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Pygmy dipole resonance: Collective features and symmetry energy effects

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A very important open question related to the pygmy dipole resonance refers to its quite elusive collective nature. In this paper, within a harmonic oscillator shell model, generalizing an approach introduced by Brink, we first identify the dipole normal modes in neutron-rich nuclei and derive the energy-weighted sum rule exhausted by the pygmy dipole resonance. Then, by solving numerically the self-consistent Landau-Vlasov kinetic equations for neutrons and protons with specific initial conditions, we explore the structure of the different dipole vibrations in the ^{132}Sn system and investigate their dependence on the symmetry energy. We evidence the existence of a distinctive collective isoscalar-like mode with an energy well below the giant dipole resonance (GDR), which is very weakly dependent on the isovector part of the nuclear effective interaction. At variance with this, its corresponding strength is rather sensitive to the behavior of the symmetry energy below saturation, which rules the number of excess neutrons in the nuclear surface.

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One of the important tasks in many-body physics is to understand the emergence of collective features as well as their structure in terms of the individual motion of the constituents. Exotic collective excitations show up when one moves away from the valley of stability [1]. Experimental characterization and theoretical description of these excitations is a challenge for modern nuclear physics. Recent experiments provided several pieces of evidence about their existence, but the available information is still incomplete and their nature is a matter of debate.

An interesting exotic mode is pygmy dipole resonance (PDR), which was observed as an unusually large concentration of the dipole response at energies clearly below the values associated with the giant dipole resonance (GDR). The latter is one of the most prominent and robust collective motions, present in all nuclei whose centroid position varies for medium-heavy nuclei, as $80A^{-1/3}$ MeV. Adrich *et al.* [2] reported the observation of a resonant-like shape distribution with a pronounced peak around 10 MeV in ^{130}Sn and ^{132}Sn isotopes. A concentration of dipole excitations near and below the particle emission threshold was also observed in stable Sn nuclei, and a systematics of PDR in these systems is presented in Ref. [3]. It was concluded that the strongest transitions locate at energies between 5 and 8.5 MeV, and a sizable fraction of the energy-weighted sum rule (EWSR) is exhausted by these states. From a comparison of the available data for stable and unstable Sn isotopes, a correlation between the fraction of pygmy strength and isospin asymmetry was noted [4]. In general the exhausted sum rule increases with the proton-to-neutron asymmetry. This behavior was related to the symmetry energy properties below saturation and therefore connected to the size of the neutron skin [5–7]. However, other theoretical analyses suggest a weak connection between PDR and the skin thickness [8].

In spite of the theoretical progress in the interpretation of this mode within phenomenological studies based on hydrodynamical equations [9,10], nonrelativistic microscopic approaches using random phase approximation (RPA) with various effective interactions [11–13] or relativistic quasiparticle RPA [14,15], and new experimental information [16–19], a number of critical questions concerning the nature of the PDR have remained unanswered. These include the macroscopic picture of neutron and proton vibrations, the exact location of the PDR excitation energy, the degree of collectivity of the low-energy dipole states, and the role of the symmetry energy [20]. Some microscopic studies predict a large fragmentation of the GDR strength and the absence of collective states in the low-lying region in ^{132}Sn [21].

Our goal is to address the important issue related to the collective nature of PDR in connection with the role of the symmetry energy. Within the harmonic oscillator shell model (HOSM) for neutron-rich nuclei, we show first that the coordinates associated with the neutron excess vibration against the core and the dipole core mode are separable, and so we can derive the EWSR exhausted by each of them. Then we adopt a description based on the Fermi liquid theory with effective interactions and investigate the dynamics and the interplay between the dipole modes identified in HOSM. This self-consistent, transport-based treatment allows us to inquire on the role of the symmetry energy and its density dependence upon the dipole response.

