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Baryon fields with $U_L(3) \times U_R(3)$ chiral symmetry. IV. Interactions with chiral $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ vector and axial-vector mesons and anomalous magnetic moments

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We construct all $SU_L(3) \times SU_R(3)$ chirally invariant anomalous magnetic, i.e., involving a Pauli tensor and one-derivative, interactions of one chiral $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ meson field with chiral $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$, and $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ baryon fields and their "mirror" images. We find strong chiral selection rules; e.g., there is only one off-diagonal chirally symmetric anomalous magnetic interaction between $J = \frac{1}{2}$ fields belonging to the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ and the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplets. We also study the chiral selection rules for the anomalous magnetic interactions of the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ and the $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ baryon fields. Again, no diagonal and only one off-diagonal chiral $SU_L(3) \times SU_R(3)$ interaction of this type is allowed, that turns out also to conserve the $U_A(1)$ symmetry. We calculate the F/D ratios for the baryons' anomalous magnetic moments predicted by these interactions in the SU(3) symmetry limit and find that only the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$-$[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ one reproduces $F/D = 1/3$, in close proximity to the value extracted from experiment.

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I. INTRODUCTION

Our basic assumption is that baryons are linear combinations of three basic chiral representations ($[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}$, and $\mathbf{3})]$, $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$)[1] formed by three-quark interpolating fields. Recent studies [1,2] point towards baryon chiral mixing of $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ with either $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ or $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ chiral multiplets as a possible mechanism underlying the baryons' axial couplings. This finding is in line with the old current algebra results of Gerstein and Lee [3,4] and of Harari [5,6], updated to include the (most recent) F and D values extracted from experiment Ref. [7], and extended to include the flavor-singlet coupling $g_A^{(0)}$ of the nucleon [8,9], that was not considered in the mid-1960s at all, presumably due to the lack of data. Our own starting point were the QCD interpolating fields' $U_A(1)$ chiral properties [10–12]. In Ref. [13] it has been shown that both the Gerstein-Lee [3,4] and the Harari [5,6]) scenario survive in chiral Lagrangian models that constrain the baryon masses.

Having thus made the first step, viz. to reproduce the phenomenological mixing starting from a chiral effective model interaction, we turn to the next step, which is to look for chirally symmetric dynamics that produce anomalous magnetic moments. One such mechanism is the simplest chirally symmetric *one-derivative* one-(ρ, a)-meson interaction Lagrangian; one-derivative because only thus can one couple the baryon magnetic moment (the Pauli current) to the ρ field. Here we study vector meson couplings because photon couplings follow them under the vector meson dominance (VMD) hypothesis which has been shown to work in the low energy region. We note, however, that the VMD hypothesis is merely a convenient, but not necessary device, as Gerstein and Lee [4] have shown that the anomalous magnetic moments of the nucleons can be obtained using the current algebra, under the assumption that the photon transforms as a member of the (broken chiral symmetry) $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ multiplet, which amounts to VMD hypothesis with vector mesons belonging to the $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ chiral representation.

In this paper we construct all $SU_L(3) \times SU_R(3)$ chirally invariant one-derivative one-vector-meson-baryon interactions and then use them to calculate the baryons' magnetic moments. We derive the nonderivative Dirac terms, as well. Another, perhaps equally important and difficult problem, viz. that of the flavor-singlet anomalous magnetic moment of the nucleon, is also addressed. Thus our present paper serves to provide a dynamical model of chiral mixing that is an optimal approximation to the phenomenological solution of both the (F, D) and the flavor-singlet axial couplings, and of the anomalous magnetic moments.

In our previous publication [13] we found two solutions that fit the axial coupling data[2]: one that conserves the $U_A(1)$ symmetry (the Harari scenario) and another one that does not (the Gerstein-Lee scenario). Here we show that only the former scenario leads to nucleon anomalous magnetic moments that are in agreement with experiment extrapolated to the SU(3) symmetry limit. The latter ("Gerstein-Lee") scenario requires vanishing nucleon anomalous magnetic moments, in serious disagreement with experimental result extrapolated to the SU(3) symmetry limit.

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[1]These chiral multiplets are not limited to three-quark interpolators: for a discussion of the validity of our assumptions, see Sec. II D.

[2]This does not preclude the existence of more complicated solutions.

Here we have, for the sake of clarity, temporarily ignored the chiral mixing in the vector meson sector (that must violate the $U_A(1)$ symmetry, see Ref. [14]). This does not affect the validity of our conclusions, as in Sec. III A we show that the chiral interactions of vector mesons belonging to the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplet with the above baryon fields lead to phenomenologically incorrect values of the anomalous magnetic moment F/D. We also emphasize the fact that our results ("selection rules") hold for arbitrary chiral mixing angles, thus making the renormalization of axial couplings due to the axial-vector mesons irrelevant for this purpose.

All this goes to show that the "QCD $U_A(1)$ anomaly" probably does not play a role in the "nucleon spin problem" [8,9], as was once widely thought [15]. Rather, in all likelihood the $U_A(1)$ anomaly provides only a (relatively) small part of the solution, associated with the higher Fock space components, whereas the largest part comes from the $U_A(1)$-symmetric chiral structure of the nucleon.

In this paper we use the baryon interpolating fields to construct chirally invariant interactions. These fields have been often used in the QCD sum rule analyses [19–21] and lattice QCD calculations [22]. Most of QCD sum rule studies used the "Ioffe current" which leads to baryon masses consistent with those of the ground state baryons. In particular, Espriu, Pascual, and Tarrach [21] have studied the dependence on the field mixing parameter t of local interpolating operators and found an optimal value around $t \approx -1$, which corresponds to the "Ioffe current". The Ioffe interpolating field is the same as N_- in our notation, which belongs to the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral representation [23]. This is the only local interpolating field that appears in our chiral admixture results, which is consistent with the optimal choice in the QCD sum rule analyses.

This paper consists of four parts: after the present section as Introduction. In Sec. II we construct the $SU_L(3) \times SU_R(3)$ chirally invariant interactions of nonderivative Dirac type. In Sec. III we apply chiral mixing formalism to the hyperons' vector-current form factors. Finally, in Sec. IV we discuss the results and present a summary and an outlook on the future developments.

II. NONDERIVATIVE DIRAC TYPE INTERACTION

In this section we propose a method for the construction of $N_f = 3$ chiral invariant vector meson-baryon interactions with $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ meson fields. Both the diagonal and off-diagonal terms are possible, which we shall study in the following.

We have classified the baryon interpolating fields in our previous paper [10]. Our conventions for them are the same as in Refs. [1,13]. Next we shall briefly define the conventions for the vector and axial-vector mesons.

A. Preliminaries: Chiral transformations of vector and axial-vector mesons

We define the vector and axial-vector mesons in the SU(3) space:

$$\rho^a_\mu = \bar{q}_A \lambda^a_{AB} \gamma_\mu q_B, \quad a^a_{1\mu} = \bar{q}_A \lambda^a_{AB} \gamma_\mu \gamma_5 q_B, \tag{1}$$

where the index a goes from 1 to 8. They belong to the chiral representation $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$. Their linear combinations, $M^b_\mu = \rho^b_\mu + \gamma_5 a^b_{1\mu}$ and $M^{[\mathrm{mir}]b}_\mu = \gamma_0 M^{+b}_\mu \gamma_0 = \rho^b_\mu - \gamma_5 a^b_{1\mu}$, are the right-handed spin-one-meson vector current and the left-handed vector current, respectively. They transform as $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ and $(\mathbf{1}, \mathbf{8}) \oplus (\mathbf{8}, \mathbf{1})$, respectively:

$$\delta_5^{\vec{b}} M^b_\mu = \gamma_5 b^a f_{abc} M^c_\mu, \quad \delta_5^{\vec{b}} M^{[\mathrm{mir}]b}_\mu = -\gamma_5 b^a f_{abc} M^{[\mathrm{mir}]c}_\mu. \tag{2}$$

To proceed our calculations sometimes we use the "physical" basis, of which the definitions are

$$\begin{pmatrix} M^1_\mu \\ M^2_\mu \\ M^3_\mu \\ M^4_\mu \\ M^5_\mu \\ M^6_\mu \\ M^7_\mu \\ M^8_\mu \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \rho^1_\mu + a^1_{1\mu} \\ \rho^2_\mu + a^2_{1\mu} \\ \rho^3_\mu + a^3_{1\mu} \\ \rho^4_\mu + a^4_{1\mu} \\ \rho^5_\mu + a^5_{1\mu} \\ \rho^6_\mu + a^6_{1\mu} \\ \rho^7_\mu + a^7_{1\mu} \\ \rho^8_\mu + a^8_{1\mu} \end{pmatrix}. \tag{3}$$

We can define another type of $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ chiral representation: $R^b_\mu = \rho^b_\mu + a^b_{1\mu}$ as the right-handed vector meson, and $L^b_\mu = \rho^b_\mu - a^b_{1\mu}$ as the left-handed vector meson. They transform as

$$\delta_5^{\vec{b}} R^b_\mu = b^a f_{abc} R^c_\mu, \quad \delta_5^{\vec{b}} L^b_\mu = -b^a f_{abc} L^c_\mu. \tag{4}$$

We can use these two fields to write interactions that can be used in other calculations. We note here that these two fields contain both the positive- and the negative-parity components, however.

