# Relativistic effect of spin and pseudospin symmetries

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Dirac Hamiltonian is scaled in the atomic units $\hbar = m = 1$, which allows us to take the nonrelativistic limit by setting the Compton wavelength $\lambda \to 0$. The evolutions of the spin and pseudospin symmetries toward the nonrelativistic limit are investigated by solving the Dirac equation with the parameter $\lambda$. Setting the $\lambda$ transformation from the original Compton wavelength to 0, the spin splittings decrease monotonously in all spin doublets, and the pseudospin splittings increase in several pseudospin doublets, show no change, or even reduce in several other pseudospin doublets. The various energy splitting behaviors of both the spin and pseudospin doublets with $\lambda$ are well explained by the perturbation calculations of the Dirac Hamiltonian in the present units. It indicates that the origin of spin symmetry is entirely due to the relativistic effect, while the origin of pseudospin symmetry cannot be uniquely attributed to the relativistic effect.

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It is well known that the spin and pseudospin symmetries play a critical role in the shell structure and its evolution. The introduction of the spin-orbit potential to the single-particle shell model can well explain the experimentally observed existence of magic numbers for nuclei close to the valley of $\beta$ stability [1,2]. To understand the near degeneracy observed in heavy nuclei between two single-particle states with the quantum numbers $(n-1, l+2, j = l+3/2)$ and $(n, l, j = l+1/2)$, the pseudospin symmetry (PSS) was introduced by defining the pseudospin doublets $(\tilde{n} = n-1, \tilde{l} = l+1, j = \tilde{l} \pm 1/2)$ [3,4], which has explained numerous phenomena in nuclear structure including deformation [5], superdeformation [6], identical bands [7], and magnetic moment [8]. Because of these successes, there have been comprehensive efforts to understand their origins as well as the breaking mechanisms. For the spin symmetry (SS), the spin-orbit potential can be obtained naturally from the solutions of the Dirac equation. Thus, the SS can be regarded as a relativistic symmetry. For the PSS, its origin has not been fully clarified until now. It is worth reviewing some of the major progresses in understanding the underlying mechanism of PSS. In Ref. [9], a helicity unitary transformation of a nonrelativistic single-particle Hamiltonian was introduced to discuss the PSS in the nonrelativistic harmonic oscillator. The particular condition between the coefficients of spin-orbit and orbit-orbit terms was indicated in the corresponding nonrelativistic single-particle Hamiltonian for the requirement of PSS. The same kind of unitary transformation was considered in Ref. [10], where the application of the helicity operator to the nonrelativistic single-particle wave function maps the normal state $(l, s)$ onto the pseudostate $(\tilde{l}, \tilde{s})$, while keeping all other global symmetries. A substantial progress was achieved in Ref. [11], where the relativistic feature of PSS was recognized. The pseudo-orbital angular momentum $\tilde{l}$ is nothing but the orbital angular momentum of the lower component of the Dirac spinor, and the equality in magnitude but difference in sign of the scalar potential $S$ and vector potential $V$ was suggested as the exact PSS limit. Meng *et al.* showed that exact PSS occurs in the Dirac equation when the sum of the scalar $S$ and vector $V$ potentials is equal to a constant [12]. Unfortunately, the exact PSS cannot be met in real nuclei, much effort has been devoted to the cause of splitting. In Refs. [13–15], it was pointed out that the observed pseudospin splitting arises from a cancellation of the several energy components, and the PSS in nuclei has a dynamical character. A similar conclusion was reached in Refs. [16,17]. In addition, it was noted that, unlike the spin symmetry, the pseudospin breaking cannot be treated as a perturbation of the pseudospin-symmetric Hamiltonian [18]. The nonperturbation nature of PSS has also been indicated in Ref. [19]. Regardless of these pioneering studies, the origins of the spin and pseudospin symmetries have not been fully understood in the relativistic framework. Recently, we have checked the PSS by use of the similarity renormalization group and shown explicitly the relativistic origin of this symmetry [20]. However, the dependence of the quality of PSS on the relativistic effect has not been checked until now. In this paper, we study the evolution of the spin and pseudospin symmetries from the relativistic to the nonrelativistic to explore the relativistic relevance of this symmetries.

