Determination of the ^{3}He + $\alpha \to {}^7$Be asymptotic normalization coefficients, the nuclear vertex constants, and their application for the extrapolation of the ^{3}He(α, γ)^{7}Be astrophysical S factors to the solar energy region

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A new analysis of the modern astrophysical S factors for the direct-capture ^{3}He(α, γ)^{7}Be reaction, precisely measured in recent works [B.S. Nara Singh *et al.*, Phys. Rev. Lett. **93**, 262503 (2004); D. Bemmerer *et al.*, *ibid.* **97**, 122502 (2006); F. Confortola *et al.*, Phys. Rev. C **75**, 065803 (2007), Gy. Gyrky *et al.*, *ibid.* **75**, 035805 (2007), T. A. D. Brown *et al.*, *ibid.* **76**, 055801 (2007), and A. Di Leva, *et al.*, Phys. Rev. Lett. **102**, 232502 (2009)], has been carried out within the modified two-body potential approach. New estimates are obtained for the "indirectly determined" values of the asymptotic normalization constants and the respective nuclear vertex constants for ^{3}He + $\alpha \to {}^7$Be(g.s.) and ^{3}He + $\alpha \to {}^7$Be(0.429 MeV) as well as the astrophysical S factors $S_{34}(E)$ at $E \leqslant 90$ keV, including $E = 0$. The values of asymptotic normalization constants have been used to obtain the values of the ratio of the α-particle spectroscopic factors for the mirror (^{7}Li^{7}Be) pair.

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I. INTRODUCTION

The ^{3}He(α,γ)^{7}Be reaction is one of the critical links in the *pp* chain of solar hydrogen burning [1–4]. Its rate at a stellar temperature ($T_6 \sim 15$ K) determines how much the ^{7}Be and ^{8}B branches of the *pp* chain contribute to solar hydrogen burning. The predicted flux of solar neutrinos from ^{7}Be, which is propotional to $[S_{34}(0)]^{0.8}$ [3], depends noticeably on the accurancy of the cross sections (or the astophysical S factors) of the ^{3}He(α,γ)^{7}Be reaction at experimentally inaccessible solar energies ($E \lesssim 25$ keV). It is found out that the uncertainty in the extrapolation of the astrophysical S factors to the Gamow solar energy E_G ($E_G = 23$ keV, $T_6 = 0.1417E_G^{3/2} = 15.6$ K [5]) contributes significantly to the uncertainty in the predicted fluxes for solar ^{7}Be and ^{8}B neutrinos [6,7].

Despite the impressive improvements in our understanding of the the nuclear-astrophysical ^{3}He(α,γ)^{7}Be reaction made in recent decades (see Refs. [6–14] and references therein), some ambiguities still exist, connected with both the extrapolation of the measured astrophysical S factors for the aforementioned reaction to the solar energy region and the theoretical predictions for $\sigma_{34}(E)$ [or $S_{34}(E)$], and they may influence the predictions of the standard solar model [2,4].

Experimentally, there are two types of data for the ^{3}He(α,γ)^{7}Be reaction at extremely low energies: (i) six measurements based on detection of γ-ray capture (see Refs. [6,7] and references therein) from which the extracted astrophysical S factor $S_{34}(0)$ changes within the range $0.470 \leqslant S_{34}(0) \leqslant 0.598$ keV b; and (ii) six measurements based on detection of ^{7}Be (see Refs. [6,7] references therein, as well as [9–14]) from which the extracted $S_{34}(0)$ changes within the range $0.53 \leqslant S_{34} \leqslant 0.63$ keV b. All of these measured data show a similar energy dependence of the astrophysical S factor $S_{34}(E)$. However, in Refs. [10–14] the adaptation of the available energy dependencies predicted in Refs. [15,16] for the extrapolation of each of the measured data to experimentally inaccessible low-energy regions, including $E = 0$, leads to a value of $S_{34}(0)$ that differs from others by more than the experimental uncertainty. Nevertheless, the value of $S_{34}(0) = 0.56 \pm 0.02(\text{exp}) \pm 0.02(\text{theor})$ keV b, which has been combined from the results of Refs. [9–14], is recommended in a recent work [7].

The theoretical calculations of $S_{34}(0)$ performed with different methods also show considerable spread [15,17–24], and the result depends on the specific model used. Nevertheless, in most cases theoretical calculations also show practically the same energy dependence for the calculated $S_{34}(E)$, but they have different normalizations, exceeding noticeably the absolute experimental errors of the modern experimental data [9–14]. For example, calculations of $S_{34}(0)$ performed by the resonating-group method in Ref. [15] show considerable sensitivity to the form of the effective NN interaction used, and the obtained estimates are within the range $0.312 \leqslant S_{34}(0) \leqslant 0.841$ keV b.

The estimate of $S_{34}(0) = 0.52 \pm 0.03$ keV b [25] also should be noted. It was obtained from analysis of older experimental astrophysical S-factor calculations [26], performed within the framework of the standard two-body potential model with the assumption that the dominant contribution to the peripheral reaction comes from the surface and external regions of the nucleus ^{7}Be [27]. In Ref. [25] the contribution from the nuclear interior ($r < r_{\text{cut}}$, $r_{\text{cut}} = 4$ fm) to the amplitude is ignored. In this case, the astrophysical S factor is directly expressed in terms of the nuclear vertex constants (NVCs) for the virtual decays ^{7}Be $\to \alpha + {}^3$He (or in terms of the respective asymptotic normalization coefficients (ANCs), which determine the amplitude of the tail of the bound-state wave function for the nucleus ^{7}Be in the binary ($\alpha + {}^3$He) channel denoted by ^{3}He + $\alpha \to {}^7$Be everywhere below) [28,29]. As a result, in Ref. [25] the NVC values for the virtual decays ^{7}Be(g.s.) $\to \alpha + {}^3$He and ^{7}Be(0.429 MeV) $\to \alpha + {}^3$He were

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obtained, which were then used for calculations of the astrophysical S factors at $E < 180$ keV, including $E = 0$. However, the values of the NVCs (or ANCs) for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}$ and the $S_{34}(0)$ obtained in Ref. [25] may not be accurate enough, due to the aforementioned assumption of the contribution from the nuclear interior ($r < r_{\mathrm{cut}}$) and the spread in the experimental data [26] used for the analysis. Regarding the available values of these ANCs obtained in Refs. [16,19,22], they depend noticeably on the specific model used (see Sec. II A). Therefore, determination of precise experimental values of the ANCs for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be(g.s.)}$ and $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}(0.429$ MeV) is highly encouraged, since it has direct effects on the correct extrapolation of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ astrophysical S factor at solar energies. For this purpose, use of the modern experimental S factors precisely measured in Refs. [9–14] for the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction is desirable.

Recently, a modified two-body potential approach (MTBPA) was proposed in Ref. [30] for the peripheral direct-capture A(a,γ)B reaction, which is based on the idea proposed in Ref. [27] that low-energy direct radiative captures of particle a by light nuclei A proceed mainly in regions well outside the range of the internuclear a-A interactions. In the MTBPA, the direct astrophysical S factor is expressed in terms of ANC for $\mathrm{A} + a \to \mathrm{B}$ rather than through the spectroscopic factor for the nucleus B in the $(\mathrm{A} + a)$ configuration, as it is made within the standard two-body potential method [31,32]. In Refs. [30,33,34], the MTBPA has been successfully applied to the radiative proton and α-particle capture by some light nuclei. Therefore, it is of great interest to apply the MTBPA for analysis of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction.

In this work, new analysis of the modern precise experimental astrophysical S factors for the direct-capture $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction at extremely low energies ($\geqslant$92.9 keV) [9–14] is performed within the MTBPA [30] to obtain "indirectly determined" values of the ANCs (or NVCs) for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be(g.s.)}$ and $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}(0.429$ MeV) and of $S_{34}(E)$ at $E \leqslant 90$ keV, including $E = 0$. Here we quantitatively show that the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction within the aforementioned energy region is mainly peripheral, and one can extract ANCs for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}$ directly from the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction. The ambiguities inherent in the standard two-body potential model calculation of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction, which are connected with the choice of the geometric parameters (the radius R and the diffuseness a) for the Woods–Saxon potential [31,32] and the spectroscopic factors [21–24] can be reduced in the physically acceptable limit, being within the experimental errors for the $S_{34}(E)$.

The contents of this paper are as follows. Section II presents the results of the analysis of the precise measured astrophysical S factors for the direct radiative capture $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction (Secs. II A–II C). The conclusion is given in Sec. III. In the Appendix, basic formulas of the modified two-body potential approach to the direct radiative capture $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction are given.

II. ANALYSIS OF THE $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ REACTION

Let us write l_f (j_f) for the relative orbital (total) angular moment of $^3\mathrm{He}$ and an α particle in nucleus $^7\mathrm{Be}$ ($\alpha + {}^3\mathrm{He}$), and l_i (j_i) for the orbital (total) angular moment of the relative motion of the colliding particles in the initial state. For the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction populating the ground ($E^* = 0.0$; $J^\pi = 3/2^-$) and first excited ($E^* = 0.429$ MeV; $J^\pi = 1/2^-$) states of $^7\mathrm{Be}$, the values of j_f are taken to be equal to $3/2$ and $1/2$, respectively, the value of l_f is taken to be equal to 1, and $l_i = 0, 2$ for the $E1$ transition and $l_i = 1$ for the $M1$ and $E2$ transitions.

The basic formulas used for the analysis are presented in the Appendix.

A. The asymptotic normalization coefficients for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}$

To determine the ANC values for $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be(g.s)}$ and $^3\mathrm{He} + \alpha \to {}^7\mathrm{Be}(0.429$ MeV), the modern experimental astrophysical S factors $S^{\exp}_{l_f j_f}(E)$ [or $S^{\exp}_{34}(E)$], for the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction populating the ground ($l_f = 1$ and $j_f = 3/2$) and first excited ($l_f = 1$ and $j_f = 1/2$) states [9–14] are reanalyzed based on the relations (A1)–(A7) given in the Appendix. The experimental data analyzed by us cover the energy ranges $E = 92.9$–170.1 keV [10–12], 420–950 keV [9], 327–1235 keV [13], and 701–1203 keV [14], for which only the external capture is substantially dominant [27]. Also, the total experimental astrophysical S factors measured in Ref. [12] have been separated up to the astrophysical S factors corresponding to the ground and first excited states of the residual $^7\mathrm{Be}$ nucleus only for three experimental points of energy E ($E = 93.3$, 106.1 and 170.1 keV). In contrast, in Ref. [13] the total experimental astrophysical S factors have been separated for all experimental points of E from the aforementioned energy region by means of detecting the prompt γ emission ("the prompt") and by counting the $^7\mathrm{Be}$ activations ("the activation").