B. Diagonal interactions

1. Chiral $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ baryons diagonal interactions

To start with, it is useful to look at the chiral group structure of the vector meson-baryon interaction $\bar{N} M N'$, where N and N' denote two baryon fields and M denotes the vector meson fields with the Lorentz index μ contracted either with the Dirac matrix γ_μ or with the Pauli tensor $\sigma^{\mu\nu}$.

The Dirac current $\bar{N}\gamma_\mu N$ contains two γ matrices, γ_μ and γ_0, the latter of which comes from the Dirac conjugate of the baryon field. Therefore, it is diagonal in the chiral base, in other words, it takes the form

$$\begin{aligned}\bar{N}MN' &\sim \bar{N}_L M N'_L + \bar{N}_R M N'_R \\ &\sim (\bar{N}_L M_L N'_L + \bar{N}_R M_R N'_R) \text{ and} \\ &\times \left(\bar{N}_L M_L^{[\text{mir}]} N'_L + \bar{N}_R M_R^{[\text{mir}]} N'_R\right), \qquad (5)\end{aligned}$$

when decomposed into the left and right helicity components. Then the diagonal interaction has the structure in group representation notation

$$\bar{N}_L(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \times M_L(\mathbf{8}, \mathbf{1}) \times N_L(\mathbf{6}, \mathbf{3}) + \bar{N}_R(\bar{\mathbf{3}}, \bar{\mathbf{6}}) \times M_R(\mathbf{1}, \mathbf{8}) \times N_R(\mathbf{3}, \mathbf{6}),$$

and

$$\bar{N}_L(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \times M_L^{[\text{mir}]}(\mathbf{1}, \mathbf{8}) \times N_L(\mathbf{6}, \mathbf{3}) + \bar{N}_R(\bar{\mathbf{3}}, \bar{\mathbf{6}}) \times M_R^{[\text{mir}]}(\mathbf{8}, \mathbf{1}) \times N_R(\mathbf{3}, \mathbf{6}),$$

where in the second structure the mirror field $M_{L,R}^{[\text{mir}]}$ transforms as $M_{R,L}$.

In the first term the product $\bar{N}_L M_L N_L$ is decomposed as

$$\begin{aligned}(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \otimes (\mathbf{8}, \mathbf{1}) \otimes (\mathbf{6}, \mathbf{3}) &\sim [(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \otimes (\mathbf{6}, \mathbf{3})] \otimes (\mathbf{8}, \mathbf{1}) \\ &\ni (\mathbf{8}, \mathbf{1}) \otimes (\mathbf{8}, \mathbf{1}) \ni (\mathbf{1}, \mathbf{1}), \qquad (6)\end{aligned}$$

and the one of $\bar{N}_L M_L^{[\text{mir}]} N_L$ is decomposed as

$$\begin{aligned}(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \otimes (\mathbf{1}, \mathbf{8}) \otimes (\mathbf{6}, \mathbf{3}) &\sim [(\bar{\mathbf{6}}, \bar{\mathbf{3}}) \otimes (\mathbf{6}, \mathbf{3})] \otimes (\mathbf{1}, \mathbf{8}) \\ &\ni (\mathbf{1}, \mathbf{8}) \otimes (\mathbf{1}, \mathbf{8}) \ni (\mathbf{1}, \mathbf{1}). \qquad (7)\end{aligned}$$

Therefore, there are two chiral invariant combinations for the left chirality. The situation is the same for the right chirality. We also do this for other diagonal and off-diagonal interactions in the following subsections.

Now we shall construct their explicit forms as

$$\overline{N}^a_{(18)}\gamma^\mu M^c_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)},$$

and/or

$$\overline{N}^a_{(18)}\gamma^\mu M^{[\text{mir}]c}_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)},$$

where the indices a and b run from 1 to 18, and the index c just runs from 1 to 8. By applying the chiral transformation to this Lagrangian and demanding that this variation vanishes, we obtain hundreds of equations, such as

$$\begin{aligned}\delta_5^3\left(\overline{N}^a_{(18)}\gamma^\mu M^c_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)}\right) &= \left(-\mathbf{C}^{1,1,1}_{(18)} - \frac{2\sqrt{2}}{3}\mathbf{C}^{1,10,1}_{(18)} + \frac{2\sqrt{2}}{3}\mathbf{C}^{10,1,1}_{(18)}\right)\bar{p}(i\gamma_5 b_3)\gamma^\mu M^1_\mu p \\ &\quad + \left(\frac{2\sqrt{2}}{3}\mathbf{C}^{10,1,3}_{(18)} - \frac{2\sqrt{2}}{3}\mathbf{C}^{1,10,3}_{(18)}\right)\bar{p}(i\gamma_5 b_3)\gamma^\mu M^3_\mu p + \cdots = 0. \qquad (8)\end{aligned}$$

Solving these equations together with the Hermiticity condition, we find that there are two solutions:

(1) One solution can be written out using $\mathbf{A}^c_{(18)}$ in the following form $(\bar{N}^a_{(18)}\gamma^\mu M^c_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)})$:

$$\mathcal{L}^A_{(18)} = g^A_{(18)}\bar{N}^a_{(18)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{A}^c_{(18)}\right)_{ab}N^b_{(18)}, \qquad (9)$$

where $g^A_{(18)}$ is the coupling constant, and the solution is

$$\begin{aligned}\mathbf{A}^c_{(18)} &= \begin{pmatrix}\frac{\sqrt{3}}{2}\mathbf{D}^c_{(8)} + \frac{5}{2\sqrt{3}}\mathbf{F}^c_{(8)} & \mathbf{T}^c_{(8/10)} \\ \mathbf{T}^{\dagger c}_{(8/10)} & \frac{2}{\sqrt{3}}\mathbf{F}^c_{(10)}\end{pmatrix} \\ &= \frac{\sqrt{3}}{2}\left(\mathbf{V}^c_{(18)} + \mathbf{F}^c_{(18)}\right). \qquad (10)\end{aligned}$$

The chiral group structure for this interaction is shown in Eq. (6). The matrices $\mathbf{V}^c_{(18)}$ and $\mathbf{F}^c_{(18)}$ has been defined by

$$\begin{aligned}\mathbf{V}^c_{(18)} &= \begin{pmatrix}\mathbf{F}^c_{(8)} & 0 \\ 0 & \mathbf{F}^c_{(10)}\end{pmatrix}, \\ \mathbf{F}^c_{(18)} &= \begin{pmatrix}\mathbf{D}^c_{(8)} + \frac{2}{3}\mathbf{F}^c_{(8)} & \frac{2}{\sqrt{3}}\mathbf{T}^c_{(8/10)} \\ \frac{2}{\sqrt{3}}\mathbf{T}^{\dagger c}_{(8/10)} & \frac{1}{3}\mathbf{F}^c_{(10)}\end{pmatrix} \qquad (11)\end{aligned}$$

with $\mathbf{D}^c_{(8)}$, $\mathbf{F}^c_{(8)}$, $\mathbf{T}^c_{(8/10)}$ given in Ref. [1]

(2) The other solution can be written out using $\mathbf{B}^c_{(18)}$ in the following form $(\bar{N}^a_{(18)}\gamma^\mu M^{[\text{mir}]c}_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)})$:

$$\mathcal{L}^B_{(18)} = g^B_{(18)}\bar{N}^a_{(18)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{B}^c_{(18)}\right)_{ab}N^b_{(18)}, \qquad (12)$$

where $g^B_{(18)}$ is the coupling constant, and the solution is

$$\begin{aligned}\mathbf{B}^c_{(18)} &= \begin{pmatrix}\frac{\sqrt{3}}{2}\mathbf{D}^c_{(8)} - \frac{1}{2\sqrt{3}}\mathbf{F}^c_{(8)} & \mathbf{T}^c_{(8/10)} \\ \mathbf{T}^{\dagger c}_{(8/10)} & -\frac{1}{\sqrt{3}}\mathbf{F}^c_{(10)}\end{pmatrix} \\ &= -\frac{\sqrt{3}}{2}\left(\mathbf{V}^c_{(18)} - \mathbf{F}^c_{(18)}\right). \qquad (13)\end{aligned}$$

The chiral group structure for this interaction is shown in Eq. (7).

