PHYSICAL REVIEW C **85**, 057602 (2012)

Transition from subbarrier to deep-subbarrier regimes in heavy-ion fusion reactions

Ei Shwe Zin Thein,[1] N. W. Lwin,[1] and K. Hagino[2]
[1]*Department of Physics, Mandalay University, Myanmar*
[2]*Department of Physics, Tohoku University, Sendai 980-8578, Japan*
(Received 3 April 2012; published 16 May 2012)

We analyze recent experimental data of heavy-ion fusion cross sections available up to deep-subbarrier energies in order to discuss the threshold incident energy for a deep-subbarrier fusion hindrance phenomenon. To this end, we employ a one-dimensional potential model with a Woods-Saxon internuclear potential. Fitting the experimental data in two different energy regions with different Woods-Saxon potentials, we define the threshold energy as an intersection of the two fusion excitation functions. We show that the threshold energies so extracted are in good agreement with the empirical systematics as well as with the values of the Krappe-Nix-Sierk (KNS) potential at the touching point. We also discuss the asymptotic energy shift of fusion cross sections with respect to the potential model calculations, and show that it decreases with decreasing energies in the deep-subbarrier region, although it takes a constant value at subbarrier energies.

DOI: 10.1103/PhysRevC.85.057602 PACS number(s): 25.70.Jj, 24.10.Eq

Heavy-ion fusion reactions at low incident energies are intimately related to the quantum tunneling phenomena of many-body systems. Because of a strong cancellation between the repulsive Coulomb interaction and an attractive short-range nuclear interaction between the colliding nuclei, a potential barrier, referred to as a Coulomb barrier, is formed, which has to be surmounted in order for fusion to take place. In heavy-ion reactions, because of a strong absorption inside the Coulomb barrier, it has been usually assumed that a compound nucleus is automatically formed once the Coulomb barrier has been overcome. The simplest approach to heavy-ion fusion reactions based on this idea, that is, a one-dimensional potential model, has been successful in reproducing experimental fusion cross sections at energies above the Coulomb barrier [1]. A one-dimensional potential model fitted to reproduce fusion cross sections above the Coulomb barrier, however, has been found to underestimate fusion cross sections at lower energies. It has been well recognized by now that subbarrier fusion enhancement is caused by couplings of the relative motion between the colliding nuclei with other degrees of freedom, such as collective vibrational and rotational motions in the colliding nuclei [2,3].

The behavior of fusion cross sections at extremely low energies is a critical issue for estimating reaction rates of astrophysical interest. One of the current interests in heavy-ion fusion reactions is a steep fall-off phenomenon of fusion cross sections at deep-subbarrier energies. Recently, fusion cross sections for several colliding systems have been measured down to extremely low cross sections, up to several nb [4–8]. These experimental data have shown that fusion cross sections fall off much more steeply at deep-subbarrier energies with decreasing energy, compared to the expectation of the energy dependence of cross sections at subbarrier energies. That is, experimental fusion cross sections appear to be hindered at deep-subbarrier energies compared to standard coupled-channels calculations, although fusion cross sections are still enhanced with respect to a prediction of a single-channel potential model. Although a few theoretical models have been proposed [9,10], the origin of deep-subbarrier fusion hindrance has not yet been fully understood.

In Refs. [4,5,11], deep-subbarrier fusion hindrance has been analyzed using the astrophysical S factor. It has been claimed [4,5,11] that deep-subbarrier fusion hindrance sets in at the energy at which the astrophysical S factor reaches its maximum. The authors of Refs. [4,5,11] even parametrized the threshold energy as

$$E_s = 0.356\left(Z_1 Z_2\sqrt{\frac{A_1A_2}{A_1+A_2}}\right)^{2/3} \quad \text{(MeV)}. \qquad (1)$$

Notice that the S-factor representation provides a useful tool only when the penetration of the Coulomb repulsive potential is a dominant contribution, such as in fusion reactions of light systems at low energies. In fact, the relation between the threshold for deep-subbarrier hindrance and the maximum of the S factor is not clear physically, and thus it is not trivial how to justify theoretically the identification of the threshold energy with the astrophysical S factor. Nevertheless, it has turned out that the threshold energy so obtained closely follows the values of phenomenological internucleus potentials, such as the Krappe-Nix-Sierk (KNS) [12], the Bass [13], the proximity [14], and the Akyüz-Winther [15] potentials, at the touching configuration [16]. This clearly implies that the dynamics which takes place after the colliding nuclei touch each other is responsible for deep-subbarrier fusion hindrance, making at the same time the astrophysical S factor decrease as the incident energy decreases.