The real potential in the Woods–Saxon form, split with a parity (l dependence) proposed by the authors of Refs. [35–37], is used here for calculations of both the bound-state radial wave function $\varphi_{l_f j_f}(r)$ and scattering one $\psi_{l_i j_i}(r)$. For the s and d waves, the potential depths for the central and spin-orbital terms are given as $V_{o;c} = 87$ MeV and $V_{o;sl} = 2.61$ MeV, respectively. For the p wave the depth values are given as $V_{o;c} = 110$ MeV and $V_{o;sl} = 6.60$ MeV. For this, the values of the geometric potential parameters are recommended in Refs. [36,37] to be $R = 1.80$ fm and $a = 0.70$ fm (the standard values). Such a choice of the potential is based on the following considerations. First, this potential form is justified from the microscopic point of view because it takes into account the Pauli exclusion principle between nucleons in $^3\mathrm{He}$ and α clusters in the ($\alpha + {}^3\mathrm{He}$) bound state by means of inclusion of deeply bound forbidden states. The latter imitates the additional node (n) arising in the wave functions of α-$^3\mathrm{He}$ relative motion in $^7\mathrm{Be}$ similar to the result of the microscopic resonating-group method [15]. Second, this potential describes well the phase shifts for α-$^3\mathrm{He}$ scattering in a wide energy range [36,37] and reproduces the energies of low-lying states of the $^7\mathrm{Be}$ nucleus [38].

We vary the geometric parameters (R and a) of the adopted Woods–Saxon potential in the physically acceptable ranges

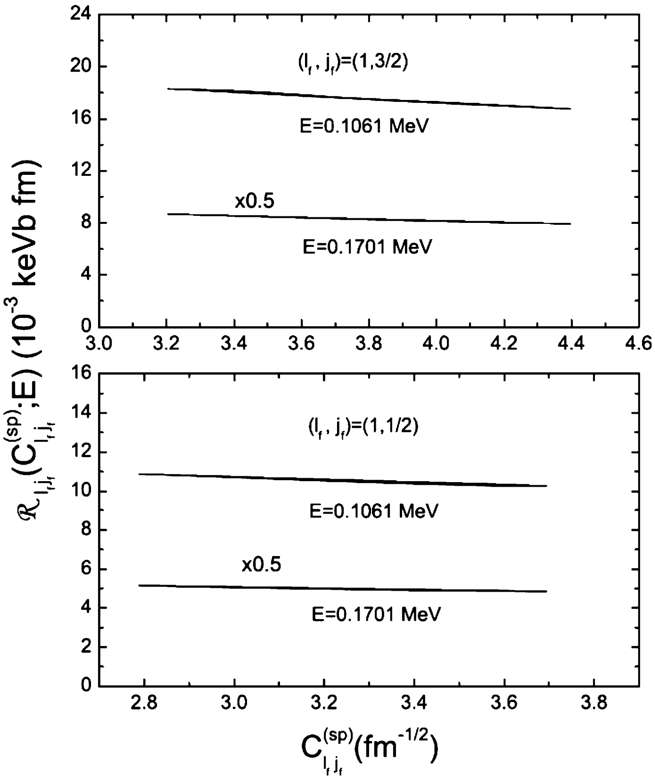

FIG. 1. The dependence of $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ as a function of the single-particle ANC $C^{(sp)}_{l_f j_f}$ for the ^{3}He(α,γ)^{7}Be(g.s.) [$(l_f, j_f) = (1, 3/2)$] and ^{3}He(α,γ)^{7}Be(0.429 MeV [$(l_f, j_f) = (1, 1/2)$] reactions at different energies E. The width of the bands for fixed values of the single-particle ANCs corresponds to the variation of the parameters R and a of the adopted Woods–Saxon potential within the intervals from $R = 1.62$ to 1.98 fm and $a = 0.63$ to 0.77 fm.

(R in 1.62–1.98 fm and a in 0.63–0.77 fm [30]) with respect to the aforementioned standard values using the procedure of adjusting the depth to fit the binding energies. As an illustration, Fig. 1 shows plots of the $\mathcal{R}_{1 j_f}(E, C^{(sp)}_{1 j_f})$ dependence on the single-particle ANC $C^{(sp)}_{1 j_f}$ for $j_f = 3/2$ and $1/2$ within the ranges $3.204 \leqslant C^{(sp)}_{1\,3/2} \leqslant 4.397$ fm$^{-1/2}$ and $2.777 \leqslant C^{(sp)}_{1\,1/2} \leqslant 3.763$ fm$^{-1/2}$ for two values of energy E. The width of the band for these curves is the result of the weak "residual" (R, a) dependence of $\mathcal{R}_{1 j_f}(E, C^{(sp)}_{1 j_f})$ on the parameters R and a (up to $\pm 2\%$) for $C^{(sp)}_{1 j_f} = C^{(sp)}_{1 j_f}(R, a) = \text{const}$ [30,39]. The same dependence is also observed at other energies. For example, in Fig. 1 plotted for $E = 0.1061$ and 0.1701 MeV, the overall uncertainty ($\Delta_{\mathcal{R}}$) of the function $\mathcal{R}_{1\,3/2}(E, C^{(sp)}_{1\,3/2})$ with respect to the central value $\mathcal{R}_{1\,3/2}(E, C^{(sp)}_{13/2})$ corresponding to those of $C^{(sp)}_{1\,3/2}(1.80, 0.70) = 3.768$ fm$^{-1/2}$ comes to $\Delta_{\mathcal{R}} = \pm 4.5\%$ for the ground state of ^{7}Be. The values of $\Delta_{\mathcal{R}}$ for the same two energies are equal to $\pm 3.4\%$ and $\pm 2.9\%$, respectivley, for the excited state of ^{7}Be for which $C^{(sp)}_{1\,1/2}(1.80, 0.70) = 3.250$ fm$^{-1/2}$. It is seen that the ^{3}He(α,γ)^{7}Be(0.429 MeV) reaction is slightly more peripheral than the ^{3}He(α,γ)^{7}Be(g.s.) reaction, since the binding energy for ^{7}Be(0.429 MeV) is less than that for ^{7}Be(g.s). A similar dependence of $\mathcal{R}_{1 j_f}(E, C^{(sp)}_{1 j_f})$ on the $C^{(sp)}_{1 j_f}$ values is observed for other energies E, and the value of $\Delta_{\mathcal{R}}$ is no more than $\sim\pm 5.0\%$, within the experimental uncertainties for $S^{\exp}_{1 j_f}(E)$. Such dependence is apparently associated also with indirectly taking into account the Pauli principle within the nuclear interior in the adopted nuclear α-^{3}He potential, leading as a whole to reduction of the contribution from the interior part of the radial matrix element to the $\mathcal{R}_{1 j_f}(E, C^{(sp)}_{1 j_f})$ function, which is typical for peripheral reactions (see the Appendix). Nevertheless, the analysis shows that values of $\Delta_{\mathcal{R}}$ become larger as the energy E increases (for E more than 1.3 MeV). Therefore, for the considered reaction, condition (A2) over the energy region $92.9 \leqslant E \leqslant 1235$ keV is satisfied within uncertainties not exceeding the experimental errors of $S^{\exp}_{1 j_f}(E)$. Besides, one notes that the contributions of the $M1$ and $E2$ transitions are too small in the aforementioned energy region and below, including solar energies, and they change from about 0.4% up to about 2.2% as the energy E increases.

We also calculated phase shifts of α-^{3}He elastic scattering by variation of the parameters R and a in the same range for the adopted Woods–Saxon potential. As an illustration, the results of the calculations corresponding to the $s_{1/2}$ and $p_{3/2}$ waves only are presented in Fig. 2, in which the band width corresponds to a change of the calculated values of phase shifts with respect to variation of the R and a parameters. As seen from Fig. 2, the experimental phase shifts [40,41] are well reproduced within an uncertainty of about $\pm 5\%$. The same results are also obtained for the $p_{1/2}$, $d_{3/2}$, and $d_{5/2}$ waves.

This circumstance allows us to test the condition (A3) at the energies $E = 93.3$, 106.1, and 170.1 keV, for which the ^{3}He(α,γ)^{7}Be(g.s.) and ^{3}He(α,γ)^{7}Be(0.429 MeV) astrophysical S factors were separately measured in Ref. [12]. As an

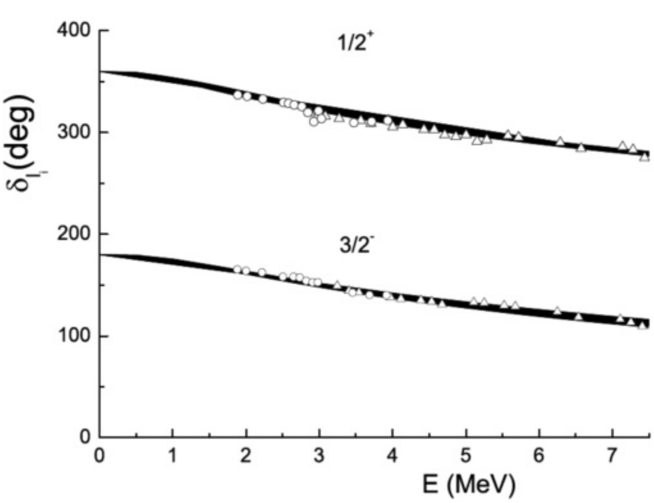

FIG. 2. The energy dependence of the α-^{3}He elastic-scattering phase shifts for the $s_{1/2}$ and $p_{3/2}$ partial waves. The experimental data are taken from Refs. [40] (open circles) and [41] (open triangles). The bands are our calculated data. The width of the bands for fixed energies corresponds to the variation of the parameters R and a of the adopted Woods–Saxon potential within the intervals from $R = 1.62$ to 1.98 fm and $a = 0.63$ to 0.77 fm.

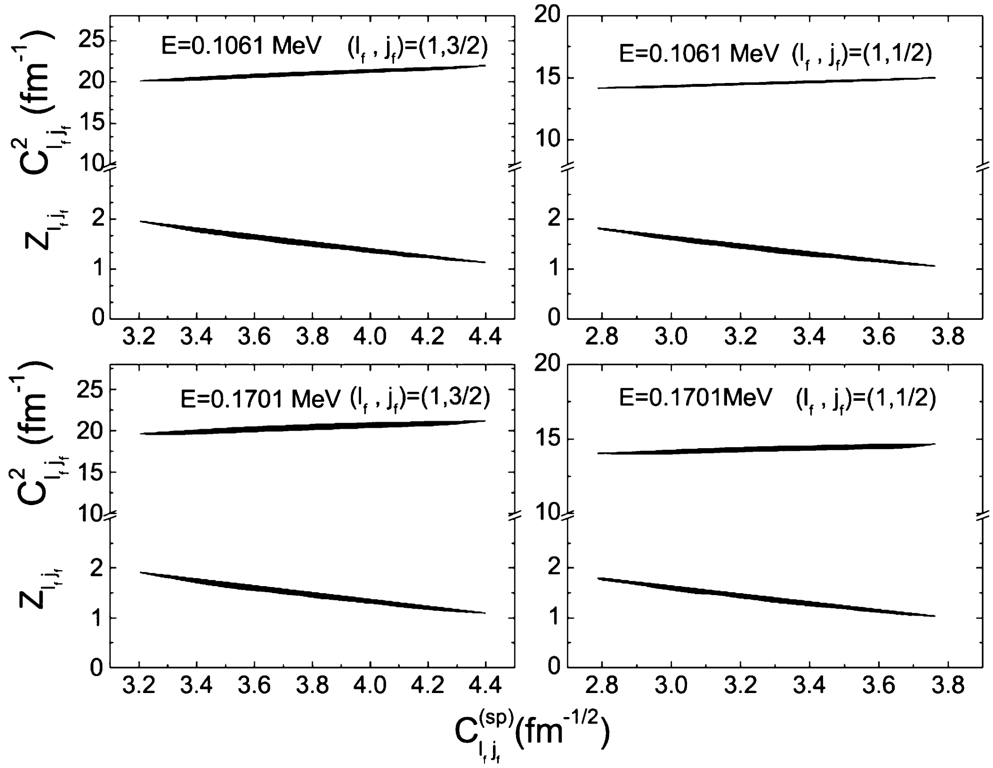

FIG. 3. The dependence of the ANCs $C_{l_f j_f}$ (upper band) and the spectroscopic factors $Z_{l_f j_f}$ (lower band) on the single-particle ANC $C^{(sp)}_{l_f j_f}$ for the ^{3}He$(\alpha,\gamma)^7$Be(g.s.) [left column, $(l_f, j_f) = (1, 3/2)$] and ^{3}He$(\alpha,\gamma)^7$Be(0.429 MeV) [right column, $(l_f, j_f) = (1, 1/2)$] reactions at different energies E. The width of the bands for fixed values of the single-particle ANCs corresponds to the variation of the parameters R and a of the adopted Woods–Saxon potential within the intervals from $R = 1.62$ to 1.98 fm and $a = 0.63$ to 0.77 fm.

