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New Geiger-Nuttall law for α decay of heavy nuclei

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Recent α-decay data of heavy nuclei are collected and systematic analysis shows that there is a sudden change between the logarithm of decay half-life and the reciprocal of the square root of decay energy across the $N = 126$ shell closure. In order to reproduce this sudden change, the new Geiger-Nuttall law is proposed where the effects of the quantum numbers of α-core relative motion are naturally embedded in the law. The remedy achieved by a very simple parametrization of these effects is remarkable. By adding terms to the Geiger-Nuttall law, the parameters in the formula of decay half-lives need not be changed, except for some odd nuclei. This is an important development to the original Geiger-Nuttall law, which is valid for the ground-state transitions of even-even nuclei with $N \geqslant 128$. The law is generalized to the favored and hindered transitions of the $N \leqslant 128$ nuclei and of high-spin isomers. The results of this article point to the simplicity of the underlying mechanism of the decay.

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I. INTRODUCTION

The Geiger-Nuttall law is a famous formula written in many textbooks of modern physics and nuclear physics. It states that there is a linear relationship between the logarithm of α-decay half-lives and the reciprocal of the square root of decay energies for ground-state transitions of even-even nuclei with $N \geqslant 128$ [1,2]. Although it was proposed a century ago, it is still widely used to systematize the data of α decay and to predict the half-lives of unknown nuclei. Based on the Geiger-Nuttall law and quantum tunneling theory, various formulas [3–8] and models [9–23] have been developed to calculate the decay half-lives and they are useful for current researches of α decay of heavy and superheavy nuclei [24,25]. One of the authors of this article and his collaborators have produced an extension of the original Geiger-Nuttall law to cluster decay other than α decay [26]. With the α-decay data of ground-state and high-spin isomers accumulating, it is interesting to see whether the relationship between half-lives and decay energies deviates systematically from the original Geiger-Nuttall law. By analyzing the behavior of systematic deviation, some quantum-mechanical effects can be observed. By including some quantum-number effects, it is also important to investigate whether this law can be generalized to hindered α decays with the change of parity between parent nuclei and daughter nuclei and to the α decays from high-spin isomers. These are the purposes of this article.

II. NUMERICAL RESULTS AND DISCUSSIONS

We start from a unified three-parameter formula between half-lives and decay energies of α decay and cluster radioactivity [8],

$$lg\, T_{1/2} = a\sqrt{\mu}\, Z_c Z_d/\sqrt{Q} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c. \quad (1)$$

Here the values of three parameters are $a = 0.39961$, $b = -1.31008$, and $c_{e\text{-}e} = -17.00698$ for even-even (e-e) nuclei [8]. $T_{1/2}(s)$ is the half-life of α decay and Q(MeV) is the corresponding decay energy. Z_c and Z_d are the charge numbers of the cluster and daughter nucleus. $\mu = A_c A_d/(A_c + A_d)$ is the reduced mass and A_c, A_d are the mass numbers of the cluster and daughter nucleus, respectively. For α decay, $Z_c = 2$ and $A_c = 4$. The three parameters, a, b, c, are obtained by fitting the data of even-even nuclei with $Z \geqslant 84$ and $N \geqslant 128$ [8].

