Electromagnetic transition strengths in heavy nuclei

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We calculate reduced $B(E2)$ and $B(M1)$ electromagnetic transition strengths within and between K bands in support of a recently proposed model for the structure of heavy nuclei. Previously, only spectra and a rough indication of the largest $B(E2)$ strengths were reported. The present more detailed calculations should aid the experimental identification of the predicted 0^+, 1^+, and 2^+ bands and, in particular, act to confirm or refute the suggestion that the model 0^+ and 2^+ bands correspond to the well known and widespread beta and gamma bands. Furthermore they pinpoint transitions that can indicate the presence of a so-far elusive 1^+ band by feeding relatively strongly into or out of it. Some of these transitions may already have been measured in ^{230}Th, ^{232}Th, and ^{238}U.

I. INTRODUCTION

We have recently proposed a model for excited states of heavy nuclei involving the coupling of a 2^+ excitation to a rotor [1]. An important characteristic of this model is the generation of excited 0^+, 1^+, and 2^+ K bands, at similar excitation energies, as a generic feature. Such triplets of K bands have been seen in the light nuclei ^{16}O [2], ^{24}Mg [3], and ^{40}Ca [4], and there is no obvious reason why they should not also occur in heavier rare-earth and actinide nuclei.

It is difficult to make a conclusive identification of the three predicted K bands from the calculated excitation energies alone. Their precise ordering depends on the details of the cluster-core interaction and the excitation energy of the 2^+ core state. If that interaction is overwhelmingly a quadrupole-quadrupole coupling, with model parameters appropriate to the actinide region, then our previous calculation [1] has the bands, in order of increasing excitation energy above the ground state band, as $K^\pi = 1^+$, then 0^+, and finally 2^+. However, even if we restrict ourselves to this simplest form, we are unable to predict the absolute values of the bandhead excitation energies with any confidence because we do not know the true strength of the noncentral part of the cluster-core interaction.

Since 0^+ beta and 2^+ gamma bands are a generic feature of the spectra of heavy nuclei, at about 1 MeV above the ground state we have chosen our interaction strength so as to place the excited K bands in this region. The value required to do this is not particularly large and indicates an intermediate strength, rather than a truly strong coupling regime. Nevertheless, the suggestion that our calculated 0^+ and 2^+ bands might be identifiable with the beta and gamma bands needs to be confirmed or refuted.

An obvious form of confirmation would be that an accompanying 1^+ band should be detected in the same region of excitation energies as the other two. Telltale signs of such a band would include the discovery of a 1^+ state (the band head) and also pairs of odd-J states (members of the proposed 1^+ and 2^+ bands) at similar excitation energies. However, as pointed out previously [1], the population of the 1^+ bandhead is likely

to be experimentally difficult. In addition, the intermediate-strength interaction leads us to predict a staggering of the energies of the states in the 1^+ band, which would make it unclear that they belong in a common band from casual comparison of their excitation energies, even if they were successfully populated. (This feature of staggering is common to the 1^+ bands of ^{16}O [2], ^{24}Mg [3], and ^{40}Ca [4], as well as to the 1^- band of ^{238}U [5,6].) Hence, additional information such as predicted electromagnetic transition strengths within and between the proposed band members is needed to enable experimental groups to recognize the states of the 1^+ band if they do succeed in exciting them.

Here, we seek to improve on our previous rough indications of where strong $E2$ transitions should be expected by calculating in-band and cross-band reduced $B(E2)$ and $B(M1)$ transition strengths for the ground and excited 0^+, 1^+, and 2^+ K bands in far greater detail. We cannot provide a definitive and unambiguous account of the $E2$ and $M1$ transitions because not only does their direction depend on the details of the ordering of the states in the spectrum, but also the wave functions are sensitive to how close in energy the states they represent lie. Subject to these provisos, this paper presents a detailed account of the calculated $E2$ and $M1$ transition strengths appropriate to the states generated previously [1] which should be of considerable assistance to experimental groups searching for the predicted triplet of K bands.

Theoretical studies of positive-parity bands in the actinide nuclei including calculation of some electromagnetic transition rates (but generally only in-band rather than cross-band) have also been presented within the cranked random phase approximation (RPA) [7–10], the collective model [11–13], the interacting-boson approximation [14,15], the variable moment-of-inertia model [16], and the alpha-particle cluster model [17].

In the next section we briefly outline the structure model leading to the generation of K bands. Then, we calculate reduced $B(E2)$ and $B(M1)$ electromagnetic transition strengths within and between these K bands. Insofar as possible, we

compare our calculated $B(E2)$ strengths with experimental values for some isotopes of Th and U. Finally we summarize our conclusions.

II. CLUSTER MODEL TO GENERATE K BANDS

We model a nucleus (Z,A) as a core (Z_1,A_1) and a cluster (Z_2,A_2) interacting via a deep, local potential $V(R)$ consisting of nuclear and Coulomb terms $V_N(R)$ and $V_C(R)$, respectively, where R is the separation of their centers. We parametrize the nuclear term as [18]

$$V_N(R) = -V_0 \left\{ \frac{x}{[1 + \exp{((R - R_0)/a)}]} + \frac{1 - x}{[1 + \exp{((R - R_0)/3a)}]^3} \right\} \quad (1)$$

and take $V_C(R)$ to represent a cluster point charge interacting with a uniformly charged spherical core of radius R_0. If the cluster and core were both restricted to their 0^+ ground states, a single band of states would be produced by solving the Schrödinger equation with this potential for a fixed value of the global quantum number $G = 2n + L$ (n is the number of internal nodes in the radial wave function and L is the orbital angular momentum). The value of G must be chosen large enough to satisfy the major requirements of the Pauli exclusion principle by excluding the cluster nucleons from states occupied by the core nucleons. This can be achieved in the actinide region by taking $G = 5A_2$. This program leads to a band of states $L^\pi = 0^+, 2^+, 4^+, \ldots, G^+$ and excitation energies E_L.

