-1}(p_2) \\ &\quad + t_1^{-1}(p_1) \circ \left[Q_2 t_2^{-1}(p_2) - t_2^{-1}(p_2 - k)Q_2\right] \end{aligned} \tag{19}$$

for a two-nucleon system, where p_1 and p_2 are the initial four-momenta of nucleons 1 and 2, respectively; i.e., the first term results from the photon being emitted by nucleon 1 and the second term results from it being emitted by nucleon 2.

In the same schematic notation, the four-divergence of the interaction current V^μ then is simply [23,29]

$$\begin{aligned}k_\mu V^\mu &= V\hat{Q} - \hat{Q}V \\ &= V(p_1', p_2'; p_1 - k, p_2)Q_1 + V(p_1', p_2'; p_1, p_2 - k)Q_2 \\ &\quad - Q_1 V(p_1' + k, p_2'; p_1, p_2) \\ &\quad - Q_2 V(p_1', p_2' + k; p_1, p_2),\end{aligned} \tag{20}$$

where the arguments of V are nucleon momenta and the momentum dependence of the interaction current is

$$V^\mu = V^\mu(k, p_1', p_2'; p_1, p_2), \text{ with } \quad p_1' + p_2' + k = p_1 + p_2; \tag{21}$$

i.e., the momenta p_1, p_2 and p_1', p_2' are those of the incoming and outgoing nucleons, respectively. Whether the charge operators Q_i in Eq. (20) pertain to incoming or outgoing nucleons is clear from where the Q_i are placed in the equation. In other words, if placed on the right of V, Q_i describes the charge of the incoming nucleon i, and if placed on the left, it describes the charge of the outgoing nucleon i. Placing the charge operators in this manner is necessary since they interact with the isospin dependence of the interaction V.

The four-divergence of J^μ then follows as

$$\begin{aligned}k_\mu J^\mu &= \left(\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}\right) G_0 V + V G0 \left(\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}\right) \\ &\quad + V\hat{Q} - \hat{Q}V - VG_0(\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q})G_0 V \\ &= -G_0^{-1}\hat{Q}G_0 V + VG_0\hat{Q}G_0^{-1} - VG_0\hat{Q}V + V\hat{Q}G_0 V.\end{aligned} \tag{22}$$

For the entire current, we then find

$$\begin{aligned}k_\mu B^\mu = (1 + TG_0)\big(&VG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0 V \\ &- VG_0\hat{Q}V + V\hat{Q}G_0 V\big)(G_0 T + 1),\end{aligned} \tag{23}$$

and thus, finally, using (1),

$$k_\mu B^\mu = TG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0 T. \tag{24}$$

This is the correct generalized Ward-Takahashi identity [30] for the bremsstrahlung current providing a conserved current for external on-shell nucleons. In a more explicit notation, using the same arguments for T and B^μ as for V and V^μ, respectively, in Eqs. (20) and (21), this reads

$$\begin{aligned}k_\mu B^\mu &= T(p_1', p_2'; p_1 - k, p_2)\, t_1(p_1 - k)\, Q_1\, t_1^{-1}(p_1) \\ &\quad + T(p_1', p_2'; p_1, p_2 - k)\, t_2(p_2 - k)\, Q_2\, t_2^{-1}(p_2) \\ &\quad - t_1^{-1}(p_1')\, Q_1\, t_1(p_1' + k)\, T(p_1' + k, p_2'; p_1, p_2) \\ &\quad - t_2^{-1}(p_2')\, Q_2\, t_2(p_2' + k)\, T(p_1', p_2' + k; p_1, p_2).\end{aligned} \tag{25}$$

The inverse nucleon propagators $t_i^{-1}(p)$ appearing here ensure that this four-divergence vanishes (i.e., that the bremsstrahlung current is conserved) if all external nucleon legs are on-shell.

III. APPROXIMATION: COVARIANT THREE-DIMENSIONAL REDUCTION

To calculate any reaction amplitude in a full four-dimensional framework is a daunting numerical task. In practical applications of relativistic reaction theories, therefore, one often employs three-dimensional reductions that eliminate the energy variable from loop integrations in a covariant manner, leaving only integrations over the components of three-momenta. Many such reduction schemes can be found in the literature [31–34]. Our results presented below hold true for any reduction scheme that puts both nucleons in loops on their respective energy shells.

For hadronic reactions, the primary technical constraint to be satisfied by any three-dimensional reduction is the preservation of covariance and (relativistic) unitarity. For reactions involving electromagnetic interactions, there is the additional constraint of gauge invariance. This is a nontrivial constraint since the reduction scheme, in general, will destroy gauge invariance as a matter of course. Hence, to restore it, one must introduce additional current mechanisms as part of the reduction prescription for photoprocesses. As we shall see, this cannot be done in a unique manner because gauge invariance does not constrain transverse current contributions.

For the *NN* problem, three-dimensional reductions result from replacing the free two-nucleon propagator G_0 by one containing a δ function that eliminates the energy integration in loops. In the following, we make the replacement

$$G_0 \to g_0 \tag{26}$$

to indicate that the internal integration is a three-dimensional one over the three-momentum of the loop. To obtain on-the-energy-shell integral equations from the Bethe-Salpeter equations (1) when using this reduction, the external nucleon legs must be taken on shell as well. This provides the reduced integral equations

$$t = v + vg_0 t = v + tg_0 v, \tag{27}$$

where lowercase letters v and t (instead of V and T, respectively) signify that all nucleons—internal and external—are on their energy shell. However, when considering below the gauge invariance of the bremsstrahlung current that results from the reduction (26), we require fully off-shell T matrices. They can be obtained from iterated versions of Eq. (1), which are then subjected to the reduction (26), producing

$$T = V + V(g_0 + g_0 t g_0)V, \tag{28}$$

where all external nucleons may be considered off-shell. This off-shell T, thus, is obtained by quadratures from the integral-equation on-shell solution t. However, this T is *not* the same as the solution of the full four-dimensional Bethe-Salpeter scattering equation (1) even though we use the same notation to keep matters simple, even at the risk of inviting confusion. The T's appearing in the following context always refer to fully off-shell or half off-shell versions of the T matrix defined by the quadrature formula (28).

The previous proof of gauge invariance of the full four-dimensional formalism given in Sec. II B shows that gauge invariance depends on an intricate interplay of all hadronic reaction mechanisms (which will also discussed in more detail in Sec. IV). Any approximation, therefore, will in general destroy gauge invariance. Hence, we expect that simply subjecting the full bremsstrahlung current B^μ of Eq. (14) to the reduction prescription (26) will not retain gauge invariance, and that, therefore, additional steps will be necessary to ensure gauge invariance. Since, from a formal point of view, any modification of a given current can, without lack of generality, always be expressed by adding an extra current, we may write the gauge-invariant current B_r^μ that results from a judicious adaptation of the three-dimensional reduction procedure to B^μ of Eq. (14) in the form

$$B^\mu \to B_r^\mu = [(TG_0+1)J^\mu(1+G_0T)]_{\rm red} + X^\mu_{\rm GIP}. \quad (29)$$

The first term on the right-hand side, $[\cdots]_{\rm red}$, schematically denotes the necessary modifications of the hadronic mechanisms in B^μ itself and the last term, $X^\mu_{\rm GIP}$, is the additional gauge-invariance-preserving (GIP) current that is to be determined to make B_r^μ gauge invariant. In other words, we demand that

$$k_\mu B_r^\mu = TG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0T, \quad (30)$$

i.e., that the four-divergence of the reduced current B_r^μ remains identical in form to the generalized WTI of Eq. (24), and we are going to ensure this by choosing $X^\mu_{\rm GIP}$ accordingly after having determined $[\cdots]_{\rm red}$ in Eq. (29). [We repeat here that—completely consistent with the covariant three-dimensional reduction—the off-shell T's of Eq. (30), and of the subsequent equations in this section, are those defined by the off-shell extension (28) of (27), and not the solutions of the original Bethe-Salpeter equations (1) that appear in Eq. (24); see also the discussion surrounding Eq. (28).]