This three-parameter formula is a natural realization of both the Geiger-Nuttall law and the famous Viola-Seaborg formula [3] toward the unified description of α decay and cluster radioactivity. When this formula is used to calculate the half-lives of ground-state transitions of even-even nuclei with N stepping over the $N = 126$ shell closure, a strong effect is observed. A dramatic deviation occurs between calculated half-lives and experimental ones for $N \leqslant 126$ nuclei on the isotopic chains of $Z = 84$–92. For even-even Po nuclei, it is plotted in Fig. 1 and the points denoted with original law correspond to the deviation between the experimental value and the value calculated with Eq. (1). The points denoted with new law are our results and will be explained later. In Fig. 1, the x axis is the mass number of parent nuclei and the y axis is the logarithm of the ratio between experimental half-lives and calculated values. It is seen from Fig. 1 that the values calculated with Eq. (1) (denoted by original law) agree well with the data of $N \geqslant 128$ but the agreement is very bad for $N \leqslant 126$ because the ratio between experimental value and the value calculated with the original law is beyond a factor of ten for some Po nuclei ($lg10 = 1$). These are also the common case for $Z = 86, 88, 90, 92$ isotopic chains and a similar figure for even-even Rn isotopes is drawn in Fig. 2. From Fig. 2, a similar deviation is observed between the results calculated with Eq. (1) (denoted by original law) and experimental half-lives. Therefore the abnormal deviation is systematic behavior for isotopic chains $Z = 84$–92. Why does this phenomenon happen when N passes across $N = 126$? What is the underlying physics behind it? At first we will make a general analysis based on quantum theory.

It is well known that α decay is a quantum tunneling phenomenon and there is no correspondence to this phenomenon in classical mechanics. Therefore the process of α decay can be described by quantum theory. The quantum motion of an α

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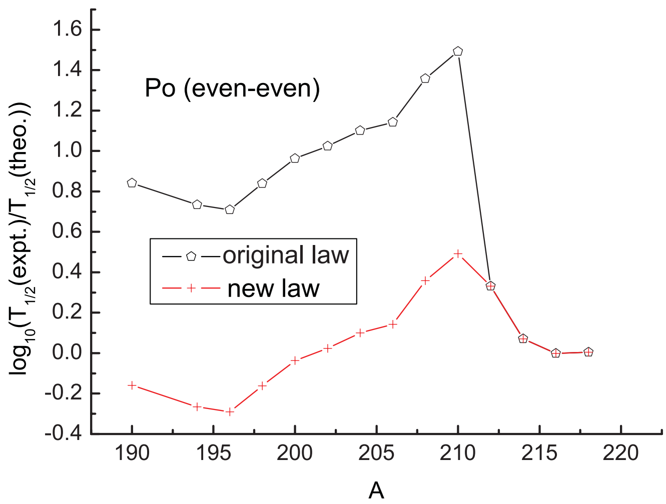

FIG. 1. (Color online) Logarithms of the ratios between experimental α-decay half-lives and theoretical ones for even-even Po nuclei with the original law [Eq. (1)] and with the new law [Eq. (3)]. The original law and new law go together in the range of $N \geqslant 128$.

cluster in nuclei is determined by a wave function that is the solution of the Schrödinger equation for a quasibound state [22]. However, a review of Eq. (1) clearly shows that the information of the wave function is not included in the original Geiger-Nuttall law or Eq. (1). This is not strange because the Geiger-Nuttall law was proposed before the quantum theory was founded. Although the complete information of the wave function can not be included in a simple formula such as Eq. (1), some basic observables such as quantum numbers can be absorbed in the formula for a better description of α-decay data.

Some important quantum numbers of an α cluster in spherical potentials are the global quantum numbers G, the radial quantum number n, the angular momentum l, and parity. Usually $G = 2n + l$ [9,11,20–22] is used for the motion of an α cluster in many models of α-decay half-lives where a parent nucleus is considered to be a quantum two-body system consisting of an α cluster and a daughter nucleus (a core). The meaning of the radial quantum number is the number of radial nodes of wave functions of the α cluster.