The situation becomes a little more complicated if we accept the possibility of the core being in either its ground state or an excited state having spin parity I^π (we restrict attention to 2^+ excitations here) and excitation energy ϵ. The cluster-core potential may now contain noncentral terms and the system must be described in terms of coupled basis states $|(IL)JM\rangle$ formed by combining the core spin I with the relative orbital angular momentum L to obtain a total angular momentum J. The simplest form of noncentral potential compatible with considerations of time-reversal and parity invariance is a quadrupole-quadrupole interaction. The matrix elements of such a noncentral potential between our coupled basis states are

$$V_{LL'II'}^J = i^{L'-L+I'-I}(-1)^{J+L+L'}\beta\hat{I}\hat{I}'\hat{L}\hat{L}' \\ \times \langle L0L'0|20\rangle\langle I0I'0|20\rangle W(L'I'LI;J2), \quad (2)$$

where $\hat{L} = \sqrt{(2L + 1)}$, etc. By combining these with the diagonal elements of the cluster-core rotational motion, simplified to $\alpha L(L + 1)$, and the core excitation energy, zero for $I = 0$ and ϵ for $I = 2$, and assuming that all the radial integrals can be parametrized by a single strength β, we can obtain eigenvalues and eigenvectors by diagonalization of low-dimensional matrices. Figure 1 shows the spectrum resulting from such a calculation using the parameters of Ref. [1], where more details of the calculation are given.

III. CALCULATION OF ELECTROMAGNETIC TRANSITION STRENGTHS

We calculate reduced $B(E2)$ and $B(M1)$ electromagnetic transition strengths appropriate to the spectrum shown in Fig. 1. The general features of these results are expected to be common to all those nuclei for which the triplet of K bands produced by coupling a 2^+ excitation to a rotor is an appropriate description of part of the spectrum. There should be strong in-band $E2$ transitions (typically a few hundred Weisskopf units in the actinide region), related to one another rather closely by Clebsch-Gordan coefficients, and usually much weaker cross-band $E2$ transitions of typically a few Weisskopf units (although there turn out to be a few interesting exceptions to this general statement).

Magnetic dipole transitions are possible between some of the levels due to mixing of the relative-motion L values (e.g., the 3^+ states are a mixture of $L = 2, 4 \otimes I = 2$ while the 2^+ states contain mixtures of $L = 0, 2, 4 \otimes I = 2$ as well as $L = 2 \otimes I = 0$) which allow a transition to take place between certain components of these states without changing L or I. These $B(M1)$ strengths should be rather small, typically 0.01 Weisskopf units, but the simple existence of such transitions between 0^+ and 1^+ states, where $E2$ transitions are impossible, can provide a strong indication of our proposed band structure.

The precise details of the transition rates will vary from nucleus to nucleus. Accidental near-degeneracy of excited states in a particular nucleus can lead to exceptionally large mixing with accompanying strong $E2$ transition strengths. It must also be borne in mind that any given real nucleus may have some of the predicted levels in a different order from those exhibited in Fig. 1 so that the transitions go in the opposite direction. Indeed, the excited bandhead ordering may differ from that shown in Fig. 1. Even then, our results can still be useful because it is straightforward to transform the calculated strength for $J_i \to J_f$ into a corresponding value for $J_f \to J_i$ on multiplication by $(2J_i + 1)/(2J_f + 1)$.

To maintain generality as far as possible we give the calculated $B(E2)$ transition strengths as ratios to the value for $B(E2; 2^+ \to 0^+)$ in the ground-state band. However, we give the $B(M1)$ strength estimates directly in Weisskopf units because they only depend on the Z/A ratios (which are all very similar in the actinide region) with the appropriate Weisskopf unit independent of the charge and mass of the nucleus in question.

A. Reduced $B(E2)$ strengths

In the limit of strong coupling there would be no cross-band $E2$ transitions at all, and the in-band transitions would be given by (see for example Ref. [19])

$$B(E2; J_i \to J_f) = \frac{5}{4\pi} Q_2^2 \langle J_i K20|J_f K\rangle^2, \quad (3)$$

where Q_2 is the constant intrinsic electric quadrupole transition strength. We have checked that by increasing the strength of our quadrupole-quadrupole interaction to very large values this situation does indeed emerge from our calculation. Even with a much reduced interaction strength, Eq. (3) still provides a reasonable zeroth-order approximation
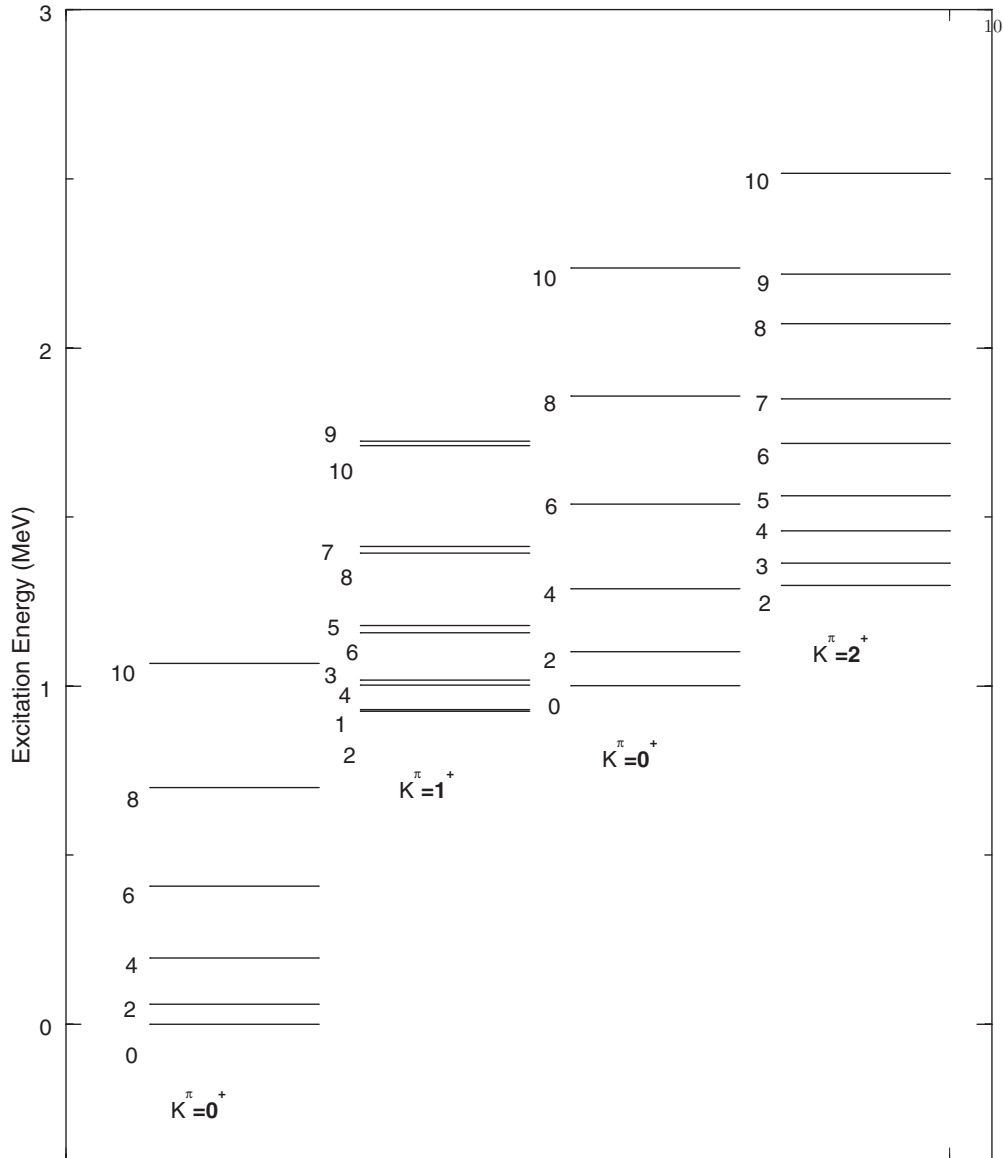
FIG. 1. Generic positive-parity spectrum obtained by coupling a 0^+ ground state and an excited 2^+ state to a rotor. We use a rotational parameter $\alpha = 0.01$ MeV, a core excitation energy $\epsilon = 0.8$ MeV, and a radial integral parameter $\beta = -0.178$ MeV, exactly as in Ref. [1].