As a first step, we employ the reduction for the external G_0 factors and write

$$B_r^\mu = (Tg_0+1)\tilde{J}_r^\mu(1+g_0T) + X^\mu_{\rm GIP}, \quad (31)$$

where $\tilde{J}_r^\mu$ is the reduced form of the basic production current J^μ of (11). For its determination, we note that one cannot simply employ the reduction (26) for every G_0 appearing in Eq. (11) since this produces unphysical mechanisms. The box-graph contribution

$$b^\mu = VG_0d^\mu G_0V \quad (32)$$

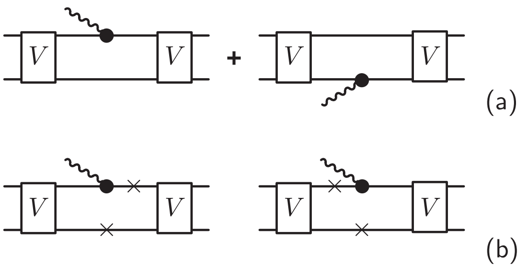


FIG. 4. (a) Sum of box graphs describing Eq. (32), with rectangles labeled V subsuming all mechanisms of the NN interaction. All internal nucleon lines are off-shell, in general. (b) The two possibilities of putting nucleons on-shell in the internal loop of the first diagram of (a), as indicated by "×" on the corresponding nucleon line, leaving one nucleon at the photon vertex off-shell. Analogous diagrams can be drawn for the corresponding second diagram.

shown in Fig. 4(a), which appears as a subtraction in Eq. (12), cannot be reduced in the form

$$VG_0d^\mu G_0V \to Vg_0d^\mu g_0V \quad \text{(unphysical)} \quad (33)$$

since this would have the bremsstrahlung photon emerging from intermediate on-shell nucleons, which is not possible for a physical photon. At least one of the nucleon legs at the photon vertex—either the incoming or the outgoing one—must remain off-shell. The two possibilities of doing that and allowing the production of physical photons are

$$VG_0d^\mu G_0V \to \begin{cases} VG_0d^\mu g_0V, \\ Vg_0d^\mu G_0V, \end{cases} \quad (34)$$

as shown in Fig. 4(b). Since there is nothing that suggests that one choice is to be preferred over the other, we allow for both and thus make the replacement

$$b^\mu \to b_r^\mu = \lambda_i VG_0d^\mu g_0V + \lambda_f Vg_0d^\mu G_0V, \quad (35)$$

where by symmetry we would have $\lambda_i = \lambda_f = 1/2$, of course, but we want to allow here more flexibility for reasons given below.

In detail, the reduced bremsstrahlung current thus reads

$$\begin{aligned} B_r^\mu =& (Tg_0+1)\big[d^\mu G_0V + VG_0d^\mu + V^\mu + D_r^\mu\big] \\ &\times(1+g_0T) + X^\mu_{\rm GIP}, \end{aligned} \quad (36)$$

where

$$D_r^\mu = T_r^\mu - b_r^\mu \quad (37)$$

is the reduced form of Eq. (12), with T_r^μ denoting the reduced form of the transverse current T^μ satisfying $k_\mu T_r^\mu = 0$. Note that the G_0's appearing in the d^μ terms cannot be reduced to g_0's for the same reason that the reduction (33) is not possible. Evaluating the four-divergence of this expression gives

$$\begin{aligned} k_\mu B_r^\mu &= (Tg_0+1)\big[\big(\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}\big)G_0V + VG_0\big(\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}\big) + V\hat{Q} - \hat{Q}V - k_\mu b_r^\mu\big)\big](1+g_0T) + k_\mu X^\mu_{\rm GIP} \\ &= -(Tg_0+1)G_0^{-1}\hat{Q}G_0T + TG_0\hat{Q}G_0^{-1}(1+g_0T) - (Tg_0+1)k_\mu b_r^\mu(1+g_0T) + k_\mu X^\mu_{\rm GIP} \\ &= TG_0\hat{Q}G_0^{-1} - G_0^{-1}\hat{Q}G_0T - (Tg_0+1)k_\mu b_r^\mu(1+g_0T) + k_\mu X^\mu_{\rm GIP}, \end{aligned} \quad (38)$$

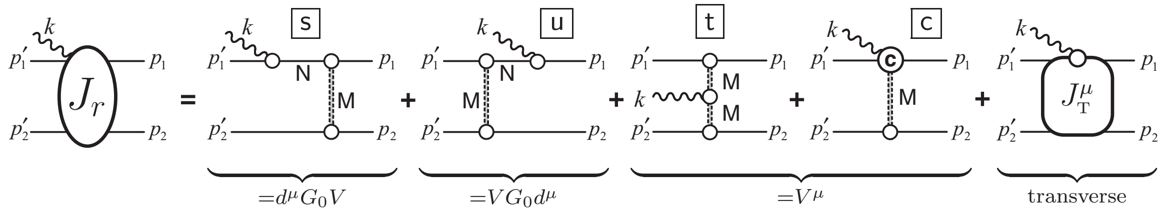


FIG. 5. Reduced basic bremsstrahlung production current J_r^μ of Eq. (47) resulting from the covariant three-dimensional reduction (see Sec. III) for nucleon-nucleon interactions V based on single-meson exchanges only. Time proceeds from right to left. (These diagrams correspond to the current employed in Ref. [3], where the exchanged mesons are $M = \pi, \eta, \rho, \omega, \sigma$, and a_0.) The nucleonic current corresponds to the first two diagrams on the right-hand side and the meson-exchange current is depicted by the third diagram. The first four diagrams, respectively labeled s, u, t, and c, correspond to the *complete* gauge-invariant description for the process $NM \to N\gamma$ for the upper nucleon line, with the labels s, u, and t alluding to the kinematic situations described by the corresponding Mandelstam variables (see Fig. 6 and the corresponding discussion in Sec. IV A). The fourth diagram contains the $NM \to N\gamma$ four-point contact current M^μ_{int} discussed in Sec. IV A, labeled "c" in the diagram. The correct gauge-invariance treatment of this diagram was found to be crucial in reproducing the KVI data in Ref. [3]. The diagrams corresponding to s, u, and c for the lower nucleon line are suppressed. The last diagram (labeled J^μ_{T}) subsumes the transverse five-point currents of (45). Transverse transition-current contributions are also subsumed in this diagram; examples are the $\gamma\rho\pi$, $\gamma\omega\pi$, and $\gamma N\Delta$ transition currents depicted in Figs. 3 and 9, respectively. Antisymmetrization of identical nucleons is implied.

