Properties of the lowest $1/2^+$, $T = 3/2$ states in $A = 11$ nuclei

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Abstract. Analysis of energies and widths of the lowest $1/2^+$ $T = 3/2$ states in $A = 11$ nuclei suggests that the excitation energy in ^{11}C should be about 200 keV below the energy in the literature, and the width should be 4 to 5 times the literature value. Properties of the state in ^{11}B and ^{11}N are in agreement with the present model.

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I. INTRODUCTION

The lowest $T = 3/2$ states in ^{11}B and ^{11}C [1] have been a puzzle for a long time. In a study of the low-lying levels of the $A = 11$ isospin quartet [2], we found that the $1/2^-$ and $5/2^+$ states behaved appropriately in the four nuclei, but there was a problem for the $1/2^+$ states. For the ground state (g.s.) of ^{11}N, the various experimental determinations of its energy and width did not agree within the assigned uncertainties. And, for the known (^{11}B) and supposed (^{11}C) $1/2^+$ states the experimental widths were only 1/3 (or less) of the values expected. Barker disagreed [3], arguing that these states could have lost width by mixing with $T = 1/2$ states—ignoring the fact that if these states lost width by mixing, then some nearby states would have acquired this missing width. No such states are known.

We have previously used a simple potential model [4] to compute energies of 0^+, $T = 2$ states using a nuclear plus Coulomb potential to couple states in nuclei $A - 1$ to single nucleons to produce the $T = 2$ nuclei A. The model has worked reasonably well. The present situation for $A = 11$, 12 is similar to that for $A = 15$, 16 [5]. In the latter, the 0^+, $T = 2$ state was not known in ^{16}F [6], and the $1/2^+$, $T = 3/2$ state was unknown in ^{15}O [7]. Also, the energy of the $1/2^+$ g.s. of ^{15}F was poorly defined because of its large width [8,9]. For $A = 11$, 12 the 0^+, $T = 2$ state in ^{12}N has not been identified [1], and the $1/2^+$, $T = 3/2$ state in ^{11}C is questionable. Similar to ^{15}F, the g.s. of ^{11}N [10,11] is too wide to provide a precise energy for it.

For $A = 15$, 16, we were able to use the known masses and relationships among the masses in our model to put constraints on the unknown energies and on the percentage of s^2 component in the 0^+, $T = 2$ state (assumed equal for all five $T = 2$, $A = 16$ nuclei) [12]. That procedure also provided "best" values for the energy of the g.s. of ^{15}F. Here, we have attempted to apply that technique to $A = 11$, 12.

II. ^{11}B

The best evidence for the lowest $T = 3/2$ state in ^{11}B comes from the ^{10}Be(p,γ) reaction [13]. Those authors found a $T = 3/2$ state with $J^\pi = 1/2^+$ or $(3/2^+)$ at an excitation energy of 12.55(3) MeV and with a width of 230(65) keV. This width persisted in the compilations for more than 30 years [1], until those data were refit [14] and it was found that the data required a broad peak in order to explain the cross section. And this width could come only from the $1/2^+$, $T = 3/2$ state. The resulting excitation energy and width were 12.61(5) MeV and 640(33) keV, respectively, rather than the 210(20) keV width listed in the compilations [1]. Alternative fits with various assumptions (e.g., energy-dependent width vs constant width, four states vs three) gave widths of about 730 and 700(100) keV. Barker later refit the (p,γ) data and provided a width "of order 600 keV" [15]. So, the ^{11}B puzzle was solved, but the ^{11}C problem remained.

III. ^{11}C

Here, a state at $E_x = 12.16(4)$ MeV has been assigned $T = 3/2$ in several reactions [1], but J^π has never been assigned. But, of the known states, it is the only candidate to be the required $1/2^+$ state. This state was observed in the reactions ^{11}B(^{3}He,t), ^{9}Be(^{3}He,n), and in ^{10}B(p,p') resonance inelastic scattering [16]. In the latter the width is all for decay to the 0^+, $T = 1$ state of ^{10}B. The (^{3}He,t) data are especially compelling because they were compared to results of the inelastic reaction ^{11}B(^{3}He, ^{3}He$'$) leading to the ^{11}B state discussed above. All these reactions found a small width (as did the inelastic reaction for the ^{11}B state).