In a seminal paper [22], Brink has shown that for a system of $A = N + Z$ nucleons moving in a harmonic oscillator well with the Hamiltonian $H_{sm} = \sum_{i=1}^{A} \frac{\vec{p}_i^2}{2m} + \frac{K}{2} \sum_{i=1}^{A} \vec{r}_i^2$, it is possible to perform a separation in four independent parts $H_{sm} = H_{n\text{int}} + H_{p\text{int}} + H_{\text{c.m.}} + H_D$. The first two terms determine the internal motion of protons and neutrons respectively, depending only on the proton-proton and neutron-neutron relative coordinates. $H_{\text{c.m.}} = \frac{1}{2Am} \vec{P}_{\text{c.m.}}^2 + \frac{KA}{2} \vec{R}_{\text{c.m.}}^2$ characterizes the nucleus center of mass (c.m.) motion, while $H_D = \frac{A}{2mNZ} \vec{P}^2 + \frac{KNZ}{2A} \vec{X}^2$ describes a Goldhaber-Teller (G-T) [23] vibration of protons against neutrons. The oscillator constant K can be determined by fitting the nuclear size [24]. $\vec{X}$ and $\vec{R}_{\text{c.m.}}$ denote the neutron-proton relative coordinate and the center of mass position, and we have introduced the conjugate momenta

$\vec{P} = \frac{NZ}{A}(\frac{1}{Z}\vec{P}_Z - \frac{1}{N}\vec{P}_N)$ and $\vec{P}_{\mathrm{c.m.}} = \vec{P}_Z + \vec{P}_N$, where $\vec{P}_Z$ ($\vec{P}_N$) are proton (neutron) total momenta. Correspondingly, the eigenstates of the nucleus are represented as a product of four wave functions $\Psi = \psi_{n\mathrm{int}}\chi_{p\mathrm{int}}\alpha(\vec{R}_{\mathrm{c.m.}})\beta(\vec{X})$, which are the eigenvectors of the four Hamiltonians constructed above. For an $E1$ absorption a G-T collective motion with a specific linear combination of single-particle excitations is produced and the wave function $\beta(\vec{X})$ is changing from the ground state to the one-GDR phonon state. Denoting by E_i the energy eigenvalues of the system and by D the dipole operator, with the use of the Thomas-Runke-Kuhn (TRK) sum rule, the total absorption cross section is given by

$$\sigma_D = \int_0^\infty \sigma(E)dE = \frac{4\pi^2 e^2}{\hbar c}\sum_i E_i|\langle i|D|0\rangle|^2$$
$$= \frac{4\pi^2 e^2}{\hbar c}\frac{1}{2}\langle 0|[D,[H_{sm},D]]|0\rangle = 60\frac{NZ}{A}\text{mb MeV}.$$

Now let us turn to the physical situation, corresponding to the case of very neutron-rich nuclei, where the system is conveniently described in terms of a bound core containing all protons and N_c neutrons, plus some (less bound) excess neutrons N_e. Thus the total neutron number N is split into the sum $N = N_c + N_e$ and we denote by $A_c = Z + N_c$ the number of nucleons contained in the core. In this case we have worked out an exact separation of the HOSM Hamiltonian in a sum of six independent (commuting) quantities: $H_{sm} = H_{n_c\mathrm{int}} + H_{p_c\mathrm{int}} + H_{e\mathrm{int}} + H_{\mathrm{c.m.}} + H_c + H_y$. The first three terms contain only relative coordinates and momenta among nucleons of each ensemble (i.e., core neutrons, core protons, and excess neutrons) and, as before, describe their internal motion. $H_c = \frac{A_c}{2ZN_cm}\vec{P}_c^2 + \frac{KN_cZ}{2A_c}\vec{X}_c^2$ characterizes the core dipole vibration, while the relative motion of the excess neutrons against the core, usually associated with the pygmy mode, is determined by $H_y = \frac{A}{2A_cN_em}\vec{P}_y^2 + \frac{KN_eA_c}{2A}\vec{Y}^2$. Here $\vec{X}_c$ denotes the distance between neutron and proton centers of mass in the core, while $\vec{Y}$ is the distance between the core center of mass and the center of mass of the excess neutrons. The corresponding canonically conjugate momenta are $\vec{P}_c = \frac{N_cZ}{A_c}(\frac{1}{Z}\vec{P}_Z - \frac{1}{N_c}\vec{P}_{N_c})$ and $\vec{P}_y = \frac{N_eA_c}{A}[\frac{1}{A_c}(\vec{P}_Z + \vec{P}_{N_c}) - \frac{1}{N_e}\vec{P}_{N_e}]$. The eigenstates of H_c and H_y are describing two independent collective excitations, and both of them will contribute to the dipole response since the total dipole momentum can be expressed as $\vec{D} = \frac{NZ}{A}\vec{X} = \frac{ZN_c}{A_c}\vec{X}_c + \frac{ZN_e}{A}\vec{Y} \equiv \vec{D}_c + \vec{D}_y$. In this picture the PDR results in a collective motion of G-T type with the excess neutrons oscillating against the core. The $E1$ absorption leads also to the change of the wave function associated with the coordinate $\vec{Y}$. The total cross section for the PDR is $\sigma_y = \frac{4\pi^2e^2}{\hbar c}\frac{1}{2}\langle 0|[D_y,[H_{sm},D_y]]|0\rangle = \frac{N_eZ}{NA_c}\sigma_D$. This shows that a fraction $f_y = \frac{N_eZ}{NA_c}$ of the EWSR is exhausted by the pygmy mode. It is worth mentioning that this result is consistent with the molecular sum rule introduced by Alhassid *et al.* [25] and an identical expression was obtained within other approaches, such as the sum-rule model [26]. For the tin isotope ^{132}Sn, if the excess neutrons were simply defined as the difference between neutron and proton numbers, that is, $N_e = 32$, one would expect $f_y = 19.5\%$. This is greater than the value estimated experimentally, which is around 5%. A possible explanation for this difference is that only a part of the excess neutrons, N_y, with $N_y < N_e$, contribute to PDR, the rest being still bound to the core. [1]