where we have used here that

$$TG_0\hat{Q}G_0^{-1}g_0T = 0 \quad \text{and} \quad Tg_0G_0^{-1}\hat{Q}G_0T = 0 \tag{39}$$

vanish identically. For the proof, consider

$$g_0G_0^{-1}\hat{Q}G_0 \to \frac{\Lambda_1\Lambda_2}{s-(\varepsilon_1+\varepsilon_2)^2} \circ \left[t_1^{-1}(p)Q_1t_1(p+k)\right], \tag{40}$$

where the right-hand side here provides one generic contribution (stripped of all extraneous factors) contained in the expression on the left. The "$\circ$" symbol indicates the remaining three-momentum loop integration. The variable s is the squared total energy of the system and the ε_i are the individual on-shell energies of the two on-shell nucleons in the loop [indicated by the symbol "$\times$" in Fig. 4(b)]. The Λ_i are the positive-energy projectors of the nucleons. The energy component of $p = (\varepsilon_1, \mathbf{p})$ is on-shell and thus

$$\Lambda_1(\mathbf{p})t_1^{-1}(p) = \frac{p^2-m^2}{2m}\Lambda_1(\mathbf{p}) = 0. \tag{41}$$

Because $p^2 = m^2$, this term always vanishes for any $\mathbf{p}$. Therefore, even if $s-(\varepsilon_1+\varepsilon_2)^2$ should vanish as well for some $\mathbf{p}$, this cannot be compensated; i.e., the limit of the corresponding $\frac{0}{0}$ situation is zero. This proves that $Tg_0G_0^{-1}\hat{Q}G_0T = 0$; the proof for $TG_0\hat{Q}G_0^{-1}g_0T = 0$ follows in a similar fashion.

The first two terms in the last line of (38) already provide the complete four-divergence (30) necessary for the gauge-invariance condition to hold true. It follows then that the last two terms must vanish,

$$k_\mu X^\mu_{\mathrm{GIP}} - (Tg_0+1)k_\mu b^\mu_r(1+g_0T) \stackrel{!}{=} 0. \tag{42}$$

The four-divergence of X^μ_{GIP} thus is constrained by

$$k_\mu X^\mu_{\mathrm{GIP}} = (Tg_0+1)k_\mu b^\mu_{\mathrm{L}}(1+g_0T), \tag{43}$$

where b^μ_{L} describes the longitudinal pieces of the reduced box-graph current b^μ_r. Hence, without lack of generality, we may write

$$X^\mu_{\mathrm{GIP}} + (Tg_0+1)D^\mu_r(1+g_0T) = (Tg_0+1)J^\mu_{\mathrm{T}}(1+g_0T), \tag{44}$$

where J^μ_{T} is the purely transverse current,

$$J^\mu_{\mathrm{T}} = T^\mu_r - \lambda_i VG_0d^\mu_{\mathrm{T}}g_0V - \lambda_f Vg_0d^\mu_{\mathrm{T}}G_0V; \tag{45}$$

d^μ_{T} here only contains the transverse pieces of the nucleon currents as they appear in d^μ. As far as gauge invariance of B^μ_r is concerned, the current J^μ_{T} is irrelevant. Therefore, the parameters λ_i and λ_f may now also be treated as independent, unconstrained parameters.

To summarize the present results obtained for the three-dimensional reduction, in this approximation the gauge-invariant bremsstrahlung current reads

$$B^\mu_r = (Tg_0+1)J^\mu_r(1+g_0T), \tag{46}$$

where the reduced basic production current is given as

$$J^\mu_r = d^\mu G_0V + VG_0d^\mu + V^\mu + J^\mu_{\mathrm{T}}. \tag{47}$$

How one chooses J^μ_{T} in an application is not fixed by the formalism, beyond the generic form given in Eq. (45). Note that the generic graphical structure depicted in Fig. 2 remains valid also for J^μ_r, with the last graph labeled D^μ on the right-hand side of the figure depicting now the transverse five-point current J^μ_{T}. For the specific case of NN interactions based on single-meson exchanges only, the reduced basic production current J^μ_r of Eq. (47) is illustrated diagrammatically in more detail in Fig. 5.

Let us add some remarks here. Even though the procedure to preserve gauge invariance was presented here in terms of an additional *ad hoc* current X^μ_{GIP}, the derivation shows that, rather than adding a current, the application of the three-dimensional reduction procedure to the current B^μ really amounts to dropping (at least part of) the reduced contribution from b^μ that was necessary in the full four-dimensional treatment to prevent double counting of the $TG_0d^\mu G_0T$ contribution. The particular form of J^μ_{T} of Eq. (45) follows from exploiting the constraint (43) to the extent to which it is possible since the gauge invariance cannot constrain transverse contributions. Our applications (those of Ref. [3] for the KVI data and the recent application to the proton-proton data at 310 MeV incident energy taken at the PROMICE-WASA facility at Uppsala [36] and others whose results will be reported elsewhere) suggest that the choice $\lambda_i = \lambda_f = 0$ yields by far the best numerical results. In view of the fact that the double-counted term $TG_0 d^\mu G_0 T$ in the full four-dimensional formulation of Sec. II A now appears in Eq. (46) as two distinctly different contributions $TG_0 d^\mu g_0 T + T g_0 d^\mu G_0 T$, there are now no longer any terms counted doubly and thus the corresponding subtraction is no longer necessary either. The finding that $\lambda_i = \lambda_f = 0$ yields the best numerical results is completely consistent with this fact. $J^\mu_{\rm T}$ then contains only transverse contributions from electromagnetic transitions described explicitly in terms of the corresponding transition Lagrangians, like those for $\gamma\rho\pi$, $\gamma\omega\pi$, and $\gamma N\Delta$ (for the latter, see the discussion in Sec. V).

IV. LOCAL GAUGE INVARIANCE

The gauge-invariance constraints in terms of the generalized Ward-Takahashi identities, either in the form (24) for the full amplitude or as (30) for the reduced one, are just formal constraints, of course, since the only physically relevant—i.e., measurable—ramification is current conservation when all external nucleons of the bremsstrahlung process are on their respective energy shells. It is for this reason that the current-conservation constraint,

$$k_\mu B^\mu = 0 \quad \text{(external nucleons on-shell)}, \tag{48}$$

called *global* gauge invariance, is the only constraint that is implemented in many reaction models of photoprocesses. We would like to advocate, however, that using this as the sole constraint is not enough if one aims at providing a consistent *microscopic* description of the photoreaction at hand. It was shown in the preceding sections that the gauge invariance of the total bremsstrahlung amplitude hinges in an essential way on each subprocess providing its correct current share to ensure the gauge invariance of the entire current—and thus ultimately provide a conserved current. This is only possible if the current associated with each subprocess satisfies its own generalized Ward-Takahashi identity, as exemplified here by Eq. (16) for the propagators and by Eq. (20) for the interaction current. Thus, imposing *local* gauge invariance, i.e., imposing consistent off-shell constraints of this kind for all subprocesses in a microscopic description of the reaction at hand, will then automatically ensure that the total amplitude will satisfy a generalized WTI of its own. Not only does this mean that it will indeed satisfy the physical constraint of a conserved current, but beyond that it will ensure that if the process at hand will be used as a subprocess for another, larger reaction, it will automatically provide the correct contribution to make the larger process gauge invariant as well.