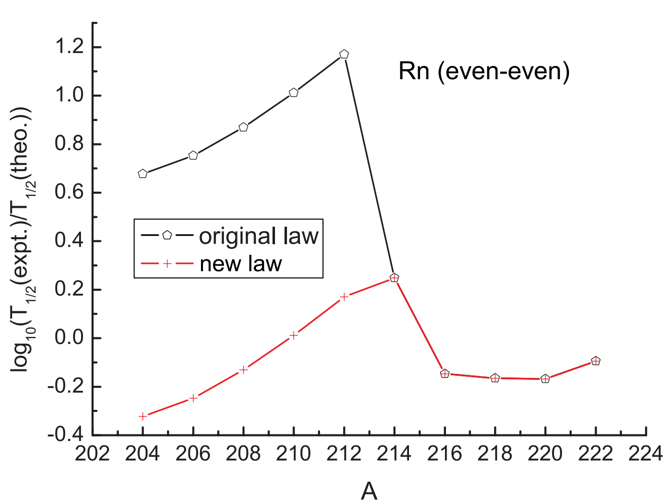

FIG. 2. (Color online) Logarithms of the ratios between experimental α-decay half-lives and theoretical ones for even-even Rn nuclei with the original law [Eq. (1)] and with the new law [Eq. (3)]. The original law and new law go together in the range of $N \geqslant 128$.

For ground-state transitions of even-even nuclei, the ground-state spin and parity of parent and daughter nuclei are both 0^+. In this case, the angular momentum and parity carrying by an α particle is also 0^+, and the radial quantum number n (or equivalently the global quantum number G) affects the decay half-lives. When the neutron number N goes across the shell closure at $N = 126$ (from $N \leqslant 126$ to $N \geqslant 128$ for even-even nuclei), it is expected that the change of the global quantum number is $\Delta G = 2$ [9,11,20–22] and it corresponds to $\Delta n = 1$. Therefore we introduce a quantum number $S = -\Delta G/2 = -\Delta n$ to mock up their effect on α-decay half-lives. We define the value of S for ground-state transitions of even-even nuclei: $S = 0$ for $N \geqslant 128$ and $S = 1$ for $N \leqslant 126$. This is consistent with the fact that the parameters in Eq. (1) are based on the fitting of the data with $N \geqslant 128$ even-even nuclei.

As a result, for the ground-state transitions of even-even nuclei, a new version of Eq. (1) or the Geiger-Nuttall law is

$$lgT_{1/2} = a\sqrt{\mu}\, Z_c Z_d/\sqrt{Q} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S. \qquad (2)$$

By including the possible effect of angular momentum and parity of α particle on half-lives of various nuclei, a general expression derived from quantum tunneling theory can be approximately written in the following way:

$$lgT_{1/2} = a\sqrt{\mu}\, Z_c Z_d/\sqrt{Q} + b\sqrt{\mu}\sqrt{Z_c Z_d} + c + S + P\, l(l+1), \qquad (3)$$

$S = 0$ for $N \geqslant 127$ and $S = 1$ for $N \leqslant 126$. We call this formula [Eq. (3)] the new Geiger-Nuttall law. The last term $l(l+1)$ can be approximately derived based on the quantum tunneling theory when the centrifugal potential is taken into account and this is similar to the derivation of Eq. (1) [8,23,27]. Here a, b, c have the same values as before [Eqs. (1) and (2)]. The value of P can be parity dependent. When the parity of the α-core relative motion is odd, the value of P will be larger, which will be discussed later.

With Eq. (3), we first calculate the half-lives of the ground state of even-even nuclei with $84 \leqslant Z \leqslant 92$ isotopic chains [in this case, Eq. (3) is naturally back to Eq. (2) as $l = 0$]. The numerical results of $Z = 84$–92 isotopic chains are listed in Table I and the deviations of the logarithm of half-lives for Po and Rn are drawn in Figs. 1 and 2, which are denoted as new law. It is seen from Fig. 1 that the systematic deviation between experimental half-lives and calculated ones decreases rapidly for $N \leqslant 126$ nuclei. Figures 1 and 2 show the same effect. This clearly shows that the introduction of S based on the quantum theory eliminates the discrepancy. The remedy achieved by a very simple parametrization of these effects is remarkable. A lot of numerical calculations with different potentials also show that the change of the global number $\Delta G = 2$ can lead to a change of $lgT_{1/2}$ with $\Delta lgT_{1/2} = 1$ [20–22]. They provide further support for the introduction S in Eqs. (2) and (3).