The Dirac equation of a particle of mass $m$ in external scalar $S$ and vector $V$ potentials is given by

$$H = c\vec{\alpha} \cdot \vec{p} + \beta(mc^2 + S) + V, \tag{1}$$

where $\vec{\alpha}$ and $\beta$ are the usual Dirac matrices. For a spherical system, the Dirac spinor $\psi$ has the form

$$\psi = \frac{1}{r} \begin{pmatrix} iG_{n\kappa}(r)\phi_{\kappa m_j}(\vartheta, \varphi) \\ F_{n\kappa}(r)\vec{\sigma} \cdot \hat{r}\phi_{\kappa m_j}(\vartheta, \varphi) \end{pmatrix}, \tag{2}$$

where $n$ is the radial quantum number and $m_j$ is the projection of angular momentum on the third axis. $\kappa = \pm(j + 1/2)$ with "$-$" for aligned spin ($s_{1/2}$, $p_{3/2}$, etc.), and "$+$" for unaligned spin ($p_{1/2}$, $d_{3/2}$, etc.). Splitting off the angular part and leaving the radial functions satisfy the following equation:

$$\begin{pmatrix} mc^2 + \Sigma(r) & -c\frac{d}{dr} + \frac{c\kappa}{r} \\ c\frac{d}{dr} + \frac{c\kappa}{r} & -mc^2 + \Delta(r) \end{pmatrix} \begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \varepsilon \begin{pmatrix} G(r) \\ F(r) \end{pmatrix}, \tag{3}$$

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where $\Sigma(r) = V(r) + S(r)$ and $\Delta(r) = V(r) - S(r)$. Based on Eq. (3), a lot of work has been carried out to check the origins of the spin and pseudospin symmetries [11,13–15,18,19]. Although they are recognized as the symmetries of the Dirac Hamiltonian, the important role of the relativistic effect is still not very clear. In order to explore the relativistic effects of this symmetries, the atomic units $\hbar = m = 1$ are adopted instead of the conventional relativistic units $\hbar = c = 1$ in the present system. For simplicity, the operator $H$ is measured in unit of the rest mass, $mc^2$. Then the equation (3) is presented as

$$\begin{pmatrix} 1+\lambda^2\Sigma & \lambda\left(-\frac{d}{dr}+\frac{\kappa}{r}\right) \\ \lambda\left(\frac{d}{dr}+\frac{\kappa}{r}\right) & -1+\lambda^2\Delta \end{pmatrix}\begin{pmatrix} G(r) \\ F(r) \end{pmatrix} = \varepsilon \begin{pmatrix} G(r) \\ F(r) \end{pmatrix}, \quad (4)$$

where the Compton wavelength $\lambda = \hbar/mc = 1/c$. In these units, the result in the nonrelativistic limit can be obtained in a very simple, intuitive, and straightforward manner by taking the speed of light $c \to \infty$ or the Compton wavelength $\lambda \to 0$, which is not possible in the latter units since $c = 1$.

In order to investigate the evolution from the relativistic to the nonrelativistic, $\lambda$ is regarded as a parameter and the original Compton wavelength $\lambda = \hbar/mc$ is labeled as $\lambda_0$. The relativistic result corresponds to the solution of Eq. (4) with $\lambda = \lambda_0$. The result in the nonrelativistic limit can be obtained from Eq. (4) by setting $\lambda \to 0$. Thus, the evolution from the relativistic to the nonrelativistic can be checked by transforming $\lambda$ from $\lambda_0$ to 0. Then, the relativistic effects of the spin and pseudospin symmetries can be investigated by extracting the energy splittings between the spin or pseudospin doublets, and these symmetries that develop toward the nonrelativistic limit can be checked, and vice versa.

In order to make this clear, we have solved Eq. (4) for a Woods-Saxon type potential for $\Sigma(r)$ and $\Delta(r)$, i.e., $\Sigma(r) = \Sigma_0 f(a_\Sigma, r_\Sigma, r)$ and $\Delta(r) = \Delta_0 f(a_\Delta, r_\Delta, r)$ with

$$f(a_0, r_0, r) = \frac{1}{1+\exp\left(\frac{r-r_0}{a_0}\right)}. \quad (5)$$

The corresponding parameters are determined by fitting the energy spectra from the RMF calculations for $^{208}$Pb (see Ref. [21]). The energy spectra of Eq. (4) are calculated by expansion in harmonic oscillator basis.