to the calculated in-band reduced $E2$ transition strengths, as can be seen from Table I. The stretched $E2$ transitions between states with J differing by 2 are uniformly strong and much more prominent than most of the transitions between states whose J values differ by 1. This simple distinction does not hold at low spins, where many states lie close together in energy and consequently their wave functions are thoroughly mixed. There we see equally strong $1^+ \rightarrow 2^+$ and $3^+ \rightarrow 2^+$ transitions within the $K^\pi = 1^+$ band and $3^+ \rightarrow 2^+, 4^+ \rightarrow 3^+$, and $5^+ \rightarrow 4^+$ transitions in the $K^\pi = 2^+$ band. The general pattern of strong $J \rightarrow J - 2$ and much weaker $J \rightarrow J \pm 1$ transitions reasserts itself at higher values of J as state mixing gradually decreases and individual states become more widely separated in energy.

In view of the staggering of energy levels predicted for the $K^\pi = 1^+$ band, this general preference for stretched $E2$

transitions could easily lead to the perception that the $K^\pi = 1^+$ band consists of two separate bands with angular momentum sequences $2^+, 4^+, 6^+, \ldots$ and $1^+, 3^+, 5^+, \ldots$ such that their common origin is not apparent. See also Fig. 3 of Ref. [1].

In general, the $E2$ transitions in our model are produced by a sum of three terms. One is due to intrinsic excitations of the core, another is due to intrinsic excitations of the cluster, and the third is due to the relative motion of cluster about core. Since we restrict the cluster to its 0^+ ground state there is no contribution from intrinsic cluster excitations and the electromagnetic operator $M(E2)$ reduces to

$$M(E2) = a_{\text{core}} r^2 Y_2(\hat{\mathbf{r}}) + a_{\text{rel}} R^2 Y_2(\hat{\mathbf{R}}), \tag{4}$$

where r is a core internal coordinate and R the cluster core separation, and a_{core} and a_{rel} are the relevant charge factors,

TABLE I. Calculated in-band reduced $E2$ strengths $B(E2; J_i \rightarrow J_f)$ for $K^\pi = 0^+_{gs}$, 1^+, 0^+_{ex}, and 2^+ bands in heavy nuclei. Values are given as ratios relative to the strength of the $2^+ \rightarrow 0^+$ transition in the ground-state band. See text for details.

Transition	K^π			
	0^+_{gs}	1^+	0^+_{ex}	2^+
$2^+ \rightarrow 0^+$	1.00		0.60	
$4^+ \rightarrow 2^+$	1.43	0.93	1.06	0.38
$6^+ \rightarrow 4^+$	1.58	1.09	1.22	0.74
$8^+ \rightarrow 6^+$	1.67	1.14	1.30	0.92
$10^+ \rightarrow 8^+$	1.72	1.15	1.34	1.03

Transition	K^π 1^+	Transition	K^π 2^+
$1^+ \rightarrow 2^+$	1.45	$3^+ \rightarrow 2^+$	1.21
$3^+ \rightarrow 4^+$	0.22	$4^+ \rightarrow 3^+$	0.83
$3^+ \rightarrow 1^+$	0.80	$5^+ \rightarrow 4^+$	0.70
$3^+ \rightarrow 2^+$	0.63	$5^+ \rightarrow 3^+$	0.64
$5^+ \rightarrow 6^+$	0.07	$6^+ \rightarrow 5^+$	0.37
$5^+ \rightarrow 3^+$	1.25	$7^+ \rightarrow 6^+$	0.42
$5^+ \rightarrow 4^+$	0.27	$7^+ \rightarrow 5^+$	0.89
$7^+ \rightarrow 8^+$	0.02	$8^+ \rightarrow 7^+$	0.18
$7^+ \rightarrow 5^+$	1.39	$9^+ \rightarrow 8^+$	0.29
$7^+ \rightarrow 6^+$	0.17	$9^+ \rightarrow 7^+$	1.03
$9^+ \rightarrow 10^+$	0.01	$10^+ \rightarrow 9^+$	0.10
$9^+ \rightarrow 7^+$	1.45		
$9^+ \rightarrow 8^+$	0.12		

with $a_{rel} = \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2}$. We follow the convention of Brink and Satchler [20] in defining reduced matrix elements of rank-k tensor operators through the Wigner-Eckart theorem by

$$\langle J_f M_f | T_k^q | J_i M_i \rangle = (-1)^{2k} \langle J_f M_f | J_i M_i k q \rangle \langle J_f || T_k || J_i \rangle \tag{5}$$

so that we have to modify Eq. (3C-17) of Ref. [21] and write the reduced transition strengths as

$$B(E2; J_i \rightarrow J_f) = \frac{2J_f + 1}{2J_i + 1} |\langle J_f || M(E2) || J_i \rangle|^2, \tag{6}$$

and the corresponding Weisskopf unit is [22]

$$B(E2)_{W.u.} = \frac{0.747}{4\pi} (A_1 + A_2)^{4/3} \; e^2 \, \text{fm}^4 \tag{7}$$