In Table I, the first column denotes the parent nucleus and the second column represents the α-decay energy of the

TABLE I. The logarithm of α-decay half lives of even-even $Z = 84 - 92$ isotopes calculated with new Geiger-Nuttall law ($lgT_{\rm theo}$) and the corresponding experimental ones ($lgT_{\rm expt}$). The experimental decay energies of nuclei [Q (MeV)] are also listed in the table.

Nuclei	Q (MeV)	$lgT_{\rm expt}(s)$	$lgT_{\rm theo}(s)$	Nuclei	Q (MeV)	$lgT_{\rm expt}(s)$	$lgT_{\rm theo}(s)$
^{218}Po	6.115	2.27	2.27	^{218}Ra	8.546	–4.59	–4.46
^{216}Po	6.906	–0.84	–0.84	^{216}Ra	9.526	–6.74	–6.93
^{214}Po	7.833	–3.78	–3.86	^{214}Ra	7.273	0.39	0.45
^{212}Po	8.954	–6.52	–6.86	^{206}Ra	7.415	–0.62	–0.05
^{210}Po	5.407	7.08	6.59	^{204}Ra	7.636	–1.22	–0.78
^{208}Po	5.215	7.96	7.60	^{202}Ra	8.020	–2.58	–1.97
^{206}Po	5.327	7.14	7.00	^{232}Th	4.082	17.65	17.56
^{204}Po	5.485	6.28	6.18	^{230}Th	4.770	12.38	12.38
^{202}Po	5.701	5.15	5.12	^{228}Th	5.520	7.78	7.88
^{200}Po	5.981	3.79	3.83	^{226}Th	6.451	3.26	3.43
^{198}Po	6.309	2.27	2.43	^{224}Th	7.298	0.02	0.15
^{196}Po	6.657	0.77	1.06	^{222}Th	8.127	–2.69	–2.56
^{194}Po	6.987	–0.41	–0.14	^{220}Th	8.953	–5.01	–4.87
^{190}Po	7.693	–2.61	–2.45	^{218}Th	9.849	–6.96	–7.04
^{222}Rn	5.590	5.52	5.61	^{216}Th	8.071	–1.57	–1.39
^{220}Rn	6.405	1.75	1.91	^{214}Th	7.826	–1.00	–0.63
^{218}Rn	7.263	–1.46	–1.29	^{212}Th	7.952	–1.44	–1.03
^{216}Rn	8.200	–4.35	–4.20	^{238}U	4.270	17.15	17.17
^{214}Rn	9.208	–6.57	–6.82	^{236}U	4.573	14.87	14.84
^{212}Rn	6.385	3.16	2.99	^{234}U	4.858	12.89	12.85
^{210}Rn	6.159	3.95	3.94	^{232}U	5.414	9.34	9.44
^{208}Rn	6.261	3.37	3.50	^{230}U	5.993	6.25	6.40
^{206}Rn	6.384	2.74	2.99	^{228}U	6.803	2.74	2.82
^{204}Rn	6.546	2.01	2.33	^{226}U	7.701	–0.57	–0.47
^{226}Ra	4.871	10.70	10.67	^{224}U	8.620	–3.03	–3.29
^{224}Ra	5.789	5.50	5.56	^{222}U	9.500	–5.85	–5.59
^{222}Ra	6.679	1.58	1.65	^{218}U	8.786	–2.22	–2.75
^{220}Ra	7.592	–1.75	–1.62				

nucleus. The third column and the fourth column represent the logarithm of experimental half-life and the logarithm of theoretical half-life, respectively. The experimental data are from the nuclear mass table by Audi *et al.* [28,29]. In the table, some nuclei are missing on an isotopic chain because there are no α-decay data on them or some data are uncertain by a symbol (?) when Audi and his collaborators edit the table of nuclear properties [28,29]. Columns 5–8 have similar meanings as those of columns 1–4.

