Experimental constraint on the ρ-meson form factors in the time-like region

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The annihilation reaction $e^+ + e^- \to \bar{\rho} + \rho$ is considered. The constraint on time-like ρ-meson form factors from the measurement done by the BaBar Collaboration at $\sqrt{s} = 10.58$ GeV is analyzed.

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Introduction. Hadron and meson electromagnetic form factors (FFs) provide important information about the structure and internal dynamics of these systems. They have been the object of extended experimental studies for many decades. Presently, new facilities and detectors allow one to reach high precision and to access new kinematical regions. Assuming P and T invariance, a particle of spin S is characterized by $2S + 1$ electromagnetic FFs. This number corresponds to the maximum number of independent scalars, which can be built on the available momenta and spin vectors, for the vertex $\gamma^* HH$, where H is a nonpointlike hadron.

The case of deuteron, which has spin 1, has been largely discussed in the literature. The individual determination of the three deuteron FFs requires the measurement of the differential cross section and at least one polarization observable, usually the tensor polarization, t_{20}, of the scattered deuteron in unpolarized ed scattering. Data on the three deuteron FFs, charge G_C, quadrupole G_Q, and magnetic G_M, are available up to a momentum transfer squared $Q^2 = 1.9$ GeV^2 [1]. They are best described by a model based on a six-quark hard core and a meson cloud [2]. They contradict, surprisingly, QCD predictions, even at the largest Q^2 value experimentally reached which corresponds to internal distances smaller than the nucleon dimension.

The time-like (TL) region, accessible through annihilation reactions, is expected to bring a new insight into FFs. As the measurement of deuteron FFs in the TL region is beyond the present experimental possibilities, it is interesting to measure the electromagnetic FFs of the ρ meson, which has also spin 1. The most simple reaction which contains information on TL ρ-meson FFs is the annihilation of an electron-positron pair into a $\rho^+\rho^-$ pair. This question has been discussed in a previous work [3]. Following a model-independent formalism developed for spin-1 particles in Ref. [4], the differential (and total) cross sections and various polarization observables were calculated in terms of the electromagnetic FFs of the corresponding $\gamma^*\rho\rho$ current. The elements of the spin-density matrix of the ρ meson were also calculated.

The estimation of the observables was done on the basis of a simple vector dominance (VMD) parametrization for ρ-meson FFs. The parameters were adjusted in order to reproduce the existing theoretical predictions in the SL region [5] where the ρ-meson electromagnetic FFs were calculated, both in covariant and light-front formalisms with constituent quarks. The parametrization was then analytically extended to the TL region. Since that time, the BaBar Collaboration has detected four pions identifying the $e^+ + e^- \to \rho^+ + \rho^-$ reaction [6]. The results have been given in terms of helicity amplitudes. The purpose of this work is to give the correspondence between our formalism and the helicity amplitudes and to evaluate the constraint that this unique experimental data point sets on our parameters. Relating our description of the $\gamma^* \to \rho^+\rho^-$ vertex in terms of the electromagnetic FFs of the ρ meson with the helicity amplitudes for this vertex, we can obtain the absolute values of FF moduli at the q^2 value where the experiment was done.

Formalism. Let us consider the transition

$$\gamma^*(q) \to \rho^-(p_1) + \rho^+(p_2), \tag{1}$$

where $q = p_1 + p_2$, $p_1^2 = p_2^2 = M^2$, and M is the ρ-meson mass. We consider this transition in the center of mass system (CMS) of the two ρ mesons. The 4-momenta of the considered particles are

$$q = (W, 0), \quad p_1 = (E, \vec{p}), \quad p_2 = (E, -\vec{p}),$$

where $W = \sqrt{q^2}$, $E(\vec{p})$ is the ρ-meson energy (momentum). Let us choose the z axis along the vector $\vec{p}$, i.e., along the ρ^+-meson momentum. Then the polarization states in the helicity basis are

$$\epsilon_\mu^{(+)} = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad \epsilon_\mu^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0),$$
$$\epsilon_\mu^{(0)} = (0, 0, 0, 1),$$
$$U_{1\mu}^{(+)} = -\frac{1}{\sqrt{2}}(0, 1, i, 0), \quad U_{1\mu}^{(-)} = \frac{1}{\sqrt{2}}(0, 1, -i, 0),$$
$$U_{1\mu}^{(0)} = \frac{1}{M}(p, 0, 0, E),$$