The single-particle energy varying with the parameter $\lambda$ is displayed in Fig. 1, where it can be seen that the energy decreases monotonously with $\lambda$ decreasing for all the levels available. The trend of energy with $\lambda$ is toward the direction of the nonrelativistic limit. With the decreasing of $\lambda$, the calculation is closer to the nonrelativistic result. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the solution of Eq. (4) is almost the same as the nonrelativistic result. Furthermore, for the different single-particle states, the sensitivity of energy to $\lambda$ is different. For the spin unaligned states, the decreasing of energy is faster than that for the spin aligned states, which leads to the energy splittings of the spin doublets which reduce with decreasing $\lambda$. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the spin-orbit splittings almost disappear for all the spin doublets. These indicate that the spin symmetry becomes better as $\lambda$ decreases, and

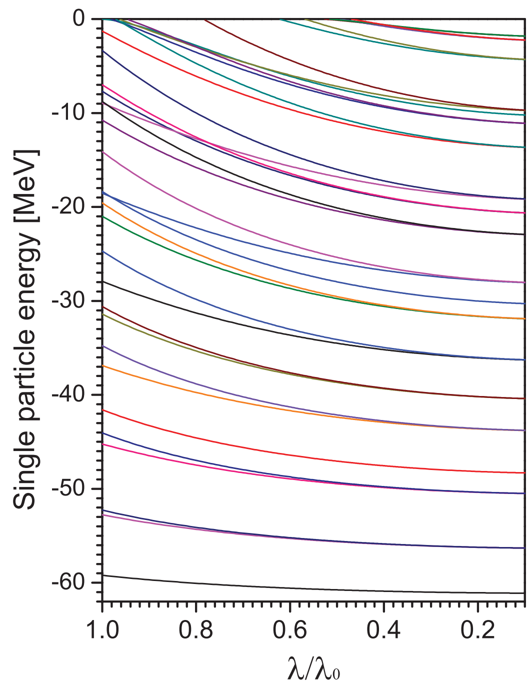


FIG. 1. (Color online) Variation of single particle energy with $\lambda/\lambda_0$.

the spin symmetry breaking is entirely due to the relativistic effect.

To better understand the preceding claim, the energies in several $\lambda$ values are listed in Table I for all single-particle levels. For comparison, Table I does also display the data of the nonrelativistic calculations (the last column), which are obtained by solving the Schrödinger equation $H\psi(r) = E\psi(r)$ with $H = -\frac{\hbar^2}{2m}(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}) + \Sigma(r)$. From Table I, it can be seen that the relativistic spin-orbit splitting ($\lambda = \lambda_0$) is considerably large. This splitting decreases with the decreasing of $\lambda$. When $\lambda/\lambda_0 = 0.001$, the energy of the spin unaligned state in conjunction with that of the spin aligned state degenerates to the nonrelativistic result. These indicate that Eq. (4) reproduces well the process of development from the relativistic to the nonrelativistic, and both results of the relativistic and nonrelativistic can be obtained well from Eq. (4) with an appropriate value of $\lambda$. Hence, the relativistic effects of the spin and pseudospin symmetries can be checked from the solutions of the Dirac equation with the parameter $\lambda$.

In order to recognize clearly the relativistic effect of spin symmetry, the energy splittings of spin doublets varying with $\lambda$ are plotted in Fig. 2, where it is shown that the energy splittings decrease monotonously with reducing $\lambda$ for all the spin partners. When $\lambda$ is reduced to $\lambda/\lambda_0 = 0.1$, the energy splittings of all the spin doublets are almost reduced to zero. The detailed observation shows that the energy splittings are more sensitive to $\lambda$ for the states with higher orbital angular momentum in the same radial quantum number. For the states with the same orbital angular momentum, the energy splittings appear as crosses in the different radial quantum numbers. These reflect that the relativistic sensitivity