Besides the Lagrangians (9) and (12), their mirror parts

$$\begin{aligned}&g^A_{(18m)}\bar{N}^a_{(18m)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{A}^c_{(18)}\right)_{ab}N^b_{(18m)}, \quad \text{and} \\ &g^B_{(18m)}\bar{N}^a_{(18m)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{B}^c_{(18)}\right)_{ab}N^b_{(18m)},\end{aligned}$$

are also chiral invariant. Using these solutions, and performing the chiral transformation, we can obtain the following relations:

$$\begin{aligned}-\mathbf{F}^{a\dagger}_{(18)}\mathbf{A}^b_{(18)} + \mathbf{A}^b_{(18)}\mathbf{F}^a_{(18)} + if_{abc}\mathbf{A}^c_{(18)} &= 0, \\ -\mathbf{F}^{a\dagger}_{(18)}\mathbf{B}^b_{(18)} + \mathbf{B}^b_{(18)}\mathbf{F}^a_{(18)} - if_{abc}\mathbf{B}^c_{(18)} &= 0.\end{aligned} \qquad (14)$$

Note that the generators $\mathbf{F}^a_{(18)}$ in Eq. (11) are Hermitian matrices, i.e., $\mathbf{F}^{a\dagger}_{(18)} = \mathbf{F}^a_{(18)}$. Therefore, Eqs. (14) turn into the familiar SU(3) × SU(3) Lie algebra commutators that have already been proven in Ref. [1]. This confirms the consistency of our present calculation with that in Ref. [1], as expected.

The solution in the physical basis $(\bar{N}^a_{(18)}\gamma^\mu M^c_\mu N^b_{(18)}\mathbf{C}^{abc}_{(18)})$ can be obtained by the following relations:

$$\begin{aligned}
\mathbf{C}^{ab3}_{(18)} &= (\mathbf{A}^3_{(18)})_{ab},\ \mathbf{C}^{ab8}_{(18)} = (\mathbf{A}^8_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab1}_{(18)} + \mathbf{C}^{ab2}_{(18)}) &= (\mathbf{A}^1_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab1}_{(18)} + \mathbf{C}^{ab2}_{(18)}) &= (\mathbf{A}^2_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab4}_{(18)} + \mathbf{C}^{ab5}_{(18)}) &= (\mathbf{A}^4_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab4}_{(18)} + \mathbf{C}^{ab5}_{(18)}) &= (\mathbf{A}^5_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab6}_{(18)} + \mathbf{C}^{ab7}_{(18)}) &= (\mathbf{A}^6_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab6}_{(18)} + \mathbf{C}^{ab7}_{(18)}) &= (\mathbf{A}^7_{(18)})_{ab}.
\end{aligned} \tag{15}$$

Another strategy for finding the two chiral interactions ("solutions") is to study the two parity-violating baryon currents interacting with the left- and right-handed vector mesons and then to combine them to obtain the parity conserving and parity violating Lagrangians. The explicit forms of interactions that we obtained by using this strategy appear to be the most convenient ones for practical use.

To get the first solution, we use the following right-handed and left-handed currents:

(1) The right-handed current solution can be written in the following form:

$$\begin{aligned}
\mathcal{L}^R_{(18)} &= g^R_{(18)} R^c_\mu \mathbf{R}^c_{\mu(18)}\\
&= g^R_{(18)}(\rho^c_\mu + a^c_{1\mu})\overline{N}^a_{(18)}\gamma^\mu(\mathbf{R}^c_{(18)})_{ab}N^b_{(18)},
\end{aligned} \tag{16}$$

where $g^R_{(18)}$ is the right-handed current coupling constant and we have used

$$\begin{aligned}
\mathbf{R}^c_{(18)} &= (\mathbf{V}^c_{(18)} + \gamma_5\mathbf{F}^c_{(18)})\\
&= \begin{pmatrix} \gamma_5\mathbf{D}^c_{(8)} + (1+\frac{2}{3}\gamma_5)\mathbf{F}^c_{(8)} & \frac{2}{\sqrt{3}}\gamma_5\mathbf{T}^c_{(8/10)}\\ \frac{2}{\sqrt{3}}\gamma_5\mathbf{T}^{\dagger c}_{(8/10)} & (1+\frac{1}{3}\gamma_5)\mathbf{F}^c_{(10)}\end{pmatrix},
\end{aligned}$$

and

$$\begin{aligned}
&\mathbf{R}^c_{\mu(18)}\\
&= (\mathbf{J}^c_{\mu(18)} + \mathbf{J}^c_{\mu5(18)})\\
&= \overline{N}^a_{(18)}\gamma^\mu\gamma_5\begin{pmatrix} \mathbf{D}^c_{(8)} + (\frac{2}{3}+\gamma_5)\mathbf{F}^c_{(8)} & \frac{2}{\sqrt{3}}\mathbf{T}^c_{(8/10)}\\ \frac{2}{\sqrt{3}}\mathbf{T}^{\dagger c}_{(8/10)} & (\frac{1}{3}+\gamma_5)\mathbf{F}^c_{(10)}\end{pmatrix}_{ab} N^b_{(18)},
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{J}^c_{\mu(18)} &= \overline{N}^a_{(18)}\gamma^\mu(\mathbf{V}^c_{(18)})_{ab}N^b_{(18)},\\
\mathbf{J}^c_{\mu5(18)} &= \overline{N}^a_{(18)}\gamma^\mu\gamma_5(\mathbf{F}^c_{(18)})_{ab}N^b_{(18)}.
\end{aligned} \tag{17}$$

(2) The left-handed current solution can be written in the following form:

$$\begin{aligned}
\mathcal{L}^L_{(18)} &= g^L_{(18)} L^c_\mu \mathbf{L}^c_{\mu(18)}\\
&= g^L_{(18)}(\rho^c_\mu - a^c_{1\mu})\overline{N}^a_{(18)}\gamma^\mu(\mathbf{L}^c_{(18)})_{ab}N^b_{(18)},
\end{aligned} \tag{18}$$

where $g^L_{(18)}$ is the left-handed current coupling constant and we have used

$$\begin{aligned}
\mathbf{L}^c_{(18)} &= (\mathbf{V}^c_{(18)} - \gamma_5\mathbf{F}^c_{(18)})\\
&= \begin{pmatrix} -\gamma_5\mathbf{D}^c_{(8)} + (1-\frac{2}{3}\gamma_5)\mathbf{F}^c_{(8)} & -\frac{2}{\sqrt{3}}\gamma_5\mathbf{T}^c_{(8/10)}\\ -\frac{2}{\sqrt{3}}\gamma_5\mathbf{T}^{\dagger c}_{(8/10)} & (1-\frac{1}{3}\gamma_5)\mathbf{F}^c_{(10)}\end{pmatrix},
\end{aligned}$$

and

$$\begin{aligned}
&\mathbf{L}^c_{\mu(18)}\\
&= (\mathbf{J}^c_{\mu(18)} - \mathbf{J}^c_{\mu5(18)})\\
&= -\overline{N}^a_{(18)}\gamma^\mu\gamma_5\begin{pmatrix} \mathbf{D}^c_{(8)} + (\frac{2}{3}-\gamma_5)\mathbf{F}^c_{(8)} & \frac{2}{\sqrt{3}}\mathbf{T}^c_{(8/10)}\\ \frac{2}{\sqrt{3}}\mathbf{T}^{\dagger c}_{(8/10)} & (\frac{1}{3}-\gamma_5)\mathbf{F}^c_{(10)}\end{pmatrix}_{ab} N^b_{(18)}.
\end{aligned}$$