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$$U^{(+)}_{2\mu} = -\frac{1}{\sqrt{2}}(0,-1,i,0),$$
$$U^{(-)}_{1\mu} = \frac{1}{\sqrt{2}}(0,-1,-i,0), \quad U^{(0)}_{1\mu} = \frac{1}{M}(-p,0,0,E), \tag{2}$$

where $\epsilon^{(\lambda)}_\mu$ and $U^{(\lambda)}_{1\mu}(U^{(\lambda)}_{2\mu})$ are the polarization vectors of the virtual photon and of the $\rho^-(\rho^+)$ meson with helicity λ.

Because the ρ meson is a spin-1 particle, its electromagnetic current is completely described by three FFs. Assuming P and C invariance of the hadron electromagnetic interaction, this current can be written as [7]

$$J_\mu = (p_1-p_2)_\mu\bigg[-G_1(q^2)U_1^*\cdot U_2^* + \frac{G_3(q^2)}{M^2}\left(U_1^*\cdot qU_2^*\cdot q - \frac{q^2}{2}U_1^*\cdot U_2^*\right)\bigg] - G_2(q^2)(U^*_{1\mu}U_2^*\cdot q - U^*_{2\mu}U_1^*\cdot q), \tag{3}$$

where $G_i(q^2)$ $(i=1,2,3)$ are the ρ-meson electromagnetic FFs. The FFs $G_i(q^2)$ are complex functions of the variable q^2 in the region of the TL momentum transfer ($q^2>0$). They are related to the standard ρ-meson electromagnetic FFs: G_C (charge monopole), G_M (magnetic dipole), and G_Q (charge quadrupole) by

$$G_M = -G_2, \quad G_Q = G_1+G_2+2G_3,$$
$$G_C = -\frac{2}{3}\tau(G_2-G_3) + \left(1-\frac{2}{3}\tau\right)G_1, \quad \tau = \frac{q^2}{4M^2}, \tag{4}$$

or, inversely,

$$G_1 = G_Q + G_M - \frac{1}{\tau-1}\left[G_C - G_M - \left(1-\frac{2}{3}\tau\right)G_Q\right],$$
$$G_2 = -G_M.$$
$$G_3 = \frac{1}{2(\tau-1)}\left[G_C - G_M - \left(1-\frac{2}{3}\tau\right)G_Q\right]. \tag{5}$$

The standard FFs have the following normalizations:

$$G_C(0)=1, \quad G_M(0)=\mu_\rho=2.14,$$
$$G_Q(0) = -M^2Q_\rho = -0.79\,, \tag{6}$$

where $\mu_\rho(Q_\rho)$ is the ρ-meson magnetic (quadrupole) moment.

The matrix element of the $\gamma^*\to\rho^++\rho^-$ transition is

$$M = \epsilon\cdot(p_1-p_2)\bigg[-G_1(q^2)U_1^*\cdot U_2^* + \frac{G_3(q^2)}{M^2}\left(U_1^*\cdot qU_2^*\cdot q - \frac{q^2}{2}U_1^*\cdot U_2^*\right)\bigg] - G_2(q^2)(\epsilon\cdot U_1^*U_2^*\cdot q - \epsilon\cdot U_2^*U_1^*\cdot q). \tag{7}$$

Let us define the following helicity amplitudes:

$$F_{\lambda_1\lambda_2} = M^\lambda_{\lambda_1\lambda_2} = M\big(\epsilon\to\epsilon^{(\lambda)}, U_1\to U_1^{(\lambda_1)}, U_2\to U_2^{(\lambda_2)}\big),$$

where $\lambda_1=\lambda_{\rho^+}, \lambda_2=\lambda_{\rho^-}$, and $\lambda=\lambda_{\gamma^*}$. We have $\lambda=\lambda_1-\lambda_2$ and, therefore, $F_{1-1}=F_{-11}=0$ since the virtual photon has spin 1. From symmetry properties it follows that $F_{-1-1}=F_{11}$ and $F_{10}=F_{01}=F_{-10}=F_{0-1}$ and we are left with only three independent helicity amplitudes. Let us choose the following ones: F_{00}, F_{10}, and F_{11}.