illustration, for the same energies E as in Fig. 1 we present in Fig. 3 (the upper panels) the results of the $C^2_{1j_f}$-value calculation given by Eq. (A3) ($j_f = 3/2$ and 1/2) in which instead of $S_{1j_f}(E)$ the experimental $S^{\rm exp}_{1j_f}(E)$ were taken. The same dependence occurs for other considered energies. The calculation shows that the obtained $C^2_{1j_f}$ values also weakly depend (up to 5.0%) on the $C^{(sp)}_{1j_f}$ value. However, the values of the spectroscopic factors $Z_{1\,3/2}$ and $Z_{1\,1/2}$ corresponding to the $(\alpha + {}^3\text{He})$ configuration for ^{7}Be(g.s.) and ^{7}Be(0.429 keV), respectively, change strongly: about 1.7 times, since calculated single-particle astrophysical S factors $S^{(sp)}_{1j_f}(E)$ [see Eq. (A1)] also vary by 1.7 times (see the lower panels in Fig. 3). Consequently, the ^{3}He$(\alpha,\gamma)^7$Be reaction within the considered energy ranges is peripheral, and the use of parametrization in terms of the ANCs given by Eq. (A1) is adequate for the reaction physics.

At first, for each experimental point of E ($E = 93.3$, 106.1 and 170.1 keV), the values of the ANCs, $(C^{\rm exp}_{1\,3/2})^2$ and $(C^{\rm exp}_{1\,1/2})^2$, can be obtained for $\alpha + {}^3\text{He} \to {}^7\text{Be(g.s.)}$ and $\alpha + {}^3\text{He} \to {}^7\text{Be}(0.429$ MeV) by using the corresponding separated experimental astrophysical S factor [$S^{\rm exp}_{1\,3/2}(E)$ and $S^{\rm exp}_{1\,1/2}(E)$ (the prompt)] [12] in the relation (A1) instead of $S_{1j_f}(E)$, and by using the central values of $\mathcal{R}_{1j_f}(E, C^{(sp)}_{1j_f})$ corresponding to the standard values of the parameters R and a in the ratio of the right-hand side of Eq. (A1). The results of the ANCs for these three experimental points of E are presented in the second and third columns of Table I as well as in Figs. 4(a) and 4(d) (solid circles). There the uncertainties correspond to those found from Eq. (A1) [averaged square errors (a.s.e.)], which include the total experimental errors (a.s.e. from the statistical and systematic uncertainties [12]) in the corresponding experimental astrophysical S factor and the uncertainty in $\mathcal{R}_{1j_f}(E, C^{(sp)}_{1j_f})$. The same results for the ANCs are obtained when $S^{\rm exp}_{1\,3/2}(E)$ and $R^{\rm exp}(E)$ [or $S^{\rm exp}_{34}(E)$] [12] are used in Eq. (A7) [or in Eq. (A5)] instead of $S_{1\,3/2}(E)$ and $\bar{R}(E)$ [or $S_{34}(E)$]. Then, in Eq. (A6), we use the averaged means of $\lambda_C = (C_{1\,1/2}/C_{1\,3/2})^2 = 0.682$, obtained above from three of the ANCs ($C_{1\,3/2}$ and $C_{1\,1/2}$) and from the unseparated $S^{\rm exp}_{34}(E)$; for others (the four experimental points of E ($E =$ 105.6, 126.5, 147.7, and 168.9 keV) from Refs. [10,11], the four E ($E = 420.0$, 506.0, 614.5, and 950.0 keV) from Ref. [9], the three E ($E = 92.9$, 105.7, and 169.5 keV) from Ref. [12] and an additional E ($E = 701$–1203 keV) from Ref. [14]), the values of ANCs can also be obtained. The results of the ANCs are also presented in the second and third columns of Table I as well as in Fig. 4, in which the open circles and triangles were obtained from analysis of the experimental data in Refs. [9–12](see the lines 1–14 of Table I), and the solid triangles were obtained from analysis of the experimental data in Ref. [14] (see the lines 15–24 of Table I). In the same way, the values of the ANCs are obtained by using the separated

TABLE I. The "indirectly determined" values of the ANCs [$(C^{\exp}_{1\,3/2})^2$ and $(C^{\exp}_{1\,1/2})^2$] for ^{3}He + $\alpha \to {}^7$Be, the experimental astrophysical S factors [$S^{\exp}_{1\,j_f}$ and $S^{\exp}_{34}(E)$], and the branching ratio [$R^{\exp}(E)$] at different energies E.

E (keV)	$(C^{\exp}_{1\,j_f})^2$ (fm^{-1})		$S^{\exp}_{1\,j_f}$ (keV b)		$S^{\exp}_{34}(E)$ (keV b)	$R^{\exp}(E)$
	$j_f = 3/2$	$j_f = 1/2$	$j_f = 3/2$	$j_f = 1/2$		
92.9[a]	21.7±1.5	14.8±1.0	0.380 ± 0.026	0.154 ± 0.010	0.534 ± 0.023 [12]	0.407 ± 0.039
93.3[b]	21.7±1.3	13.8±1.4	0.382 ± 0.021 [12]	0.145 ± 0.014 [12]	0.527 ± 0.027 [12]	0.380 ± 0.030 [12]
105.6[a]	21.2±1.7	14.4±1.2	0.367 ± 0.029	0.149 ± 0.012	0.516 ± 0.031 [11]	0.407 ± 0.046
105.7[a]	20.2±1.4	13.8±0.9	0.350 ± 0.024	0.143 ± 0.010	0.493 ± 0.021 [12]	0.406 ± 0.039
106.1[b]	21.0±1.2	14.6±1.3	0.366 ± 0.018 [12]	0.152 ± 0.013 [12]	0.518 ± 0.024 [12]	0.415 ± 0.029 [12]
126.5[a]	21.2±0.9	14.6±0.6	0.366 ± 0.023	0.148 ± 0.009	0.514 ± 0.019 [10,11]	0.404 ± 0.020
147.7[a]	21.1±1.3	14.4±0.9	0.354 ± 0.022	0.145 ± 0.009	0.499 ± 0.017 [10,11]	0.407 ± 0.036
168.9[a]	20.6±0.8	14.1±0.6	0.343 ± 0.010	0.139 ± 0.006	0.482 ± 0.017 [10,11]	0.405 ± 0.020
169.5[a]	21.8±1.4	14.9±1.0	0.360 ± 0.023	0.147 ± 0.009	0.507 ± 0.018 [12]	0.407 ± 0.037
170.1[b]	21.6±1.1	15.1±1.0	0.360 ± 0.015 [12]	0.150 ± 0.010 [12]	0.510 ± 0.021 [12]	0.417 ± 0.020 [12]
420.0[a]	21.4±1.7	14.7±1.1	0.297 ± 0.020	0.123 ± 0.009	0.420 ± 0.032 [9]	0.414 ± 0.050
506.0[a]	20.9±1.9	14.3±1.3	0.266 ± 0.020	0.113 ± 0.010	0.379 ± 0.031 [9]	0.424 ± 0.050
614.5[a]	21.5±1.4	14.7±0.9	0.254 ± 0.020	0.108 ± 0.006	0.362 ± 0.018 [9]	0.425 ± 0.040
950.0[a]	22.7±1.2	15.6±0.8	0.220 ± 0.010	0.096 ± 0.005	0.316 ± 0.009 [9]	0.436 ± 0.030
701.0[c]	24.4±4.5	16.7±3.0	0.227 ± 0.050	0.117 ± 0.021	0.339 ± 0.071 [14]	0.424 ± 0.110
802.0[c]	25.3±2.0	17.3±1.3	0.269 ± 0.021	0.115 ± 0.009	0.384 ± 0.028 [14]	0.427 ± 0.047
902.0[c]	22.5±1.7	16.1±1.2	0.236 ± 0.017	0.101 ± 0.007	0.338 ± 0.025 [14]	0.430 ± 0.044
1002[c]	25.4±1.8	17.4±1.2	0.244 ± 0.016	0.106 ± 0.007	0.350 ± 0.021 [14]	0.433 ± 0.043
1002[c]	24.2±1.6	16.5±1.1	0.232 ± 0.015	0.100 ± 0.006	0.332 ± 0.019 [14]	0.433 ± 0.042
1102[c]	25.6±1.6	17.5±1.1	0.235 ± 0.015	0.103 ± 0.006	0.338 ± 0.018 [14]	0.437 ± 0.039
1102[c]	25.2±1.5	17.3±1.1	0.232 ± 0.014	0.101 ± 0.006	0.334 ± 0.016 [14]	0.437 ± 0.038
1103[c]	25.1±1.7	17.2±1.2	0.232 ± 0.016	0.101 ± 0.007	0.333 ± 0.019 [14]	0.437 ± 0.042
1203[c]	25.9±1.7	17.8±1.2	0.231 ± 0.015	0.102 ± 0.006	0.333 ± 0.018 [14]	0.440 ± 0.041
1203[c]	25.9±1.9	17.8±1.3	0.231 ± 0.017	0.102 ± 0.007	0.333 ± 0.020 [14]	0.440 ± 0.046

[a]The activation.
[b]The prompt.
[c]The ^{7}Be recoil.

experimental astrophysical S factors ($S^{\exp}_{1\,3/2}$ and $S^{\exp}_{1\,1/2}$) [13], and they are in good agreement with those obtained from the analysis of the experimental data in Ref. [14]. The results for these ANCs are also presented in Figs. 4(b) and 4(e) for both the activation (solid stars) and for the prompt (solid quares).

As seen from Figs. 4(a), 4(b), 4(d), and 4(e) as well as from Table I, the ANC values obtained from the analysis performed separately for the prompt and the activation of the experimental data of Refs. [9–12] are in a good agreement within the experimental errors of $S^{\exp}_{1\,3/2}(E)$ and $S^{\exp}_{1\,1/2}(E)$ [or $S^{\exp}_{34}(E)$]. This is connected with the fact that, for each of the independently measured experimental astrophysical S factors [9–12], the ratio in the right-hand side of the relation (A4) does not practically depend on the energy E within experimental uncertainties, although absolute values of the corresponding experimental astrophysical S factors depend noticeably on the energy and grow about 1.7 times when E changes from 92.6 to 950 keV. The analogous situation occurs for the ANC values obtained from the analysis of the experimental data (the prompt, the activation, and the ^{7}Be recoils) of Refs. [13,14], although absolute values of the corresponding experimental astrophysical S factors also grow about 1.5 times when E increases from 327.4 to 1235 keV. It follows from here that the energy dependence of the experimental astrophysical S factors [9–14] is well determined by the calculated function $\mathcal{R}_{1j_f}(E, C^{(sp)}_{1j_f})$ and by $\mathcal{R}_{13/2}(E, C^{(sp)}_{13/2}) + \lambda_C \mathcal{R}_{11/2}(E, C^{(sp)}_{11/2})$. Hence, the experimental astrophysical S factors presented in Refs. [9–14] can be used as an independent source of reliable information about the ANCs for $\alpha + {}^3$He $\to {}^7$Be(g.s.) and $\alpha + {}^3$He $\to {}^7$Be(0.429 MeV).