In this paper, we investigate the threshold energy for deep-subbarrier fusion hindrance using an alternative method, which is physically more transparent than the definition with the maximum of the S factor. That is, we determine the threshold energies by fitting the experimental fusion cross sections in subbarrier and deep-subbarrier energy regions separately using single-channel barrier penetration model calculations, and compare them with the systematics given by Eq. (1) as well as with the touching energy evaluated with the KNS potential. We also discuss the energy dependence of fusion cross sections

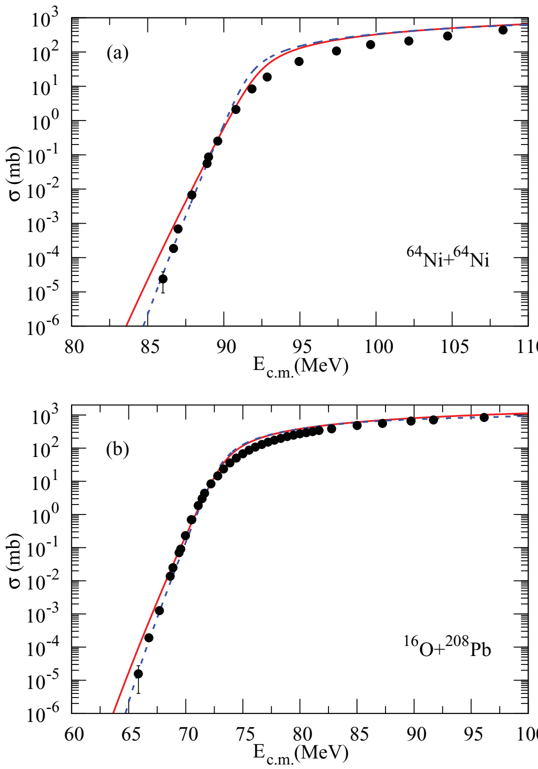

FIG. 1. (Color online) Fusion excitation functions for $^{64}Ni + {}^{64}Ni$ [Fig. 1(a)] and $^{16}O + {}^{208}Pb$ [Fig. 1(b)]. The solid and the dashed lines are results of single-channel potential model calculations which fit the experimental data in the subbarrier and the deep-subbarier energy regions, respectively. The experimental data are taken from Refs. [4,8].

at deep-subbarrier energies in terms of an asymptotic energy shift proposed by Aguiar *et al.* [17].

In order to illustrate our procedure, Figs. 1(a) and 1(b) show fusion cross sections for $^{64}Ni + {}^{64}Ni$ and $^{16}O + {}^{208}Pb$ systems, respectively. We first define the subbarrier energy region as the one in which fusion cross sections are between 10^{-2} and 10^{0} mb. We fit the experimental data in this energy region with a potential model with a Woods-Saxon potential treating the three parameters of the potential, that is, the depth V_0, the radius R_0, and the surface diffuseness a, as adjustable parameters. To this end, we numerically solve the Schrödinger equation without resorting to the parabolic approximation [18]. The fusion cross sections calculated in this way are shown by the solid lines in the figure. Of course, these calculations do not account for the fusion cross sections at higher energies as the channel-coupling effects are completely ignored. However, it is sufficient for our purpose, as we are interested only in the energy dependence of fusion cross sections at subbarrier energies, that is, the slope of fusion excitation functions. These calculations do not reproduce the experimental data at lower energies, either. In order to obtain a better fit in the lower-energy region, the surface diffuseness parameter has to be increased, as has been noticed in Refs. [8,18]. We then define the deep-subbarrier region as the one in which fusion cross sections are below 10^{-3} mb. The dashed lines in the figure show the fusion cross sections obtained by fitting to the experimental data in this energy region. See Table I for the actual values of the surface diffuseness parameter. From the two curves, we finally define the threshold energy for deep-subbarrier fusion hindrance as the energy at which the two fusion excitation functions intersect with each other.