To keep our results as general as possible we need to avoid explicit dependence on charge and mass values. To this end we note that in ^{208}Pb, a typical core for our model, the $B(E2 \downarrow)$ from the excited 2^+ state at 4.08524 MeV to the 0^+ ground state is measured as 8.4 ± 0.5 W.u.. The strengths of the corresponding $E2$ transitions in our parent nuclei are of order 200–300 W.u. (which we model as a combination of intrinsic core and relative motion contributions). We use this information to estimate the relative strengths of the orbital and core excitation terms in the $M(E2)$ operator. This is sufficient because we are not concerned with the absolute values of the $B(E2)$ strengths, and intend to calculate them only as ratios to the strength of the ground-state band $2^+ \rightarrow 0^+$ transition. If we write $M(E2) = a_{rel} R^2 Y_2(\hat{\mathbf{R}}) + a_{core} r^2 Y_2(\hat{\mathbf{r}})$, then we can estimate a_{rel}/a_{core} by fitting to a ratio of a mid-mass-range

actinide $B(E2)$ value and the ^{208}Pb $B(E2)$ value which, on ignoring the niceties of angular momentum coupling, yields

$$\frac{(a_{rel} + a_{core})^2}{a_{core}^2} \approx \frac{250 \text{ W.u.}}{8.4 \text{ W.u.}}. \tag{8}$$

We therefore expect relative coefficients of the corresponding terms in the $M(E2)$ matrix elements to be roughly in the ratio $[\sqrt{(250/8.4)} - 1]:1$, which is to say roughly between 4:1 and 5:1. In view of this, we use an $M(E2)$ operator

$$M(E2) = 0.25 r^2 Y_2(\hat{\mathbf{r}}) + R^2 Y_2(\hat{\mathbf{R}}) \tag{9}$$

and present our results in Tables I and II as ratios to the strength of the $2^+ \rightarrow 0^+$ transition in the ground-state band.

Table I shows our results for in-band $E2$ transitions. Stretched $E2$ transitions are all strong and due principally to the relative motion term in the $M(E2)$ operator. Their values are not generally far from the expectations of the rotational model given in Eq. (3), despite the strength of our quadrupole-quadrupole interaction being so weak that only intermediate-strength coupling is produced. Therefore, the measurement of in-band stretched $E2$ transitions will not distinguish between the two models to any convincing degree. The weaker values between states with J differing by less than 2 can largely be attributed to angular-momentum coupling effects (i.e., Clebsch-Gordan coefficients).

On the other hand, our predictions for the cross-band $E2$ transitions, shown in Table II, are very different from the strong-coupling rotational model, according to which they should all be zero. It is interesting to note that if we had only included the relative motion term in the $M(E2)$ operator, we would have obtained very weak cross-band transitions and so reached an indistinguishable conclusion, despite not having attained a truly strong-coupling limit. However, the presence of the core excitation term in $M(E2)$ gives rise to small, but nevertheless significant cross-band $E2$ transition strengths—a few hundreths of those seen for the in-band cases. These strengths typically amount to a few W.u. and should be experimentally measurable. We also note that in a few cases, where the state mixing is especially favorable, there are some transitions between the excited 0^+ and 1^+ bands whose strengths are about one tenth (or even larger) than characteristic in-band values. These cross-band features differ substantially from the expectations of the strong-coupling rotational model and thus distinguish our model from it. At present, the experimental data in the actinide region (which we examine later) are too sparse to draw any firm conclusion. However, similar effects are expected in the rare-earth region, and we hope to turn our attention to some of those nuclei in due course.

B. Reduced $B(M1)$ strengths

In principle, magnetic dipole transitions in our model are mediated by contributions from the cluster-core relative motion and also from the magnetic dipole moments of the cluster and core. Since the cluster remains in its 0^+ ground state there is no contribution from this to the $M1$ transition rates. Furthermore, we intend to take the magnetic dipole moment of the excited 2^+ core state as zero as well (its 0^+ ground state is unable to make any contribution). This is mainly a matter of

TABLE II. Calculated cross-band reduced $E2$ strengths $B(E2; J_i \rightarrow J_f)$ between $K^\pi = 0^+_{gs}$, 1^+, 0^+_{ex}, and 2^+ bands in heavy nuclei. Values are given as ratios relative to the strength of the $2^+ \rightarrow 0^+$ transition in the ground-state band. See text for details.

Transition	$1^+ \rightarrow 0^+_{gs}$	$0^+_{ex} \rightarrow 0^+_{gs}$	$0^+_{ex} \rightarrow 1^+$	$2^+ \rightarrow 0^+_{gs}$	$2^+ \rightarrow 1^+$	$2^+ \rightarrow 0^+_{ex}$	$0^+_{ex} \rightarrow 2^+$
$0^+ \rightarrow 2^+$		0.0533	0.6878				
$2^+ \rightarrow 0^+$	0.0299	0.0002		0.0076		0.0166	
$1^+ \rightarrow 2^+$	0.0479						
$2^+ \rightarrow 1^+$			0.2301		0.0019		
$2^+ \rightarrow 2^+$	0.0143	0.0060	0.0184	0.0154	0.0051	0.0841	
$2^+ \rightarrow 3^+$			0.1898		0.0000		
$2^+ \rightarrow 4^+$	0.0022	0.0471	0.0070	0.0027	0.0029	0.0129	
$3^+ \rightarrow 1^+$					0.0032		
$3^+ \rightarrow 2^+$	0.0179			0.0125	0.0051	0.0253	
$3^+ \rightarrow 3^+$					0.0007		
$3^+ \rightarrow 4^+$	0.0288			0.0132	0.0116	0.0975	
$3^+ \rightarrow 5^+$					0.0021		
$4^+ \rightarrow 2^+$	0.0354	0.0000	0.0010	0.0026	0.0054	0.0045	
$4^+ \rightarrow 3^+$			0.0776		0.0001		
$4^+ \rightarrow 4^+$	0.0065	0.0118	0.0013	0.0140	0.0084	0.0934	
$4^+ \rightarrow 5^+$			0.0737		0.0017		
$4^+ \rightarrow 6^+$	0.0010	0.0399	0.0001	0.0108	0.0108		
$5^+ \rightarrow 3^+$					0.0038		
$5^+ \rightarrow 4^+$	0.0232			0.0075	0.0047	0.0012	
$5^+ \rightarrow 5^+$					0.0002		
$5^+ \rightarrow 6^+$	0.0217			0.0185	0.0183	0.1461	
$5^+ \rightarrow 7^+$					0.0071		
$6^+ \rightarrow 4^+$	0.0334	0.0001	0.0000	0.0012	0.0026	0.0240	0.0156
$6^+ \rightarrow 5^+$			0.0389		0.0001		
$6^+ \rightarrow 6^+$	0.0045	0.0180	0.0081	0.0085	0.0059	0.0683	
$6^+ \rightarrow 7^+$			0.0295		0.0039		
$6^+ \rightarrow 8^+$	0.0007	0.0292	0.0001	0.0212	0.0192		
$7^+ \rightarrow 5^+$					0.0029		
$7^+ \rightarrow 6^+$	0.0250			0.0047	0.0032	0.0031	
$7^+ \rightarrow 7^+$					0.0001		
$7^+ \rightarrow 8^+$	0.0178			0.0213	0.0220		
$7^+ \rightarrow 9^+$					0.0131		
$8^+ \rightarrow 6^+$	0.0293	0.0001	0.0002	0.0006	0.0012	0.0216	0.0057
$8^+ \rightarrow 7^+$			0.0242		0.0001		0.1330
$8^+ \rightarrow 8^+$	0.0035	0.0217	0.0134	0.0045	0.0033	0.0381	
$8^+ \rightarrow 9^+$			0.0115		0.0042		
$8^+ \rightarrow 10^+$	0.0006	0.0206	0.0000	0.0288	0.0261		
$9^+ \rightarrow 7^+$					0.0020		
$9^+ \rightarrow 8^+$	0.0249			0.0031	0.0022	0.0097	
$9^+ \rightarrow 9^+$					0.0000		
$9^+ \rightarrow 10^+$	0.0156			0.0223	0.0246		
$10^+ \rightarrow 8^+$	0.0241	0.0001	0.0003	0.0003	0.0006	0.0106	0.0006
$10^+ \rightarrow 9^+$			0.0170		0.0001		0.1219
$10^+ \rightarrow 10^+$	0.0029	0.0228	0.0168	0.0022	0.0018	0.0200	