Earlier [4] we found that the value of α^2 (the s^2 component) in ^{12}O(g.s.) needed to explain its Coulomb energy was 53(3)%. And, as noted above, our model assumes this component is the same in the five $A = 12$ nuclei. Here, we present our results for various values of this parameter. We use the symbol of a nucleus to represent the mass excess of that nucleus, and an asterisk to denote the lowest $T = 3/2$ state in a $T_z = \pm 1/2$ nucleus. We define Δ_B and Δ_C so that ^{11}B* = ^{11}B* (Ref. [1]) + Δ_B, ^{11}C* = ^{11}C* (Ref. [1]) + Δ_C. As noted above, refitting the ^{10}Be(p,γ) data provided $\Delta_B = 50(50)$ keV [14]—a small correction.

The ^{11}B* and ^{11}C* masses are needed as input to compute the energy of the lowest 0^+, $T = 2$ state of ^{12}C. If we require that the model fits this energy exactly within the uncertainties, we arrive at a constraint connecting Δ_B, Δ_C, and α^2 represented in Fig. 1. We have temporarily suppressed the uncertainties in the figure, but we return to them shortly. First, we note that the small correction Δ_B from the (p,γ) refit [14] (horizontal dashed lines) is consistent with a wide range of values of α^2. Secondly, the required value of Δ_C is negative; that is, the "best" excitation energy in ^{11}C is below the one in the compilation [1]. For α^2 in the previously mentioned

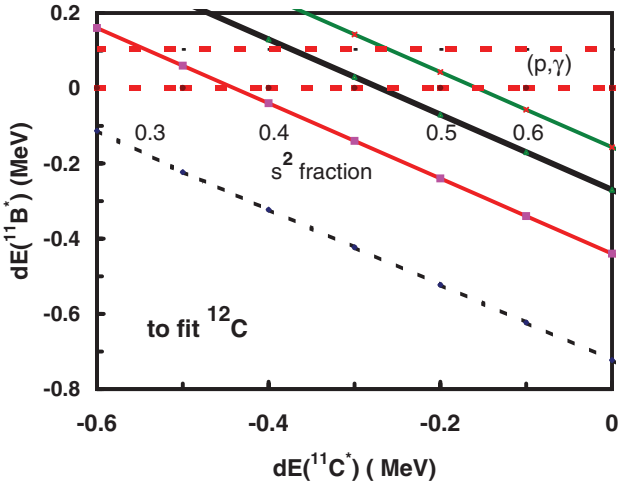

FIG. 1. (Color online) Plot of energy correction in ^{11}B vs the correction in ^{11}C needed to fit the 0^+, $T = 2$ energy in ^{12}C for various values of α^2, the s^2 fraction in the 0^+ state. Horizontal dashed lines represent the 50(50) keV ^{11}B correction from Ref. [14].

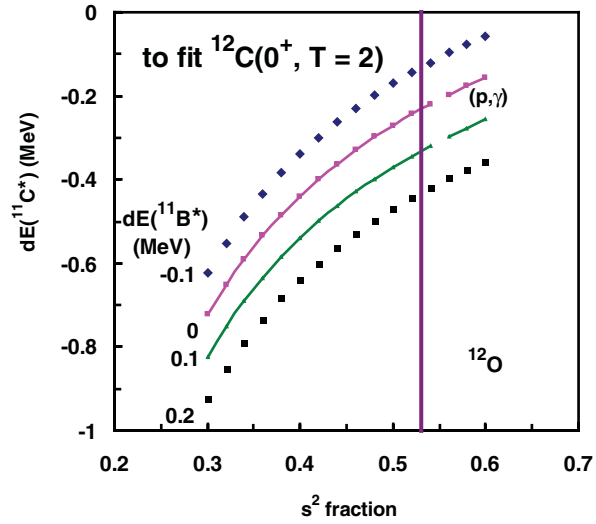

FIG. 2. (Color online) Same information as in Fig. 1, but plot of the ^{11}C correction vs α^2, for various ^{11}B corrections. Vertical line at $\alpha^2 = 0.53$ is from Ref. [4].