The following relation between these amplitudes and the ρ-meson FFs holds:

$$F_{00} = -\frac{\sqrt{q^2-4M^2}}{2M^2}[q^2(G_1+G_2+G_3)-2M^2G_1],$$
$$F_{11} = \sqrt{q^2-4M^2}(G_1+2\tau G_3), \tag{8}$$
$$F_{10} = -\sqrt{\tau(q^2-4M^2)}G_2.$$

The value of the total cross section was evaluated in Ref. [6] at $\sqrt{s}=10.58$ GeV, after extrapolating beyond the experimental acceptance: $\sigma=[19.5\pm1.6(\text{stat})\pm3.21(\text{syst})]$ fb. The BaBar experiment measured also the ratio of the moduli squared of three independent amplitudes at $\sqrt{s}=10.58$ GeV:

$$\left|F^B_{00}\right|^2 : \left|F^B_{10}\right|^2 : \left|F^B_{11}\right|^2 = 0.51\pm0.14(\text{stat})\pm0.07(\text{syst}) : 0.10\pm0.04(\text{stat}) \pm0.01(\text{syst}) : 0.04\pm0.03(\text{stat})\pm0.01(\text{syst}), \tag{9}$$

where the following normalization was used:

$$\left|F^B_{00}\right|^2 + 4\left|F^B_{10}\right|^2 + 2\left|F^B_{11}\right|^2 = 1. \tag{10}$$

We are left with three unknown FFs and three independent values (two independent ratios measured in the experiment and the normalization condition). Thus, we can constrain the values of three FFs (moduli) at one q^2 value where the experiment was done.

The total cross section of $e^++e^-\to\rho^++\rho^-$ [3] can be written in terms of helicity amplitudes as

$$\sigma = \frac{\pi\alpha^2\beta^3}{3q^2}\frac{1}{4M^2(\tau-1)}(|F_{00}|^2+4|F_{10}|^2+2|F_{11}|^2). \tag{11}$$

The value of the total cross section extracted from Ref. [3] is one order of magnitude larger: $\sigma=201$ fb. This gives an overall rescaling factor of 0.011 ± 0.002 GeV^2, which should be applied to the amplitudes extracted from parametrization [3]. The error is calculated by propagating the experimental error on the cross section extracted from the experiment.

In our notations, Eq. (10) reads as

$$(q^2-4M^2)[4\tau|G_2|^2 + 2|G_1+2\tau G_3|^2 + |2\tau(G_1+G_2+G_3)-G_1|^2] = 0.011. \tag{12}$$

In Ref. [3], the electromagnetic FFs for the ρ-meson were parametrized in order to reproduce the predictions from Ref. [5] in the space-like region:[1]

$$G_C(q^2) = \frac{G_C(0)(A+Bq^2)m_C^4}{\left(m_C^2-q^2\right)^2},$$
$$G_M(q^2) = \frac{G_M(0)m_M^4}{\left(m_M^2-q^2\right)^2},$$
$$G_Q(q^2) = \frac{G_Q(0)m_Q^4}{\left(m_Q^2-q^2\right)^2}. \tag{13}$$

[1]Note that the square in the denominator of the expression for G_C was missing in Ref. [3], Eq. (38).

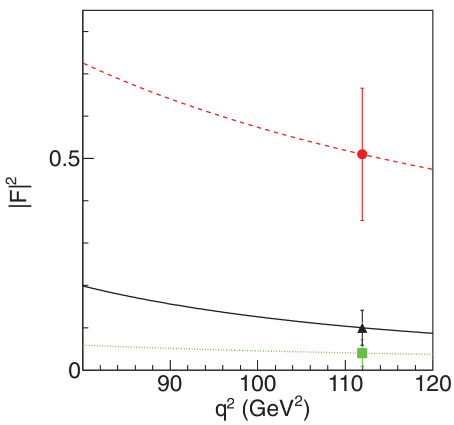

FIG. 1. (Color online) Helicity amplitudes (moduli squared) of $e^+ + e^- \to \rho^+ + \rho^-$ from Ref. [6]. The lines are the parametrization of the present work. Data are from Ref. [6], lines from parametrization (I): $|F_{00}|^2$ (red circle and dashed line), $|F_{10}|^2$ (black triangle and solid line), and $|F_{11}|^2$ (green square and dotted line).