TABLE I. The relativistic bound energies ($E = \varepsilon - m$, in MeV) of a Dirac particle for the Woods-Saxon potential with $\lambda/\lambda_0 = 1, 0.5, 0.1, 0.01, 0.001$. The last column represents the nonrelativistic results, which are obtained from the solutions of the Schrödinger equation $H\psi(r) = E\psi(r)$ with $H = -\frac{\hbar^2}{2m}(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2}) + \Sigma(r)$.

| $\lambda/\lambda_0$ | 1 | 0.5 | 0.1 | 0.01 | 0.001 | non |
|---|---|---|---|---|---|---|
| $1s_{1/2}$ | −59.206 | −60.756 | −61.109 | −61.123 | −61.123 | −61.123 |
| $2s_{1/2}$ | −41.592 | −47.056 | −48.305 | −48.353 | −48.353 | −48.353 |
| $3s_{1/2}$ | −18.358 | −27.968 | −30.287 | −30.376 | −30.377 | −30.377 |
| $1p_{3/2}$ | −52.763 | −55.631 | −56.306 | −56.332 | −56.332 | −56.332 |
| $1p_{1/2}$ | −52.263 | −55.578 | −56.304 | −56.332 | −56.332 | |
| $2p_{3/2}$ | −31.401 | −38.680 | −40.396 | −40.462 | −40.463 | −40.463 |
| $2p_{1/2}$ | −30.611 | −38.576 | −40.393 | −40.462 | −40.463 | |
| $3p_{3/2}$ | −7.694 | −17.957 | −20.633 | −20.737 | −20.738 | −20.738 |
| $3p_{1/2}$ | −6.999 | −17.822 | −20.628 | −20.737 | −20.738 | |
| $1d_{5/2}$ | −45.234 | −49.459 | −50.493 | −50.533 | −50.534 | −50.534 |
| $1d_{3/2}$ | −44.055 | −49.329 | −50.489 | −50.533 | −50.534 | |
| $2d_{5/2}$ | −20.999 | −29.752 | −31.897 | −31.980 | −31.981 | −31.981 |
| $2d_{3/2}$ | −19.573 | −29.543 | −31.890 | −31.980 | −31.981 | |
| $1f_{7/2}$ | −36.882 | −42.381 | −43.786 | −43.841 | −43.842 | −43.842 |
| $1f_{5/2}$ | −34.775 | −42.137 | −43.779 | −43.841 | −43.842 | |
| $2f_{7/2}$ | −10.759 | −20.436 | −22.933 | −23.031 | −23.032 | −23.031 |
| $2f_{5/2}$ | −8.777 | −20.102 | −22.922 | −23.031 | −23.032 | |
| $1g_{9/2}$ | −27.921 | −34.508 | −36.276 | −36.346 | −36.347 | −36.346 |
| $1g_{7/2}$ | −24.701 | −34.114 | −36.264 | −36.346 | −36.347 | |
| $1h_{11/2}$ | −18.545 | −25.944 | −28.044 | −28.127 | −28.128 | −28.128 |
| $1h_{9/2}$ | −14.117 | −25.366 | −28.025 | −28.127 | −28.128 | |
| $1i_{13/2}$ | −8.942 | −16.792 | −19.167 | −19.262 | −19.263 | −19.263 |
| $1i_{11/2}$ | −3.361 | −16.000 | −19.141 | −19.262 | −19.263 | |

is different for the states with different quantum numbers. When $\lambda$ is reduced to zero, the spin-orbit splittings disappear for all the spin partners, the nonrelativistic results are obtained in excellent agreement with those from the solutions of the Schrödinger equation. Namely, the spin-orbit splitting arises completely from the relativistic effect, and can be treated as a perturbation of the spin-symmetric Hamiltonian as indicated in Ref. [18].

