RAPID COMMUNICATIONS

PHYSICAL REVIEW C **85**, 051901(R) (2012)

Symmetric correlations as seen in central Au + Au collisions at $\sqrt{s} = 200A$ GeV

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(Received 10 August 2011; revised manuscript received 19 April 2012; published 10 May 2012)

We analyze the forward-backward multiplicity correlation coefficient as measured by STAR. We show that in the most central Au + Au collisions bins located symmetrically around $\eta = 0$ with large separation in pseudorapidity are more strongly correlated than bins located asymmetrically with smaller separation. In proton-proton collisions the opposite effect is observed. It suggests a qualitatively different behavior of the two-particle correlation as a function of pseudorapidity sum in $p + p$ and Au + Au collisions.

DOI: 10.1103/PhysRevC.85.051901 PACS number(s): 25.75.Gz, 25.75.Dw

Correlations between particles produced in different rapidity regions have been intensively studied since the early times of high-energy physics [1]. Particularly interesting are correlations between particles with large separation in rapidity. It is recognized that such correlations are born immediately after the collision, when the produced system is very small (spatial size of the order of a few femtometers) and before rapid longitudinal expansion.

One popular method to study long-range correlations is to measure the multiplicity correlation coefficient, that is, to quantify how multiplicity (number of particles) in one rapidity window influences multiplicity in another one. This problem was thoroughly studied in hadron-hadron collisions at various energies [2–14]. One important lesson from these studies is that the forward-backward correlation coefficient decreases as a function of rapidity distance between bins.

Recently the STAR Collaboration at RHIC announced the results [15] of the forward-backward multiplicity correlation coefficient measured in Au + Au collisions at $\sqrt{s} = 200$ GeV. The measurement was performed for two narrow pseudorapidity bins with the distances between them ranging from 0.2 to 1.8, covering a substantial part of the midrapidity region. Very interesting features were observed: (i) The correlation coefficient increases significantly with centrality of the collision, and (ii) it remains approximately constant (except for very peripheral collisions) across the measured midrapidity region $|\eta| < 1$. These results were interpreted in the framework of the color glass condensate [16] or the dual-parton [6] models.

Recently various mechanisms have been proposed to understand the data quantitatively [17–20]. However, in these calculations the sophistication of the STAR analysis was not fully appreciated, and the published results cannot be directly compared with data. As emphasized by Lappi and McLerran [21] in the STAR analysis, the correlation coefficient is measured at a given number of particles in an additional reference window. This procedure significantly influences the forward-backward correlations, and we come back to this problem later.

In the present Rapid Communication we analyze the STAR data and extend the discussion initiated in Ref. [21]. We describe the STAR analysis in detail and derive a general formula that relates the correlation coefficients measured with and without the step of fixing particle number in the reference window.

The main result of this study is the observation that the two-particle pseudorapidity correlation function is qualitatively different in $p + p$ and central Au + Au collisions when studied as a function of pseudorapidity sum $\eta_1 + \eta_2$. In a model-independent way we show that bins located asymmetrically around $\eta = 0$ with a small separation in pseudorapidity are significantly more weakly correlated than bins located symmetrically with much larger separation. In $p + p$ collisions the opposite effect is observed; that is, bins with smaller separation are more strongly correlated even if they are asymmetric.

The multiplicity correlation coefficient for two bins X and Y is

$$b_{XY} = \frac{D_{XY}^2}{D_{XX} D_{YY}}, \quad (1)$$

$$D_{XY}^2 = \langle n_X n_Y \rangle - \langle n_X \rangle \langle n_Y \rangle; \quad D_{YY}^2 = \langle n_Y^2 \rangle - \langle n_Y \rangle^2, \quad (2)$$

where n_X and n_Y, respectively, are event-by-event multiplicities in X and Y. Due to the Cauchy-Schwarz inequality b_{XY} varies from -1 to $+1$.

The STAR Collaboration measured the multiplicity correlation coefficient between two symmetric (with respect to $\eta = 0$ in the center-of-mass frame) pseudorapidity bins B (backward) and F (forward) of width 0.2. To reduce a trivial source of correlations coming from the impact parameter fluctuations,[1] STAR introduced the third symmetric reference bin R (see Fig. 1), and all averages $\langle n_B \rangle_{n_R}$, $\langle n_B^2 \rangle_{n_R}$, and $\langle n_B n_F \rangle_{n_R}$ were measured at a given number of particles n_R in this bin. Next they calculated the appropriate covariance and variance in the following way:

$$\begin{aligned} D_{BF}^2|_{\rm STAR} &= \sum\nolimits_{n_R} P(n_R)\left[\langle n_B n_F \rangle_{n_R} - \langle n_B \rangle_{n_R}^2\right], \\ D_{BB}^2|_{\rm STAR} &= \sum\nolimits_{n_R} P(n_R)\left[\langle n_B^2 \rangle_{n_R} - \langle n_B \rangle_{n_R}^2\right], \end{aligned} \quad (3)$$

where $P(n_R)$ is the multiplicity distribution in the reference bin R at a given centrality class that is defined by a range of

[1] Higher n_B triggers a smaller impact parameter that leads to higher n_F.