A. Constraining subprocesses

For further discussion, we draw attention to the fact that the reaction dynamics depicted by the first four diagrams on the right-hand side of Fig. 5 correspond to the capture reaction $NM \to N\gamma$, where the meson M is emitted by the spectator nucleon depicted by the lower nucleon line. The corresponding time-reversed equivalent meson-production process is shown in Fig. 6 for the example of pion production. The current amplitude for this process can always be broken down into four generic contributions (see, for example, [23]),

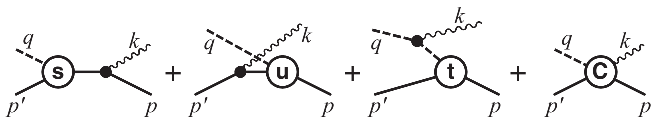


FIG. 6. Generic pion photoproduction diagrams $\gamma + N \to \pi + N$. Time proceeds from right to left. If read from left to right, this corresponds to the pion-capture reaction as it appears in the first four diagrams on the right-hand side of Fig. 5 along the upper nucleon line. The last diagram here labeled "c" corresponds to the interaction current arising from attaching the photon to the interior of the πNN vertex. The properties of this contact-type four-point current $M^\mu_{\rm int}$ are essential to render the entire amplitude gauge invariant, with the necessary constraint equation given by (51). At the tree level for undressed hadrons, this term reduces to the Kroll-Ruderman term.

$$M^\mu = M^\mu_s + M^\mu_u + M^\mu_t + M^\mu_{\rm int}, \tag{49}$$

where the indices s, u, and t allude to the Mandelstam variables describing the kinematic situations of the corresponding diagrams in the figure. The corresponding interaction current, in particular, is obtained by attaching the photon to the inner workings of the meson-nucleon-nucleon vertex F according to

$$M^\mu_{\rm int} = -\{F\}^\mu . \tag{50}$$

If we demand now local gauge invariance, the current M^μ must satisfy an off-shell generalized WTI. In addition to the trivial contributions resulting from the propagator WTIs for the currents associated with the external legs of M^μ_s, M^μ_u, and M^μ_t, similar to Eq. (18), this means, in particular, that the interaction current must satisfy [23]

$$k_\mu M^\mu_{\rm int} = Q_N F_u + Q_M F_t - F_s Q_N, \tag{51}$$

where F_x denotes the meson-nucleon-nucleon vertices F, with the subscripts $x = s, u, t$ corresponding to Mandelstam variables of the respective kinematical situation of the vertices in s-, u-, and t-channel contributions, as depicted in Fig. 6. Q_M and Q_N are the charge operators of the meson and the nucleon, respectively. Since the charge operators interact with the isospin dependence of the vertex F, their placements before or after F_x are significant to ensure charge conservation. Note that Eq. (51) is the exact analog of Eq. (20) for the present application, with Eq. (20) providing the four-divergence for a five-point interaction current for an outgoing photon and Eq. (51) constraining the four-point interaction current for an incoming photon. By using the $\hat{Q}$ notation introduced in Eq. (17), this may be made more obvious by writing Eq. (51) as

$$k_\mu M^\mu_{\rm int} = (\hat{Q}_N + \hat{Q}_M)F - F\hat{Q}_N, \tag{52}$$

where the charge operators of the final hadrons appear on the left of F and that of the initial hadron on its right, as is appropriate for the interaction current of any meson-production process $\gamma N \to NM$ similar to that depicted in Fig. 6; for the reverse process $NM \to N\gamma$, one needs to change

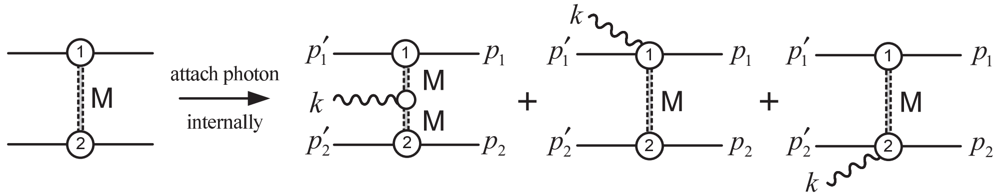


FIG. 7. Generic structure of interaction current $V^\mu_{\rm MEC}$, Eq. (54), for the single-meson exchange contribution $V_{\rm MEC}$ of the *NN* interaction depicted on the left. Time proceeds from right to left. The three diagrams on the right-hand side comprise the meson-exchange current and the two interaction-current contributions M^μ_i, $i = 1, 2$, where the photon interacts with the interiors of the respective vertices for nucleons 1 and 2.

$k \to -k$ and exploit the isospin dependence of the vertex to write $\hat{Q}_M F = -F\hat{Q}_M$ since the meson changes from being outgoing to incoming.

Demanding consistency of the microscopic dynamics across various reactions means that the entire $NM \to N\gamma$ subprocess in Fig. 5 must satisfy the same generalized Ward-Takahashi identity as the amplitude M^μ itself, except for trivial modifications arising from the fact that this is the time-reversed process. This must be true for any one of the exchanged mesons—whether scalar, pseudoscalar, or vector—in the bremsstrahlung process, not just for the pion example shown in Fig. 6.

To illustrate in more detail how the requirement of local gauge invariance ties together the various current mechanisms, let us consider a single-meson-exchange contribution $V_{\rm MEC}$ to the full *NN* potential V. Generically, we may write

$$V_{\rm MEC} = F_1 t_M F_2, \tag{53}$$

where the F_i are the meson-nucleon-nucleon vertices for nucleon $i = 1, 2$ (including all coupling operators and isospin dependencies) and t_M describes the propagator for the exchanged meson M. Graphically, the process is given in the diagram on the left-hand side of Fig. 7. If we now attach an outgoing photon to all internal mechanisms of $V_{\rm MEC}$, we obtain the three diagrams on the right-hand side of the figure that comprise the corresponding *NN* interaction current, $V^\mu_{\rm MEC} = -\{V_{\rm MEC}\}^\mu$. By employing the product rule for the gauge derivative of the right-hand side of Eq. (53), this current may be written as

$$\begin{aligned} V^\mu_{\rm MEC} &= -F_1\,\{t_M\}^\mu\,F_2 - \{F_1\}^\mu\,t_M F_2 - F_1 t_M\,\{F_2\}^\mu \\ &= F_1 t_M\,\Gamma^\mu_M\,t_M F_2 + M^\mu_1 t_M F_2 + F_1 t_M M^\mu_2, \end{aligned} \tag{54}$$

where $M^\mu_i = -\{F_i\}^\mu$ is the four-point interaction current for the vertex $i = 1, 2$ and

$$-\{t_M\}^\mu = t_M\,\Gamma^\mu_M\,t_M \tag{55}$$

produces the current operator Γ^μ_M for the exchanged meson that satisfies the single-particle WTI [27,28]

$$k_\mu \Gamma^\mu_M = \hat{Q}_M t_M^{-1} - t_M^{-1}\hat{Q}_M. \tag{56}$$

Now, by assuming without lack of generality that four-momentum and charge flow from vertex 2 to vertex 1 in Fig. 7, the analogs of (52) read