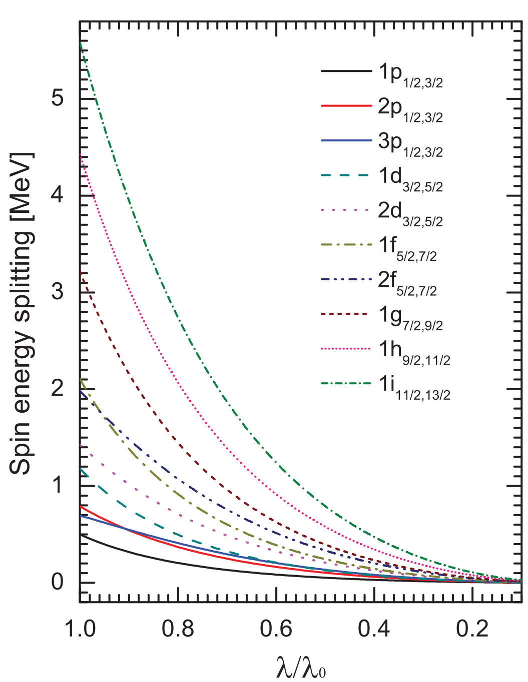


FIG. 2. (Color online) The energy splittings of spin doublets $\Delta E = E_{n,l-1/2} - E_{n,l+1/2}$ varying with $\lambda/\lambda_0$.

Different from the spin symmetry, the relativistic origin of pseudospin symmetry is more complicated. In Fig. 3, we display the energy splittings of pseudospin doublets varying with the parameter $\lambda$. From there, it can be observed that the energy splittings increase significantly with $\lambda$ decreasing for the pseudospin partners ($2g_{9/2}$,$1i_{11/2}$), ($2f_{7/2}$,$1h_{9/2}$), and ($3p_{3/2}$,$2f_{5/2}$). Especially for ($2g_{9/2}$,$1i_{11/2}$), the increasing of energy splitting is very obvious. For the doublets ($2d_{5/2}$,$1g_{7/2}$), the increasing of energy splitting with decreasing $\lambda$ is relatively small. When $\lambda/\lambda_0$ decreases below 0.6, the energy splitting goes toward a stable value. The same phenomenon also appears in the doublet ($3s_{1/2}$,$2d_{3/2}$). However for the pseudospin doublets ($2p_{3/2}$,$1f_{5/2}$) and ($2s_{1/2}$,$1d_{3/2}$), an opposite evolution of energy splitting with $\lambda$ is disclosed. It shows that the origin of pseudospin symmetry is more complicated than that of spin symmetry. The pseudospin splitting cannot be attributed uniquely to the relativistic effect. The quantum number of single-particle states and the shape of the potential have an important influence on this symmetry.

In order to better understand the relativistic effects of the spin and pseudospin symmetries, we expand perturbatively the Dirac Hamiltonian in Eq. (4) to analyze the effects of each higher-order term on the energy splitting behaviors of both spin and pseudospin doublets. Following Ref. [20], for the Dirac particle, the expanded Hamiltonian up to the order $1/m^3$

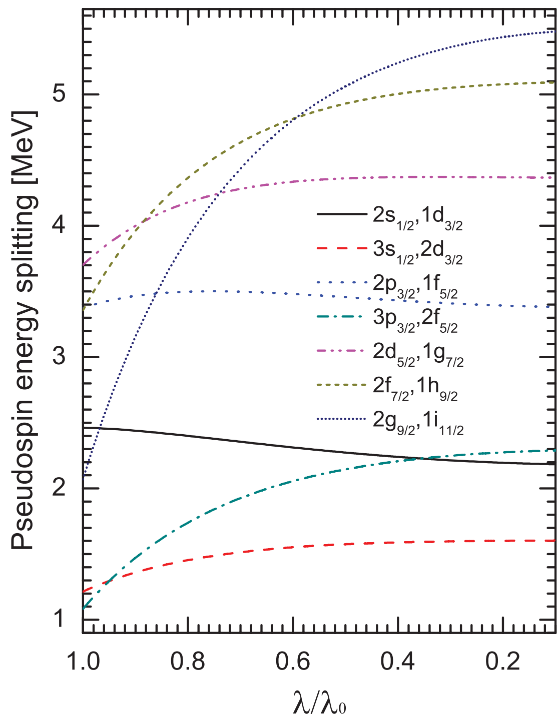


FIG. 3. (Color online) The energy splittings of pseudospin doublets $\Delta E = E_{n,\tilde{l}-1/2} - E_{n-1,\tilde{l}+1/2}$ varying with $\lambda/\lambda_0$.