These two chiral interactions generally contain both the parity-violating and the parity-conserving parts. Their sum also contains both of these terms, unless $g^L_{(18)} = \pm g^R_{(18)}$, when it is either purely parity-conserving, in the case of plus sign,

$$\begin{aligned}
\mathcal{L}^{\rm PC}_{(18)} &= g^{\rm PC}_{(18)}\frac{1}{2}\overline{N}^a_{(18)}[(\rho^c_\mu - a^c_{1\mu})(\mathbf{L}^{\mu c}_{(18)})\\
&\quad + (\rho^c_\mu + a^c_{1\mu})(\mathbf{R}^{\mu c}_{(18)})]N^b_{(18)}\\
&= g^{\rm PC}_{(18)}[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu(18)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu5(18)}],
\end{aligned} \tag{19}$$

or purely parity-violating, in the case of the minus sign.

$$\begin{aligned}
\mathcal{L}^{\rm PV}_{(18)} &= g^{\rm PV}_{(18)}\frac{1}{2}\overline{N}^a_{(18)}[-(\rho^c_\mu - a^c_{1\mu})(\mathbf{L}^{\mu c}_{(18)})\\
&\quad + (\rho^c_\mu + a^c_{1\mu})(\mathbf{R}^{\mu c}_{(18)})]N^b_{(18)}\\
&= g^{\rm PV}_{(18)}[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu5(18)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu(18)}].
\end{aligned} \tag{20}$$

Thus we have obtained the first solution, Eq. (19).

To get the second solution, we can simply multiply an extra γ_5 in front of $\mathbf{R}^c_{(18)}$ and $\mathbf{L}^c_{(18)}$, and rewrite Eqs. (16) and (18) to be

$$\begin{aligned}
\mathcal{L}'^R_{(18)} &= g'^R_{(18)} R^c_\mu \mathbf{R}'^c_{\mu(18)}\\
&= g'^R_{(18)}(\rho^c_\mu + a^c_{1\mu})\overline{N}^a_{(18)}\gamma^\mu\gamma_5(\mathbf{R}^c_{(18)})_{ab}N^b_{(18)},\\
\mathcal{L}'^L_{(18)} &= g'^L_{(18)} L^c_\mu \mathbf{L}'^c_{\mu(18)}\\
&= g'^L_{(18)}(\rho^c_\mu - a^c_{1\mu})\overline{N}^a_{(18)}\gamma^\mu\gamma_5(\mathbf{L}^c_{(18)})_{ab}N^b_{(18)}.
\end{aligned} \tag{21}$$

They are also chiral invariant. Similarly, we can use them to construct the parity-conserving and parity-violating parts:

$$\begin{aligned}
\mathcal{L}'^{\rm PC}_{(18)} &= g'^{\rm PC}_{(18)}[\boldsymbol{\rho}^\mu\cdot\mathbf{J}'_{\mu(18)} + \mathbf{a}^\mu_1\cdot\mathbf{J}'_{\mu5(18)}],\\
\mathcal{L}'^{\rm PV}_{(18)} &= g'^{\rm PV}_{(18)}[\boldsymbol{\rho}^\mu\cdot\mathbf{J}'_{\mu5(18)} + \mathbf{a}^\mu_1\cdot\mathbf{J}'_{\mu(18)}],
\end{aligned} \tag{22}$$

where

$$\begin{aligned}
\mathbf{J}'^c_{\mu(18)} &= \overline{N}^a_{(18)}\gamma^\mu(\mathbf{F}^c_{(18)})_{ab}N^b_{(18)},\\
\mathbf{J}'^c_{\mu5(18)} &= \overline{N}^a_{(18)}\gamma^\mu\gamma_5(\mathbf{V}^c_{(18)})_{ab}N^b_{(18)}.
\end{aligned} \tag{23}$$

Thus we have obtained the second solution, Eq. (22).

Using these solutions, and performing the chiral transformation, we can obtain the following relations:

$$-\mathbf{F}^{a\dagger}_{(18)}\gamma_5\mathbf{L}^b_{(18)} + \mathbf{L}^b_{(18)}\mathbf{F}^a_{(18)}\gamma_5 - if_{abc}\mathbf{L}^c_{(18)} = 0,$$
$$-\mathbf{F}^{a\dagger}_{(18)}\gamma_5\mathbf{R}^b_{(18)} + \mathbf{R}^b_{(18)}\mathbf{F}^a_{(18)}\gamma_5 + if_{abc}\mathbf{R}^c_{(18)} = 0. \quad (24)$$

We can check the equivalence of these two sets of solutions, and verify the following relations:

$$\frac{\mathcal{L}^{\rm PC}_{(18)}}{g^{\rm PC}_{(18)}} = \frac{1}{\sqrt{3}}\left(\frac{\mathcal{L}^A_{(18)}}{g^A_{(18)}} - \frac{\mathcal{L}^B_{(18)}}{g^B_{(18)}}\right),$$
$$\frac{\mathcal{L}'^{\rm PC}_{(18)}}{g'^{\rm PC}_{(18)}} = \frac{1}{\sqrt{3}}\left(\frac{\mathcal{L}^A_{(18)}}{g^A_{(18)}} + \frac{\mathcal{L}^B_{(18)}}{g^B_{(18)}}\right). \quad (25)$$

2. *Chiral* $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ *baryons diagonal interactions*

The product of the first term inside the structure $(\bar{N}_L M_L N_L + \bar{N}_R M_R N_R)$ is decomposed as

$$(\bar{\mathbf{3}},\mathbf{3}) \otimes (\mathbf{8},\mathbf{1}) \otimes (\mathbf{3},\bar{\mathbf{3}}) \sim [(\bar{\mathbf{3}},\mathbf{3}) \otimes (\mathbf{3},\bar{\mathbf{3}})] \otimes (\mathbf{8},\mathbf{1}) \ni (\mathbf{8},\mathbf{1}) \otimes (\mathbf{8},\mathbf{1}) \ni (\mathbf{1},\mathbf{1}), \quad (26)$$

while the one of $(\bar{N}_L M_L^{[\rm mir]} N_L + \bar{N}_R M_R^{[\rm mir]} N_R)$ is decomposed as

$$(\bar{\mathbf{3}},\mathbf{3}) \otimes (\mathbf{1},\mathbf{8}) \otimes (\mathbf{3},\bar{\mathbf{3}}) \sim [(\bar{\mathbf{3}},\mathbf{3}) \otimes (\mathbf{3},\bar{\mathbf{3}})] \otimes (\mathbf{1},\mathbf{8}) \ni (\mathbf{1},\mathbf{8}) \otimes (\mathbf{1},\mathbf{8}) \ni (\mathbf{1},\mathbf{1}). \quad (27)$$

Therefore, there are two chiral invariant combinations. Consequently, following the same procedures as in the previous section, we find that there are two solutions:

(1) One solution can be written out using $\mathbf{A}^c_{(9)}$ in the following form $(\bar{N}^a_{(9)}\gamma^\mu M^c_\mu N^b_{(9)}\mathbf{C}^{abc}_{(9)})$:

$$\mathcal{L}^A_{(9)} = g^A_{(9)}\bar{N}^a_{(9)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)(\mathbf{A}^c_{(9)})_{ab}N^b_{(9)}, \quad (28)$$

where the solution is

$$\mathbf{A}^c_{(9)} = \begin{pmatrix} 0 & \frac{1}{\sqrt{6}}\mathbf{T}^c{}_{(1/8)} \\ \frac{1}{\sqrt{6}}\mathbf{T}^{\dagger c}_{(1/8)} & \frac{1}{2}\mathbf{D}^c_{(8)} + \frac{1}{2}\mathbf{F}^c_{(8)} \end{pmatrix} = \frac{1}{2}\left(\mathbf{V}^c_{(9)} + \mathbf{F}^c_{(9)}\right). \quad (29)$$

The chiral group structure for this interaction is shown in Eq. (26). The matrices $\mathbf{F}^c_{(9)}$ have been defined in Eq. (30) and

$$\mathbf{V}^c_{(9)} = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{F}^c_{(8)} \end{pmatrix}. \quad (30)$$

(2) The other solution can be written out using $\mathbf{B}^c_{(9)}$ in the following form $(\bar{N}^a_{(9)}\gamma^\mu M^{[\rm mir]c}_\mu N^b_{(9)}\mathbf{C}^{abc}_{(9)})$:

$$\mathcal{L}^B_{(9)} = g^B_{(9)}\bar{N}^a_{(9)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)(\mathbf{B}^c_{(9)})_{ab}N^b_{(9)}, \quad (31)$$

where the solution is

$$\mathbf{B}^c_{(9)} = \begin{pmatrix} 0 & \frac{1}{\sqrt{6}}\mathbf{T}^c{}_{(1/8)} \\ \frac{1}{\sqrt{6}}\mathbf{T}^{\dagger c}_{(1/8)} & \frac{1}{2}\mathbf{D}^c_{(8)} - \frac{1}{2}\mathbf{F}^c_{(8)} \end{pmatrix} = -\frac{1}{2}\left(\mathbf{V}^c_{(9)} - \mathbf{F}^c_{(9)}\right). \quad (32)$$

The chiral group structure of this interaction is shown in Eq. (27).