range, the value of Δ_C is about -0.23 MeV. The uncertainty in this value can perhaps be seen better in Fig. 2, where we replot this constraint differently. Here we plot Δ_C vs α^2 for various values of Δ_B. Recall that the earlier estimate for Δ_B is 0 to 0.1 MeV [14]. The vertical line at $\alpha^2 = 0.53$ is the value required to fit the ^{12}O Coulomb energy. The best-fit value for Δ_C from this analysis is $\Delta_C = -0.27(10)$ MeV, where the uncertainty contains contributions from uncertainties in the various energies and in the value of α^2. The result for Δ_C is negative, but with a disappointingly large uncertainty. Smaller uncertainties in the relevant excitation energies would be a great help.

IV. ^{11}N

For ^{11}N(g.s.), the experimental $p + {}^{10}$C resonance energies [10,11,17–20] cover the range from 1.27 to 1.63 MeV, with widths ranging from 0.24(24) to 1.44(2) MeV. Theoretical resonance energies [4,21–23] span a similar range, but calculated widths are all about 0.8 MeV or larger. These are summarized in Table I and Figs. 3 and 4. In Ref. [21], the large uncertainty in the predicted width comes largely from the uncertainty in predicted energy. We also list three "averages" of the experimental results. The most recent $A = 11$ compilation [20] averaged results of the three experiments with the best resolution to get $E_p = 1.49(6)$ MeV, $\Gamma = 0.83(3)$ MeV. The mass evaluation [24] has an average of $E_p = 1.315(46)$ MeV. If we average all five experimental values, the results are $E_p = 1.41(10)$ MeV, $\Gamma = 0.78(11)$ MeV. Our predictions [4] were 1.35(7) and 0.87(10) MeV, respectively.

So far here, we have not made use of the isobaric multiplet mass equation (IMME). If we use the uncorrected energies for $A = 11$, we can compute the value of d—the coefficient of a possible cubic term in the IMME. For $A = 12$, $T = 2$, the result was $d = -8.4(17)$ keV. Thus, those masses do not require a nonzero value for d. With $d = 0$ in $A = 11$, $T = 3/2$, the masses obey a simple relation: ^{11}N = ^{11}Be -3 ^{11}B* + 3 ^{11}C*. The mass tables [24] list a ^{11}N mass

TABLE I. Resonance energies and widths (both in MeV) of ^{11}N(g.s.).

	Label	Method	E_r	Γ	Ref.
Expt.	1	$p + {}^{10}$C elastic	1.30(4)	$0.99^{+0.10}_{-0.20}$	[17]
	2	$p + {}^{10}$C elastic	$1.27^{+0.18}_{-0.05}$	1.44(2)	[10]
	3	^{10}B(^{14}N,^{13}B)	1.63(5)	0.4(1)	[18]
	4	^{14}N(^{3}He,^{6}He)	1.31(5)	0.24(24)	[19]
	5	$p + {}^{10}$C elastic	1.54(2)	0.83(3)	[11]
Ave.	7	Compilation	1.49(6)	0.83(3)	[20]
	8	Mass evaluation	1.315(46)		[24]
	9	Present	1.41(10)	0.78(11)	Present
Calc.	11	Mirror of ^{11}Be	1.35(7)	0.87(10)	4
	12	Mirror of ^{11}Be	1.60(22)	$1.58^{+0.75}_{-0.52}$	21
	13	Mirror of ^{11}Be	1.2	1.1	22
	14	Mirror of ^{11}Be	1.34	1.47	23