is

$$H = \Sigma(r) + \frac{p^2}{2m} - \lambda^2 \frac{1}{2m^2}\left(Sp^2 - S'\frac{d}{dr}\right) - \lambda^2 \frac{\kappa}{r}\frac{\Delta'}{4m^2} + \lambda^4 \frac{S}{2m^3}\left(Sp^2 - 2S'\frac{d}{dr}\right) + \lambda^4 \frac{\kappa}{r}\frac{S\Delta'}{2m^3} + \lambda^2 \frac{\Sigma''}{8m^2} - \lambda^2 \frac{p^4}{8m^3} - \lambda^4 \frac{\Sigma'^2 - 2\Sigma'\Delta' + 4S\Sigma''}{16m^3}, \tag{6}$$

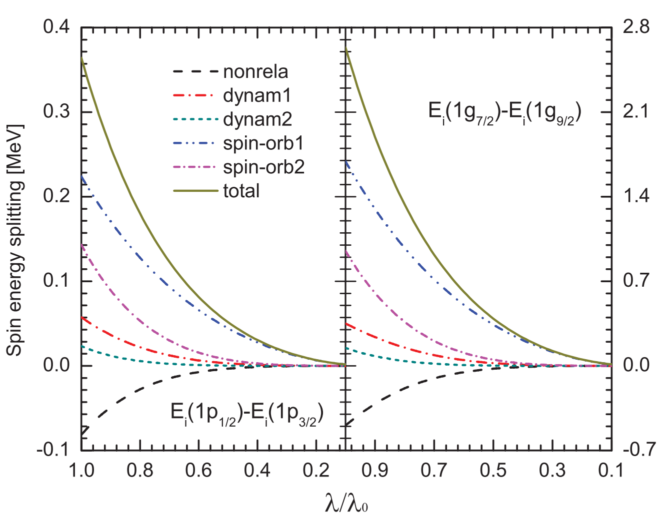


FIG. 4. (Color online) The spin energy splittings of each component $O_i$ $(i = 1, 2, \ldots, 5)$ varying with $\lambda$, where the splittings caused by the $O_1$, $O_2$, ..., $O_5$ are respectively labeled as "nonrela," "dynam1," "spin-orb1," "dynam2," "spin-orb2," and the total energy splitting is labeled as "total."

where $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2}$. Based on the same considerations as Ref. [20], $H$ is decomposed into the eight components: $\Sigma(r) + \frac{p^2}{2m}$, $-\lambda^2\frac{1}{2m^2}(Sp^2 - S'\frac{d}{dr})$, $-\lambda^2\frac{\kappa}{r}\frac{\Delta'}{4m^2}$, $+\lambda^4\frac{S}{2m^3}(Sp^2 - 2S'\frac{d}{dr})$, $+\lambda^4\frac{\kappa}{r}\frac{S\Delta'}{2m^3}$, $+\lambda^2\frac{\Sigma''}{8m^2}$, $-\lambda^2\frac{p^4}{8m^3}$, $-\lambda^4\frac{\Sigma'^2-2\Sigma'\Delta'+4S\Sigma''}{16m^3}$, which are respectively labeled as $O_1, O_2, \ldots, O_8$. $O_1$ corresponds to the Hamiltonian in the nonrelativistic limit, i.e., the Schrödinger part of $H$. $O_2(O_4)$ is the dynamical term relating to the order $1/m^2(1/m^3)$. $O_3(O_5)$ is the spin-orbit coupling corresponding to the order $1/m^2(1/m^3)$. The eigenvalues of $H$ are calculated with the fully same $\Sigma(r)$ and $\Delta(r)$ as that in calculating the exact solutions of Eq. (4).