Besides the Lagrangians (28) and (31), their mirror parts,

$$g^A_{(9m)}\bar{N}^a_{(9m)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{A}^c_{(9)}\right)_{ab}N^b_{(9m)}, \quad \text{and}$$
$$g^B_{(9m)}\bar{N}^a_{(9m)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{B}^c_{(9)}\right)_{ab}N^b_{(9m)},$$

are also chiral invariant. Using these solutions, and performing the chiral transformation, we can obtain the following relations:

$$-\mathbf{F}^{a\dagger}_{(9)}\mathbf{A}^b_{(9)} + \mathbf{A}^b_{(9)}\mathbf{F}^a_{(9)} + if_{abc}\mathbf{A}^c_{(9)} = 0,$$
$$-\mathbf{F}^{a\dagger}_{(9)}\mathbf{B}^b_{(9)} + \mathbf{B}^b_{(9)}\mathbf{F}^a_{(9)} - if_{abc}\mathbf{B}^c_{(9)} = 0. \quad (33)$$

Note that the generators $\mathbf{F}^a_{(9)}$ defined in Ref. [1] are hermitian matrices, i.e., $\mathbf{F}^{a\dagger}_{(9)} = \mathbf{F}^a_{(9)}$. Therefore, Eqs. (33) turn into the familiar SU(3) × SU(3) Lie commutators that have already been proven in Ref. [1]. This, once again, proves the consistency of our present calculation with that in Ref. [1].

We can use the other strategy which has been discussed in the previous section to obtain these two interactions. One solution is

$$\mathcal{L}^{\rm PC}_{(9)} = g^{\rm PC}_{(9)}\left[\boldsymbol{\rho}^\mu \cdot \mathbf{J}_{\mu(9)} + \mathbf{a}^\mu_1 \cdot \mathbf{J}_{\mu5(9)}\right], \quad (34)$$

where

$$\mathbf{J}^c_{\mu(9)} = \overline{N}^a_{(9)}\gamma^\mu\left(\mathbf{V}^c_{(9)}\right)_{ab}N^b_{(9)}, \quad \mathbf{J}^c_{\mu5(9)} = \overline{N}^a_{(9)}\gamma^\mu\gamma_5\left(\mathbf{F}^c_{(9)}\right)_{ab}N^b_{(9)}, \quad (35)$$

and the other solution is (similarly obtained by adding an extra γ_5):

$$\mathcal{L}'^{\rm PC}_{(9)} = g'^{\rm PC}_{(9)}\left[\boldsymbol{\rho}^\mu \cdot \mathbf{J}'_{\mu(9)} + \mathbf{a}^\mu_1 \cdot \mathbf{J}'_{\mu5(9)}\right], \quad (36)$$

where

$$\mathbf{J}'^c_{\mu(9)} = \overline{N}^a_{(9)}\gamma^\mu\left(\mathbf{F}^c_{(9)}\right)_{ab}N^b_{(9)}, \quad \mathbf{J}'^c_{\mu5(9)} = \overline{N}^a_{(9)}\gamma^\mu\gamma_5\left(\mathbf{V}^c_{(9)}\right)_{ab}N^b_{(9)}. \quad (37)$$

The relevant purely parity-violating partners are

$$\mathcal{L}^{\rm PV}_{(9)} = g^{\rm PV}_{(9)}\left[\boldsymbol{\rho}^\mu \cdot \mathbf{J}_{\mu5(9)} + \mathbf{a}^\mu_1 \cdot \mathbf{J}_{\mu(9)}\right].$$
$$\mathcal{L}'^{\rm PV}_{(9)} = g'^{\rm PV}_{(9)}\left[\boldsymbol{\rho}^\mu \cdot \mathbf{J}'_{\mu5(9)} + \mathbf{a}^\mu_1 \cdot \mathbf{J}'_{\mu(9)}\right], \quad (38)$$

We can check the equivalence of these two sets of solutions, and verify the following relations:

$$\frac{\mathcal{L}^{\rm PC}_{(9)}}{g^{\rm PC}_{(9)}} = \frac{\mathcal{L}^A_{(9)}}{g^A_{(9)}} - \frac{\mathcal{L}^B_{(9)}}{g^B_{(9)}}, \quad \frac{\mathcal{L}'^{\rm PC}_{(9)}}{g'^{\rm PC}_{(9)}} = \frac{\mathcal{L}^A_{(9)}}{g^A_{(9)}} + \frac{\mathcal{L}^B_{(9)}}{g^B_{(9)}}. \quad (39)$$

3. *Chiral* $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ *baryons diagonal interactions*

The product of the first term inside the structure $(\bar{N}_L M_L N_L + \bar{N}_R M_R N_R)$ is decomposed as

$$(\mathbf{8},\mathbf{1}) \otimes (\mathbf{8},\mathbf{1}) \otimes (\mathbf{8},\mathbf{1}) \ni [(\mathbf{8},\mathbf{1}) \oplus (\mathbf{8},\mathbf{1})] \otimes (\mathbf{8},\mathbf{1}) \ni (\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1}). \quad (40)$$

Therefore, there are two chiral invariant combinations. Consequently, following the same procedure as in the previous section(s), we find that there are two solutions. They can be written out using $\mathbf{F}^c_{(8)}$ and $\mathbf{D}^c_{(8)}$ in the following form ($\bar{N}^a_{(8)}\gamma^\mu M^c_\mu N^b_{(8)}\mathbf{C}^{abc}_{(8)}$):

$$\mathcal{L}^F_{(8)} = g^F_{(8)}\bar{N}^a_{(8)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(8)}\right)_{ab}N^b_{(8)}, \quad (41)$$

$$\mathcal{L}^D_{(8)} = g^D_{(8)}\bar{N}^a_{(8)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{D}^c_{(8)}\right)_{ab}N^b_{(8)}. \quad (42)$$

The chiral group structure for these two interactions is just shown in Eq. (40).

Besides the Lagrangians (41) and (42), their mirror parts

$$g^F_{(8m)}\bar{N}^a_{(8m)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(8)}\right)_{ab}N^b_{(8m)} \quad \text{and}$$
$$g^D_{(8m)}\bar{N}^a_{(8m)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{D}^c_{(8)}\right)_{ab}N^b_{(8m)},$$

are also chiral invariant. Using these solutions, and performing the chiral transformation, we can obtain the following relations:

$$\begin{aligned} -\mathbf{F}^{a\dagger}_{(8)}\mathbf{F}^b_{(8)} + \mathbf{F}^b_{(8)}\mathbf{F}^a_{(8)} + if_{abc}\mathbf{F}^c_{(8)} &= 0,\\ -\mathbf{F}^{a\dagger}_{(8)}\mathbf{D}^b_{(8)} + \mathbf{D}^b_{(8)}\mathbf{F}^a_{(8)} + if_{abc}\mathbf{D}^c_{(8)} &= 0. \end{aligned} \quad (43)$$

Note that the generators $\mathbf{F}^a_{(8)}$ defined in Ref. [1] are Hermitian matrices, i.e., $\mathbf{F}^{a\dagger}_{(8)} = \mathbf{F}^a_{(8)}$. Therefore, Eqs. (43) turn into the familiar SU(3) × SU(3) Lie commutators that have already been proven in Ref. [1]. This, once again, proves the consistency of our present calculation with that in Ref. [1].