For many even-even nuclei on $Z = 84$–92 chains, the calculated values agree with the experimental data within a factor of 2, which corresponds to a deviation of the logarithm of half-life with a value 0.3 ($lg2 \approx 0.3$). Only for a few nuclei such as ^{210}Po and ^{218}U, the ratio between the experimental half-life and the theoretical one is approximately a factor of 3 ($lg3.2 \approx 0.5$). This shows again the reliability of the new Geiger-Nuttall law [Eq. (3)] for half-lives of α decay.

When we extend the calculations with Eq. (3) to ground-state transitions of even-even nuclei of other mass region such as $Z = 60$–74, we find that the calculated half-lives are in good agreement with the experimental data. The numerical results of some isotopes ($Z = 60$–74) are listed in Table II. It confirms again that the introduction of S in Eq. (2) or Eq. (3) brings the expected result. We would like to mention that the introduction of the terms to the Geiger-Nuttall law is not only valid for Eq. (3), but also useful for other similar formulas of α-decay half-lives such as the Viola-Seaborg formula. By adding terms to the law, the parameters need not be changed, except for some odd nuclei.

After we successfully reproduce the ground-state half-lives of even-even nuclei by the new Geiger-Nuttall law, it is interesting to investigate the case of odd-A nuclei and the case of isomers. It is well known that the situations of odd-A and odd-odd nuclei are very complicated. Usually the half-lives of odd nuclei can be calculated with a similar formula with that of even-even nuclei but the parameter c in Eq. (1) will be different from that of even-even nuclei [8]. For example, $c_{e\text{-}o} = -16.40484$ is used for even-Z and odd-A nuclei [8]. But if the changes of quantum numbers are disregarded in the calculations for odd nuclei, the deviation between calculated and experimental half-lives can be as large as a factor of 10–1000 in some cases. In this article, we try to treat the transitions of odd nuclei in different ways. For the favored ground-state transitions of odd-A nuclei, we calculate their half-lives in exactly the same way as even-even nuclei because there is no change of spin and parity when parent nuclei and daughter nuclei have same spin and parity (this is a dominant branch in α decay). For the transitions of unfavored ones, the change of spin and parity between parent and daughter nuclei can be included by the last term of Eq. (3) and the last term will

TABLE II. The logarithm of α-decay half-lives of even-even $Z = 60$–74 isotopes calculated with new Geiger-Nuttall law ($lgT_{\rm theo}$) and the corresponding experimental ones ($lgT_{\rm expt}$). The experimental decay energies of nuclei [Q (MeV)] are also listed in the table.

Nuclei	Q (MeV)	$lgT_{\rm expt}(s)$	$lgT_{\rm theo}(s)$
^{168}W	4.506	6.20	6.50
^{166}W	4.856	4.74	4.53
^{164}W	5.2785	2.22	2.41
^{162}W	5.6773	0.48	0.64
^{160}W	6.065	–0.99	–0.91
^{158}W	6.613	–2.86	–2.86
^{162}Hf	4.417	5.69	5.95
^{160}Hf	4.9024	3.29	3.28
^{158}Hf	5.4047	0.80	0.90
^{156}Hf	6.028	–1.62	–1.62
^{158}Yb	4.172	6.63	6.36
^{156}Yb	4.811	2.42	2.75
^{154}Yb	5.4742	–0.35	–0.31
^{156}Er	3.487	9.84	10.04
^{154}Er	4.2799	4.68	4.61
^{152}Er	4.9344	1.06	1.15
^{154}Dy	2.946	13.98	13.56
^{152}Dy	3.726	6.93	7.04
^{150}Dy	4.3513	3.08	3.13
^{152}Gd	2.203	21.53	21.09
^{150}Gd	2.808	13.75	13.56
^{148}Gd	3.27121	9.37	9.26
^{148}Sm	1.9861	23.34	22.81
^{146}Sm	2.5284	15.51	15.17
^{144}Nd	1.9052	22.86	22.40

be smaller when the change of angular momentum is smaller such as the cases of $l = 1, 2$. The last term will be important for some hindered transitions from high-spin isomers and for the transition with large change of spin and with the change of parity. These ideas are based on the experimental facts and quantum theory.