Figure 2 shows the threshold energies thus obtained as a function of $Z_1 Z_2 \sqrt{A_1 A_2/(A_1 + A_1)}$. The figure also shows the threshold energy for $^{28}Si + {}^{64}Ni$ [19], $^{64}Ni + {}^{64}Ni$ [4], $^{16}O + {}^{208}Pb$ [8], $^{60}Ni + {}^{89}Y$ [4], $^{90}Zr + {}^{90}Zr$ [20], $^{90}Zr + {}^{92}Zr$ [20], and $^{90}Zr + {}^{89}Y$ [20] systems. For comparison, the figure also shows the empirical systematics given by Eq. (1) with the solid line, and the "experimental" data defined as the maximum energy of the S factor [11] by the stars. These values are summarized in Table I, together with the potential energy at the touching point [16] estimated with the KNS potential. One can see that the values of the threshold energy defined in our way are in good agreement with those defined as the maximum of the astrophysical S factor as well as with the potential energy at the touching configuration.

Let us next discuss briefly the asymptotic energy shift for deep-subbarrier fusion reactions. This quantity was introduced by Aguiar *et al.* [17] as a measure of subbarrier enhancement of fusion cross sections. It was defined as the extra energy needed to fit the experimental fusion cross sections with respect to a single-channel potential model calculation. It has

TABLE I. The threshold energy E_s for deep-subbarrier fusion hindrance for several systems, obtained with the two-slope fit to the experimental fusion cross sections. $a_>$ and $a_<$ are the diffuseness parameters in the Woods-Saxon potential used to fit the subbarrier and the deep-subbarrier regions of fusion cross sections. ζ is defined as $\zeta = Z_1 Z_2 \sqrt{A_1 A_2/(A_1 + A_2)}$, in which Z_i and A_i $(i = 1, 2)$ are the charge and the mass numbers of the nucleus i. $E_s^{(\mathrm{exp})}$ and $E_s^{(\mathrm{emp})}$ are the "experimental" threshold energy [11] and the empirical energies given by Eq. (1), respectively. V_{KNS} is the potential energy at the touching configuration [16] estimated with the KNS potential. All the energies are shown in units of MeV, while the lengths are in units of fm.

Systems	ζ	$a_>$	$a_<$	E_s	$E_s^{(\mathrm{exp})}$	$E_s^{(\mathrm{emp})}$	V_{KNS}
$^{28}Si + {}^{64}Ni$	1730.05	0.71	0.99	46.2	47.3 ± 0.9	51.3	43.9
$^{16}O + {}^{208}Pb$	2528.55	0.87	0.94	71.1	69.6	66.1	70.5
$^{64}Ni + {}^{64}Ni$	4434.97	0.76	0.9	88.92	87.3 ± 0.9	96.1	89.0
$^{60}Ni + {}^{89}Y$	6537.33	0.74	0.815	124.5	123 ± 1.2	124.5	125.4
$^{90}Zr + {}^{89}Y$	10435.5	0.76	0.87	171.8	171 ± 1.7	170.3	175.2
$^{90}Zr + {}^{90}Zr$	10733.1	0.56	0.76	176.1	175 ± 1.8	173.2	179.9
$^{90}Zr + {}^{92}Zr$	10791.9	0.53	0.78	171.7	171 ± 1.7	173.9	179.1