The parameters $A = 1$ and $B = 0.33$ have been fixed in order to reproduce the node of $G_C(q^2 = -3\ \text{GeV}^2)$ predicted in Ref. [5], and $m_C = 1.34$ GeV, $m_M = 1.42$ GeV, $m_Q = 1.51$ GeV have been determined by fitting the theoretical calculation. They have the meaning of masses for the particles (mesons) carrying the interaction.

The extension of the model to the TL region was made by analytical continuation, introducing an imaginary part through widths for the particles. This leads to the following parametrization:

$$G_C(t) = \frac{(A + Bt)m_C^4}{\left(m_C^2 - t - im_C\Gamma_C\right)^2},$$
$$G_M(t) = \frac{G_M(0)m_M^4}{\left(m_M^2 - t - im_M\Gamma_M\right)^2}, \quad (14)$$
$$G_Q(t) = \frac{G_Q(0)m_Q^4}{\left(m_Q^2 - t - im_Q\Gamma_Q\right)^2},$$

with the following result for $\sqrt{s} = 10.58$ GeV: $|G_C|^2 = 1.017 \times 10^{-4}$, $|G_Q|^2 = 1.167 \times 10^{-7}$, and $|G_M|^2 = 5.186 \times 10^{-7}$. The effect of the width was illustrated in Ref. [3] by comparing two values: 1% and 10% of the corresponding mass. At large q^2 this effect is negligible.

Keeping A and B fixed, we can readjust the mass parameters such that the helicity amplitudes obtained from this parametrization coincide with those measured in the BaBar experiment (Fig. 1).

The difference between the old and the present parametrization due to the experimental constraint is shown in Fig. 2, where the moduli of the three FFs are illustrated as a function of q^2. The overall relative effect is small and essentially lower than an order of magnitude. In the present case we used 10% width.

Note that two solutions are possible for m_C and m_Q, which cannot be disentangled: the two sets of parameters denoted as (I) and (II) in Table I, are strictly equivalent as far as the values of the amplitudes ratio and the cross section at $\sqrt{s} = 10.58$ GeV are concerned, although the corresponding FFs may be different as illustrated in Fig. 3 in an extended q^2 range.

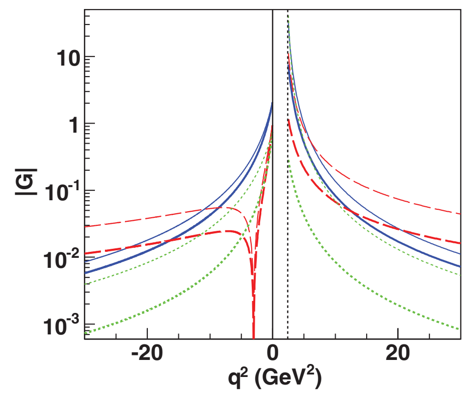

FIG. 2. (Color online) Absolute value of ρ-meson FFs $|G_i|$, $i = C, Q, M$, for the parametrization (I) of the present work (thick lines) and from Ref. [3] (thin lines) as functions of q^2 in space- and time-like regions: magnetic $|G_M|$ (blue solid line), charge $|G_C|$ (red dashed line) and quadrupole $|G_Q|$ (green dotted line). The black dotted line indicates the kinematical threshold for the considered reaction.

The error on the parameters is related, on one side to the choice of the function chosen for the fit, and on the other side to the experimental error on the point. The error on each point is obtained by summing in quadrature the statistical and the systematic experimental errors. Contrary to Ref. [3] where the errors of parameters were determined by the fitting procedure, in the present case we do not fit the data because we have equal numbers of parameters and constraints. The errors on the "effective" masses in Table I are obtained propagating this experimental error. In the limits of the errors, the results of the different parametrizations look quite consistent, validating the extension of this approach.

Conclusion. Using the parametrization of the electromagnetic current for $\gamma^*\rho\rho$ vertex in terms of three complex FFs, we compared the helicity amplitudes with the experimental value given by the BaBar Collaboration. We used a simple model for the ρ-meson FFs, based on vector meson dominance, which reproduces a calculation in the SL region based on covariant and light-front frameworks with constituent quarks [5] and analytically continued to the TL region. In VMD-like models, the virtual photon behaves as superposition of vector meson resonances, and in principle, all vector mesons may contribute. This has been discussed in Ref. [8] and references therein for the case of the pion TL form factor.

The present model may contain such additional contributions. However, the experimental constraint is given on helicity amplitudes and it translates into a constraint for a combination

TABLE I. Parameters of the model for ρ-meson electromagnetic FFs.