We can use the other strategy which has been discussed in the previous section(s) to obtain these two interactions. One solution is

$$\mathcal{L}^{\rm PC}_{(8)} = g^{\rm PC}_{(8)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu(8)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu5(8)}\right], \quad (44)$$

where

$$\mathbf{J}^c_{\mu(8)} = \overline{N}^a_{(8)}\gamma^\mu\left(\mathbf{F}^c_{(8)}\right)_{ab}N^b_{(8)}, \quad \mathbf{J}^c_{\mu5(8)} = \overline{N}^a_{(8)}\gamma^\mu\gamma_5\left(\mathbf{F}^c_{(8)}\right)_{ab}N^b_{(8)}. \quad (45)$$

Here, we might think that the other solution can be obtained similarly by adding an extra γ_5 which we have done in the previous section(s). However, we find that the solution obtained in this way is same as the original solution. Therefore, in order to get the second solution we need to find another different set of $\mathbf{J}'^c_{\mu(8)}$ and $\mathbf{J}'^c_{\mu5(8)}$:

$$\mathbf{J}'^c_{\mu(8)} = \overline{N}^a_{(8)}\gamma^\mu\left(\mathbf{D}^c_{(8)}\right)_{ab}N^b_{(8)}, \quad \mathbf{J}'^c_{\mu5(8)} = \overline{N}^a_{(8)}\gamma^\mu\gamma_5\left(\mathbf{D}^c_{(8)}\right)_{ab}N^b_{(8)}, \quad (46)$$

and the second solution is

$$\mathcal{L}'^{\rm PC}_{(8)} = g'^{\rm PC}_{(8)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}'_{\mu(8)} + \mathbf{a}^\mu_1\cdot\mathbf{J}'_{\mu5(8)}\right]. \quad (47)$$

The relevant parity-violating partners are

$$\mathcal{L}^{\rm PV}_{(8)} = g^{\rm PV}_{(8)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu5(8)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu(8)}\right], \quad (48)$$

$$\mathcal{L}'^{\rm PV}_{(8)} = g'^{\rm PV}_{(8)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}'_{\mu5(8)} + \mathbf{a}^\mu_1\cdot\mathbf{J}'_{\mu(8)}\right]. \quad (49)$$

We can check the equivalence of these two sets of solutions, and verify the following relations:

$$\frac{\mathcal{L}^{\rm PC}_{(8)}}{g^{\rm PC}_{(8)}} = \frac{\mathcal{L}^F_{(8)}}{g^F_{(8)}}, \quad \frac{\mathcal{L}'^{\rm PC}_{(8)}}{g'^{\rm PC}_{(8)}} = \frac{\mathcal{L}^D_{(8)}}{g^D_{(8)}}. \quad (50)$$

4. Chiral [(10, 1) ⊕ (1, 10)] baryons diagonal interactions

The product of the first term inside the structure $(\bar{N}_L M_L N_L + \bar{N}_R M_R N_R)$ is decomposed as

$$\begin{aligned}(\bar{\mathbf{10}},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})\otimes(\mathbf{10},\mathbf{1}) &\sim [(\bar{\mathbf{10}},\mathbf{1})\otimes(\mathbf{10},\mathbf{1})]\otimes(\mathbf{8},\mathbf{1})\\ &\ni (\mathbf{8},\mathbf{1})\otimes(\mathbf{8},\mathbf{1}) \ni (\mathbf{1},\mathbf{1}). \end{aligned} \quad (51)$$

Therefore, there is only one chiral invariant combination. Consequently, following the same procedure as in the previous section(s), we find that there is only one solution, which can be written out using $\mathbf{F}^c_{(10)}$ in the following form ($\bar{\Delta}^a_{(10)}\gamma^\mu M^c_\mu \Delta^b_{(10)}\mathbf{C}^{abc}_{(10)}$):

$$\mathcal{L}_{(10)} = g_{(10)}\bar{\Delta}^a_{(10)}\gamma^\mu\left(\rho^c_\mu + \gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(10)}\right)_{ab}\Delta^b_{(10)}. \quad (52)$$

The chiral group structure for these two interactions is just shown in Eq. (51).

Besides the Lagrangian Eq. (52), its mirror part,

$$g_{(10m)}\bar{\Delta}^a_{(10m)}\gamma^\mu\left(\rho^c_\mu - \gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(10)}\right)_{ab}\Delta^b_{(10m)},$$

is also chiral invariant. Using these solutions, and performing the chiral transformation, we can obtain the following relations:

$$-\mathbf{F}^{a\dagger}_{(10)}\mathbf{F}^b_{(10)} + \mathbf{F}^b_{(10)}\mathbf{F}^a_{(10)} + if_{abc}\mathbf{F}^c_{(10)} = 0. \quad (53)$$

Note that the generators $\mathbf{F}^a_{(10)}$ defined in Ref. [1] are Hermitian matrices, i.e., $\mathbf{F}^{a\dagger}_{(10)} = \mathbf{F}^a_{(10)}$. Therefore, Eq. (53) turns into the familiar SU(3) × SU(3) Lie commutators that have already been proven in Ref. [1]. This, once again, proves the consistency of our present calculation with that in Ref. [1].

We can also use the other strategy which has been discussed in the previous section(s) to obtain this interaction:

$$\mathcal{L}^{\rm PC}_{(10)} = g^{\rm PC}_{(10)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu(10)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu5(10)}\right], \quad (54)$$

where

$$\begin{aligned}\mathbf{J}^c_{\mu(10)} &= \overline{N}^a_{(10)}\gamma^\mu\left(\mathbf{F}^c_{(10)}\right)_{ab}N^b_{(10)},\\ \mathbf{J}^c_{\mu5(10)} &= \overline{N}^a_{(10)}\gamma^\mu\gamma_5\left(\mathbf{F}^c_{(10)}\right)_{ab}N^b_{(10)}.\end{aligned} \quad (55)$$

The relevant parity-violating partner is

$$\mathcal{L}^{\rm PV}_{(10)} = g^{\rm PV}_{(10)}\left[\boldsymbol{\rho}^\mu\cdot\mathbf{J}_{\mu5(10)} + \mathbf{a}^\mu_1\cdot\mathbf{J}_{\mu(10)}\right]. \quad (56)$$

C. Chiral mixing interactions

To construct chiral invariant off-diagonal interactions, we need to consider the following off-diagonal terms in the chiral base, in other words, it can also take the form

$$\begin{aligned}\bar{N}MN' &\sim \bar{N}_L M N'_R + \bar{N}_R M N'_L\\ &= (\bar{N}_L M_L N'_R + \bar{N}_R M_R N'_L)\\ &\quad + (\bar{N}_L M_R N'_R + \bar{N}_R M_L N'_L),\end{aligned} \quad (57)$$

when decomposed into the left and right components. However, to arrive at this form we need to use the mirror field $N'^{[\rm mir]}$ to have the correct helicity structure:

$$\bar{N}_L M N'_R + \bar{N}_R M N'_L \sim \bar{N}_L M N'^{[\rm mir]}_L + \bar{N}_R M N'^{[\rm mir]}_R. \quad (58)$$

TABLE I. Allowed chiral invariant Dirac type interaction terms with one $(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$ vector meson field $\bar{N}\gamma^{\mu}M_{\mu}N$. In the first column we show the chiral representation of N, and the first row the chiral representation of $\bar{N}$. We use "[mir]" to denote the relevant mirror fields.