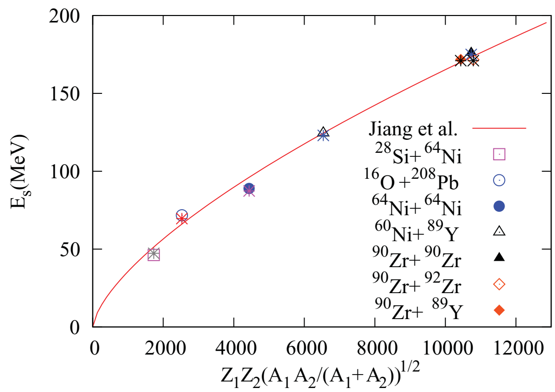

FIG. 2. (Color online) The threshold energy E_s for deep-subbarrier hindrance for several systems, determined with the two-slope fit to the experimental fusion cross sections, as a function of the parameter $Z_1 Z_2 \sqrt{A_1 A_2/(A_1 + A_2)}$. The solid curve is the empirical function given by Eq. (1), while the stars denote the "experimental" values defined as the maximum energy of the astrophysical S factors [11].

been argued that the calculated fusion cross sections have approximately the same exponential energy dependence as the experimental data in the subbarrier energy region, but are shifted in energy by a constant amount [17]. In connection to deep-subbarrier fusion hindrance, it may be interesting to revisit this representation.

In order to define the asymptotic energy shift, we first adjust the value of V_0 and R_0 in the Woods-Saxon potential, keeping the same value for the diffuseness parameter a as the one which has been obtained to fit to the subbarrier fusion cross sections (see $a_>$ in Table I), so that the experimental fusion cross sections at high energies, that is, those above $\sigma > 100$ mb, can be approximately reproduced (see Fig. 3). We then define the asymptotic energy shift as a difference between the solid line in Fig. 3 and the experimental data for a fixed value of fusion cross section. Figure 4 shows the asymptotic energy shift so extracted for several systems as a function of corresponding

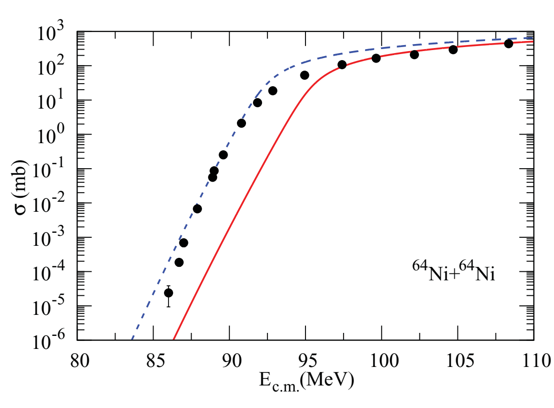

FIG. 3. (Color online) Fusion excitation functions for the $^{64}Ni + ^{64}Ni$ system. The dashed and the solid lines are results of single-channel potential model calculations which fit the experimental data in the subbarrier region and at energies above the Coulomb barrier, respectively. The experimental data are taken from Ref. [4].

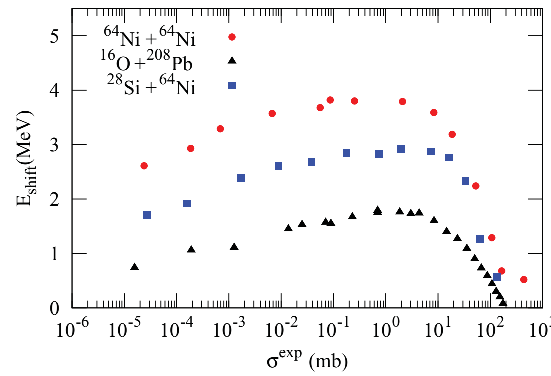

FIG. 4. (Color online) The asymptotic energy shift as a function of fusion cross section for $^{28}Si + ^{64}Ni$ (filled squares), $^{16}O + ^{208}Pb$ (filled triangles), and $^{64}Ni + ^{64}Ni$ (filled circles) systems.

fusion cross section. As one can see, the asymptotic energy shift is nearly constant in the range of $0.1 \lesssim \sigma \lesssim 1$ mb, in accordance to the previous conclusion by Aguiar *et al.* [17]. However, in the deep-subbarrier region, the asymptotic energy shift starts decreasing as the fusion cross sections decrease, reflecting the fact that the fusion cross sections have a different exponential slope than that in the subbarrier region, as shown in Fig. 1.