Nevertheless, the ANC values for $\alpha + {}^3$He $\to {}^7$Be(g.s.) and $\alpha + {}^3$He $\to {}^7$Be(0.429 MeV) obtained from the analysis of Refs. [9–12] [see Figs. 4(a) and 4(d) and Table I], differ from those obtained from the analysis of Refs. [13,14][see Figs. 4(b) and 4(e) and Table I] and this difference is about 10%. The main reason for this difference is in the systematic discrepancy observed in absolute values of the experimental astrophysical S factors measured in Refs. [9–14] at $E \leqslant 614.6$ keV (see Fig. 5). This discrepancy is also about 10%; slightly more than the experimental errors. It can influence the result of an extrapolation of the experimental astrophysical S factors [9–14] to the experimentally inaccessible energy regions ($E < 90$ keV) and the estimates of their true uncertainties obtained from Eqs. (A1), (A4), and (A5). The accuracy of the extrapolated result depends mainly on that of the ANC values used in Eq. (A1). Therefore, the true ANC values and their uncertainties should indeed be determined correctly, taking into account the discrepancy mentioned above. To this end, it is convenient to split formally the experimental S factors presented in Refs. [9–14], and used here for determination of

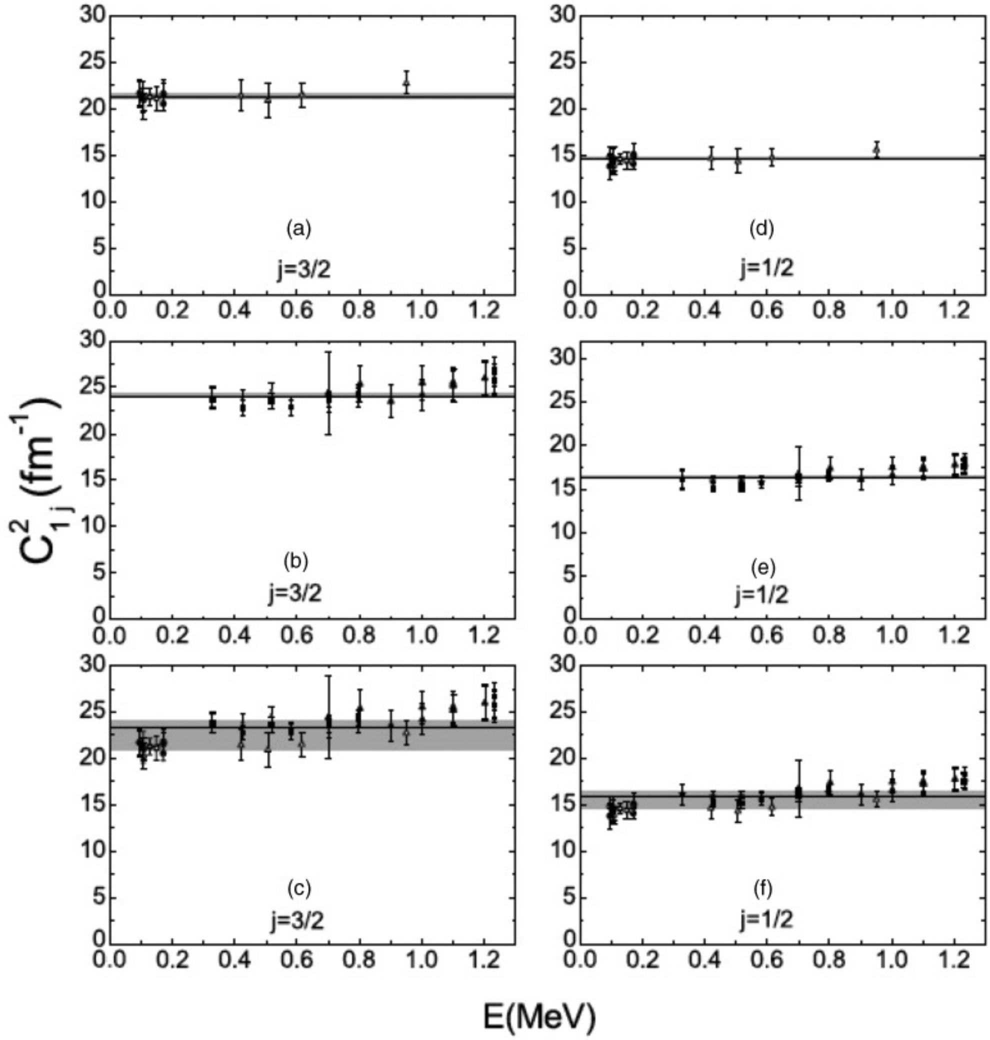

FIG. 4. The values of the ANCs, $C^2_{1\,3/2}$ and $C^2_{1\,3/2}$, for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be(g.s.)}$ [(a)–(c)] and $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}(0.429\ \mathrm{MeV})$ [(d)–(f)] for each experimental point E. In (a) and (d), data denoted by △ and ○ (•) are obtained from the analysis of the unseparated (separated) $S^{(\mathrm{exp})}_{34}(E)$ of Refs. [9] (activation) and [10–12] (activation) (Ref. [12]; prompt), respectively, from data set I. In (b) and (e), data denoted by ⋆ and ■ (▲) are obtained from the analysis of the separated (unseparated) $S^{(\mathrm{exp})}_{34}(E)$ of Refs. [13] (activation) and [13] (prompt) (Ref. [14]; ^{7}Be recoils), respectively, from data set II. Data in (c) and (f) are obtained from the analysis of all of the data in sets I and II. The solid lines present our results for the weighted means. Everywhere the width of each of the bands is the corresponding weighted uncertainty.

the ANC values, into two sets. The first set of experimental data is denoted data set I [9–12] and the second one is denoted data set II [13,14]. Each set provides mutually agreeing results for the ANCs, $(C^{\mathrm{exp}}_{1\,3/2})^2$ and $(C^{\mathrm{exp}}_{1\,1/2})^2$, within the corresponding uncertainties of $S^{\mathrm{exp}}_{1\,3/2}(E)$ and $S^{\mathrm{exp}}_{1\,1/2}(E)$ [or $S^{\mathrm{exp}}_{34}(E)$] used for each experimental point of E. Therefore, in order to estimate an accuracy of determination of the ANCs derived from the analysis of data sets I and II both for the prompt and for the activation (the ^{7}Be recoils) measurements, for both data sets first the weighted means of the ANC values and their uncertainties should be deduced from the ANC values derived separately from data sets I and II for each experimental point E. Both sets are presented in Table II. As seen from the first, second, and third lines of Table II, the weighted means [8] of the ANC values for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be(g.s.)}$ and $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}(0.429\ \mathrm{MeV})$ obtained by analyses performed separately for the activation and the prompt of data set I are in a good agreement with one another, and their uncertainties do not exceed the experimental errors for $S^{\mathrm{exp}}_{1\,3/2}(E)$ and $S^{\mathrm{exp}}_{1\,1/2}(E)$. The same situation occurs for the weighted means of the ANC values, which are derived from the analysis of data set II (the prompt, the ^{7}Be activation, and the ^{7}Be recoils). They are also presented in the seventh and eighth lines of Table II. But, as seen also from Table II, the weighted means of the ANC values obtained by using jointly the activation and the prompt of data set I (see the parenthetical figures in the fourth line of Table II) as well as using jointly the prompt and the activation (the ^{7}Be recoils) of data set II (see the parenthetical figures in the tenth line of Table II) noticeably differ from one another [1.13 times for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be(g.s.)}$ and 1.12 times for $\alpha + {}^3\mathrm{He} \to {}^7\mathrm{Be}(0.429\ \mathrm{MeV})$]. This also is a result of the aforementioned systematical discrepancy

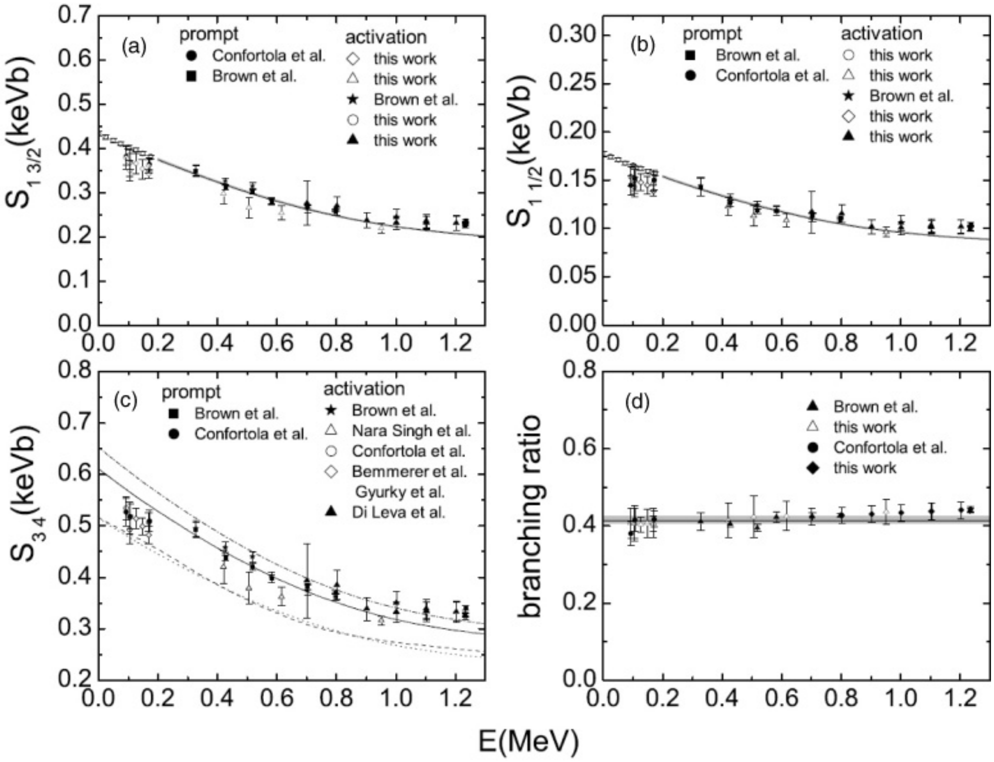

FIG. 5. The experimental and calculated astrophysical S factors for $^3\text{He}(\alpha,\gamma)^7\text{Be}$ populating the ground (a) and first exited (b) states of ^{7}Be as well as their sum (c) and the branching ratio (d). In (a) and (b), the open circles are our results of the extrapolation. The solid lines are our results of the calculation with the standard values of geometric parameters ($R = 1.80$ fm and $a = 0.70$ fm). In (c), the dotted (dash-dotted) and dashed lines are the results of the present work obtained with the lower (upper) values of the ANCs and those of Refs. [23,24], respectively. In (d), the straight line is our result for the weighted mean. Everywhere the width of each of the band is the corresponding weighted uncertainty.

between absolute values of the experimental astrophysical S factors of data set I and data set II. Also, the central values of the weighted means for the ANC values for $^3\text{He} + \alpha \to {}^7\text{Be(g.s.)}$ and $^3\text{He} + \alpha \to {}^7\text{Be}(0.429\ \text{MeV})$ obtained from all of the experimental data (data set I [9–12] and data set II [13,14]), which are presented in the eleventh line of Table II, differ up to 10% more and 3% less than those deduced only from the analyses of data set I and data set II, respectively. As, at present, there is no reasonable argument to adhere to either of these experimental data (either data set I or data set II); it seems reasonable to obtain the weighted means of the ANCs derived from all these real experimental ANCs with upper ($\Delta_1^{(C)}$) and lower ($\Delta_2^{(C)}$) limits corresponding to data set II and data set I, respectively. This leads to an asymmetric uncertainty for the weighted means of ANCs, which involves the uncertainty arising from the experimental errors of data sets I and II, the uncertainty of the MTBPA used, and the aforementioned discrepancy between absolute values.