For recognizing the relativistic effect of SS, we analyze the reason why the energies of the spin unaligned states decrease faster than those of the spin aligned states when $\lambda$ decreases. As an illustrated example, we display the energy splittings of every component $O_i(i = 1, 2, \ldots, 5)$ varying with $\lambda$ for the spin doublets $(1p_{1/2}, 1p_{3/2})$ and $(1g_{7/2}, 1g_{9/2})$ in Fig. 4, where we neglect the results of $O_6$, $O_7$, and $O_8$ because their contributions to the energy splitting are minor and do not influence on the total energy splitting behavior with $\lambda$. From Fig. 4 it can be seen that the contributions of all the $O_i(i = 2, 3, 4, 5)$ to the energy splittings between the spin unaligned states and the spin aligned states are positive, and the positive energy splittings decrease with decreasing $\lambda$. It is for this reason that the energies of the spin unaligned states decrease faster than those of the spin aligned states with decreasing $\lambda$. Compared with $O_3$ (the spin-orbit coupling corresponding to the order $1/m^2$), and the contributions of $O_2$, $O_4$, and $O_5$ to the spin energy splittings are relatively minor. The total energy splittings are dominated by the contribution of $O_3$ when $\lambda$ is sufficiently small. This means that, as the relativistic effect becomes weak, the spin splittings are almost entirely due to the spin-orbit coupling. For the different spin partners, the energy splitting behaviors with $\lambda$ are same except for the extent of splittings, as displayed in Fig. 4 for the spin doublets $(1p_{1/2}, 1p_{3/2})$ and $(1g_{7/2}, 1g_{9/2})$. These indicate that the spin symmetry originates completely from the relativistic effect, and possesses the perturbation attribute claimed in Ref. [18]. To understand the relativistic effect of PSS, we analyze the cause of the various energy splitting behaviors of pseudospin doublets. In Fig. 5, we show the energy splittings of each component $O_i(i = 1, 2, \ldots, 5)$ varying with $\lambda$ for the pseudospin partners $(2s_{1/2}, 1d_{3/2})$ and $(2f_{7/2}, 1h_{9/2})$. From there, it can be seen that the pseudospin energy splittings caused by the Schrödinger part of $H$ are dominated. This splittings are reduced by the contribution of spin-orbit coupling, and added by the contribution of dynamical terms. For the pseudospin partner $(2s_{1/2}, 1d_{3/2})$, with the decreasing of $\lambda$, the contribution of the pseudospin breaking (the dynamical terms) declines faster than that of the pseudospin improvement (the spin-orbit coupling), which results in better PSS when $\lambda$ decreases. However for the $(2f_{7/2}, 1h_{9/2})$, the energy splittings caused by the pseudospin breaking varying with $\lambda$ are relatively slower than that by the spin-orbit coupling, which leads to the PSS becoming worse with decreasing $\lambda$. These cause the different energy splitting behaviors of pseudospin doublets with $\lambda$. Hence, the

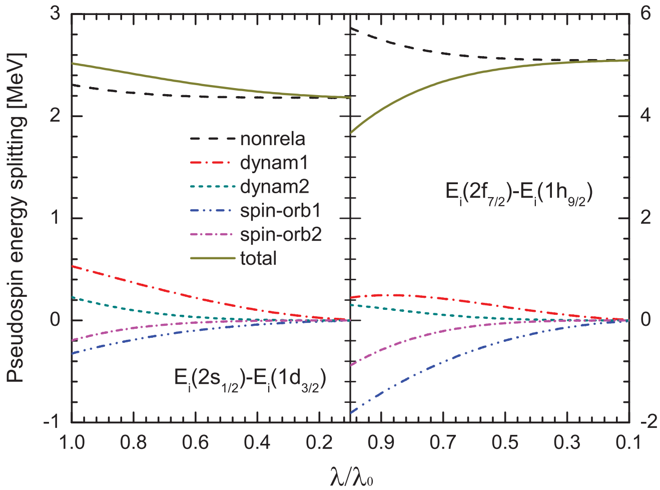


FIG. 5. (Color online) The same as Fig. 4, but for the pseudospin energy splittings.

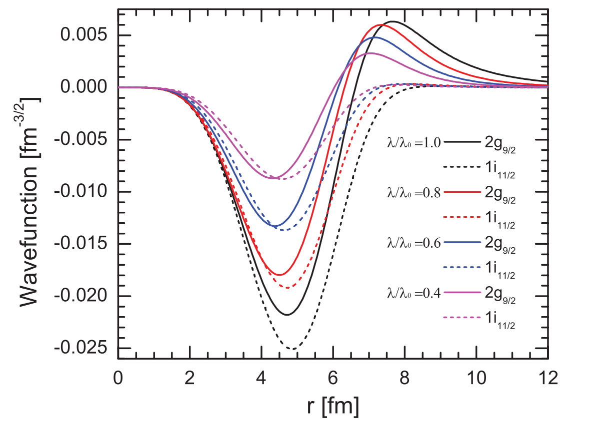


FIG. 7. (Color online) The lower component of the Dirac spinor $F(r)/r$ for the pseudospin doublet $(2g_{9/2},1i_{11/2})$ with $\lambda/\lambda_0 = 1.0, 0.8, 0.6, 0.4$.

pseudospin splitting cannot be regarded as a perturbation in agreement with the claim in Ref. [18].