	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$2\times M_{\mu}$			M_{μ}
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$		M_{μ}, $M_{\mu}^{\dagger}$		
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$			M_{μ}, $M_{\mu}^{\dagger}$	
$(\overline{\mathbf{10}},\mathbf{1})\oplus(\mathbf{1},\overline{\mathbf{10}})$	M_{μ}			M_{μ}
	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$		
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$[mir]		M_{μ}		
$(\bar{\mathbf{3}},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\bar{\mathbf{3}})$[mir]	$M_{\mu}^{\dagger}$			

1. Chiral mixing interaction $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$

The product of the first term inside $(\bar{N}_L M_L N_L'^{[\mathrm{mir}]}+\bar{N}_R M_R N_R'^{[\mathrm{mir}]})$ is decomposed as

$$(\bar{\mathbf{6}},\bar{\mathbf{3}})\otimes(\mathbf{8},\mathbf{1})\otimes(\bar{\mathbf{3}},\mathbf{3})\sim[(\bar{\mathbf{6}},\bar{\mathbf{3}})\otimes(\bar{\mathbf{3}},\mathbf{3})]\otimes(\mathbf{8},\mathbf{1})$$
$$\ni(\mathbf{8},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})\ni(\mathbf{1},\mathbf{1}). \quad (59)$$

Therefore, there is one chiral invariant combination, and we find that the mixing of $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ with $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]_{[\mathrm{mir}]}$ baryon fields together with an $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ chiral multiplet of vector and axial-vector meson fields can form a chiral singlet. We find the following form of the chiral invariant interaction

$$\overline{N}^a_{(9m)}\gamma^{\mu}M^c_{\mu}N^b_{(18)}\mathbf{C}^{abc}_{(9/18)}+\mathrm{H.c.} \quad (60)$$

The coefficients $\mathbf{C}^{abc}_{(9/18)}$ can be similarly obtained as in Eq. (15), and once again we find a parity-conserving interaction

$$\mathcal{L}^{\mathrm{PC}}_{(9/18)}=g^{\mathrm{PC}}_{(9/18)}\big[\overline{N}^a_{(9m)}\gamma^{\mu}\big(\rho^c_{\mu}+\gamma_5 a^c_{1\mu}\big)$$
$$\times\big(\mathbf{T}^c_{(9/18)}\big)_{ab}N^b_{(18)}+\mathrm{H.c.}\big], \quad (61)$$

and a parity-violating partner

$$\mathcal{L}^{\mathrm{PV}}_{(9/18)}=g^{\mathrm{PV}}_{(9/18)}\big[\overline{N}^a_{(9m)}\gamma^{\mu}\gamma_5\big(\rho^c_{\mu}+\gamma_5 a^c_{1\mu}\big)$$
$$\times\big(\mathbf{T}^c_{(9/18)}\big)_{ab}N^b_{(18)}+\mathrm{H.c.}\big], \quad (62)$$

with

$$\mathbf{T}^c_{(9/18)}=\begin{pmatrix}\frac{1}{2}\mathbf{T}^c_{(1/8)} & \mathbf{0}_{1\times 10}\\ -\frac{\sqrt{3}}{2\sqrt{2}}\mathbf{D}^c_{(8)}-\frac{1}{2\sqrt{6}}\mathbf{F}^c_{(8)} & \frac{1}{\sqrt{2}}\mathbf{T}^c_{(8/10)}\end{pmatrix}, \quad (63)$$

that satisfies the following relation:

$$\mathbf{F}^{a\dagger}_{(9)}\mathbf{T}^b_{(9/18)}+\mathbf{T}^b_{(9/18)}\mathbf{F}^a_{(18)}+if_{abc}\mathbf{T}^c_{(9/18)}=0. \quad (64)$$

2. Chiral mixing interaction $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ – $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$

The product of the first term inside $(\bar{N}_L M_L N_L+\bar{N}_R M_R N_R)$ is decomposed as

$$(\bar{\mathbf{10}},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})\sim[(\bar{\mathbf{10}},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})]\otimes(\mathbf{8},\mathbf{1})$$
$$\ni(\mathbf{8},\mathbf{1})\otimes(\mathbf{8},\mathbf{1})\ni(\mathbf{1},\mathbf{1}). \quad (65)$$

Therefore, there is one chiral invariant combination, and we find that the mixing of $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ with $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ baryon fields together with an $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ chiral multiplet of vector and axial-vector meson fields can form a chiral singlet. We find the following form of the chiral invariant interaction:

$$\overline{N}^a_{(8)}\gamma^{\mu}M^c_{\mu}N^b_{(10)}\mathbf{C}^{abc}_{(8/18)}+\mathrm{H.c.} \quad (66)$$

The coefficients $\mathbf{C}^{abc}_{(8/18)}$ can be similarly obtained as in Eq. (15), and once again we find a parity-conserving interaction

$$\mathcal{L}^{\mathrm{PC}}_{(8/18)}=g^{\mathrm{PC}}_{(8/18)}\big[\overline{N}^a_{(8)}\gamma^{\mu}\big(\rho^c_{\mu}+\gamma_5 a^c_{1\mu}\big)\big(\mathbf{T}^c_{(8/10)}\big)_{ab}N^b_{(10)}+\mathrm{H.c.}\big], \quad (67)$$

and a parity-violating partner

$$\mathcal{L}^{\mathrm{PV}}_{(8/18)}=g^{\mathrm{PV}}_{(8/10)}\big[\overline{N}^a_{(8)}\gamma^{\mu}\gamma_5\big(\rho^c_{\mu}+\gamma_5 a^c_{1\mu}\big)$$
$$\times\big(\mathbf{T}^c_{(8/10)}\big)_{ab}N^b_{(10)}+\mathrm{H.c.}\big]. \quad (68)$$

We find that the only solution is formed by the $\mathbf{T}^c_{(8/10)}$ matrices defined in Ref. [1] that satisfy the following relation:

$$-\mathbf{F}^{a\dagger}_{(8)}\mathbf{T}^b_{(8/10)}+\mathbf{T}^b_{(8/10)}\mathbf{F}^a_{(10)}+if_{abc}\mathbf{T}^c_{(8/10)}=0. \quad (69)$$

D. Brief summary of the Dirac type interactions

Note that all of the diagonal Dirac type interactions that were shown here also appear in a *local* $\mathrm{SU}_L(3)\times\mathrm{SU}_R(3)$ chiral symmetry Yang-Mills interaction. We have found more, however: there are chiral off-diagonal interaction terms that cannot be obtained by a minimal substitution in the kinetic energy, of the Yang-Mills type because *per definition* the kinetic energies are diagonal operators in the chiral representation space. These off-diagonal terms show up in the flavor-decimet channel, and therefore are physically less accessible than the diagonal flavor-octet ones, which are easier to access by way of elastic scattering. Here, as well, there are strong chiral selection rules.

III. ONE-DERIVATIVE PAULI TYPE INTERACTIONS

Now let us look at one-derivative Pauli type interactions. They lead to the anomalous magnetic moments through vector meson dominance. The interaction terms take in general the following form:

$$\bar{N}^a\sigma^{\mu\nu}\partial_{\nu}M^c_{\mu}N^b\mathbf{C}_{abc}, \quad (70)$$

which has the helicity flip structure, i.e., $\bar{N}_L\mathcal{O}N_R$. Due to this structure, the chiral selection rules are far more restrictive than

TABLE II. Allowed chiral invariant Pauli type interaction terms with one $(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$ vector meson field $\bar{N}\sigma^{\mu\nu}\partial_\nu M_\mu N$. In the first column we show the chiral representation of N, and the first row the chiral representation of $\bar{N}$. We use "[mir]" to denote the relevant mirror fields.

	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$[mir]	$2\times M_\mu$			M_μ
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$[mir]		M_μ, $M_\mu^\dagger$		
$(\bar{\mathbf{3}},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\bar{\mathbf{3}})$[mir]			M_μ, $M_\mu^\dagger$	
$(\mathbf{1},\overline{\mathbf{10}})\oplus(\overline{\mathbf{10}},\mathbf{1})$[mir]	M_μ			M_μ
	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$		
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$		M_μ		
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$	$M_\mu^\dagger$			

otherwise. As first noted by Dashen and Gell-Mann [16], all of the diagonal anomalous magnetic interactions must vanish due to such chiral symmetry restrictions.