The ANC values recommended by us are presented in the eleventh line of Table II as well as in Figs. 4(c) and 4(f), where the solid lines and the band widths correspond to the weighted means and their asymmetric uncertainties, being equal to $\Delta_1^{(C)} = 4.3\%$ and $\Delta_2^{(C)} = 9.9\%$ for $C_{1\,3/2}^{(\text{exp})}$ and $\Delta_1^{(C)} = 3.8\%$ and $\Delta_2^{(C)} = 9.4\%$ for $C_{1\,1/2}^{(\text{exp})}$. One notes that the uncertainty for these weighted means is about 7% on average [$\Delta^{(C)} = (\Delta_1^{(C)} + \Delta_2^{(C)})/2$], within the experimental errors of data sets I and II. The corresponding values of NVCs obtained by using Eq. (A8) are given in Table II.

The results of the present work differ noticeably from the values obtained in Refs. [16,19,25], which are also presented in Table II. In this connection one notes the following. The results of Ref. [16] for $C_{1\,3/2}$ and $C_{1\,1/2}$ were obtained from the analysis of the older experimental $^3\text{He}(\alpha,\gamma)^7\text{Be}$ astrophysical S factors [26] (see Ref. [6] and references therein) performed within the R-matrix method, where the contribution from the internal part of the amplitude was simulated by the background for a single pole with free resonant parameters. But there the assumption about equality of the ANCs ($C_{1\,3/2} = C_{1\,1/2}$) was used to reduce the number of adjusted parameters, and the best fitting of the calculated $S_{34}(E)$ to the experimental data was reached at $C_{1\,3/2} = 3.79\ \text{fm}^{-1/2}$ and a channel radius $r_c = 3.0$ fm. But, following the present work, in reality the values of the ANCs $C_{1\,3/2}$ and $C_{1\,1/2}$ should not be equal. Moreover, the calculation shows that the asymptotic behavior of the bound ($\alpha + {}^3$He) state and α-^{3}He scattering wave functions is reached simultaneously only at $r_c \gtrsim 5.0$ fm, so at $r_c \geqslant 3.0$ fm their substitution for these wave functions in the external part of the amplitude in Ref. [16] is not correct. In Ref. [19] the bound-state wave functions, which correspond to the calculated value of the binding energy for ^{7}Be(g.s.) in the ($\alpha + {}^3$He) channel differing niticeably from the experimental

TABLE II. The weighted means of the ANC values $(C^{\text{exp}})^2$ for ^{3}He + $\alpha \to {}^7$Be, NVCs $|G|^2_{\text{exp}}$, and the calculated values of $S_{34}(E)$ at energies $E = 0$ and 23 keV. Lines 1–3, 7, and 8 are the results obtained from the analysis of data of works pointed out in the first column. The parenthetical values presented in lines 4 and 10 are the results for the weighted means obtained from the data given in lines 1–3 (data set I [9-12]) and lines 7 and 8 (data set II [13,14]).

Experimental data for the astrophysical S factors	$(C^{\text{exp}}_{1\,3/2})^2$ (fm^{-1})	$\lvert G_{1\,3/2}\rvert^2_{\text{exp}}$ (fm)	$(C^{\text{exp}}_{1\,1/2})^2$ (fm^{-1})	$\lvert G_{1\,1/2}\rvert^2_{\text{exp}}$ (fm)	$S_{34}(0)$ (keV b)	$S_{34}(23$ keV) (keV b)
[9–11] (the activation)	21.2 ± 0.4	1.00 ± 0.02	14.5 ± 0.3	0.688 ± 0.013	0.560 ± 0.011	0.551 ± 0.011
[12] (the prompt)	21.4 ± 0.7	1.02 ± 0.04	14.6 ± 0.5	0.697 ± 0.033	0.564 ± 0.021	0.558 ± 0.021
[12] (the activation)	21.2 ± 0.8	1.01 ± 0.04	14.5 ± 0.5	0.690 ± 0.026	0.560 ± 0.021	0.550 ± 0.020
data set I ([9–12])	(21.3 ± 0.3)	(1.01 ± 0.02)	(14.6 ± 0.2)	(0.694 ± 0.011)	(0.562 ± 0.008)	(0.552 ± 0.008)
					0.53 ± 0.02 [9]	
					0.560 ± 0.017 [12]	
[13] (the activation)	24.0 ± 0.4	1.13 ± 0.02	16.2 ± 0.2	0.768 ± 0.011	0.630 ± 0.008	0.619 ± 0.008
[13,14] (the prompt) and [14] (^{7}Be recoils)	24.1 ± 0.3	1.14 ± 0.02	16.4 ± 0.2	0.773 ± 0.011	0.624 ± 0.010	0.612 ± 0.010
data set II ([13,14])	(24.1 ± 0.2)	(1.14 ± 0.01)	(16.3 ± 0.2)	(0.771 ± 0.008)	(0.628 ± 0.006)	(0.616 ± 0.006)
[7–12] (data sets I and II)	$23.3^{+1.0}_{-2.3}$	$1.10^{+0.05}_{-0.11}$	$15.9^{+0.6}_{-1.5}$	$0.751^{+0.028}_{-0.069}$	$0.613^{+0.026}_{-0.063}$	$0.601^{+0.030}_{-0.072}$ y
					0.596 ± 0.021 [13]	
					0.57 ± 0.04 [14]	
[9–14]					0.580 ± 0.043 [47]	
[9–14]					0.56 ± 0.02(exp) ± 0.02(th) [7]	
[26]	14.4 [16]	0.680 [16]	14.4 [16]	0.680 [16]	0.51 ± 0.04 [16]	
[26]	17.1 [25]	0.81 [25]	13.1 [25]	0.62 [25]	0.52 ± 0.03 [25]	
	12.6 ± 1.1 [19]	0.596 ± 0.052 [19]	8.41 ± 0.58 [19]	0.397 ± 0.030 [19]	$\approx$0.40 [19]	
					0.56 [17] 0.593 [20]	
					0.516 [23]0.53 [24]	

one (see Table I in Ref. [19]), and the initial-state wave functions were computed with different potentials, so these calculations were not self-consistent. Since the ANCs for ^{3}He + $\alpha \to {}^7$Be are sensitive to the form of the NN potential, it is desirable, first, to calculate the wave functions of the bound state using other forms of the NN potential, and, second, in order to guarantee self-consistency, the same forms of the NN potential should be used for calculation of the initial wave functions, as was done in Ref. [20]. Besides, a comparison of the present result and that obtained in Ref. [25] shows that the underestimation of the contributions from both the nuclear interior and exterior indeed occurs in Ref. [25], since the contribution of the nuclear interior ($r < 4.0$ fm) to the calculated astrophysical S factors and the use of experimental data [9-14] that are more accurate than those in Ref. [25] can influence the extracted values of the ANCs. The present results differ strongly also from the values $C^2_{1\,3/2} = 7.95$–14.3 fm^{-1} and $C^2_{1\,1/2} = 6.45$–7.45 fm^{-1} [42], which were obtained by the analytical continuation of the corresponding bound (^{3}He + α) state pole of the Coulomb-modified effective-range theory expansion for α-^{3}He scattering. But this method has faced difficulties in correctly estimating the extrapolation errors [43]. Nevertheless, the resulting ANC value for $\alpha + {}^3$He $\to {}^7$Be(g.s.) obtained by us is in a good agreement with the value $C^2_{1\,3/2} = 25.3$ fm^{-1}, and that for $\alpha + {}^3$He $\to {}^7$Be(0.429 MeV) differs noticeably from the value $C^2_{1\,1/2} = 22.0$ fm^{-1}, which were obtained in Ref. [44] with the ($\alpha + {}^3$He)-channel resonating-group method.

We would like to note that, in Ref. [45], the ANC values recommended in the present work have already been applied to the analysis of low-energy experimental phase shifts for α-^{3}He scattering within the effective-range expansions restricted to terms up to k^4, and the "indirectly determined" values of the effective-range expansion parameters were obtained, which reproduce fairly well the corresponding experimental phase shifts up to energies E of about 5 MeV. Therefore, the self-consistent result obtained in Ref. [45] between the fundamental characteristics of the bound ($\alpha + {}^3$He) state of the nuclear ^{7}Be in the form of the ANCs, the effective-range expansion parameters, and the low-energy phase shifts for α-^{3}He scattering, confirms the reliability of the ANC values recommended in the present work. Therefore, at present the ANC values recommended in this work can be considered as straightforward "best indirectly determined" values of the ANCs (NVCs) by means of the analysis of the modern precisely measured astrophysical S factors [9–14], and they are the most important result of this work. Nevertheless, since the discrepancy between data sets I and II occurs at $E \leqslant 614.6$ keV and is more than the experimental errors, we recommend decisive measurement of the astrophysical S factors for the ^{3}He$(\alpha,\gamma)^7$Be reaction in this energy range. It would allow determination the ANC (NVC) values with a higher accuracy.

Below, we will use the ANC values to obtain information about the α-particle spectroscopic factors for the (^{7}Li^{7}Be) mirror nuclei and to extrapolate the astrophysical S factors at lower energies, including $E = 0$.

B. α-particle spectroscopic factors for the mirror (^{7}Li^{7}Be) pair

The indirectly determined values of the ANCs for ^{3}He + $\alpha \to {}^7$Be presented in the eleventh line of Table II and those for $\alpha + t \to {}^7$Li deduced in Ref. [30] can be used to obtain information on the ratio $R_{Z;j_f} = Z_{1j_f}(^7\mathrm{Be})/Z_{1j_f}(^7\mathrm{Li})$ for the virtual α decays of the bound mirror (^{7}Li^{7}Be) pair, where $Z_{1j_f}(^7\mathrm{Be})$ and $Z_{1j_f}(^7\mathrm{Li})$ are the spectroscopic factors for ^{7}Be and ^{7}Li in the ($\alpha + {}^3$He) and ($\alpha + t$) configurations, respectively. For this purpose, from $C_{1\,j_f}(\mathrm{B}) = Z^{1/2}_{1\,j_f}(\mathrm{B})C^{(sp)}_{1\,j_f}(\mathrm{B})$ (B = ^{7}Li and ^{7}Be) [28] we form the relation

$$R_{Z;\,j_f} = \frac{R_{C;\,j_f}}{R_{C^{(sp)};\,j_f}}, \tag{1}$$

where $R_{C;\,j_f} = (C_{1\,j_f}(^7\mathrm{Be})/C_{1\,j_f}(^7\mathrm{Li}))^2$ and $R_{C^{(sp)};\,j_f} = (C^{(sp)}_{1\,j_f}(^7\mathrm{Be})/C^{(sp)}_{1\,j_f}(^7\mathrm{Li}))^2$ are the ratios of squares of the ANCs and the single-particle ANCs for the bound mirror (^{7}Li^{7}Be) pair for the ground ($j_f = 3/2$) and first excited ($j_f = 1/2$) states of the mirror nuclei, respectively. The relation (1) makes it possible to verify the validity of the approximation ($R_{C;j_f} \approx R_{C^{(sp)};\,j_f}$, i.e. $R_{Z;\,j_f} \approx 1$) used in Ref. [46] for the mirror (^{7}Li^{7}Be) conjugated α decays.