Therefore it is important to test this assumption within a more sophisticated analysis of the dipole response. Indeed, a more accurate picture of the GDR in nuclei corresponds to an admixture of G-T and Stenweidel-Jensen (S-J) vibrations. The latter, in symmetric nuclear matter, is a volume-type oscillation of the isovector density $\rho_i = \rho_n - \rho_p$ keeping the total density $\rho = \rho_n + \rho_p$ constant [27]. A microscopic, self-consistent study of the collective features and the role of the nuclear effective interaction upon the PDR can be performed within the Landau theory of Fermi liquids. This is based on two coupled Landau-Vlasov kinetic equations for neutron and proton one-body distribution functions $f_q(\vec{r},\vec{p},t)$ with $q = n, p$:

$$\frac{\partial f_q}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f_q}{\partial \mathbf{r}} - \frac{\partial U_q}{\partial \mathbf{r}}\frac{\partial f_q}{\partial \mathbf{p}} = I_{\mathrm{coll}}[f], \quad (1)$$

and was applied quite successfully in describing various features of the GDR, including pre-equilibrium dipole excitation in fusion reactions [28]. However, it should be noticed that within such a semiclassical description, the shell effects, which are certainly important in shaping the fine structure of the dipole response [29], are absent. Based on a linear response theory in a semiphenomenological model, some properties of the PDR were investigated with the use of Eq. (1) [30]. By solving numerically the Vlasov equation in the absence of Coulomb interaction, Urban [31] evidenced from the study of the total dipole moment D a collective response around 8.6 MeV, which was identified as a pygmy mode. It was pointed out, from the properties of transition densities and velocities, that the PDR can be related to one of the low-lying modes associated with isoscalar toroidal excitations, providing indications about its isoscalar character. Here, by considering in the transport simulations and the Coulomb interaction, we investigate in a complementary way the collective nature of PDR by studying the dynamics of the pygmy degree of freedom D_y suggested by HOSM. Moreover, we explore the isoscalar character of the mode by a comparative analysis employing three different density parametrizations of the symmetry energy.

We neglect the two-body collision effects, and hence the main ingredient of the dynamics is the nuclear mean-field, for which we consider a Skyrme-like (SKM^*) parametrization $U_q = A\frac{\rho}{\rho_0} + B(\frac{\rho}{\rho_0})^{\alpha+1} + C(\rho)\frac{\rho_n-\rho_p}{\rho_0}\tau_q + \frac{1}{2}\frac{\partial C}{\partial\rho}\frac{(\rho_n-\rho_p)^2}{\rho_0}$, where $\tau_q = +1(-1)$ for $q = n(p)$ and ρ_0 denotes the saturation density. The saturation properties of symmetric nuclear matter are reproduced with the values of the coefficients $A = -356.8$ MeV, $B = 303.9$ MeV, $\alpha = 1/6$, leading to a compressibility modulus $K = 201$ MeV. For the isovector sector we employ three different parametrizations of $C(\rho)$ with the density, termed asysoft, asystiff, and asysuperstiff respectively; see Ref. [32] for a detailed description. The values of the symmetry energy, $E_{\mathrm{sym}}/A = \frac{\epsilon_F}{3} + \frac{C(\rho)}{2}\frac{\rho}{\rho_0}$, at