expediency since we do not have sufficient information to make any other choice. Very few 2^+ state magnetic dipole moments have been measured in the ^{208}Pb region [23]. However, one of the few that has is for the 2^+ state in ^{206}Pb at ~ 800 keV which has a measured value of $\mu < 0.03$ nm [23], which tends to support our proposed assignment of zero. One other piece of evidence in support of a negligible $\mu(2^+)$ value is provided by our ability to describe the magnetic dipole moment of the lowest-lying 2^+ state of ^{224}Ra with the simplest form of

our model. This means taking the state as a ^{210}Pb core in its ground state orbited by a ^{14}C cluster with relative orbital angular momentum $L = 2$. The magnetic dipole moment is then given by

$$\langle \mu_J \rangle = \mu_0 \left(\frac{A_1^2 Z_2 + A_2^2 Z_1}{A_1 A_2 (A_1 + A_2)} \right)$$
$$\times \left\{ \frac{J(J+1) + L(L+1) - I(I+1)}{2(J+1)} \right\}, \quad (10)$$

TABLE III. Calculated cross-band reduced $M1$ strengths $B(M1; J_i \to J_f)$ in W.u. between $K^\pi = 0^+_{gs}$, 1^+, 0^+_{ex}, and 2^+ bands in heavy nuclei. See text for details.

Transition	$1^+ \to 0^+_{gs}$	$0^+_{ex} \to 0^+_{gs}$	$0^+_{ex} \to 1^+$	$2^+ \to 0^+_{gs}$	$2^+ \to 1^+$	$2^+ \to 0^+_{ex}$	$0^+_{ex} \to 2^+$
$0^+ \to 1^+$			0.1028				
$1^+ \to 0^+$	0.0084						
$1^+ \to 2^+$	0.0022						
$2^+ \to 1^+$			0.0045		0.0271		
$2^+ \to 2^+$	0.0052	0.0023	0.0064	0.0001	0.0124	0.0112	
$2^+ \to 3^+$			0.0480		0.0042		
$3^+ \to 2^+$	0.0066			0.0001	0.0118	0.0081	
$3^+ \to 3^+$					0.0217		
$3^+ \to 4^+$	0.0067			0.0001	0.0033	0.0122	
$4^+ \to 3^+$			0.0069		0.0106		
$4^+ \to 4^+$	0.0112	0.0023	0.0007	0.0005	0.0111	0.0256	
$4^+ \to 5^+$			0.0466		0.0055		
$5^+ \to 4^+$	0.0058			0.0002	0.0050	0.0049	
$5^+ \to 5^+$					0.0196		
$5^+ \to 6^+$	0.0079			0.0002	0.0034	0.0287	
$6^+ \to 5^+$			0.0072		0.0040		
$6^+ \to 6^+$	0.0123	0.0018	0.0000	0.0009	0.0080	0.0267	
$6^+ \to 7^+$			0.0508		0.0029		
$7^+ \to 6^+$	0.0052			0.0002	0.0028	0.0014	
$7^+ \to 7^+$					0.0158		
$7^+ \to 8^+$	0.0091			0.0003	0.0031		
$8^+ \to 7^+$			0.0061		0.0014		0.0371
$8^+ \to 8^+$	0.0139	0.0012	0.0003	0.0012	0.0057	0.0193	
$8^+ \to 9^+$			0.0542		0.0011		
$9^+ \to 8^+$	0.0048			0.0003	0.0018	0.0001	
$9^+ \to 9^+$					0.0124		
$9^+ \to 10^+$	0.0106			0.0004	0.0027		
$10^+ \to 9^+$			0.0047		0.0005		0.0453
$10^+ \to 10^+$	0.0159	0.0009	0.0005	0.0014	0.0043	0.0124	

which for $I = 0$ and $J = L = 2$ reduces to

$$\langle \mu_2 \rangle = 2\mu_0 \left(\frac{A_1^2 Z_2 + A_2^2 Z_1}{A_1 A_2 (A_1 + A_2)} \right). \tag{11}$$

For ^{224}Ra treated as ^{210}Pb $+$ ^{14}C this yields a numerical value of 0.852 nm, which compares favorably with the measured value of 0.9 ± 0.2 nm [23].

Using the relative-motion term alone leads to a magnetic dipole operator in our model of

$$M(M1) = \sqrt{\frac{3}{4\pi}} \frac{A_1^2 Z_2 + A_2^2 Z_1}{A_1 A_2 (A_1 + A_2)} \mathbf{L}. \tag{12}$$

To avoid explicit use of charge and mass values we use an average value appropriate to ^{208}Pb $+$ ^{14}C, ^{20}O, ^{24}Ne, and ^{28}Mg of 0.42 for the charge-mass dependent factor above. Thus

$$M(M1) \approx 0.42 \times \sqrt{\frac{3}{4\pi}} \mathbf{L}. \tag{13}$$

The Weisskopf unit in these same units is $45/8\pi$ [22].