For the ground and first excited states of the mirror (^{7}Li^{7}Be) pair, the values of $C^{(sp)}_{1\,j_f}(^7\mathrm{Be})$ and $C^{(sp)}_{1\,j_f}(^7\mathrm{Li})$ change by a factor of 1.3 under variation of the geometric parameters (R and a) of the adopted Woods–Saxon potential [36,37] within the aforementioned ranges, while the ratios $R_{C^{(sp)};3/2}$ and $R_{C^{(sp)};1/2}$ for the bound and first excited states of the mirror (^{7}Li^{7}Be) pair change only about 1.5% and 6%, respectively. It is seen that the ratios do not depend practically on variation of the free parameters R and a, which are equal to $R_{C^{(sp)};3/2} = 1.37 \pm 0.02$ and $R_{C^{(sp)};1/2} = 1.40 \pm 0.09$. They are in good agreement with those calculated in Ref. [46] within the microscopic cluster and two-body potential models (see Table I there). The ratios for the ANCs are $R_{C;3/2} = 1.83^{+0.18}_{-0.25}$ and $R_{C;1/2} = 1.77^{+0.19}_{-0.24}$. From Eq. (1) the $R_{Z;\,j_f}$ ratio values are equal to $R_{Z;3/2} = 1.34^{+0.13}_{-0.18}$ and $R_{Z;1/2} = 1.26^{+0.16}_{-0.19}$ for the ground and the first excited states, respectively. Within their uncertainties, these values differ slightly from $R_{Z;3/2} = 0.995 \pm 0.005$ and $R_{Z;1/2} = 0.990$ calculated in Ref. [46] within the microscopic cluster model, and indeed are sensitive to the model assumptions (the choice of the oscillation radius b and the effective NN potential form). Such a model dependence may actually influence the mirror symmetry for the α-particle spectroscopic factors. The mirror-symmetry breakup for the α-particle spectroscopic factors can also be signalled by the results for the ratio of $S_{34}(^7\mathrm{Be})/S_{34}(^7\mathrm{Li})$ at zero energy for the mirror (^{7}Li^{7}Be) pair obtained in Ref. [15] within the resonating-group method by using seven different forms for the effective NN potential. As seen in Ref. [15], this ratio is sensitive to the form of the effective NN potential used, and changes from 1.0 to 1.18. One of the possible reasons for the sensitivity observed in Ref. [15] can apparently be associated with a sensitivity of the ratio $R_{Z;\,j_f}$ to the form of the effective NN potential used. In contrast to such model dependence observed in Refs. [15,46], here the problem of the ambiguity connected with the model (R, a) dependence for the ratios $R_{Z;\,j_j}$ found by us from Eq. (1) is reduced to a minimum within the experimental uncertainty.

It is seen from here that the empirical values of $R_{Z;\,j_f}$ exceed unity both for the ground state and for the first excited state of the mirror (^{7}Li^{7}Be) pair. This result is not accidental and can be explained qualitatively by the following consideration: The spectroscopic factors $Z_{1\,j_f}(^7\mathrm{Li})$ and $Z_{1\,j_f}(^7\mathrm{Be})$ are determined as a norm of the radial overlap function of the bound-state wave functions of the t, α, and ^{7}Li as well as ^{3}He, α, and ^{7}Be nuclei, respectively, and are given by Eqs. (100) and (101) from Ref. [28]. The interval of integration ($0 \leqslant r < \infty$) in Eq. (101) can be divided in two parts. In the first integral denoted by $Z^{(1)}_{1\,j_f}(^7\mathrm{Li})$ for ^{7}Li and $Z^{(1)}_{1\,j_f}(^7\mathrm{Be})$ for ^{7}Be, the integration over r covers the internal region $0 \leqslant r \leqslant r_c$, where nuclear α-t and α-^{3}He interactions dominate over the Coulomb interactions. In the second integral

$$Z^{(2)}_{1\,j_f}(\mathrm{B}) = C^2_{1\,j_f}(\mathrm{B}) \int_{r_c}^{\infty} dr\, W^2_{-\eta_\mathrm{B};\,3/2}(2\kappa_{\alpha a} r), \tag{2}$$

where in the external region the radial overlap function in the integrand is replaced by the appropriate Whittaker function (see, for example, Eq. (108) of Ref. [28]), interaction between a and the α particle, where $a = t$ for B = ^{7}Li and $a = {}^3$He for B = ^{7}Be, is governed by Coulomb forces only. In Eq. (2), $\kappa_{\alpha a} = \sqrt{2\mu_{\alpha a}\varepsilon_{\alpha a}}$ and $W_{-\eta_\mathrm{B};\,3/2}(x)$ is the Whittaker function. One notes that the magnitudes $Z^{(1)}_{1\,j_f}(^7\mathrm{Li})$ and $Z^{(1)}_{1\,j_f}(^7\mathrm{Be})$ as well as $Z^{(2)}_{1\,j_f}(^7\mathrm{Li})$ and $Z^{(2)}_{1\,j_f}(^7\mathrm{Be})$ define the probability of finding t and ^{3}He in the ($\alpha + t$) and ($\alpha + {}^3$He) configurations at distances of $r \leqslant r_c$ as well as of $r > r_c$, respectively. Obviously $Z_{1\,j_f}(^7\mathrm{Li}) = Z^{(1)}_{1\,j_f}(^7\mathrm{Li}) + Z^{(2)}_{1\,j_f}(^7\mathrm{Li})$ and $Z_{1\,j_f}(^7\mathrm{Be}) = Z^{(1)}_{1\,j_f}(^7\mathrm{Be}) + Z^{(2)}_{1\,j_f}(^7\mathrm{Be})$.

Values of $Z^{(2)}_{1\,j_f}(^7\mathrm{Li})$ and $Z^{(2)}_{1\,j_f}(^7\mathrm{Be})$ can be obtained from Eq. (2) by using the values of the ANCs for $\alpha + t \to {}^7$Li and $\alpha + {}^3$He $\to {}^7$Be recommended in Ref. [30] and in the present work, respectively. For example, for $r_c \approx 4.0$ fm [the surface regions for the mirror (^{7}Li^{7}Be) pair], the calculation shows that the ratio $R^{(2)}_{Z;\,j_f} = Z^{(2)}_{1\,j_f}(^7\mathrm{Be})/Z^{(2)}_{1\,j_f}(^7\mathrm{Li})$ equals $1.43^{+0.13}_{-0.18}$ and $1.31^{+0.14}_{-0.18}$ for the ground and excited states of the ^{7}Li and ^{7}Be nuclei, respectively; i.e., the ratio $R^{(2)}_{Z;\,j_f} > 1$. Owing to the principle of equivalency of nuclear interactions between nucleons of the α-t pair in the ^{7}Li nucleus and the α-^{3}He pair in the ^{7}Be nucleus [46], the values of $Z^{(1)}_{1\,j_f}(^7\mathrm{Li})$ and $Z^{(1)}_{1\,j_f}(^7\mathrm{Be})$ should not differ noticeably. If one suggests that $R^{(1)}_{Z;\,j_f} \approx 1$, then the ratio $R_{Z;\,j_f} > 1$.

C. The ^{3}He(α,γ)^{7}Be astrophysical S factor at solar energies

Here, Eqs. (A1) and (A5), and the corresponding weighted means of the ANCs obtained for ^{3}He + $\alpha \to {}^7$Be(g.s) and $^3\mathrm{He}+\alpha\to{}^7\mathrm{Be}(0.429$ MeV) are used to extrapolate $S_{1\,3/2}(E)$, $S_{1\,1/2}(E)$, and $S_{34}(E)$ at solar energies ($E\leqslant 25$ keV). At first, we tested again the fulfillment of condition (A2) in the same way as was done above for $E\geqslant 90$ keV, and similar results plotted in Fig. 1 are also observed at energies of $E<90$ keV.

The separated experimental and calculated astrophysical S factors are presented in Figs. 5(a) and 5(b), where the solid triangles and the open diamonds and triangles show our results for $S^{\exp}_{1\,3/2}(E)$ and $S^{\exp}_{1\,1/2}(E)$, which are obtained from the analysis of the total (unseparated) experimental astrophysical S factors of Refs. [9–12,14], respectively, by using the corresponding values of the ANCs from Table I for each energy E experimental point. They are also presented in the fourth and fifth columns of Table I and are the second result of the present work. The solid lines present our calculations performed with the standard values of geometric parameters ($R=1.80$ fm and $a=0.70$ fm) and the weighted ANC values given in the fourth and tenth lines of Table II, and the open circles are the results of the extrapolation, where each of the quoted uncertainties is associated with that of the corresponding ANC. In Fig. 5(c), the solid line presents our calculations for $S_{34}(E)$ performed also with the standard values of geometric parameters by using the weighted means of the ANCs ($C^2_{1\,3/2}$ and $C^2_{1\,1/2}$) presented in the eleventh line of Table II. There the dashed and dot-dashed lines are the results of calculations using the aforementioned lower and upper limit values of the ANCs pointed out in the eleventh line of Table II, respectively, and the dotted line is the result of Refs. [23,24], showing the presence of the noticeable systematic underestimation with respect to the experimental data [9–14]. As seen in these figures, Eqs. (A1), (A4), and (A5) allow one to perform a correct extrapolation of $S_{1\,3/2}(E)$, $S_{1\,1/2}(E)$, and $S_{34}(E)$ at solar energies.

The weighted means of the total astrophysical S factor $S_{34}(E)$ at solar energies ($E=0$ and 23 keV) are presented in Table II. There, as a comparision, the results recommended by other authors are also presented. As seen in Table II, the weighted means of $S_{34}(0)$, deduced in the present work separately from each activation and prompt of data set I (the first, second, and third lines) and of data set II (the seventh and eighth lines), agree well within their uncertainties with each other and with those recommended in Refs. [12–14], respectively. Besides, these weighted means of $S_{34}(0)$ obtained from the independent analysis of data sets I and II differ also noticeably from one another (about 12%; see the parenthetical figures), and this distinction is mainly associated with the aforementioned difference observed in magnitudes of the corresponding ANCs presented in the fourth and tenth lines of Table II. Nevertheless, the weighted mean of $S_{34}(0)$ and its uncertainty, $S_{34}(0)=0.613^{+0.026}_{-0.063}$ keV b, recommended in the present work and presented also in the the eleventh line of Table II, within the asymmetric uncertainty (upper $\Delta^{(S)}_1=4.2\%$ and lower $\Delta^{(S)}_2=10.3\%$), agrees with that recommended in Refs. [7,12–14,47]. One notes that our result for the weighted mean of $S_{34}(0)$ is obtained from Eq. (A5) by means of using the weighted means of the ANCs presented in the eleventh line of Table II. The asymmetric uncertainty of $S_{34}(0)$ is caused by the uncertainty for the ANCs mentioned above, which is also about $\pm 7\%$ on average [$\Delta_S=(\Delta^{(S)}_1+\Delta^{(S)}_2)/2$]. It is interesting to note also that the central value of $S_{34}(0)$ is closer to that given in the tenth line of Table II than to the central value of the weighted mean given in the fourth line of Table II.