[1] If naively we assume the core neutrons to be ones of the most bound Sn isotopes (^{118}Sn and ^{120}Sn), we get as excess neutrons $N_e \simeq 12 \div 14$. The HOSM-EWSR fraction is then reduced to $f_y \simeq 7.3 \div 8.5\%$.

TABLE I. The symmetry energy at saturation, slope parameters, neutron rms radius, protons rms radius, and neutron skin thickness for the three asy-EoS.

asy-EoS	E_{sym}/A (MeV)	L(MeV)	R_n(fm)	R_p(fm)	ΔR_{np} (fm)
asysoft	29.9	14.4	4.90	4.65	0.25
asystiff	28.3	72.6	4.95	4.65	0.30
asysuperstiff	28.3	96.6	4.96	4.65	0.31

saturation, as well as the slope parameter, $L = 3\rho_0 \frac{dE_{\text{sym}}/A}{d\rho}|_{\rho=\rho_0}$, are reported in Table I for each of these asy-equations of state (EoS). Just below the saturation density the asysoft mean field has a weak variation with the density while the asysuperstiff shows a rapid decrease. Then, due to surface contributions to the collective oscillations, we expect to see some differences in the energy position of the dipole response of the system.

The numerical procedure to integrate the transport equations is based on the test-particle (t.p.) method. For a good spanning of the phase space, we work with 1200 t.p. per nucleon. We consider the neutron-rich nucleus ^{132}Sn and we determine its ground-state configuration as the equilibrium (static) solution of Eq. (1). Then the proton and neutron densities $\rho_q(\vec{r},t) = \int \frac{2d^3\mathbf{p}}{(2\pi\hbar)^3} f_q(\vec{r},\vec{p},t)$ can be evaluated. The radial density profiles for two asy-EOS are reported in Fig. 1. As an additional check of our initialization procedure, the neutron and proton mean square radii $\langle r_q^2\rangle = \frac{1}{N_q}\int r^2 \rho_q(\vec{r},t) d^3\mathbf{r}$, as well as the skin thickness $\Delta R_{np} = \sqrt{\langle r_n^2\rangle} - \sqrt{\langle r_p^2\rangle}$, were also calculated in the ground state and are shown in Table I. The values obtained with our semiclassical approach are in a reasonable agreement with those reported by employing other models for similar interactions [33]. The neutron skin thickness increases with the slope parameter, as expected from a faster reduction of the symmetry term on the surface [5,32]. This feature has been discussed in detail in Ref. [7].

To determine the collective properties of the pygmy dipole, we excite the nuclear system at the initial time $t = t_0 =$ 30 fm/c by boosting along the z direction all the excess neutrons and in the opposite direction all the core nucleons, while keeping the c.m. of the nucleus at rest (Pygmy-like initial conditions). The excess neutrons were identified as the most distant $N_e = 32$ neutrons from the nucleus c.m. Then the system is left to evolve, and the evolution of the collective coordinates Y, X_c, and X, associated with the different dipole modes is followed for 600 fm/c by solving numerically Eq. (1). During the time evolution, the number of t.p. escaping from the system corresponds, on average, to less than half a neutron, while the total energy conservation is satisfied within 1.5%. The stability of the ground state was also checked, with the mass number and energy even better preserved in this case. The simple estimate of the EWSR provided by the HOSM suggests, when compared with the experiments, that some of the N_e neutrons boosted in the initial conditions are still bound to the core. This is confirmed by the transport simulations. Indeed, apart from the quite undamped oscillations of the Y coordinate, we also remark that the core does not remain inert. In Fig. 2 we plot the time evolution of the dipole D_y, the total dipole D, and the core dipole D_c moments, for two asy-EoS. As observed, while D_y approaches its maximum value, an oscillatory motion of the dipole D_c initiates and this response is symmetry energy dependent: the larger is the slope parameter L and the more