Ref.	m_C (GeV)	m_M (GeV)	m_Q (GeV)
[3]	1.34 ± 2	1.42 ± 0.5	1.51 ± 0.1
This work (I)	$1.05^{+0.05}_{-0.09}$	$1.28^{+0.06}_{-0.08}$	$0.97^{+0.02}_{-0.01}$
This work (II)	$0.77^{+0.05}_{-0.02}$	$1.28^{+0.06}_{-0.08}$	$1.12^{+0.05}_{-0.08}$

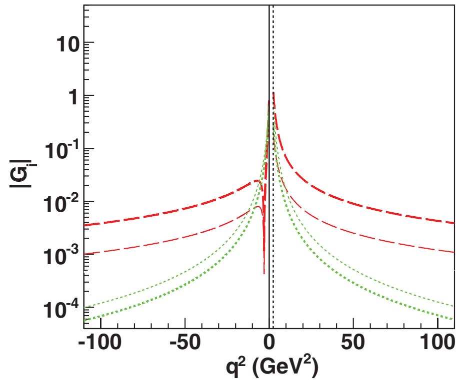

FIG. 3. (Color online) Absolute value of ρ-meson FFs $|G_i|$, $i = C, Q, M$, for the parametrizations (I) (thick lines) and (II) (thin lines) from the present work as functions of q^2 in space- and time-like regions: charge $|G_C|$ (red dashed line) and quadrupole $|G_Q|$ (green dotted line). The black dotted line indicates the kinematical threshold for the considered reaction.

of FFs which is independent from their parametrization. Including other vector mesons may change the form of the function that parametrizes G_C, G_Q, and G_M. However, this implies more (unknown) constants and/or parameters. Therefore we limit ourselves to the smallest number of contributions which give a good description of the available data. The masses and widths should be considered as "effective" masses and widths. Moreover, the absolute value of the amplitudes is very sensitive to the presence, position, and width of the included resonances. Therefore it is important to compare and update the previous calculation with new experimental results.

Note that the dominance of helicity conserving amplitudes in gauge theory [9] implies the following ratios for the FFs of spin-1 bound states: $G_C : G_M : G_Q = (1 - \frac{2}{3}\tau) : 2 : -1$. In the considered case ($\tau = 46.53$), it implies: $G_C : G_M : G_Q = -30 : 2 : -1$ which is consistent with the parametrization from Ref. [3]. However, after applying the normalization factor to the amplitudes, the following ratios have been extracted, at the corresponding q^2 in the space-like region: $G_C : G_M : G_Q = -63 : 8 : -1$ for parametrization (I) and $G_C : G_M : G_Q = -10 : 5 : -1$ for parametrization (II). Therefore, as pointed out in Ref. [6], the experimental value suggests that either helicity conservation does not apply or different reaction mechanisms contribute to the ρ production in the present kinematical range.

Whereas we cannot draw any conclusion on the validity of the Q^2 dependence of our parametrization, the present comparison validates our simple approach as far as the absolute value of the cross section is concerned. Moreover, the individual helicity amplitudes can be constrained.

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[1] D. Abbott *et al.* (JLAB t(20) Collaboration), Phys. Rev. Lett. **84**, 5053 (2000).
[2] E. Tomasi-Gustafsson, G. I. Gakh, and C. Adamuscin, Phys. Rev. C **73**, 045204 (2006).
[3] C. Adamuscin, G. I. Gakh, and E. Tomasi-Gustafsson, Phys. Rev. C **75**, 065202 (2007).
[4] G. I. Gakh, E. Tomasi-Gustafsson, C. Adamuscin, S. Dubnicka, and A. Z. Dubnickova, Phys. Rev. C **74**, 025202 (2006).
[5] J. P. B. C. de Melo and T. Frederico, Phys. Rev. C **55**, 2043 (1997).
[6] B. Aubert *et al.* (BaBar Collaboration), Phys. Rev. D **78**, 071103 (2008).
[7] A. Akhiezer and M. P. Rekalo, *Electrodynamics of Hadrons* (Naukova Dumka, Kiev, 1977).
[8] J. P. B. C. de Melo, T. Frederico, E. Pace, and G. Salme, Phys. Rev. D **73**, 074013 (2006).
[9] S. J. Brodsky and J. R. Hiller, Phys. Rev. D **46**, 2141 (1992).