The astrophysical S factors, calculated by using the values of the ANCs obtained separately from data sets I and II and from both of them (see Table II), are fitted independently using a second-order polynomial within three energy intervals ($0\leqslant E\leqslant 500$ keV, $0\leqslant E\leqslant 1000$ keV, and $0\leqslant E\leqslant 1200$ keV). The resulting slopes of $S'_{34}(0)/S_{34}(0)$ are -0.711 MeV^{-1}, -0.734 MeV^{-1}, and -0.726 MeV^{-1}, depending on the aforementioned intervals, respectively, and they do not depend on the values of the ANCs used. One notes that they also are about in agreement with the values -0.695 MeV^{-1} [20], -0.73 MeV^{-1} [24], and -0.92 ± 0.18 MeV^{-1} [47]. The first of them can be derived from the result of Ref. [20] for $S_{34}(E)$ (see Fig. 3 there) by using the second-order polynomial approximation for $S_{34}(E)$ within the energy range $0\leqslant E\leqslant 560$ keV. It is seen from here that the $S_{34}(E)$ calculated in the present work [the solid line in Fig. 5(c)] and those obtained in Refs. [20,24,47] have practically the same energy dependence, but they differ with each other mainly by a normalization. Nevertheless, our result for $S'_{34}(0)/S_{34}(0)$ differs slightly from the value of -0.64 MeV^{-1} derived in Ref. [7] from the result of Ref. [19]. Besides, the noticeable contribution of the partial s wave to the slope of $S'_{34}(0)/S_{34}(0)$ is observed. Now, the slope defined only for the pure partial s wave is found to be equal to -0.855 MeV^{-1}; i.e., it becomes less steep than the slope defined by taking into account the contributions from all partial (s, p, and d) waves, as the slopes for the pure p and d waves have a positive sign and are equal to 1.651 MeV^{-1} for the p wave and 6.841 MeV^{-1} for the d wave. One notes that the slopes obtained in the present work for the partial s and d waves of $S_{1\,3/2}(0)$ and $S_{1\,1/2}(0)$ only, which are respectively equal to -0.839 and -0.882 MeV^{-1} for the s wave and 6.860 and 6.703 MeV^{-1} for the d wave, are in a good agreement with those calculated in Ref. [48] (see Table IV of Ref. [48]) in which the Gaussian form of the nuclear α-^{3}He interaction was used. It is seen from here that the energy dependence of the calculated S factors is practically similar for both potential forms.

Our result for $S_{34}(0)$ differs noticeably from those recommended in Refs. [16,19,25] as well as [23,24] (see Table II). This circumstance is apparently connected with the underestimation of the contributions from the external and interior parts in the amplitude in those works. These underestimations mainly arise from the underestimated values of the ANCs used in Refs. [16,19,25] (see Sec. II A) and from the assumption admitted in Refs. [23,24] that a value of the ratio $R_{Z;\,j_f}$ ($j=1/2$ and 3/2) for the bound mirror (^{7}Li^{7}Be) pair is taken to be equal to unity. One notes that the values of $Z_{1\,j_f}(^7\mathrm{Be})=Z_{1\,j_f}(^7\mathrm{Li})=1.17$ [49] were used in Refs. [23,24]. But, as shown in Sec. II B, the values of $R_{Z;\,j_f}$ for the ground and first excited states of the mirror (^{7}Li^{7}Be) pair are larger than unity. Therefore, the underestimated values of $Z_{1\,j}(^7\mathrm{Be})$ used in Refs. [23,24] also result in the underestimated value of $S_{34}(0)$ for the direct-capture $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction.

Perhaps the assumption about equal values of the spectroscopic factors is correct only for the spectroscopic factors $Z_{1\,j}(^7\mathrm{Li})$, since the values of $S_{34}(0)$ obtained in Refs. [23,30] for the direct-capture $t(\alpha,\gamma)^7$Li reaction agree excellently with each other. It should be noted that in Ref. [30] the analysis of the $t(\alpha,\gamma)^7$Li experimental astrophysical S factors [50] has also been performed within the MTBPA, and the ANC for $\alpha + t \to {}^7$Li(g.s.), which has been deduced there from the data of Refs. [23,25], also is in good agreement with the values recommended by authors of Ref. [30]. The recent estimate of $S_{34}(20\,\mathrm{keV}) = 0.593$ keV b recommended in Ref. [22] should also be noted. This estimate was obtained within the standard two-body potential model by using the improved Gaussian form for the nuclear α-^{3}He interaction and with the assumption that $Z_{1j}(^7\mathrm{Li}) = Z_{1j}(^7\mathrm{Be}) = 1.0$ (see Ref. [21] also). In this assumption, the total astrophysical S factor $S_{34}(E)$ in Ref. [22] is presented as the sum of the single-particle astrophysical S factors $S^{(sp)}_{1\,j_f}(E)$ [8] with $j_f = 3/2$ and $1/2$. But, as shown in Sec. II A, the calculated values of $S^{(sp)}_{1\,j_f}(E)$ become strongly model dependent from the single-particle ANCs $C^{(sp)}_{1\,j_f}$. Therefore, in reality, the value of $S_{34}(20\,\mathrm{keV})$ recommended in Ref. [22] is also model dependent both from the spectroscopic factor values used and from the choice of the free model parameter of the single-particle ANCs. Besides, the result for $S_{34}(0)$ recommended in the present work differs noticeably from that derived in Ref. [9] from the joint analysis of the data for $S^{\mathrm{exp}}_{34}(E)$ measured in Refs. [9,51,52]. An analogous situation occurs for the value of $S'_{34}(0)/S_{34}(0) = -0.662$ MeV^{-1}, which can be obtained from the result of Ref. [9] for $S_{34}(E)$ by fitting it with a second-order polynomial. One notes that the experimental data of Ref. [51] have a noticeable spread and large uncertainties within the most important energy range, $E < 400$ keV. Perhaps that is one of the possible reason why the value for $S_{34}(0)$ recommended in Ref. [9] may be underestimated and the aforementioned $S'_{34}(0)/S_{34}(0)$ value derived from the $S_{34}(E)$ calculated in Ref. [9] differs slightly from that obtained in the present work. This can be confirmed also by the result for $S_{34}(0)$ recommended in Ref. [12], which was also obtained from the joint analysis of the modern precise experimental astrophysical S factor $S^{\mathrm{exp}}_{34}(E)$ of Refs. [9,12]; the $S_{34}(0)$ value of Ref. [12] is noticeably more than that in Ref. [9]. Therefore, the central value of $S_{34}(0)$ recommended by us is more than that recommended in Ref. [7], since the underestimated value of $S_{34}(0)$ derived in Ref. [9] was used in Ref. [7].

Nevertheless, our result is in agreement with that obtained in Refs. [17,20] within the microscopic single-channel $(\alpha + {}^3\mathrm{He})$-cluster approach and the microscopic fermionic dynamics approach using the realistic effective NN potential (an *ab initio* type calculation), respectively (see Table II also). But, the calculation performed in Ref. [18] within the microscopical two-channel $[(\alpha + {}^3\mathrm{He})$ and $(p + {}^7\mathrm{Li})]$ approach gave a value of $S_{34}(0) = 0.83$; that is, the estimate of the value of the $S_{34}(0)$ strongly changes when the model space is expanded. Our result is also in excellent agreement with the values $S_{34}(0) = 0.609$ keV b [15] and $S_{34}(0) = 0.598$ keV b [44] obtained for the with-distortion case within the $(\alpha + {}^3\mathrm{He})$ channel of the version of the resonating-group method that uses modified Wildermuth-Tang (MWT) and near-Serber exchange mixture forms for the effective NN potential. It follows from here that the mutual agreement between the results obtained in the present work and in Refs. [15,17,20,44], which are based on the common approximation regarding the cluster $(\alpha + {}^3\mathrm{He})$ structure of ^{7}Be, allows one to draw conclusions about the dominant contribution of the $(\alpha + {}^3\mathrm{He})$ clusterization to the low-energy ^{3}He$(\alpha,\gamma)^7$Be cross section both in the absolute normalization and in the energy dependence [9–14]. Therefore, a single-channel $(\alpha + {}^3\mathrm{He})$ approximation for ^{7}Be [15,17,20,44] is quite appropriate for this reaction in the considered energy range.

Also, the ratios of the $S_{1\,3/2}(0)$ and $S_{1\,1/2}(0)$ values indirectly determined here for the ^{3}He$(\alpha,\gamma)^7$Be reaction populating the ground and first excited states of ^{7}Be, to those for the mirror the $t(\alpha,\gamma)^7$Li reaction deduced in Ref. [30], are equal to $R^{(\mathrm{g.s.})}_S = 6.87^{+0.70}_{-0.87}$ and $R^{(\mathrm{exc})}_S = 6.11^{+0.67}_{-0.86}$, respectively. These values are in a good agreement with the values of $R^{(\mathrm{g.s.})}_S = 6.6$ and $R^{(\mathrm{exc})}_S = 5.9$ deduced in Ref. [46] within the microscopic cluster model. This result also confirms directly our estimate for the ratio $R_{C;\,j_f}$ obtained above, since the ANCs for $t + \alpha \to {}^7$Li(g.s) and $t + \alpha \to {}^7$Li(0.478 MeV) as well as those for $^3\mathrm{He} + \alpha \to {}^7$Be(g.s) and $^3\mathrm{He} + \alpha \to {}^7$Be(0.429 MeV) determine the astrophysical S factors for the $t(\alpha,\gamma)^7$Li and ^{3}He$(\alpha,\gamma)^7$Be reactions at zero energy and, consequently, the ratios $R^{(\mathrm{g.s.})}_S$ and $R^{(\mathrm{exc})}_S$ are proportional to $R_{C;\,3/2}$ and $R_{C;\,1/2}$, respectively.

Figure 5(d) shows a comparison between the branching ratio $\bar{R}^{\mathrm{exp}}(E)$ obtained in the present work (open triangles and solid diamonds) and that recommended in Refs. [12] (solid circles) and [13] (solid triangles). The open triangles and solid diamonds are the results obtained from the analysis of the total (unseparated) experimental S factors of works [9–12] and [14], respectively, by using the corresponding values of the ANCs deduced for each experimental point of E. The weighted mean $\bar{R}^{\mathrm{exp}}$ of the $\bar{R}^{\mathrm{exp}}(E)$ recommended by us is equal to $\bar{R}^{\mathrm{exp}} = 0.41 \pm 0.01$. As seen in Fig. 5(d), the branching ratio obtained here and in Refs. [12,13] is in a good agreement with that recommended in Ref. [51] (0.43 ± 0.02) although the $S^{\mathrm{exp}}_{34}(E)$ obtained in Ref. [51] is underestimated. Such a good agreement between two of the experimental data for the $\bar{R}^{\mathrm{exp}}(E)$ can apparently be explained by the fact that there is a reduction factor in Ref. [51] overall for the ^{3}He$(\alpha,\gamma)^7$Be(g.s.) and ^{3}He$(\alpha,\gamma)^7$Be(0.429 MeV) astrophysical S factors. The present result for $\bar{R}^{\mathrm{exp}}$ is in excellent agreement with the value 0.43 [23,53] but is noticeably larger than 0.37 [19] and 0.32 ± 0.01 [54].