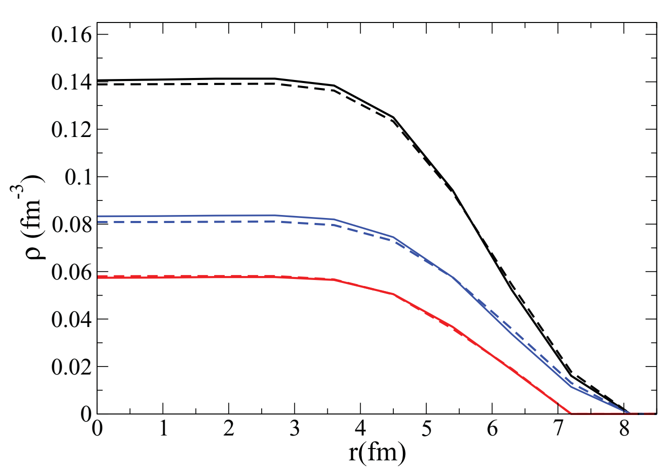

FIG. 1. (Color online) The total [the black (upper) lines], neutron [the blue (middle) lines], and proton [the red (lower) lines] radial density profiles for asysoft (the solid lines) and asysuperstiff (the dashed lines).

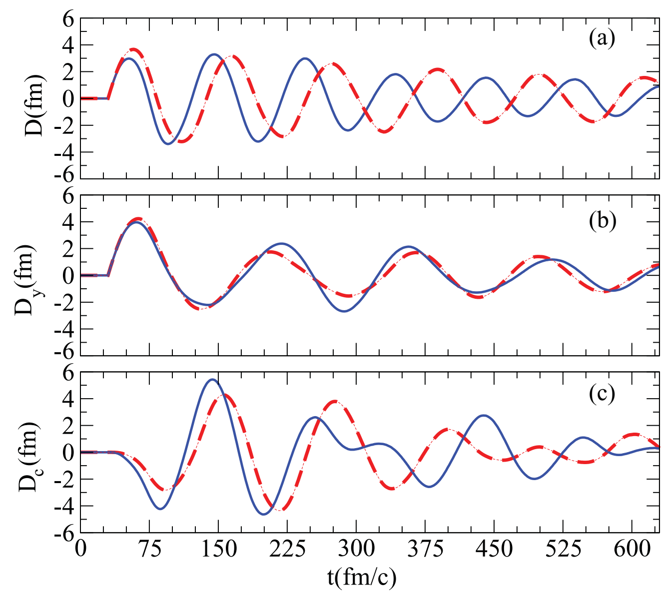

FIG. 2. (Color online) The time evolutions of the total dipole D (a), the dipole D_y (b), and core dipole D_c (c) for asysoft [the blue (solid) lines] and asysuperstiff [the red (dashed) lines] EoS. Pygmy-like initial excitation.

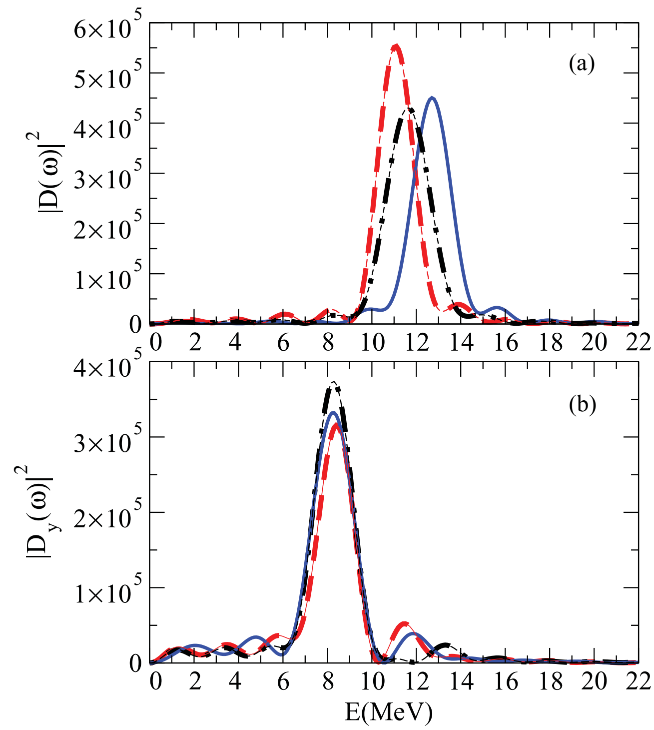

FIG. 3. (Color online) The power spectra of total dipole (a) and dipole D_y (b) (in fm^4/c^2) for asysoft [the blue (solid) lines], asystiff [the black (dot-dashed) lines], and asysuperstiff [the red (dashed) lines] EoS. Pygmy-like initial conditions.