All of the off-diagonal anomalous magnetic interactions can be easily obtained from the off-diagonal and diagonal Dirac type ones in Sec. II in many cases by simply substituting one of the baryon fields with its mirror one. Mixing of various combinations of chiral multiplets gives the following chiral invariant interactions:

(1) For $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})]_{[\mathrm{mir}]}$ the chiral invariant interactions are

$$\left(\frac{\kappa^A_{(18)}}{2M}\right)\overline{N}^a_{(18)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{V}^c_{(18)}+\mathbf{F}^c_{(18)}\right)_{ab}N^b_{(18m)} + \mathrm{H.c.},$$
$$\left(\frac{\kappa^B_{(18)}}{2M}\right)\overline{N}^a_{(18)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu+\gamma_5 a^c_{1\mu}\right)\left(\mathbf{V}^c_{(18)}-\mathbf{F}^c_{(18)}\right)_{ab}N^b_{(18m)} + \mathrm{H.c.} \tag{71}$$

(2) For $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$-$[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]_{[\mathrm{mir}]}$ the chiral invariant interactions are

$$\left(\frac{\kappa^A_{(9)}}{2M}\right)\overline{N}^a_{(9)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{V}^c_{(9)}+\mathbf{F}^c_{(9)}\right)_{ab}N^b_{(9m)} + \mathrm{H.c.},$$
$$\left(\frac{\kappa^B_{(9)}}{2M}\right)\overline{N}^a_{(9)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu+\gamma_5 a^c_{1\mu}\right)\left(\mathbf{V}^c_{(9)}-\mathbf{F}^c_{(9)}\right)_{ab}N^b_{(9m)} + \mathrm{H.c.} \tag{72}$$

(3) For $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$-$[(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})]_{[\mathrm{mir}]}$ the chiral invariant interactions are

$$\left(\frac{\kappa^A_{(8)}}{2M}\right)\overline{N}^a_{(8)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(8)}\right)_{ab}N^b_{(8m)} + \mathrm{H.c.},$$
$$\left(\frac{\kappa^B_{(8)}}{2M}\right)\overline{N}^a_{(8)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{D}^c_{(8)}\right)_{ab}N^b_{(8m)} + \mathrm{H.c.} \tag{73}$$

(4) For $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$-$[(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})]_{[\mathrm{mir}]}$ the chiral invariant interactions are

$$\left(\frac{\kappa_{(10)}}{2M}\right)\overline{\Delta}^a_{(10)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{F}^c_{(10)}\right)_{ab}\Delta^b_{(10m)} + \mathrm{H.c.} \tag{74}$$

(5) For $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ the chiral invariant interactions are

$$\left(\frac{\kappa_{(9/18)}}{2M}\right)\overline{N}^a_{(9)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu+\gamma_5 a^c_{1\mu}\right)\left(\mathbf{T}^c_{(9/18)}\right)_{ab}N^b_{(18)} + \mathrm{H.c.} \tag{75}$$

(6) For $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$-$[(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})]_{[\mathrm{mir}]}$ the chiral invariant interactions are

$$\left(\frac{\kappa_{(8/10)}}{2M}\right)\overline{N}^a_{(8)}\sigma^{\mu\nu}\partial_\nu\left(\rho^c_\mu-\gamma_5 a^c_{1\mu}\right)\left(\mathbf{T}^c_{(8/10)}\right)_{ab}N^b_{(10m)} + \mathrm{H.c.} \tag{76}$$

From the above summary of the interactions, we note that for the normal-normal (also known as the "naive-naive") combination, only $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ survives, whereas the naive-mirror one $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]_{[\mathrm{mir}]}$ vanishes, which is a selection rule due to SU(3) × SU(3) chiral symmetry. Furthermore, as the first mixing term preserves the $\mathrm{U}_A(1)$ symmetry, while the second one does not, we are forced to conclude that the selection rule leads to the Harari scenario where the $\mathrm{U}_A(1)$ symmetry is maintained as the only viable one in this three-quark baryon field and no chiral mixing in vector mesons approximation, whereas the Gerstein-Lee one is effectively ruled out by the chiral selection rule [13].

In order to have a realistic anomalous magnetic moment it may be necessary to include the flavor-singlet, chiral-singlet vector meson ϕ_μ. Once again, there are chiral selection rules that strongly prefer the Harari scenario.

A. The anomalous magnetic moment results: Comparison with experiment in the SU(3) symmetry limit

Thus far we have studied the anomalous magnetic moments of baryon fields and found specific constraints due to chiral symmetry. The basic quantity that we address is the D/F ratio for the baryon anomalous magnetic moments, whose "experimental value" has been extrapolated to $D/F \simeq 3$ in the SU(3) symmetry limit. Note that that is precisely the value that shows up in the $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ baryon mixing interaction. Indeed, all of the other chiral interactions have a vanishing D components.

Of course, it is not one, but a linear combination ("admixture") of three chiral representations that describe the physical baryon states, as explained in the Introduction and in Refs. [1,2,10,13] where we found two candidate chiral mixing scenarios: a) the Harari one, i.e., $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$-$[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$; and b) the Lee-Gerstein one, i.e., $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$-$[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$. As the $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ multiplet shows up only in the Harari scenario, this is a "smoking gun" evidence supporting it, and overturning the Gerstein-Lee one, subject to the no-chiral-mixing assumption in the vector meson sector.

Next we may consider the chiral mixing for the $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ vector mesons with the $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ component. One may use our old results, Refs. [13,14,17], to do so and relax this last assumption: the $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ chiral component of the vector mesons couples magnetically to the baryons chiral multiplets in exactly the same fashion as the spinless $[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ mesons treated in Ref. [13]. So we may use Eq. (35) in Ref. [13] to read off the F and D values of the anomalous magnetic moments in such a scheme: they are $F=1$ and $D=0$, thus leading to $D/F=0$, again in stark contrast to the "experimental" value $D/F\simeq 3$ in the SU(3) symmetry limit. This eliminates the chiral mixing of vector meson as a viable explanation of the baryons' magnetic moments. Indeed, it seems to imply certain limits on the amount of such chiral mixing, that will be explored elsewhere.

Note that these results hold even in the chiral limit and have nothing to do with the value of the pion-nucleon Σ-term as suggested in Ref. [18]. Moreover, the chiral/flavor-singlet vector meson field couples with arbitrary strength to baryons, which introduces arbitrary "strange" anomalous magnetic moment, again even in the chiral limit.

IV. SUMMARY AND CONCLUSIONS

We have used the results of our previous papers [1,10] to construct the $SU_L(3)\times SU_R(3)$ chiral invariant interactions of baryon fields with vector mesons. This approach is based on the chiral $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ multiplet mixing with the chiral $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ and $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ multiplets and is constrained by the well known phenomenological facts regarding the baryon axial currents.

The results of the three-field ("two-angle") mixing were ambiguous insofar as all phenomenologically permissible combinations of interpolating fields lead to the same F,D values, in reasonable agreement with the result extrapolated from experiment in the SU(3) symmetry limit. That led to two permissible scenarios: a) the Gerstein-Lee [3] and b) the Harari scenario [5,6], neither of which could be eliminated on the basis of axial currents and baryon masses alone.

What was left unfinished were the magnetic moments of the baryon octet. Here we attacked that problem by first constructing all $SU_L(3)\times SU_R(3)$ chirally symmetric baryon-one-vector-meson interactions that mix the three basic baryon chiral multiplets (and their mirror counterparts). All of these chiral interactions obey the $U_A(1)$ symmetry, as well.

We used the resulting interactions' chiral selection rules to select the only scenario that can reproduce the observed anomalous magnetic moments: the Harari scenario. Moreover, the magnetic moment $F/D\simeq 1/3$ predicted by the chiral interaction, has the same value as in the $SU(6_{FS})$ symmetry limit, or as in the nonrelativistic quark model. This last fact is curious and requires further investigation.

The next step, left for the future, is to investigate the $SU_L(3)\times SU_R(3)\to SU_L(2)\times SU_R(2)$ symmetry breaking and the study of the chiral $SU_L(2)\times SU_R(2)$ properties of hyperons. Then one may consider explicit chiral symmetry breaking corrections to the axial and the vector currents, which are related to the $SU_L(3)\times SU_R(3)$ symmetry breaking meson-nucleon derivative interactions, not just the explicit SU(3) symmetry breaking ones that have been considered thus far (see Ref. [7] and the previous subsection).

We finish on a historical note: even though chiral mixing has been known for more than 40 years [24–27], the $SU_L(3)\times SU_R(3)$ chiral interactions necessary to describe the anomalous magnetic moments have not been discussed in print, only the problems associated with them [28]. Moreover, it ought to be noted that Gerstein and Lee [4] had calculated anomalous magnetic moments of the nucleons that were in agreement with experiment in their chiral mixing scheme. These authors apparently did not try to extend their scheme to hyperons, however, nor did they construct a chiral Lagrangian that reproduces such chiral mixing.

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