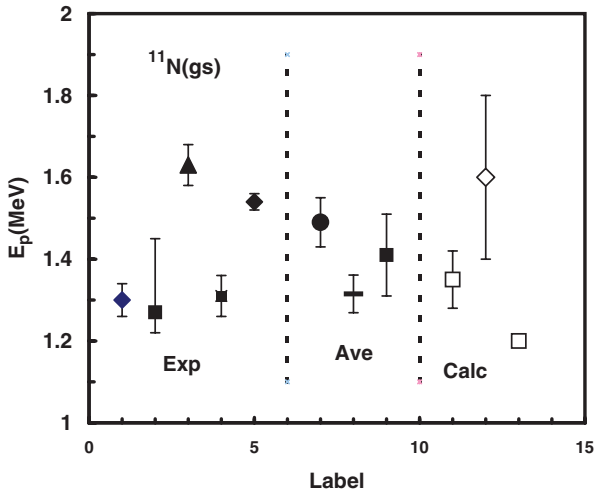

FIG. 3. Resonance energies for ^{11}N(g.s.) from various sources. Labels are as in Table I.

excess that translates to $E_p = 1.312(50)$ MeV. With a ^{10}C mass excess [24] of 15.699 MeV, and our definition of Δ_C, the IMME provides $E_p[^{11}\text{N(g.s.)}] = 1.79(19)$ MeV $+ 3\Delta_C$. (Without the ^{11}B correction, this value was 1.94(13) MeV.) Without Δ_C this value is far higher than any previous values for ^{11}N(g.s.), although with a large uncertainty. Still, this is some confirmation of the need for a nonzero, negative, value of Δ_C.

Recall from above that the IMME, with $d = 0$, requires $E_p\ [^{11}\text{N(g.s.)}] = 1.79(19)$ MeV $+ 3\Delta_c$. The three averages in Table I for ^{11}N would then yield $\Delta_c = -0.10(7)$ to $-0.16(7)$ MeV—smaller (in absolute value) than, but approximately consistent with, the other analysis presented in Sec. III above. These two proposed energy corrections for ^{11}C are summarized in Table II.

We make no further use of the IMME, but we do note that our model automatically satisfies the IMME with $d = 0$. The recent correction to the ^{11}Be(g.s.) mass [25] is too small to have a noticeable effect on the energies discussed here.

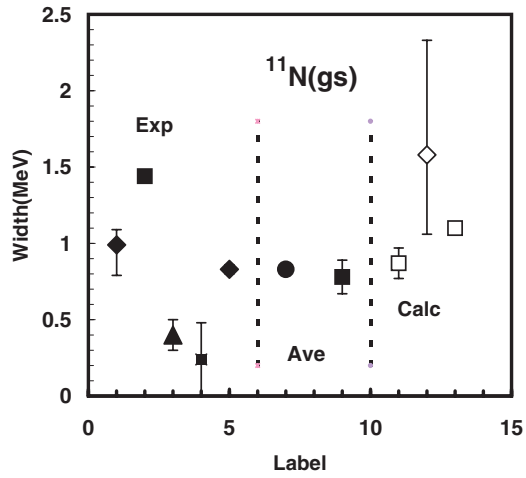

FIG. 4. As Fig. 3, but for the widths.

TABLE II. Proposed energy correction in ^{11}C.

Fit using	Δ_C (MeV)
^{11}Be, ^{11}B*; ^{12}Be, ^{12}C**, ^{12}O	−0.27(10)
IMME: ^{11}Be, ^{11}B*, ^{11}N	−0.13(7)
Weighted average	−0.18(6)

V. DISCUSSION

The spectroscopic factors for the four $1/2^+$, $T = 3/2$ states are listed in Table III. For all but ^{11}Be, these are obtained from the expression $C^2S = \Gamma_{\text{expt}}/\Gamma_{sp}$, where $C^2 = 1/3$, $2/3$, and 1 for ^{11}B, ^{11}C, and ^{11}N, respectively. Estimates of Γ_{sp} are listed in the table. They were calculated using a Woods-Saxon potential (plus Coulomb), with $r_0 = 1.25$ fm and $a = 0.65$ fm. The depths were adjusted to reproduce the observed energies.