In addition to the energy splittings associated with the relativistic effects, the wave function splittings between the (pseudo)spin doublets are also associated with the relativistic effects. An illustrated example is displayed in Fig. 6, where the upper component of the Dirac spinor for the states $1g_{7/2,9/2}$ is depicted in several $\lambda$ values. From Fig. 6, it can be seen that the wave function splitting of the spin doublet is obvious for a relativistic particle ($\lambda/\lambda_0 = 1$). With the development toward the nonrelativistic direction (to reduce $\lambda$), the wave function splitting of the spin doublet decreases, which is in agreement with the case of level splitting. For the pseudospin symmetry, the lower component of the Dirac spinor for the pseudospin doublet $(2g_{9/2},1i_{11/2})$ is drawn in Fig. 7 in several $\lambda$ values. The wave function splitting of the pseudospin doublet is obvious when $\lambda/\lambda_0 = 1$. Different from the spin splitting, we cannot see that the pseudospin splitting reduces with $\lambda$ decreasing, which is consistent with the case of level splitting.

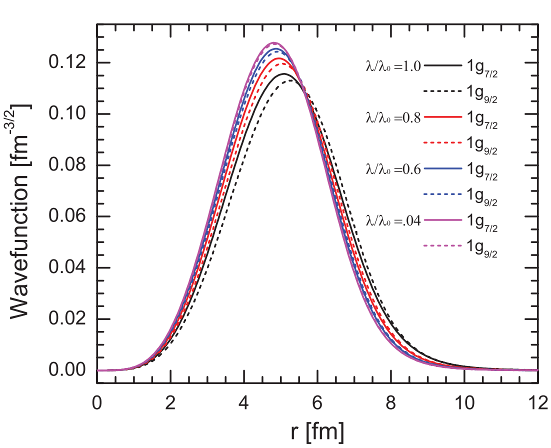


FIG. 6. (Color online) The upper component of the Dirac spinor $G(r)/r$ for the spin doublet $(1g_{7/2},1g_{9/2})$ with $\lambda/\lambda_0 = 1.0, 0.8, 0.6, 0.4$.

In summary, the Dirac Hamiltonian is scaled in the atomic units $\hbar = m = 1$, which allows us to take the nonrelativistic limit by setting the speed of light $c \to \infty$ or the Compton wavelength $\lambda \to 0$. The evolution toward the nonrelativistic limit is investigated from the solutions of the Dirac equation by a continuous transformation of the parameter $\lambda$. The solutions of the Dirac equation corresponding to $\lambda = \hbar/mc$ and $\lambda = 0$ represent, respectively, the relativistic result and that in the nonrelativistic limit. To transform the parameter $\lambda$ from $\hbar/mc$ to 0, the solutions of the Dirac equation show the evolution from the relativistic to the nonrelativistic limit. The relativistic effects of the spin and pseudospin symmetries are checked from the solutions of the Dirac equation with the parameter $\lambda$. It shows that the spin splittings decrease monotonously with reducing $\lambda$ for all the spin partners. When $\lambda$ is reduced to zero, the spin-orbit splittings disappear, which is in agreement with the result in the nonrelativistic calculations. For the pseudospin symmetry, the energy splittings increase in several partners, show no change, or even decrease in another set of partners. Compared with the spin symmetry, the origin of pseudospin symmetry is more complicated, and cannot be attributed uniquely to the relativistic effect. The quantum number of single-particle states and the shape of the potential have an important influence on this symmetry. By the perturbation calculations of the Dirac Hamiltonian, the various energy splitting behaviors of both spin and pseudospin doublets with $\lambda$ are explained, which originates from the different contributions of each component to the energy splittings. The result supports the claim in Ref. [18], the spin splitting can be treated as a perturbation, while the pseudospin splitting cannot be regarded as a perturbation. The same conclusion can also be obtained from the wave function.

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