Table III shows our results for $B(M1)$ reduced transition strengths between states in the spectrum of Fig. 1 with J values differing by 1 or 0. The values are only meant to be indicative of typical strengths, subject to the same provisos mentioned in discussing the $E2$ transitions, that the level ordering might be different from that illustrated, and accidental near-degeneracies can give rise to strong mixing which results in fortuitously strong predicted transitions. The calculated values are generally rather small, scarcely exceeding 0.01 W.u. Their possible importance lies in the existence of transitions between the heads of the 1^+ and 0^+ bands, which could not be mediated by gamma rays of any other multipolarity.

IV. COMPARISON WITH EXISTING DATA

On examining the even-even actinide nuclei for experimental $B(E2)$ transitions involving any states other than those within the ground state band, we have found surprisingly few measurements [24]. Only the four nuclei ^{230}Th, ^{232}Th, ^{234}U, and ^{238}U can act as present testing grounds for our model. We defer discussion of ^{238}U until later because its structure is so complicated, and concentrate initially on the first three of these nuclei.

Table IV compares the measured cross-band $B(E2)$ strengths with our calculated values, in W.u. In each case we accept the evaluators' identifications of beta and gamma

TABLE IV. Comparison of some measured cross-band reduced $E2$ strengths $B(E2; J_i \rightarrow J_f)$ for ^{230}Th, ^{232}Th, and ^{234}U with possible theoretical equivalents from Table II. See text for details. Data are from Refs. [25–27].

J_i (E_i keV)	J_f (E_f keV)	$B(E2)$ Expt. (W.u.)	$J_i(K_i) \rightarrow J_f(K_f)$	$B(E2)$ Theor. (W.u.)
		^{230}Th		
2^+(677.515)	4^+(174.111)	10(4)	$2^+(0_{ex}) \rightarrow 4^+(0_{gs})$	9.2
2^+(677.515)	0^+(0.0)	2.7(9)	$2^+(0_{ex}) \rightarrow 0^+(0_{gs})$	0.04
2^+(781.375)	4^+(174.111)	0.37(14)	$2^+(2) \rightarrow 4^+(0_{gs})$	0.53
2^+(781.375)	2^+(53.227)	5.5(18)	$2^+(2) \rightarrow 2^+(0_{gs})$	3.0
2^+(781.375)	0^+(0.0)	2.9(9)	$2^+(2) \rightarrow 0^+(0_{gs})$	1.5
2^+(1009.601)	2^+(677.515)	<27	$2^+(1) \rightarrow 2^+(0_{ex})$	3.6
2^+(1009.601)	4^+(174.111)	<0.38	$2^+(1) \rightarrow 4^+(0_{gs})$	0.43
2^+(1009.601)	2^+(53.227)	<5.2	$2^+(1) \rightarrow 2^+(0_{gs})$	2.8
2^+(1009.601)	0^+(0.0)	<2.7	$2^+(1) \rightarrow 0^+(0_{gs})$	5.9
		^{232}Th		
2^+(774.15)	4^+(162.12)	≈3.3	$2^+(0_{ex}) \rightarrow 4^+(0_{gs})$	9.2
2^+(774.15)	2^+(49.369)	≈0.52	$2^+(0_{ex}) \rightarrow 2^+(0_{gs})$	1.2
2^+(774.15)	0^+(0.0)	2.8(12)	$2^+(0_{ex}) \rightarrow 0^+(0_{gs})$	0.04
2^+(785.25)	4^+(162.12)	≈0.13	$2^+(2) \rightarrow 4^+(0_{gs})$	0.53
2^+(785.25)	2^+(49.369)	7.2(7)	$2^+(2) \rightarrow 2^+(0_{gs})$	3.0
2^+(785.25)	0^+(0.0)	2.9(4)	$2^+(2) \rightarrow 0^+(0_{gs})$	1.5
2^+(1387.1)	4^+(162.12)	0.51(18)	$2^+(1) \rightarrow 4^+(0_{gs})$	0.44
4^+(1413.8)	3^+(829.6)	<12	$4^+(1) \rightarrow 3^+(2)$	3.8
4^+(1413.8)	2^+(785.25)	23(7)	$4^+(1) \rightarrow 2^+(2)$	1.9
		^{234}U		
0^+(809.907)	2^+(43.4981)	>0.067	$0^+(0_{ex}) \rightarrow 2^+(0_{gs})$	1.3
2^+(851.74)	4^+(143.352)	<1	$2^+(0_{ex}) \rightarrow 4^+(0_{gs})$	11.1
2^+(851.74)	2^+(43.4981)	<0.23	$2^+(0_{ex}) \rightarrow 2^+(0_{gs})$	1.4
2^+(851.74)	0^+(0.0)	<1.3	$2^+(0_{ex}) \rightarrow 0^+(0_{gs})$	0.05
2^+(926.720)	4^+(143.352)	0.28(5)	$2^+(2) \rightarrow 4^+(0_{gs})$	0.64
2^+(926.720)	2^+(43.4981)	4.9(8)	$2^+(2) \rightarrow 2^+(0_{gs})$	3.6
2^+(926.720)	0^+(0.0)	2.9(5)	$2^+(2) \rightarrow 0^+(0_{gs})$	1.8

bands. We identify the former with our $K^\pi = 0_{ex}^+$ band and the latter with our $K^\pi = 2^+$ band. This allows us to compare the measured values for transitions from these bands to the ground-state band with our corresponding calculated transition strengths. More interesting is the observation that in ^{230}Th and ^{232}Th there are other transitions, not emanating from the beta or gamma bands, which feed into the other labeled bands, for which measured $B(E2)$ strengths are available. In the table we tentatively identify these gamma rays as coming from states in our predicted $K^\pi = 1^+$ band. We thus include a comparison of the strengths calculated for these putative transitions with the otherwise unassigned experimental values.

Overall, we then obtain quite a good description of all the cross-band transitions in these three nuclei for which measured $B(E2)$ strengths are available. Agreement is generally rather good, although we have to admit that our values for decays from the 2^+ state of the beta band to the 0^+ ground state are generally too small (although the measured values come with rather large uncertainties). We note also that the transitions assumed to proceed from states belonging to a $K^\pi = 1^+$

band are more speculative than the others in that we have no direct evidence that these initial states indeed belong to a $K^\pi = 1^+$ band. It is thus possible that a more complex residual interaction than the simple quadrupole-quadrupole one assumed here would shift the $K^\pi = 1^+$ band to higher energies than those inferred from an inspection of Fig. 1. Nevertheless, given the uncertainties surrounding our input parameters, we feel that the results of Table IV represent a highly satisfactory first step.