In Fig. 3, we draw the deviation between calculated ones and experimental data for favored transitions of odd-A Po isotopes where the meaning of Fig. 3 is similar to those of Figs. 1 and 2. In Fig. 3, the points denoted with original law correspond to the calculations with Eq. (1) and the points denoted by new law are those calculated with Eq. (2). A phenomenon similar to Figs. 1 and 2 is observed and this supports our idea to treat favored transitions of odd-nuclei in exactly the same way as even-even nuclei. The numerical results of half-lives of odd-A Po nuclei with the new law [Eq. (2) or Eq. (3)] are listed in column 4 of Table III and we also list the numerical results by the original formula of Ref. [8] in column 5 for comparison. It is seen that the results with the new law agree with experimental data better. This shows again that the treatment in this article is an improvement over the traditional way. Finally, it is mentioned that the data of ^{211}Po is not included in Fig. 3 because it is a hindered transition and it will be studied together with other hindered transitions as follows.

Hindered transitions are very complicated, but we show that they can also be treated by our simple parametrization. The examples to be shown are very illuminating. They contain

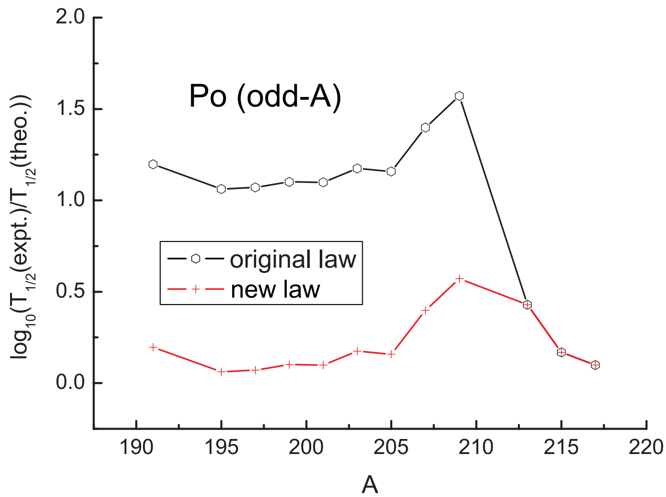

FIG. 3. (Color online) Logarithms of the ratios between experimental α-decay half-lives and theoretical ones for odd-A Po nuclei with the original law and with the new law. The original law and new law go together in the range of $N \geqslant 129$.

isomeric transitions from an 18^+ state and from an 8^+ state of the same nucleus, ^{212}Po, which are reproduced just as the transition from the 0^+ ground state [28–30]. The examples include the isotonic sequence $N = 127$ with $l = 5$ and odd parity [28–30]. The original formula, Eq. (1), fails for the complicated cases, but, with the $l(l+1)$ term, the new formula proves to be successful. The numerical results of half-lives with the new law [Eq. (3)] are listed in Table IV where the angular momentum and parity of parent nuclei and daughter nuclei and the angular momentum of the α-particle are also listed in the Table. For the calculations of the isomers of ^{212}Po, the value of P in Eq. (3) is 0.04143. For the $N = 127$ isotones, the value of P in Eq. (3) is 0.0840 and this value is large because there is a change of parity. Table IV shows that the calculated half-lives agree with experiment within a factor of two.

TABLE III. The logarithm of α-decay half-lives of odd-A Po isotopes calculated with new Geiger-Nuttall law ($lgT_{\rm theo1}$)(column 4) and the original formula in Ref. [8] ($lgT_{\rm theo2}$) (column 5) where favored transitions ($\delta l = 0$) are assumed. The corresponding experimental values ($lgT_{\rm expt}$) and decay energies of nuclei [Q (MeV)] are also listed in the table.