$$k_\mu M^\mu_1 = F_1(\hat{Q}_1 + \hat{Q}_M) - \hat{Q}_1 F_1, \tag{57a}$$

$$k_\mu M^\mu_2 = F_2\hat{Q}_2 - (\hat{Q}_M + \hat{Q}_2)F_2, \tag{57b}$$

and thus

$$\begin{aligned} k_\mu V^\mu_{\rm MEC} &= F_1 t_M\,\hat{Q}_M F_2 - F_1\hat{Q}_M t_M F_2 \\ &\quad + F_1(\hat{Q}_1 + \hat{Q}_M)t_M F_2 - \hat{Q}_1 F_1 t_M F_2 \\ &\quad + F_1 t_M F_2\hat{Q}_2 - F_1 t_M(\hat{Q}_M + \hat{Q}_2)F_2 \\ &= F_1 t_M F_2\hat{Q}_1 - \hat{Q}_1 F_1 t_M F_2 \\ &\quad + F_1 t_M F_2\hat{Q}_2 - \hat{Q}_2 F_1 t_M F_2 \\ &= V_{\rm MEC}\hat{Q} - \hat{Q}V_{\rm MEC}, \end{aligned} \tag{58}$$

which is precisely the gauge-invariance constraint (20) for this particular contribution to V. In this consistent microscopic treatment of all subprocesses, therefore, the constraints *local* gauge invariance places on the four-point interaction currents $M^\mu_{\rm int}$ of meson-production processes translate seamlessly into the corresponding constraints on five-point currents of the bremsstrahlung process. The essential step here is to ensure the validity of (57a) for the four-point current in the diagram labeled “c” in Fig. 5 and of (57b) for its counterpart for the lower nucleon line (not shown in Fig. 5). These relations must be true for each of the exchanged mesons.

The problem of how to ensure the validity of the constraints (51) or (57) has been studied extensively by the present authors and their collaborators for the equivalent photoproduction processes, and, based on the original ideas presented in Refs. [23,25], a general prescription was given in Ref. [24] that is applicable just as well for the most general case of explicit final-state interactions with completely dressed hadrons as it is for phenomenological vertex functions. The necessary extension in the context of bremsstrahlung—to account for the virtual nature of the incoming and outgoing nucleons and the exchanged meson at the four-point vertex—is accomplished following the work of Ref. [35].

The details of the four-point currents M^μ_1 and M^μ_2 and of the complete descriptions of the corresponding diagrams depicted in Fig. 7 were already given in Ref. [3]; we will not repeat them here. We emphasize, however, that the inclusion of this interaction-type current, with the correct gauge-invariance dynamics that ensure the validity of Eqs. (57), is essential to bringing about the good description of the high-precision KVI data [1,2] reported in Ref. [3]. This feature of our approach resolves a longstanding discrepancy between the data and their theoretical description and is a straightforward result of the consistent application of the off-shell gauge-invariance constraint in terms of the Ward-Takahashi identity, as outlined here.

B. Interaction current V^μ for phenomenological *NN* interactions: A systematic gauge-invariance-preserving approximation

The previous discussions show that the gauge-invariance conditions (20) and (57) for the respective interaction currents are of crucial importance to ensure an overall bremsstrahlung amplitude that satisfies gauge invariance. The understanding here is that the detailed reaction mechanisms that go into providing the details of these interaction currents are completely known and that therefore it would be straightforward to make sure that the corresponding gauge-invariance conditions are indeed satisfied. This is generally the case only for *NN* interactions based on single-meson exchanges between nucleons without any phenomenological form factors. And if one employs phenomenological form factors, one must resort to the prescriptions given in Refs. [23–25,35] to ensure Eqs. (57), as discussed in the preceding section. However, beyond problems associated with phenomenological form factors, it is conceivable that some details would not be available in some instances either because the microscopic coupling of the photon to the internal dynamics of the interaction would not be feasible or sensible or because it would be too complicated for practical applications. Examples of the former would be *NN* interactions based on position-space methods where there are no (momentum-dependent) exchange mechanisms that permit the coupling of a photon in a dynamically meaningful way. Examples of the latter might be baryon contributions beyond the nucleon since they require box-type *NN* contributions with intermediate non-nucleonic baryonic contributions that might be too cumbersome to be treated with explicit photon couplings (see Sec. V for $N\Delta$ and/or $\Delta\Delta$ systems and the resulting graphical structures depicted in Fig. 8).

Nevertheless, each of the corresponding interaction-current contributions must satisfy its appropriate gauge-invariance condition; otherwise, the gauge invariance of the entire amplitude breaks down. We will show here how one can ensure that gauge invariance is preserved for any phenomenological contribution to the *NN* interaction by constructing a

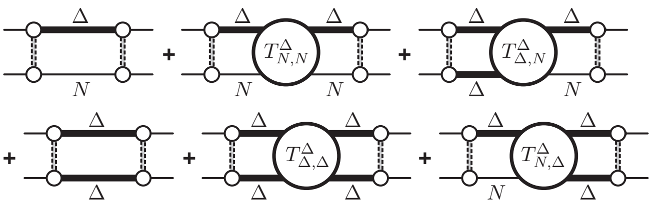


FIG. 8. Contributions to the *NN* interaction involving intermediate $N\Delta$ and $\Delta\Delta$ systems and intermediate transitions T^{Δ}_{fi} between such systems as given by V_Δ in Eq. (69). Summations over all possible exchanged mesons that mediate transitions to the Δ state are implied. Antisymmetrization of external nucleons is implied.

phenomenological interaction current that satisfies appropriate constraints.

To this end let us assume the total *NN* interaction V can be split up into n independent contributions,

$$V = V_1 + V_2 + \cdots + V_n. \tag{59}$$

By formally coupling a photon to each contribution according to $V_i^\mu = -\{V_i\}$, the total interaction current V^μ then breaks down accordingly into n independent contributions,

$$V^\mu = V_1^\mu + V_2^\mu + \cdots + V_n^\mu, \tag{60}$$

which, because of their independence, must each separately satisfy a gauge-invariance condition similar to (20), i.e.,

$$k_\mu V_i^\mu = V_i\hat{Q} - \hat{Q}V_i, \quad i = 1, 2, \ldots, n; \tag{61}$$

otherwise, the total current V^μ could not satisfy (20).

Let us assume now that one of the V_i is such that the construction $V_i^\mu = -\{V\}^\mu$ is *not* readily available. In this case, we may devise an auxiliary phenomenological current $\tilde{V}_i^\mu$ instead that satisfies exactly the same four-divergence relation (61) as would have to be satisfied by V_i^μ if it were available. To this end, we adapt the procedure used successfully in Refs. [23–25] for the four-point interaction currents M^μ_{int} of meson production to the present five-point-current case and make the ansatz

$$\begin{aligned}V_i^\mu \to \tilde{V}_i^\mu = &-[V_i(p_1', p_2'; p_1 - k, p_2) - W_i(k, p_1', p_2'; p_1, p_2)]\frac{Q_1(2p_1 - k)^\mu}{(p_1 - k)^2 - p_1^2} - [V_i(p_1', p_2'; p_1, p_2 - k)\\ &- W_i(k, p_1', p_2'; p_1, p_2)]\frac{Q_2(2p_2 - k)^\mu}{(p_2 - k)^2 - p_2^2} - \frac{Q_1(2p_1' + k)^\mu}{(p_1' + k)^2 - p_1'^2}[V_i(p_1' + k, p_2'; p_1, p_2) - W_i(k, p_1', p_2'; p_1, p_2)]\\ &- \frac{Q_2(2p_2' + k)^\mu}{(p_2' + k)^2 - p_2'^2}[V_i(p_1', p_2' + k; p_1, p_2) - W_i(k, p_1', p_2'; p_1, p_2)],\end{aligned} \tag{62}$$