The experimental situation for ^{238}U is much less transparent. Lifetimes have been measured for nine separate 2^+ states in this nucleus. The Nuclear Data Center evaluation [28] labels these as follows: 2^+(44.916) is a member of the ground state band, 2^+(966.13) and 2^+(1037.25) are members of two distinct beta bands, 2^+(1060.27) is the head of a gamma band, and the remaining five states at excitation energies of 1223.78, 1278.54, 1414.0, 1530.2, and 1782.3 keV are all seen in Coulomb excitation and assigned $J^\pi = 2^+$ through angular distribution analysis.

The allocation of two separate beta bands worries us and probably warrants further investigation. There appear to be associated 0^+ band heads at 927.21 and 997.23 keV for these proposed beta bands, although this latter state's J^π value is based on an $E0$ transition to the ground state. However, we also note that there is a 3^+ state at 1105.71 keV (assigned to the gamma band) and a (3^+) state at 1059.66 keV. The appearance of two 3^+ states, close in energy, is a signature of our three-excited-K-band model, with the two 3^+ states belonging to the excited $K^\pi = 1^+$ and 2^+ bands. We therefore compare the measured reduced $E2$ transition strengths to our calculated ones by accepting the evaluator's identification of the ground-state band, the gamma band, and the lower of the two beta bands [i.e., the one based on a 0^+ bandhead at 927.21 and containing the 2^+(966.13) state]. We treat the decays from the 2^+(1037.25) state as if that state were a member of our $K^\pi = 1^+$ band. This interpretation places the ordering of the experimental bands as $K^\pi = 0_{gs}^+$, 0_{ex}^+, 1^+, and 2^+. Since this differs from the ordering in our generic spectrum, shown in Fig. 1, we expect that our calculation would need some fine-tuning to yield a good description of ^{238}U.

Nevertheless, Table V compares the experimental cross-band reduced $E2$ strengths with the values calculated for our "typical" actinide nucleus. The level of agreement for transition strengths into the ground-state band from the beta and gamma bands is good, but our calculated values from the proposed 1^+ band are too large by a factor of 3–6. This is about as good as we could expect without fitting the model parameters specifically to ^{238}U properties.

We leave the higher 2^+ states in abeyance for the present time. Our model could accommodate (many) higher lying 2^+ states if we introduced a 2^+ cluster excitation in addition to the 2^+ core excitation. In fact, this cluster excitation could combine with the core excitation to form a total excitation "spin" value of $K_{total} = 0, 1, 2, 3,$ or 4 with associated K bands running from $K = 0$ to K_{total} in each case. Indeed, this extra excitation would introduce several extra beta bands. In this context it is worth bearing in mind that a multiplicity of beta bands can be produced by a microscopic particle-hole

TABLE V. Comparison of some measured cross-band reduced $E2$ strengths $B(E2; J_i \to J_f)$ for ^{238}U with possible theoretical equivalents from Table II. See text for details. Data are from Ref. [28].

J_i (E_i keV)	J_f (E_f keV)	$B(E2)$ Expt. (W.u.)	$J_i(K_i) \to J_f(K_f)$	$B(E2)$ Theor. (W.u.)
		^{238}U		
2^+(966.13)	4^+(148.38)	3.3(14)	$2^+(0_{ex}) \to 4^+(0_{gs})$	13.2
2^+(966.13)	2^+(44.916)	1.1(4)	$2^+(0_{ex}) \to 2^+(0_{gs})$	1.7
2^+(966.13)	0^+(0.0)	0.38(16)	$2^+(0_{ex}) \to 0^+(0_{gs})$	0.05
2^+(1037.25)	4^+(148.38)	2.28(23)	$2^+(1) \to 4^+(0_{gs})$	0.63
2^+(1037.25)	2^+(44.916)	1.23(14)	$2^+(1) \to 2^+(0_{gs})$	4.0
2^+(1037.25)	0^+(0.0)	1.47(16)	$2^+(1) \to 0^+(0_{gs})$	8.4
2^+(1060.27)	4^+(148.38)	0.33(3)	$2^+(2) \to 4^+(0_{gs})$	0.75
2^+(1060.27)	2^+(44.916)	5.3(4)	$2^+(2) \to 2^+(0_{gs})$	4.3
2^+(1060.27)	0^+(0.0)	3.04(18)	$2^+(2) \to 0^+(0_{gs})$	2.1
2^+(1223.78)	2^+(966.13)	32		
2^+(1223.78)	0^+(927.21)	27		
2^+(1223.78)	4^+(148.38)	0.017		
2^+(1223.78)	0^+(0.0)	0.29		
2^+(1278.54)	4^+(148.38)	0.29(3)		
2^+(1278.54)	2^+(44.916)	0.37(5)		
2^+(1278.54)	0^+(0.0)	0.098(9)		
2^+(1414.0)	2^+(1060.27)	36		
2^+(1414.0)	0^+(0.0)	0.125		
2^+(1530.2)	2^+(966.13)	55		
2^+(1530.2)	4^+(148.38)	3.57(43)		
2^+(1530.2)	0^+(0.0)	0.240(24)		
2^+(1782.3)	2^+(44.916)	0.57(6)		
2^+(1782.3)	0^+(0.0)	0.41(4)		

description of the excitations of the system. This point has been successfully made by Chasman in his studies of ^{232,234,236}U [29], and his conclusions are likely to hold across all nuclei discussed in the present paper.

V. CONCLUSION

Consideration of a structure model where a cluster rotates around a core having a 0^+ ground state and an excited 2^+ state leads to a spectrum containing a ground state $K^\pi = 0^+$ band and three excited bands having $K^\pi = 0^+$, 1^+, and 2^+. These features are observed in light nuclei such as ^{16}O [2], ^{24}Mg [3], and ^{40}Ca [4], and there is no obvious reason why they should not be replicated in heavier nuclei.

Neither we nor anybody else has a microscopic model capable of accurately predicting the excitation energies of the $K^\pi = 0^+$, 1^+, 2^+ bandheads. We must rely on experimental data to guide us to a correct identification. Insofar as the present calculations are concerned, we have taken a simple quadrupole-quadrupole residual interaction and observe that this places the $K^\pi = 1^+$ bandhead in the vicinity of the excited $K^\pi = 0^+$ and $K^\pi = 2^+$ bandheads as in Fig. 1. It must, however, be borne in mind that the interaction may be more complicated than the simple form above, and this could well lead to a different bandhead ordering. For example,

additional terms in the interaction could shift the $K^\pi = 1^+$ bandhead away from the $K^\pi = 0^+$ and $K^\pi = 2^+$ bandheads, but probably by no more than 1 MeV. We note that even the identification of the latter bandheads with experimental states is not absolutely clear, but we are strongly tempted to align them with the heads of the widespread beta and gamma bands. This choice then results in the considerable agreement shown in Tables IV and V between the experimental and theoretical $B(E2)$ values for transitions from the $K^\pi = 0^+$ and 2^+ excited bands to the $K^\pi = 0^+$ ground-state band. Given the circumstances the best that we can do is therefore to choose the free parameters of our model to place the excited bands in the vicinity of the beta and gamma bands in actinide nuclei. This gives the generic spectrum for heavy nuclei shown in Fig. 1, although we accept that the details, and even the precise band orderings, could change with moderate fine-tuning of the model parameters.