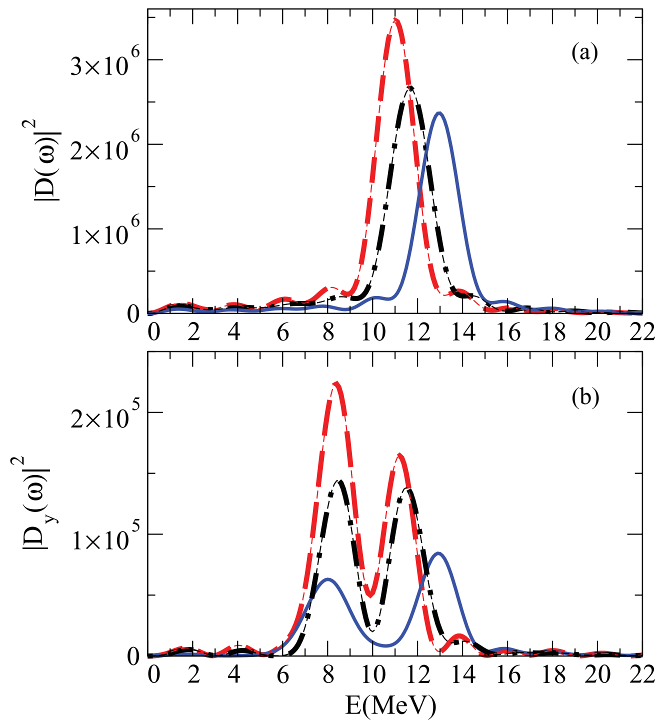

FIG. 4. (Color online) The same as in Fig. 3 but for a GDR-like initial excitation.

delayed is the isovector core reaction. This can be explained in terms of low-density (surface) contributions to the vibration and therefore of the density behavior of the symmetry energy below the normal density: a larger L corresponds to a larger neutron presence in the surface and so to a smaller coupling to the core protons. We see that the total dipole $D(t)$ is strongly affected by the presence of isovector core oscillations, mostly related to the isovector part of the effective interaction. Indeed, $D(t)$ gets a higher oscillation frequency with respect to D_y, sensitive to the asy-EOS. The fastest vibrations are observed in the asysoft case, which gives the largest value of the symmetry energy below saturation. In correspondence, the frequency of the pygmy mode seems to be not much affected by the trend of the symmetry energy below saturation (see also Fig. 3), clearly showing the different nature, isoscalar-like, of this oscillation. For each case we calculate the power spectrum of D_y: $|D_y(\omega)|^2 = |\int_{t_0}^{t_{\max}} D_y(t)e^{-i\omega t}dt|^2$ and similarly for D. The results are shown in Fig. 3. The position of the centroid corresponding to the GDR shifts toward larger values when we move from asysuperstiff (largest slope parameter L) to asysoft EoS. This evidences the importance of the volume, the S-J component of the GDR in ^{132}Sn. The energy centroid associated with the PDR is situated below the GDR peak, at around 8.5 MeV, quite insensitive to the asy-EOS, pointing to an isoscalar-like nature of this mode. A similar conclusion was reported within a relativistic mean-field approach [34]. While in the schematic HOSM all the dipole modes are degenerate, with an energy $E = 41A^{-1/3} \approx 8$ MeV for ^{132}Sn, within the Vlasov approach the GDR energy is pushed up by the isovector interaction. Hence the structure of the dipole response can be explained in terms of the development of isoscalar-like (PDR) and isovector-like (GDR) modes, as observed in asymmetric systems [35]. Both modes are excited in the considered pygmy-like initial conditions. Looking at the total dipole mode direction, which is close to the isovector-like normal mode, one observes a quite large contribution in the GDR region. On the other hand, although the pygmy mode has a more complicated structure [31], the Y direction appears closely related to it. Indeed a larger response amplitude is detected in the pygmy region; see Fig. 3(b). To check the influence of the initial conditions on the dipole response, let us consider the case of a GDR-like excitation, corresponding to a boost of all neutrons against all protons, keeping the c.m. at rest. The initial collective energy corresponds to the first GDR excited state, around 15 MeV. Now the initial excitation favors the isovector-like mode and even in the Y direction we observe a sizable contribution in the GDR region; see the Fourier spectrum of D_y in Fig. 4. From this result it clearly emerges that a part of the N_e excess neutrons is involved in a GDR-type motion and the relative weight depends on the symmetry energy: more neutrons are involved in the pygmy mode in the asysuperstiff EOS case, in connection to the larger neutron skin size. We have also checked that, if the coordinate Y is constructed by taking the N_y most distant neutrons (with $N_y < N_e$), the relative weight increases in the PDR region. In any case, since part of the excess neutrons contributes to the GDR mode, an EWSR value lower than the HOSM predictions corresponding to $N_y = N_e$ is expected. Indeed, in the Fourier power spectrum of D in Fig. 4, a weak response is seen at the pygmy frequency.