In summary, we have studied the energy dependence of heavy-ion fusion cross sections at deep-subbarrier energies using recent experimental data. To this end, we employed a one-dimensional potential model. We have shown that the asymptotic energy shift is almost a constant in the subbarrier region, but it decreases with decreasing energies in the deep-subbarrier region. This is a clear manifestation of the hindrance phenomenon of deep-subbarrier fusion. In order to see at which energy the deep-subbarrier hindrance takes place, we estimated the threshold energy with a two-slope fit procedure. That is, we defined the threshold energy as an intersection of two fusion excitation functions, which fit the experimental fusion cross sections either in the subbarrier energy region or in the deep-subbarrier energy region. We have shown that the threshold energies so defined are in good agreement with those estimated from the maximum of the astrophysical S factor.

The definition for the threshold energy proposed in this paper is complementary to the one using the maximum of the astrophysical S factor. As we have shown in this paper, both the definitions provide a similar value of threshold energy as the potential energies at the touching configuration. This strongly suggests that the dynamics after touching plays an important role in deep-subbarrier fusion reactions, changing the exponential slope of fusion cross sections and at the same time making the astrophysical S factor reach its maximum, although it is an open question why and how the dynamics after touching leads to the maximum of the astrophysical S factor.

We thank T. Ichikawa for useful discussions. This work was supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology by a Grant-in-Aid for Scientific Research under Program No. (C) 22540262.

[1] C. Y. Wong, Phys. Rev. Lett. **31**, 766 (1973).

[2] M. Dasgupta, D. J. Hinde, N. Rowley, and A. M. Stefanini, Annu. Rev. Nucl. Part. Sci. **48**, 401 (1998).

[3] A. B. Balantekin and N. Takigawa, Rev. Mod. Phys. **70**, 77 (1998).

[4] C. L. Jiang *et al.*, Phys. Rev. Lett. **89**, 052701 (2002); **93**, 012701 (2004).

[5] C. L. Jiang *et al.*, Phys. Rev. C **71**, 044613 (2005); **78**, 017601 (2008); **81**, 024611 (2010).

[6] A. M. Stefanini *et al.*, Phys. Rev. C **78**, 044607 (2008); G. Montagnoli *et al.*, *ibid.* **82**, 064609 (2010).

[7] C. L. Jiang *et al.*, Phys. Rev. C **82**, 041601(R) (2010); G. Montagnoli *et al.*, *ibid.* **85**, 024607 (2012).

[8] M. Dasgupta *et al.*, Phys. Rev. Lett. **99**, 192701 (2007).

[9] T. Ichikawa, K. Hagino, and A. Iwamoto, Phys. Rev. C **75**, 057603 (2007); Phys. Rev. Lett. **103**, 202701 (2009).

[10] S. Misicu and H. Esbensen, Phys. Rev. Lett. **96**, 112701 (2006); Phys. Rev. C **75**, 034606 (2007).

[11] C. L. Jiang, H. Esbensen, B. B. Back, R. V. F. Janssens, and K. E. Rehm, Phys. Rev. C **69**, 014604 (2004); C. L. Jiang, B. B. Back, H. Esbensen, R. V. F. Janssens, and K. E. Rehm, *ibid.* **73**, 014613 (2006).

[12] H. J. Krappe, J. R. Nix, and A. J. Sierk, Phys. Rev. C **20**, 992 (1979).

[13] R. Bass, *Nuclear Reactions with Heavy Ions* (Springer-Verlag, New York, 1980).

[14] W. D. Myers and W. J. Swiatecki, Phys. Rev. C **62**, 044610 (2000).

[15] R. A. Broglia and A. Winther, *Heavy Ion Reactions* (Addison-Wesley, Reading, MA, 1991).

[16] T. Ichikawa, K. Hagino, and A. Iwamoto, Phys. Rev. C **75**, 064612 (2007)

[17] C. E. Aguiar *et al.*, Nucl. Phys. A **500**, 195 (1989).

[18] K. Hagino, N. Rowley, and M. Dasgupta, Phys. Rev. C **67**, 054603 (2003).

[19] C. L. Jiang, B. B. Back, and H. Esbensen, Phys. Lett. B **640**, 18 (2006).

[20] J. G. Keller *et al.*, Nucl. Phys. A **452**, 173 (1986).