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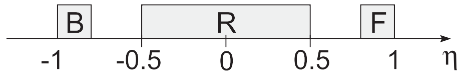

FIG. 1. Configuration with maximum pseudorapidity gap between B and F.

n_R, that is, $n_1 < n_R < n_2$. Equation (3) allows us to calculate the correlation coefficient as measured by STAR:

$$b_{BF}|_{\mathrm{STAR}} = \frac{D^2_{BF}|_{\mathrm{STAR}}}{D^2_{BB}|_{\mathrm{STAR}}}. \tag{4}$$

It is important to emphasize that if $\langle n_B\rangle$, $\langle n_B^2\rangle$, and $\langle n_B n_F\rangle$ are measured without the step of fixing n_R (namely all events are taken to directly measure D^2_{BF} and D^2_{BB} with n_R in a given centrality range) different results are obtained.[2] In the following all observables without a label "*STAR*" denote that D^2_{BF} and D^2_{BB} are calculated without fixing n_R.

The STAR procedure of measuring $b_{BF}|_{\mathrm{STAR}}$ substantially removes the impact parameter fluctuations, indeed. However, as shown in Ref. [21], it complicates the interpretation of $b_{BF}|_{\mathrm{STAR}}$ since it clearly depends (in the nontrivial way) on correlations between $B(F)$ and R. In the following we derive the relation between $b_{BF}|_{\mathrm{STAR}}$ and multiplicity correlations b_{BF} and $b_{BR} = b_{FR}$ that are obtained in the same centrality class but without the step of fixing n_R. Such calculation was performed in Ref. [21], where for simplicity the multiplicity distribution $P(n_B, n_F, n_R)$ was assumed to be in a Gaussian form. Here we show that the result derived in Ref. [21] is independent on $P(n_B, n_F, n_R)$ provided the average number of particles in B at a given n_R is a linear function of n_R:

$$\langle n_B\rangle_{n_R} = c_0 + c_1 n_R. \tag{5}$$

This relation is well confirmed by STAR [22]. It is straightforward to show that

$$c_0 = \langle n_B\rangle - \langle n_R\rangle \frac{D^2_{BR}}{D^2_{RR}}, \quad c_1 = \frac{D^2_{BR}}{D^2_{RR}}. \tag{6}$$

Indeed, to obtain Eq. (6) both sides of Eq. (5) should be multiplied first by $P(n_R)$ and second by $P(n_R)n_R$ and summed over n_R. Using an obvious relation

$$\langle O\rangle_{n_R} = \frac{1}{P(n_R)} \sum_{n_B,n_F} P(n_B, n_F, n_R) O, \tag{7}$$

two simple equations can be derived that allow us to calculate c_0 and c_1.

Taking Eqs. (3), (5), and (7) into account,

$$\begin{aligned} D^2_{BF}|_{\mathrm{STAR}} &= D^2_{BF} - c_1^2 D^2_{RR}, \\ D^2_{BB}|_{\mathrm{STAR}} &= D^2_{BB} - c_1^2 D^2_{RR}, \end{aligned} \tag{8}$$

where c_1 is defined in (6). Consequently, $b_{BF}|_{\mathrm{STAR}}$ is given by

$$b_{BF}|_{\mathrm{STAR}} = \frac{b_{BF} - b^2_{BR}}{1 - b^2_{BR}}, \tag{9}$$

where b_{BF} and b_{BR} are the appropriate correlation coefficients measured without fixing n_R. As mentioned earlier we obtain exactly the same formula as in Ref. [21]. It shows that Eq. (9) does not depend on $P(n_B, n_F, n_R)$, provided the relation (5) is satisfied.

Here we point out that the interpretation of $b_{BF}|_{\mathrm{STAR}}$ is not straightforward. For example, $b_{BF}|_{\mathrm{STAR}} = 0$ indicates only that $b_{BF} = b^2_{BR}$ but it does not mean that $b_{BF} = 0$. Moreover, $b_{BF}|_{\mathrm{STAR}}$ can be negative even if both b_{BF} and b_{BR} are positive. We conclude that the full interpretation of $b_{BF}|_{\mathrm{STAR}}$ is difficult without knowing b_{BF} and b_{BR}.