where W_i is a function to be chosen to ensure that each term here is free of propagator singularities. The four-divergence of this auxiliary current is then readily seen to produce indeed

$$k_\mu \tilde{V}_i^\mu = V_i\hat{Q} - \hat{Q}V_i \tag{63}$$

since

$$\begin{aligned}&W_i(k, p_1', p_2'; p_1, p_2)(Q_1 + Q_2)_{\text{initial}}\\ &\quad - (Q_1 + Q_2)_{\text{final}} W_i(k, p_1', p_2'; p_1, p_2) = 0\end{aligned} \tag{64}$$

vanishes because of charge conservation. The ansatz (62) thus preserves the gauge-invariance condition for the entire interaction. As to how to choose W_i, one of the simplest possibilities is

$$W_i(k, p_1', p_2'; p_1, p_2) = V_i(p_1', p_2'; p_1, p_2) + \left[(p_1 - k)^2 - p_1^2\right]\left[(p_2 - k)^2 - p_2^2\right]\left[(p_1' + k)^2 - p_1'^2\right] \times \left[(p_2' + k)^2 - p_2'^2\right] R_i(k, p_1', p_2'; p_1, p_2), \tag{65}$$

where $R_i(k, p_1', p_2'; p_1, p_2)$, except for symmetry constraints, is largely arbitrary (and may be equal to zero). This choice ensures that the resulting GIP current is free of kinematic singularities, as it must be. In detail, one thus has

$$\begin{aligned}
\tilde{V}_i^\mu = &- \frac{V_i(p_1', p_2'; p_1 - k, p_2) - V_i(p_1', p_2'; p_1, p_2)}{(p_1 - k)^2 - p_1^2} Q_1(2p_1 - k)^\mu - \frac{V_i(p_1', p_2'; p_1, p_2 - k) - V_i(p_1', p_2'; p_1, p_2)}{(p_2 - k)^2 - p_2^2} Q_2(2p_2 - k)^\mu \\
&- Q_1(2p_1' + k)^\mu \frac{V_i(p_1' + k, p_2'; p_1, p_2) - V_i(p_1', p_2'; p_1, p_2)}{(p_1' + k)^2 - p_1'^2} - Q_2(2p_2' + k)^\mu \frac{V_i(p_1', p_2' + k; p_1, p_2) - V_i(p_1', p_2'; p_1, p_2)}{(p_2' + k)^2 - p_2'^2} \\
&+ R_i(k, p_1', p_2'; p_1, p_2)\left[(p_1' + k)^2 - p_1'^2\right]\left[(p_2' + k)^2 - p_2'^2\right]\left\{\left[(p_2 - k)^2 - p_2^2\right] Q_1(2p_1 - k)^\mu \right. \\
&\left. + \left[(p_1 - k)^2 - p_1^2\right] Q_2(2p_2 - k)^\mu\right\} + \left\{Q_1(2p_1' + k)^\mu\left[(p_2' - k)^2 - p_2'^2\right] + Q_2(2p_2' + k)^\mu\left[(p_1' - k)^2 - p_1'^2\right]\right\} \\
&\times \left[(p_1 - k)^2 - p_1^2\right]\left[(p_2 - k)^2 - p_2^2\right] R_i(k, p_1', p_2'; p_1, p_2).
\end{aligned} \tag{66}$$

Note that the subtracted potential contribution $V_i(p_1', p_2'; p_1, p_2)$ is unphysical since $p_1' + p_2' \neq p_1 + p_2$.

Hence, the procedure just outlined allows a systematic hybrid treatment of all independent contributions V_i to the full interaction V where some five-point currents V_i^μ can be treated explicitly and some may be replaced by auxiliary currents $\tilde{V}_i^\mu$ according to Eq. (62). The total interaction current V^μ will satisfy the condition (20) as a matter of course and gauge invariance is not at issue.

V. EXTENSION TO COUPLED CHANNELS: NN, $N\Delta$, AND $\Delta\Delta$

The derivation of the bremsstrahlung current in Sec. II A is completely generic and will remain true regardless of the actual mechanisms taken into account in the nucleon-nucleon interaction V that drives the Bethe-Salpeter equation (1). The current mechanisms depicted in Fig. 5 assume an interaction based on single-meson exchanges between nucleons only. Here we briefly discuss the necessary modifications if such exchanges involve transitions between different baryonic states. We limit the discussion to transitions between the nucleon N and the $\Delta(1232)$ mediated by single-meson exchanges (i.e., we consider the effect of coupling the channels NN, $N\Delta$, and $\Delta\Delta$); transitions into other (resonant) baryonic states can be treated along the same lines.

It is a very simple and straightforward exercise to decouple the corresponding set of Bethe-Salpeter equations that couple the NN, $N\Delta$, and $\Delta\Delta$ channels and write the NN interaction appropriate for the single-channel Bethe-Salpeter equation (1) as

$$V = V_{\mathrm{MEC}} + V_\Delta. \tag{67}$$

The first term, V_{MEC}, describes single-meson exchanges between nucleons, as shown on the left-hand side of Fig. 7, that provide the current mechanisms depicted on the right-hand side of the figure. The second term, V_Δ, contains all intermediate $N\Delta$ or $\Delta\Delta$ contributions and their transitions. Using the notation

$$U_N: \quad \text{meson-exchange transition } NN \to N\Delta \tag{68a}$$

and

$$U_\Delta: \quad \text{meson-exchange transition } NN \to \Delta\Delta \tag{68b}$$

for the transition interactions that mediate the coupling to the primary NN channel, we obtain

$$\begin{aligned}
V_\Delta = &\, U_N^\dagger\left(G_N^\Delta + G_N^\Delta T_{N,N}^\Delta G_N^\Delta\right) U_N + U_\Delta^\dagger G_\Delta^\Delta T_{\Delta,N}^\Delta G_N^\Delta U_N \\
&+ U_\Delta^\dagger\left(G_\Delta^\Delta + G_\Delta^\Delta T_{\Delta,\Delta}^\Delta G_\Delta^\Delta\right) U_\Delta + U_N^\dagger G_N^\Delta T_{N,\Delta}^\Delta G_\Delta^\Delta U_\Delta,
\end{aligned} \tag{69}$$

where G_N^Δ and G_Δ^Δ describe the intermediate propagation of the ΔN and the $\Delta\Delta$ systems, respectively. Intermediate transitions $\Delta N \to \Delta N$, $\Delta N \to \Delta\Delta$, $\Delta\Delta \to \Delta N$, and $\Delta\Delta \to \Delta\Delta$ are subsumed in the respective T matrices $T_{N,N}^\Delta$, $T_{\Delta,N}^\Delta$, $T_{N,\Delta}^\Delta$, and $T_{\Delta,\Delta}^\Delta$.