The difficulty with ^{11}C is apparent. The spectroscopic factor derived from its width is only about 20% of S for the other three nuclei—and the S's should all be equal. As pointed out above, if the ^{11}C state loses width (spectroscopic strength) by mixing with $T = 1/2$ states, then one or more of them should exhibit this strength, and none do. We recall that the $1/2^+$, $T = 3/2$ state in ^{15}O has also never been identified. An early candidate turned out to have $T = 1/2$, as demonstrated by its large width for a decay that would be forbidden for a $T = 3/2$ state.

The problem in ^{11}C is not with the sp widths. Barker [3] used a potential model to compute Γ_{sp} for states at the experimental energies. His values (last column of Table III) are similar to ours. For ^{11}C, his sp width is actually 12% larger than ours. We thus expect a $1/2^+$, $T = 3/2$ state near 12 MeV in ^{11}C, with a width of about 1.2 MeV and $C^2S_p \sim 0.50$.

We have given considerable thought to finding a reaction to make these states in ^{11}C (and ^{15}O). The (p,t) reaction does not work, because the targets do not contain the $2s_{1/2}$ nucleon that is the main feature of these states. The (^{3}He,t) reaction populates both $T = 1/2$ and $3/2$ states, as does (^{3}He,n). Finding a state at roughly the expected energy that preferentially decays to the 0^+, $T = 1$ state of ^{10}B in the ^{10}B(p,p') reaction was encouraging, but the width reported there is also too small by about a factor of five. In the (^{3}He,n) reaction the background (both real and from $T = 1/2$ states) is a serious problem. This reaction does have the advantage that cross-section ratios for different $T = 3/2$ final states should be approximately the same in (^{3}He,n) and (t,p) on the same target and under similar kinematic conditions. Thus, for a ^{9}Be target, we expect the ratio $\sigma(1/2^+)/\sigma\ (1/2^-)$ in (^{3}He,n) to be roughly equal to the same ratio in (t,p). In the latter, the ratio at the peak angle and the ratio of angle-integrated cross sections were both about 0.22. The best candidate might be ^{10}C(d,p) in reverse kinematics. In that reaction, C^2S_n would be about 0.25.

There is one last possibility to consider—could interference between overlapping $T = 1/2$ and $T = 3/2$ states cause a broad negative dip? If so, the void between 11.44 and 12.16 MeV in ^{11}C could actually be the negative profile of the $1/2^+$,

TABLE III. Widths (in MeV) and spectroscopic factors for $1/2^+$, $T = 3/2$ states in $A = 11$ nuclei.

Nuclei	E_x (MeV)	Γ	Γ_{sp}	S	Γ_{sp} (Ref. [3])
^{11}Be	0			0.80[e]	
^{11}B	12.61(5)[a]	0.640(33)[a]	∼2.4	0.80	2.31
^{11}C	12.16(4)[b]	0.27(5)[b]	∼2.4	0.16	2.69
^{11}N	0[c]	0.83(3)[d]	∼1.3	∼0.64	1.42

[a]Includes the correction from Ref. [14].
[b]Ref. [1].
[c]$E_p = 1.32$ to 1.49 MeV (averages in Table I).
[d]Average in Table I.
[e]As averaged in Ref. [4].

$T = 3/2$ state. But, would this interference be about the same in, say, (^{3}He,t) and (p,p')?

VI. CONCLUSION

We have noted here that the global averages of the energy and width of ^{11}N(g.s.) are consistent with the calculations. We conclude that the small correction found earlier [14] for the energy of the $1/2^+$, $T = 3/2$ state in ^{11}B is consistent with the current analysis, and that the previous problem [2] with the width in ^{11}B has been solved [14]. A larger, negative energy correction [180(60) keV] is needed for ^{11}C. That finding presents a problem, because in ^{11}C there is nothing between 11.44 and 12.16 MeV. The width in ^{11}C should be 4 to 5 times the currently accepted value. From inspection of the relevant spectra, it is difficult to see how the $1/2^+$ width could be several times the estimate in the compilation [1], especially if the energy is shifted lower. Of course, inspection of the spectra also provided a small width for ^{11}B*, which we now know was a factor of three too small. It would be very useful to find a way to settle this width question in ^{11}C.

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