In this Rapid Communication we are interested in the configuration presented in Fig. 1, where the distance between B and F is a maximum one (i.e., $F = [0.8 < \eta < 1]$, B is symmetric with respect to $\eta = 0$, and $R = [-0.5 < \eta < 0.5]$). In this case the average gap between B and R is smaller by a factor of 2 than that between B and F. Assuming that the two-particle correlation function depends only on $|\eta_1 - \eta_2|$ and is not increasing as a function of $|\eta_1 - \eta_2|$ a *natural* ordering $b_{BR} \geqslant b_{BF}$ is obtained, as shown explicitly in Ref. [21]. Consequently

$$b_{BF}|_{\mathrm{STAR}} = \frac{b_{BF} - b^2_{BR}}{1 - b^2_{BR}} \leqslant \frac{b_{BR} - b^2_{BR}}{1 - b^2_{BR}} = \frac{b_{BR}}{1 + b_{BR}} \leqslant \frac{1}{2}, \tag{10}$$

since $b_{BR} \leqslant 1$. In the most central collisions STAR measured $b_{BF}|_{\mathrm{STAR}} \approx 0.58$, which violates this bound.[3] Thus we arrive at an interesting conclusion that in the midrapidity region in the most central Au + Au collisions the following inequality holds:

$$b_{BR} < b_{BF}. \tag{11}$$

It was checked by STAR that narrowing the reference bin R from $|\eta| < 0.5$ to $|\eta| < 0.1$ (so that all windows have the same widths) slightly increases the correlation coefficient $b_{BF}|_{\mathrm{STAR}}$. Also an alternative method of centrality determination was carried out using the STAR zero-degree calorimeter (measurement of forward neutrons) for the 0–10% centrality, and $b_{BF}|_{\mathrm{STAR}}$ is very close to $\frac{1}{2}$. In this case the same formula (3) applies; however, there are no explicate cuts on n_R. We conclude that the width of R and the centrality cut on n_R is not a factor in the result (11).

[2]Naively, it seems that both procedures should lead to the same result. We can always measure $\langle O\rangle_{n_R}$ at a given n_R and calculate $\langle O\rangle = \sum_{n_R} P(n_R)\langle O\rangle_{n_R}$. In this case,

$$\begin{aligned} D^2_{BF} = \langle n_B n_F\rangle - \langle n_B\rangle^2 = \sum\nolimits_{n_R} P(n_R)\langle n_B n_F\rangle_{n_R} \\ - \left(\sum\nolimits_{n_R} P(n_R)\langle n_B\rangle_{n_R}\right)^2, \end{aligned}$$

which is clearly different from Eq. (3).

[3]The STAR result has an uncertainty ±0.06. Even if one assumes that the measured $b_{BF}|_{\mathrm{STAR}}$ is slightly below 0.5, it is still difficult to understand with an assumption $b_{BR} \geqslant b_{BF}$, since it requires $b_{BR} \approx b_{BF} \approx 1$.

It is interesting to estimate the numerical values of the correlation coefficients b_{BF} and b_{BR}. As mentioned earlier we are mostly interested in the configuration where the distance between B and F is a maximum one ($\Delta\eta = 1.8$ in the STAR notation) and R is defined by $|\eta| < 0.5$.

As seen from Eq. (8) evaluation of $b_{BF} = D^2_{BF}/D^2_{BB}$ is straightforward. The covariance $D^2_{BF}|_{\rm STAR}$ and variance $D^2_{BB}|_{\rm STAR}$ are published in [15] (only for 0–10% centrality bin). From Ref. [22] one sees that $\langle n_B\rangle_{n_R}$ is a linear function of n_R with a coefficient $c_1 \approx 0.2$. To calculate $D^2_{RR} = \langle n_R^2\rangle - \langle n_R\rangle^2$ we use the uncorrected (raw) multiplicity distribution $P(n_R^{\rm raw})$ as published in Ref. [23], and take the efficiency correction to be $n_R/n_R^{\rm raw} = 1.22$ [22,23]. Performing a straightforward calculation we obtain[4] $D^2_{RR} \approx 4320$, which allows us to calculate b_{BF}. Taking Eq. (9), b_{BF}, and measured $b_{BF}|_{\rm STAR}$ into account we obtain

$$b_{BR} \approx 0.58, \quad b_{BF} \approx 0.72. \tag{12}$$

As seen from Eqs. (12) in the most central Au + Au collisions b_{BR} is significantly smaller than b_{BF}. Let us note here that the average distance between B and R (one unit of η) is smaller by a factor of two than that between B and F.

It is also interesting to see how b_{BF} depends on the distance $\Delta\eta$ between bins B and F. Taking Eq. (8) into account and repeating calculations[5] presented above we found that b_{BF} in central Au + Au collision is approximately constant as a function of $\Delta\eta$, which is consistent with the dependence of $b_{BF}|_{\rm STAR}$ on $\Delta\eta$.