Figure 8 provides a graphical representation of V_Δ. Coupling a photon to each of the mechanisms depicted here results in the interaction current

$$V_\Delta^\mu = -\{V_\Delta\}^\mu, \tag{70}$$

which, together with the interaction current V_{MEC}^μ depicted generically in Fig. 7, constitutes the total interaction current

$$V^\mu = V_{\mathrm{MEC}}^\mu + V_\Delta^\mu \tag{71}$$

(if nucleons and Δ's are the only baryon degrees of freedom considered). Of course, calculating V_Δ^μ explicitly is a formidable task. There are eight current contributions for each of the simple box graphs (where the photon can couple to each of the four intermediate hadrons and the four vertices) and eleven for each of the graphs involving an intermediate T matrix. By considering only π and ρ exchanges for these graphs involving the Δ and allowing for the four possibilities of exchanging these mesons, altogether, therefore, there are 240 current terms. This is without separately accounting for the internal mechanisms resulting from coupling the photon to the T matrices with Δ degrees of freedom, which is similar in complexity to the NN bremsstrahlung current itself. In view of this complexity, we forego drawing the corresponding diagrams for V^μ_Δ.

Each set of currents resulting from one graph in Fig. 8 corresponds to an independent interaction current, as discussed in conjunction with (60), and therefore must satisfy the gauge-invariance condition (61) independently. If an exact treatment of the corresponding interaction current is not feasible in view of the complexity of the problem, from the point of view of gauge invariance this may also be done in an approximate manner by constructing an auxiliary current for each of the graphs in Fig. 8 along the lines discussed in Sec. IV B.

In addition to the hadronic transitions into intermediate Δ states, there are also direct electromagnetic transitions $\gamma N \to \Delta$. Such contributions cannot be obtained by applying the gauge-derivative method used in Sec. II A, or by any other method based on minimal substitution. They must be added by hand, in terms of their own $\Delta N\gamma$ Lagrangian. Their full contribution to the basic production current J^μ of Eq. (11) may be written as

$$J^\mu_\Delta = d^\mu_{N\Delta}\, G^\Delta_N\, T_{\Delta N,NN} + T_{NN,\Delta N}\, G^\Delta_N\, d^\mu_{\Delta N}, \qquad (72)$$

where $T_{\Delta N,NN}$ and $T_{NN,\Delta N}$ are the T matrices resulting from summing all two-nucleon irreducible transitions $NN \to \Delta N$ and $\Delta N \to NN$, respectively; $d^\mu_{N\Delta}$ and $d^\mu_{\Delta N}$, in an obvious schematic notation borrowed from the NN contributions, contains the electromagnetic transition current $\Delta \to N$ and $N \to \Delta$, respectively, along the baryon lines that emit the photon, as depicted in Fig. 9 for the lowest-order single-meson exchange contributions to the respective T matrices. These contributions are manifestly transverse and hence have no impact on gauge invariance. Graphically, they may be subsumed in the right-most diagrams of Figs. 2 or 5, depending on whether one considers the full formalism or its three-dimensional reduction, respectively. In other words, J^μ_Δ given above is part of the transverse current T^μ, as anticipated already in Eq. (15).

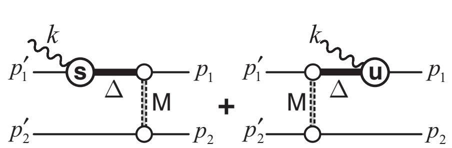


FIG. 9. Lowest-order electromagnetic $\gamma N\Delta$-transition contribution to J^μ_Δ of Eq. (72), with M subsuming all exchanged mesons compatible with the process. The full contribution involves replacing the single-meson exchanges in the two graphs by the T matrix representing the sum of all two-nucleon irreducible transitions $NN \to \Delta N$ and $\Delta N \to NN$, respectively. The transition currents are transverse and thus have no bearing on gauge invariance. Antisymmetrization of nucleons is implied.

In summary, the full basic interaction current for NN bremsstrahlung, including Δ degrees of freedom, becomes

$$J^\mu = d^\mu G_0 V + V G_0 d^\mu + V^\mu + \left[J^\mu_M + J^\mu_\Delta - V G_0 d^\mu G_0 V\right], \qquad (73)$$

where the groupings of the four terms correspond to the four graphs on the right-hand side of Fig. 2. Here, V and V^μ contain Δ degrees of freedom according to Eqs. (67) and (71), respectively, and the transverse current is taken as $T^\mu = J^\mu_M + J^\mu_\Delta$. In the reduced case, we have

$$J^\mu_r = d^\mu G_0 V + V G_0 d^\mu + V^\mu + \left[J^\mu_M + J^\mu_{r\Delta}\right], \qquad (74)$$

where the last grouping is the explicit transverse current $J^\mu_{\mathrm{T}} = J^\mu_M + J^\mu_{r\Delta}$, with λ_i and λ_f of Eq. (45) put to zero; the reduced $N\Delta$-transition current $J^\mu_{r\Delta}$ is obtained by the corresponding three-dimensional reductions of the loop integrations within J^μ_Δ.

VI. SUMMARY

We have presented a complete, rigorous formulation of the NN bremsstrahlung reaction based on a relativistic field-theory approach in which the photon is coupled in all possible ways to the underlying two-nucleon T matrix obtained from the corresponding covariant Bethe-Salpeter-type NN scattering equation using the gauge-derivative procedure of Haberzettl [23]. The resulting bremsstrahlung amplitude is unitary and analytic as a matter of course and it satisfies full local gauge invariance as dictated by the generalized Ward-Takahashi identity. The novel feature of this approach is the consistent—i.e., gauge-invariant—incorporation of interaction currents resulting from the photon coupling internally to interacting hadronic systems.

We emphasize in this respect that to achieve gauge invariance in a microscopic description of the reaction dynamics at hand, it is not sufficient to consider mere global current conservation. The interdependence of reaction mechanisms makes it necessary that each subprocess of the reaction provide its consistent contribution to ensure the overall gauge invariance of the entire process. In a microscopic description, therefore, overall gauge invariance of the reaction flows from the correct and consistent description of local off-shell gauge invariance in terms of the respective Ward-Takahashi identities of the contributing subprocesses. For the present case of NN bremsstrahlung, in particular, it was shown in Sec. IV A that an essential part of the underlying dynamics can be understood as a time-reversed meson photoproduction process in the presence of a spectator nucleon and that, therefore, the corresponding off-shell gauge-invariance results of Refs. [23–25] fully apply here, thus readily providing a description of the four-point contact-type interaction currents M^μ_i (as they appear in Fig. 7).

The formalism, in particular, is quite readily adapted to approximations and thus can be applied even in cases where the microscopic dynamical structure of the underlying interacting hadronic systems is either not known in detail or too complex to be treated in detail. We have pointed out, in Sec. IV A, how the interaction currents resulting from the photon being attached to nucleon-nucleon-meson vertices can be treated by phenomenological four-point contact currents that preserve gauge invariance following the approach of Haberzettl, Nakayama, and Krewald [24]. In an advanced application of the present formalism [3], such interaction currents had been shown to contribute significantly to reproducing the high-precision proton-proton bremsstrahlung data at 190 MeV obtained at KVI [1], thus removing a longstanding discrepancy between theory and experiment. In addition, we have provided a scheme that permits the approximate treatment of current contributions resulting from pieces of the *NN* interaction that cannot be incorporated exactly. In each case, the approximation procedure ensures gauge invariance of the entire bremsstrahlung amplitude.