Nuclei	Q (MeV)	$lgT_{\rm expt}(s)$	$lgT_{\rm theo1}(s)$	$lgT_{\rm theo2}(s)$
^{217}Po	6.660	0.17	0.07	0.82
^{215}Po	7.526	–2.75	–2.92	–2.17
^{213}Po	8.536	–5.38	–5.81	–5.06
^{209}Po	4.979	9.51	8.94	8.68
^{207}Po	5.216	8.00	7.60	7.35
^{205}Po	5.324	7.17	7.02	6.76
^{203}Po	5.496	6.30	6.13	5.87
^{201}Po	5.799	4.76	4.66	4.41
^{199}Po	6.074	3.64	3.54	3.17
^{197}Po	6.412	2.09	2.02	1.76
^{195}Po	6.746	0.79	0.73	0.48
^{191}Po	7.501	–1.66	–1.85	–2.11

TABLE IV. The α-decay half-lives of two kinds of hindered transitions with new law: the decays from the high-spin isomers of $^{212}Po^{m1,m2}$ and those from $N = 127$ isotones. The spin and parity of parent nuclei (initial state) and daughter nuclei (final state) and the angular momentum of α particle are also listed in this table. In the calculation, $P = 0.04143$ is used for decays from isomers (without a change of parity) and $P = 0.0840$ is used for decays from $N = 127$ isotones (with a change of parity).

^AZ	^AZ	I_i	I_f	L_α	Q_α (MeV)	$T_{expt}(s)$	$T_{theo}(s)$
$^{212}Po^{m2}$	^{208}Pb	18^+	0^+	18	11.884	45.13	38.60
$^{212}Po^{m1}$	^{208}Pb	8^+	0^+	8	10.431	4.07×10^{-8}	8.69×10^{-8}
^{211}Po	^{207}Pb	$9/2^+$	$1/2^-$	5	7.595	0.516	0.243
^{213}Rn	^{209}Po	$9/2^+$	$1/2^-$	5	8.243	0.0195	0.0157
^{215}Ra	^{211}Rn	$9/2^+$	$1/2^-$	5	8.864	1.55×10^{-3}	1.63×10^{-3}
^{217}Th	^{213}Ra	$9/2^+$	$1/2^-$	5	9.433	2.40×10^{-4}	2.81×10^{-4}
^{219}U	^{215}Th	$9/2^+$	$1/2^-$	5	9.860	5.5×10^{-5}	11.9×10^{-5}

III. SUMMARY

In summary, the new Geiger-Nuttall law for the calculations of α-decay half-lives is proposed where the effects of quantum numbers are naturally taken into account. This approach includes the change of the node number of the wave function of the α-core relative motion in the Geiger-Nuttall law. By including the change of quantum numbers, the available data of α-decay half-lives of ground-state transitions in even-even nuclei both $N \leqslant 126$ and $N \geqslant 128$ are well reproduced. The inclusion of the term depending on the parity and angular momentum leads to a reliable description of the hindered decays from $N = 127$ odd-A isotones and from the high-spin (high-j) isomers of ^{212}Po where the value of j can be as high as 18 for $^{212}Po^{m2}$. The existence of these terms is based on the quantum theory for the microscopic description of the α-core relative motion. By adding terms to the Geiger-Nuttall law, the parameters in the formula of decay half-lives need not be changed, except for some odd nuclei. The terms are also useful for better description of α-decay half-lives with other similar formulas. Therefore the new Geiger-Nuttall law is an important development to the original Geiger-Nuttall law or the Viola-Seaborg formula based on quantum theory and it is an extension of the original Geiger-Nuttall law to α decay of a broader range of nuclei and transitions. It is expected that the extension is useful for similar decay processes such as cluster radioactivity and proton emissions. It can be helpful to systematize experimental data of α decay and to extract information of angular momentum and parity of nuclei from experimental data. The results of this article point to the simplicity of the underlying mechanism of the decay.

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