Finally, let us notice that STAR also measured $b_{BF}|_{\rm STAR}$ in $p + p$ collisions; however, in this case the exact value of c_1 is not known. We checked that for a very broad range of c_1 we always obtain a standard ordering $b_{BR} > b_{BF}$.[6]

Several comments are warranted:

(i) To calculate the correlation coefficients b_{BF} and b_{BR} the experimental values of $D^2_{BF}|_{\rm STAR}$ and $D^2_{BB}|_{\rm STAR}$ are required as an input. Unfortunately they are provided only for the most central collisions. It would be interesting to measure the centrality dependence of the effect reported in this Rapid Communication. It is expected that in peripheral collisions the standard relation $b_{BR} > b_{BF}$ should be recovered. If so, it would indicate a qualitatively different behavior of central and peripheral Au + Au collisions.

(ii) It is worth mentioning that HIJING [24] and the Parton string model (PSM) [25] fail to describe the Au + Au data for the forward-backward multiplicity correlation coefficient. However, they are consistent with the $p + p$ data. In the most central Au + Au collisions, and for the configuration presented in Fig. 1, both models predict $b_{BF}|_{\rm STAR} < \frac{1}{2}$, which is consistent with the relation $b_{BR} > b_{BF}$.[7]

(iii) It is not straightforward to propose a realistic mechanism that more strongly correlates bins B and F than bins B and R. One possible mechanism is the formation of certain *clusters* strongly peaked at $\eta = 0$ that decay symmetrically into two particles. This mechanism obviously correlates bins B and F and introduces no (or much weaker) correlations between bins B and R. To go beyond speculations more detailed measurement of the forward-backward correlations between symmetric and asymmetric bins is warranted.

In summary, we analyzed the STAR data on the forward-backward multiplicity correlation coefficient $b_{BF}|_{\rm STAR}$ in the most central Au + Au collisions. This measurement was performed with the intermediate step of fixing the number of particles in the third reference window R, see Fig. 1, and we emphasized the importance of this step. We derived the general formula that relates $b_{BF}|_{\rm STAR}$ and the correlation coefficients b_{BF} and b_{BR} measured in $B - F$ and $B - R$ without fixing the number of particles in R.

The most important result is the observation that for the configuration presented in Fig. 1; in the most central Au + Au collisions, the correlation coefficient b_{BR} is significantly smaller than b_{BF}. This is exactly opposite of what is expected and measured in $p + p$ collisions (the distance between B and R is smaller by a factor of two than that between B and F). Moreover, we found that in central Au + Au collisions, b_{BF} is approximately constant as a function of the pseudorapidity separation between symmetrically located bins B and F. To understand these results it is necessary to assume that in central Au + Au collisions the two-particle correlation function strongly decreases as a function of $|\eta_1 + \eta_2|$. It indicates the presence of a specific mechanism of correlation that strongly correlates bins located symmetrically around $\eta = 0$ for which $|\eta_1 + \eta_2| \approx 0$, but is less effective for asymmetric bins $|\eta_1 + \eta_2| > 0$.[8]

In this Rapid Communication we solely concentrated on an analysis of the experimental results and at the moment we see no compelling explanation of this effect. It would be interesting to directly measure at RHIC and LHC the multiplicity correlation coefficient for symmetric and asymmetric bins to confirm conclusions presented in this discussion.

We thank Andrzej Bialas and Larry McLerran for enlightening discussions. Correspondence with Brijesh Srivastava is highly appreciated. This investigation was supported by the US Department of Energy under Contract No. DE-AC02-98CH10886 and by Grant No. N202 125437 of the Polish Ministry of Science and Higher Education (2009–2012).

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[4]We take $P(n_R^{\rm raw}) \propto \exp(-\frac{n_R^{\rm raw}}{370})$ for $431 \leqslant n_R^{\rm raw} \leqslant 560$ and $P(n_R^{\rm raw}) \propto \exp(-\frac{(n_R^{\rm raw}-561)^2}{2700})$ for $n_R^{\rm raw} \geqslant 561$, which gives $D^2_{RR}|_{\rm raw} = 2904$. Consequently, $D^2_{RR} = (1.22^2)D^2_{RR}|_{\rm raw}$.

[5]For small $\Delta\eta$ the reference window R is composed of two windows $0.5 < |\eta| < 1$ and we assume that $c_1^2 D^2_{RR}$ is approximately the same as with R defined by $|\eta| < 0.5$.

[6]We assume $P(n_R)$ to be given by a negative binomial distribution with standard parameters $\langle n_R\rangle = 2.3$ and $k = 2$. Taking, e.g., $c_1 = 0.1$ we obtain $b_{BR} \approx 0.28$ and $b_{BF} \approx 0.13$.

[7]In particular $b_{BF}|_{\rm STAR} \approx 0.1$ in HIJING and $b_{BF}|_{\rm STAR} \approx 0.4$ in PSM; see Ref. [15].

[8]It also indicates a strong violation of boost invariance in the midrapidity region [26].

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