We have also discussed the necessary modifications when taking into account baryonic states other than the nucleon N; in detail, we consider the $\Delta(1232)$ resonance by incorporating the couplings of the *NN* to the $N\Delta$ and $\Delta\Delta$ systems and the $\gamma N \to \Delta$ transitions. Lowest-order nonloop Δ contributions of the kind shown in Fig. 9 have been implemented in Ref. [36], where the present formalism was applied to the new proton-proton bremsstrahlung data at 310 MeV taken at Uppsala; while these results are encouraging, there is still room for improvement.

We point out in this context that the present approach in the form of the basic equation (14) results from a comprehensive field theory of the *NN* bremsstrahlung process that in principle allows for all possible hadronic degrees of freedom and all possible electromagnetic couplings. In its full implementation, therefore, this is not a model approach. The complexity of the problem, however, makes it necessary to employ certain model assumptions to render it manageable in practice, which may, on occasion, fall short of providing a complete description of all experimental data. A case in point are the truncations of the Δ contributions in the application to the Uppsala data reported in Ref. [36]. However, incorporating the current contributions resulting from Δ loops like the ones depicted in Fig. 8 requires enormous computing power not available to us at present. There is also room for improvement in our application to the KVI data discussed above [3]. While the contact-current mechanism suggested by our theory (see caption of Fig. 5) seems to provide the solution for the cross-section data puzzle that plagued earlier theoretical approaches, there is still work to be done for the KVI analyzing-power data, which in some instances is not described well in Ref. [3]. Whether resolving these discrepancies requires more refined model assumptions about the details of the contact current or other mechanisms is an open question at the moment. Further numerical applications will be reported elsewhere.

Finally, we emphasize that, despite its completeness and generality, the present approach is quite flexible and amenable to approximations, as discussed in Sec. IV B. Moreover, by following the procedures outlined in Ref. [24], it offers well-defined avenues for improving upon any approximation in a systematic manner. Ultimately, the degree of sophistication that can be implemented for any application is only limited by the available computing power. We expect, therefore, that the formalism presented here will also be useful in the ongoing investigation of hard bremsstrahlung [4,5], as well as in dilepton production processes [7] where the photon is virtual.

ACKNOWLEDGMENT

This work is supported in part by the FFE-COSY Grant No. 41788390.

[1] H. Huisman *et al.*, Phys. Rev. C **65**, 031001(R) (2002).
[2] M. Mahjour-Shafiei *et al.*, Phys. Rev. C **70**, 024004 (2004).
[3] K. Nakayama and H. Haberzettl, Phys. Rev. C **80**, 051001(R) (2009).
[4] V. Komarov *et al.*, Phys. Rev. Lett. **101**, 102501 (2008); D. Tsirkov *et al.*, J. Phys. G **37**, 105005 (2010).
[5] A. Johansson and C. Wilkin, Phys. Lett. B **673**, 5 (2009); **680**, 111(E) (2009).
[6] K. Nakayama and F. Huang, Phys. Rev. C **82**, 065201 (2010).
[7] G. Agakichiev *et al.*, Phys. Rev. Lett. **98**, 052302 (2007); Eur. Phys. J. A **41**, 243 (2009); Phys. Lett. B **690**, 118 (2010).
[8] L. P. Kaptari and B. Kämpfer, Nucl. Phys. A **764**, 338 (2006); Phys. Rev. C **80**, 064003 (2009).
[9] R. Shyam and U. Mosel, Phys. Rev. C **79**, 035203 (2009); **82**, 062201 (2010).
[10] G. H. Martinus, O. Scholten, and J. A. Tjon, Phys. Lett. B **402**, 7 (1997); Phys. Rev. C **56**, 2945 (1997); **58**, 686 (1998).
[11] M. D. Cozma, G. H. Martinus, O. Scholten, R. G. E. Timmermans, and J. A. Tjon, Phys. Rev. C **65**, 024001 (2002).
[12] V. Hermann, K. Nakayama, O. Scholten, and H. Arellano, Nucl. Phys. A **582**, 568 (1995).
[13] K. Nakayama, Phys. Rev. C **39**, 1475 (1989).
[14] A. Katsogiannis and K. Amos, Phys. Rev. C **47**, 1376 (1993); A. Katsogiannis, K. Amos, M. Jetter, and H. V. von Geramb, *ibid.* **49**, 2342 (1994); M. Jetter, H. Freitag, and H. V. von Geramb, Phys. Scr. **48**, 229 (1993).
[15] V. Herrmann and K. Nakayama, Phys. Rev. C **45**, 1450 (1992); F. de Jong and K. Nakayama, *ibid.* **52**, 2377 (1995).
[16] F. de Jong, K. Nakayama, V. Herrmann, and O. Scholten, Phys. Lett. B **333**, 1 (1994).
[17] M. Jetter and H. W. Fearing, Phys. Rev. C **51**, 1666 (1995).
[18] F. de Jong, K. Nakayama, and T.-S. H. Lee, Phys. Rev. C **51**, 2334 (1995).
[19] J. A. Eden and M. F. Gari, Phys. Lett. B **347**, 187 (1995); Phys. Rev. C **53**, 1102 (1996).
[20] S. Kondratyuk, G. H. Martinus, and O. Scholten, Phys. Lett. B **418**, 20 (1998),
[21] M. D. Cozma, O. Scholten, R. G. E. Timmermans, and J. A. Tjon, Phys. Rev. C **68**, 044003 (2003).
[22] A. N. Kvinikhidze, B. Blankleider, E. Epelbaum, C. Hanhart, and M. P. Valderrama, Phys. Rev. C **80**, 044004 (2009).
[23] H. Haberzettl, Phys. Rev. C **56**, 2041 (1997).
[24] H. Haberzettl, K. Nakayama, and S. Krewald, Phys. Rev. C **74**, 045202 (2006).

[25] H. Haberzettl, C. Bennhold, T. Mart, and T. Feuster, Phys. Rev. C **58**, R40 (1998).

[26] H. Lehmann, K. Szymanzik, and W. Zimmermann, Nuovo Cimento **1**, 205 (1955).

[27] S. Weinberg, *The Quantum Theory of Fields, Volume I—Foundations* (Cambridge University Press, New York, 1995).

[28] J. C. Ward, Phys. Rev. **78**, 182 (1950); Y. Takahashi, Nuovo Cimento **6**, 371 (1957).

[29] F. Gross and D. O. Riska, Phys. Rev. C **36**, 1928 (1987).

[30] E. Kazes, Nuovo Cimento **13**, 1226 (1959).

[31] R. Blankenbecler and R. Sugar, Phys. Rev. **142**, 1051 (1966).

[32] K. Erkelenz, Phys. Rep. C **13**, 191 (1974).

[33] E. D. Cooper and B. K. Jennings, Nucl. Phys. A **500**, 553 (1989).

[34] F. Gross, *Relativistic Quantum Mechanics and Field Theory* (Wiley, New York, 1993).

[35] K. Nakayama, Y. Oh, and H. Haberzettl, Phys. Rev. C **74**, 035205 (2006).

[36] A. Johansson, H. Haberzettl, K. Nakayama, and C. Wilkin, Phys. Rev. C **83**, 054001 (2011).