These investigations also raise the question of the appropriate way to excite the PDR. Nuclear rather than electromagnetic probes can induce neutron skin excitations closer to our first class of initial conditions [36]. In the case of the GDR-like initial excitation, we can relate the strength function to $\mathrm{Im}[D(\omega)]$ [37] and then the corresponding cross section can be calculated.[2] Our estimate of the integrated cross section over the PDR region represents 2.7% for asysoft, 4.4% for asystiff, and 4.5% for asysuperstiff, out of the total cross section. Hence the EWSR exhausted by the PDR is proportional to the skin thickness, in agreement with the results of [38]. The fraction of photon emission probability associated with the PDR region can be estimated from the total dipole acceleration, within a bremsstrahlung approach [39].[3] We obtain a percentage of 4.7% for asysoft, 7.7% for asystiff, and 9% for asysuperstiff EOS, consistent with the previous interpretation.

In summary, in this work we evidence, within both HOSM and a semiclassical Landau-Vlasov approach, the existence in neutron-rich nuclei of a collective pygmy dipole mode determined by the oscillations of some excess neutrons against the nuclear core. From the transport simulations the PDR energy centroid for ^{132}Sn appears around 8.5 MeV, well below the GDR peak, and is rather insensitive to the density dependence of the symmetry energy. This supports the isoscalar-like character of this collective motion. A complex pattern, involving the coupling of the neutron skin with the core dipole mode, is noticed. While HOSM can provide some predictions of the EWSR fraction exhausted by the pygmy mode, $f_y = \frac{N_y Z}{N A_c}$, depending on the number $N_y \leqslant N_e$ of neutrons involved, the transport model indicates that part of the excess neutrons N_e are coupled to the GDR mode and gives some hints about the number of neutrons, N_y, that are actually participating in the pygmy mode. This reduces considerably the EWSR acquired by the PDR, our numerical estimate providing values well below 10% but proportional to the symmetry energy slope parameter L,which affects the number of excess neutrons on the nuclear surface. We consider these effects as related also to the S-J component of the dipole dynamics in medium-heavy nuclei. It is therefore interesting to extend the present analysis to lighter nuclei, like Ni or Ca isotopes, where the Goldhaber-Teller component can be more important.

We mention that such self-consistent, transport approaches can be valuable in exploring the collective response of other mesoscopic systems where similar normal modes may manifest; see Ref. [40] for a time-dependent approach to the surface and volume plasmons in buckminsterfullerene and Ref. [41] for a study of out-of-phase dipolar oscillation of the thermal cloud and Bose-Einstein condensate.

[2]We consider that six oscillations in the presence of Landau damping will provide a reliable, though approximate, estimate of the strength function.

[3]The photon spectrum is estimated from $\frac{dP}{dE_\gamma} = \frac{2e^2}{3\pi\hbar c^3 E_\gamma}|d''(\omega)|^2$, where $|d''(\omega)|^2$ is the power spectrum of dipole acceleration.

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