Thus, it follows that the application of the MTBPA to the considered reaction allows one, first, to reproduce both the observed energy dependence and the absolute normalization of the modern precisely measured astrophysical S factors $S^{\mathrm{exp}}_{34}(E)$ at energies of 92.9–1235 keV [9–14] within the experimental errors, and second, to do a correct extrapolation to low-energy, experimentally inaccessible regions, including $E = 0$.

III. CONCLUSION

The scrupulous analysis of the modern experimental astrophysical S factors $S^{\mathrm{exp}}_{34}(E)$ for the ^{3}He$(\alpha,\gamma)^7$Be reaction, which were precisely measured in Refs. [9–14] at energies $E = 92.9$–1203 keV, has been performed within the MTBPA [30]. It shows quantitatively that the reaction within the considered energy ranges is mainly peripheral, and a use of the parametrization of the direct astrophysical S factors in terms of ANCs is adequate for the peripheral reaction physics.

The values of the separated experimental astrophysical S factors for the ^{3}He$(\alpha,\gamma)^7$Be reaction populating the ground and first exited states of ^{7}Be are determined by using the total (unseparated) experimental astrophysical S factors from works [9–12] and [14] as well as the values of the ANCs for $^3\text{He} + \alpha \to {}^7\text{Be(g.s.)}$ and $^3\text{He} + \alpha \to {}^7\text{Be}(0.429\,\text{MeV})$ found for each of the corresponding experimental points of E. New estimates for the weighted means of the ANCs and NVCs are obtained, which have an overall uncertainty Δ_C about 7% on the average. The found values of ANCs made it possible also to get new information about the α-particle spectroscopic factors for the mirror (^{7}Li^{7}Be) pair.

The found ANCs were also used for an extrapolation of astrophysical S factors at energies less than 90 keV, including $E = 0$. In particular, the weighted mean of the total astrophysical S factor $S_{34}(0)$, $S_{34}(0) = 0.613^{+0.026}_{-0.063}$ keV b, and the branching ratio $\bar{R}^{\text{exp}}$, $\bar{R}^{\text{exp}} = 0.41 \pm 0.01$, recommended in the present work are in agreement with the values deduced in Refs. [7,10–14] from the analysis the same experimental astrophysical S factors. The overall uncertainty Δ_S in the $S_{34}(0)$ is about 7% on average. The result for $S_{34}(0)$ within the uncertainty is in an agreement with that of Ref. [17] obtained within the microscopic single-channel ($\alpha + {}^3$He) cluster model and with that of Ref. [20] obtained recently within an *ab initio* type calculation, but it is noticeably larger than the result of Refs. [23,24] obtained within the standard two-body ($\alpha + {}^3$He) potential by using the α-^{3}He potential deduced by a double-folding procedure.

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APPENDIX: BASIC FORMULAS

Here we present only the idea and the essential formulas of the MTBPA [30] specialized for the ^{3}He$(\alpha,\gamma)^7$Be astrophysical S factor that are important for the following analysis.

According to Ref. [30], for fixed l_f and j_f we can write the astrophysical S factor $S_{l_f j_f}(E)$ for the peripheral direct-capture ^{3}He$(\alpha,\gamma)^7$Be reaction in the form

$$S_{l_f j_f}(E) = C^2_{l_f j_f} \mathcal{R}_{l_f j_f}\big(E, C^{(sp)}_{l_f j_f}\big). \tag{A1}$$

Here, $C_{l_f j_f}$ is the ANC for $^3\text{He} + \alpha \to {}^7\text{Be}$, which determines the amplitude of the tail of the ^{7}Be nucleus bound-state wave function in the $(\alpha + {}^3\text{He})$ channel, and is related to the spectroscopic factor $Z_{l_f j_f}$ for the $(\alpha + {}^3\text{He})$ configuration with the quantum numbers l_f and j_f in the ^{7}Be nucleus by the equation $C_{l_f j_f} = Z^{1/2}_{l_f j_f} C^{(sp)}_{l_f j_f}$ [28], and $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f}) = S^{(sp)}_{l_f j_f}(E)/(C^{(sp)}_{l_f j_f})^2$, where $S^{(sp)}_{l_f j_f}(E)$ is the single-particle astrophysical S factor [8] and $C^{(sp)}_{l_f j_f}$ is the single-particle ANC. The latter determines the amplitude of the tail of the single-particle shell-model wave function of the bound $(\alpha + {}^3\text{He})$ state $\varphi_{l_f j_f}(r)$ $(\equiv \varphi_{l_f j_f}(r; C^{(sp)}_{l_f j_f})$ [39]) and in turn is itself a function of the geometric parameters (radius of R and diffuseness a) of the Woods-Saxon potential, i.e., $C^{(sp)}_{l_f j_f} \equiv C^{(sp)}_{l_f j_f}(R, a)$ [39]. In Eq. (A1), the ANCs $C^2_{l_f j_f}$ and the free parameter $C^{(sp)}_{l_f j_f}$ are unknown.

In order to make the dependence of the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function on $C^{(sp)}_{l_f j_f}$ more explicit, in the radial matrix element [30,31] in the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function we split the space of interaction into two parts separated by the channel radius r_c: the interior part $(0 \leqslant r \leqslant r_c)$, where nuclear forces between the α-^{3}He pair are important, and the exterior part $(r_c \leqslant r < \infty)$, where the interaction between the α particle and ^{3}He is governed by Coulomb forces only. The exterior part of the radial matrix element in the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function does not contain explicitly the free model parameter $C^{(sp)}_{l_f j_f}$, since for $r > r_c$ the wave function $\varphi_{l_f j_f}(r; C^{(sp)}_{l_f j_f})$ can be approximated by its asymptotic behavior [28]. Consequently, the parametrization of the astrophysical S factor in the form (A1) makes it possible to fix a contribution from the exterior region $(r_c \leqslant r < \infty)$, which is dominant for the peripheral reaction, in a model-independent way if the ANCs $C^2_{l_f j_f}$ are known. It follows from here that the contribution from the interior part of the radial matrix element to the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function, which depends on $C^{(sp)}_{l_f j_f}$ through the fraction $\varphi_{l_f j_f}(r; C^{(sp)}_{l_f j_f})/C^{(sp)}_{l_f j_f}$ [39,55], must exactly determine the dependence of the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function on $C^{(sp)}_{l_f j_f}$. Since for the peripheral ^{3}He$(\alpha,\gamma)^7$Be reaction at extremely low energies this contribution into the $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ function must strongly be suppressed [27], Eq. (A1) can be used for determination of the ANCs $C_{l_f j_f}$.

For this purpose, obviously the following additional requirements [30]

$$\mathcal{R}_{l_f j_f}\big(E, C^{(sp)}_{l_f j_f}\big) = f(E) \tag{A2}$$

and

$$C^2_{l_f j_f} = \frac{S_{l_f j_f}(E)}{\mathcal{R}_{l_f j_f}\big(E, C^{(sp)}_{l_f j_f}\big)} = \text{const} \tag{A3}$$

must be fulfilled as a function of the free parameter $C^{(sp)}_{l_f j_f}$ for each experimental energy point E in the range $E_{\min} \leqslant E \leqslant E_{\max}$ and values of the function of $\mathcal{R}_{l_f j_f}(E, C^{(sp)}_{l_f j_f})$ from (A2).

The fulfillment of the relations (A2) and (A3) or their violation within the experimental uncertainty for $S^{\text{exp}}_{l_f j_f}(E)$ enables one, first, to determine an interval for energies E where the dominance of extra nuclear capture occurs and, second, to obtain the value $(C^{\rm exp}_{l_f j_f})^2$ for ^{3}He + $\alpha \to {}^7$Be using the modern precisely measured astrophysical S factors $S^{\rm exp}_{l_f j_f}(E)$ [9–14] instead of $S_{l_f j_f}(E)$, i.e.,

$$(C^{\rm exp}_{l_f j_f})^2 = \frac{S^{\rm exp}_{l_f j_f}(E)}{\mathcal{R}_{l_f j_f}\big(E, C^{(sp)}_{l_f j_f}\big)}. \tag{A4}$$

Then, the value $(C^{\rm exp}_{l_f j_f})^2$ can be used for extrapolation of the astrophysical S factor $S_{l_f j_f}(E)$ to the region of experimentally inaccessible energies $0 \leqslant E < E_{\rm min}$ by using the obtained value $(C^{\rm exp}_{l_f j_f})^2$ in Eq. (A1).

The total astrophysical S factor for the ^{3}He$(\alpha,\gamma)^7$Be(g.s. + 0.429 MeV) reaction is given by

$$S_{34}(E) = \sum_{l_f=1;\, j_f=1/2,\,3/2} S_{l_f j_f}(E) \tag{A5}$$

$$= C^2_{1\;3/2}\big[\mathcal{R}_{1\;3/2}\big(E, C^{(sp)}_{1\;3/2}\big) + \lambda_C \mathcal{R}_{1\;1/2}\big(E, C^{(sp)}_{1\;1/2}\big)\big] \tag{A6}$$

$$= C^2_{1\;3/2}\mathcal{R}_{1\;3/2}\big(E, C^{(sp)}_{1\;3/2}\big)[1 + \bar{R}(E)], \tag{A7}$$

in which $\lambda_C = (C_{1\;1/2}/C_{1\;3/2})^2$ and $\bar{R}(E)$ is a branching ratio.

One notes that, in the two-body potential model, the ANC $C_{l_f j_f}$ is related to the NVC $G_{l_f j_f}$ for the virtual decay ^{7}Be $\to \alpha + {}^3$He by the equation [28]

$$G_{l_f j_f} = -i^{l_f + \eta_{^7{\rm Be}}} \frac{\sqrt{\pi}}{\mu_{\alpha\text{-}^3{\rm He}}} C_{l_f j_f}, \tag{A8}$$

where $\eta_{^7{\rm Be}}$ is the Coulomb parameter for the ^{7}Be$(\alpha + {}^3$He) bound state and $\mu_{\alpha\text{-}^3{\rm He}}$ is the reduced mass of the α particle and ^{3}He ion. In Eq. (A8), the combinatorial factor taking into account the nucleon's identity is absorbed in $C_{l_f j_f}$, and its numerical value depends on a specific model used for describing wave functions of the ^{3}He, α, and ^{7}Be nuclei [29]. Hence, the proportionality factor in Eq. (A8), which relates NVCs and ANCs, depends on the choice of nuclear model [29]. But, as noted in Ref. [29], the NVC $G_{l_f j_f}$ is a more fundamental quantity than the ANC $C_{l_f j_f}$, since the NVC is determined in a model-independent way as the residue of the partial S matrix of the elastic α-^{3}He scattering at the pole $E = -\varepsilon_{\alpha\text{-}^3{\rm He}}$ [$\varepsilon_{\alpha\text{-}^3{\rm He}}$ is the binding energy of the bound $(\alpha + {}^3$He) state of ^{7}Be] [28,29]. Therefore, it is also of interest to obtain information about values of the NVCs from Eqs. (A4) and (A8).

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