The $K^\pi = 1^+$ band has not been widely seen in heavy nuclei, although Tables IV and V tentatively indicate its presence in ^{230}Th, ^{232}Th, and ^{238}U. However, our model suggests that it ought to be a feature common to many more heavy nuclei. The irregularly spaced state energies emerging from our intermediate strength coupling calculation for this band indicate that it will be a difficult task to identify it on the basis of J^π state excitation energies alone. To this end, we have presented a calculation of the reduced $E2$ and $M1$ electromagnetic transition strengths between the states of the spectrum of Fig. 1. Of course, only the gross features and not the details should be sought in any particular nucleus, since the $B(E2)$ and $B(M1)$ strengths will probably be even more sensitive to fine-tuning of the model parameters than the spectrum. Nevertheless, these transition-rate predictions differ sufficiently from those of the strong-coupling limit of the standard collective rotational model [19,30] that they suggest a realistic possibility of identifying the elusive $K^\pi = 1^+$ band and differentiating between that model and our own.

The in-band $E2$ transitions of both models are very similar, and related to each other by angular momentum coupling coefficients. The cross-band E2 transitions are very different. A combination of state mixing due to our intermediate strength noncentral interaction and the effects of including a 2^+ excited core lead to small but measurable cross-band transitions of typically a few W.u. in our model. These transitions are absent from the strong-coupling limit of the rotational model. We also predict small $M1$ transitions of typically a few hundredths of a W.u. The significance of the $M1$ transitions is mainly that they allow the presence of gamma rays connecting the 0^+ and 1^+ bandheads, where otherwise there would be nothing.

Our examination of the existing data in the actinide nuclei is tantalizing but not conclusive. The first comment we make is that there are surprisingly few well measured cross-band $E2$ transition strengths in these nuclei. Despite this, there are some indications of the cross-band $E2$ electromagnetic transitions predicted by our model in ^{230}Th, ^{232}Th, ^{234}U, and ^{238}U. We have considered all reported $B(E2)$ strengths in these four nuclei. In three of them there are certainly excited states, not accommodated in beta and gamma bands, that decay into the ground-state band with strengths of a few W.u., in line with

our model expectations. We also give a reasonable description of cross-band $E2$ transitions from the beta and gamma bands in all four nuclei. The only blemish is that the predicted decays from the 2^+ state of the beta band to the 0^+ ground state are generally too small.

In the near future we hope to apply our model in the rare-earth region, where qualitatively similar results are expected. We also urge our experimental colleagues to reexamine existing data and initiate new experiments to improve the overall level of spectroscopy in the actinide region and, in particular, to verify or deny the existence of the proposed $K^\pi = 1^+$ band, which is a key distinguishing characteristic of our model.

[1] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **84**, 034310 (2011).

[2] R. A. Baldock, B. Buck, and J. A. Rubio, Nucl. Phys. A **426**, 222 (1984).

[3] B. Buck, P. J. B. Hopkins, and A. C. Merchant, Nucl. Phys. A **513**, 75 (1990).

[4] A. C. Merchant, Phys. Rev. C **37**, 414 (1988).

[5] B. Buck, A. C. Merchant, and S. M. Perez, J. Phys. G: Nucl. Part. Phys. **35**, 085101 (2008).

[6] B. Buck, A. C. Merchant, S. M. Perez, T. T. Ibrahim, and S. M. Wyngaardt, J. Phys. G: Nucl. Part. Phys. **36**, 085101 (2009).

[7] K. Neergård and P. Vogel, Nucl. Phys. A **149**, 217 (1971).

[8] P. Vogel, Phys. Lett. B **60**, 431 (1976).

[9] D. Ward *et al.*, Nucl. Phys. A **600**, 88 (1996).

[10] A. A. Kuliev, E. Gugliev, F. Ertugral and S. Özkan, Eur. Phys. J. A **43**, 313 (2010).

[11] N. Minkov, S. Drenska, P. Yotov, and W. Scheid, Phys. At. Nucl. **67**, 1760 (2004).

[12] N. Minkov, P. Yotov, S. Drenska, and W. Scheid, J. Phys. G: Nucl. Part. Phys. **32**, 497 (2006).

[13] A. A. Raduta and C. M. Raduta, Nucl. Phys. A **768**, 170 (2006).

[14] N. V. Zamfir and D. Kusnezov, Phys. Rev. C **67**, 014305 (2003).

[15] P. D. Cottle and N. V. Zamfir, Phys. Rev. C **58**, 1500 (1998).

[16] D. Lenis and D. Bonatsos, Phys. Lett. B **633**, 474 (2006).

[17] T. Schneidman, G. G. Adamian, N. V. Antonenko, R. V. Jolos, and W. Scheid, Phys. Lett. B **526**, 322 (2002).

[18] B. Buck, A. C. Merchant, and S. M. Perez, Nucl. Phys. A **614**, 129 (1997).

[19] G. A. Leander, W. Nazarewicz, G. F. Bertsch, and J. Dudek, Nucl. Phys. A **453**, 58 (1986).

[20] D. M. Brink and G. R. Satchler, *Angular Momentum*, 3rd ed. (Oxford University Press, Oxford, 1993), p. 57.

[21] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol 1* (Benjamin, New York, 1969), p. 382.

[22] A. Bohr and B. R. Mottelson, *Nuclear Structure, Vol 1* (Benjamin, New York, 1969), p. 389.

[23] N. J. Stone, At. Data Nucl. Data Tables **90**, 75 (2005).

[24] Brookhaven National Nuclear Data Center at [http://www.nndc.bnl.gov/ensdf].

[25] Y. A. Akovali, Nucl. Data Sheets **69**, 155 (1993).

[26] E. Browne, Nucl. Data Sheets **107**, 2579 (2006).

[27] E. Browne, Nucl. Data Sheets **108**, 681 (2007).

[28] F. W. Chukreev, V. E. Makarenko, and M. J. Martin, Nucl. Data Sheets **97**, 129 (2002).

[29] R. R. Chasman, Phys. Rev. Lett. **42**, 630 (1979).

[30] D. J. Rowe, *Nuclear Collective Motion* (Methuen, London, 1970), p. 73.