Autocorrelation studies for the first 2^+ nuclear energy levels

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The autocorrelation function, and the correlation time are calculated for a series of the first 2^+ states of 466 even-even nuclei classified by different parameters related to nuclear structure. The results indicate enhanced values for the P factor compared with other classifying parameters to cumulate nuclei having similar properties nearby each other.

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Introduction. The introduction of random matrix theory (RMT) to describe the fluctuation properties of nuclear energy states has gained a considerable interest with many physicists [1,2]. Bohigas *et al*. [3] conjectured that RMT could be used to model the fluctuation properties of quantum systems which are fully chaotic in the classical limit. RMT provides analytical results for the spacing distributions, the level number variance, and the spectral rigidity of quantum systems. Comparing these expressions with the corresponding experimental results has been used as a signature of chaotic dynamics [4–6]. It has been realized that the nuclear dynamics interpolates between the Poisson (integrable) and Wigner (chaotic) distributions. The analysis of spectra of nuclei in the ground-state domain shows intermediate behavior [7–17].

Parameterizing isotopes of the nuclear chart according to their collective behavior in a single control parameter is an interesting field of research. Many parameters are introduced to take such a role, for example, the number of protons Z, number of neutrons N, mass number A, quadrupole deformation parameter β_2, $R_{4/2}$ ratio, and P factor.

The aim of the present work is to answer the following question: which of the previously mentioned parameters can be used to describe the collective behavior of nuclei best? In other words, we search for the parameter that is capable of arranging nuclei of similar collective behavior nearby each other.

In order to achieve this aim, we select even-even nuclei for their abundance and simplicity to be described by nuclear models. For each nucleus the first 2^+ energy state is taken—most even-even nuclei have 2^+ levels as the first excited state. In this way, we collect a series of 466 even-even nuclei ranging between $10 \leqslant A \leqslant 256$, are taken from National Nuclear Data Center [18] up to August 2010.

It is well known that—neglecting the shell closure effect—light nuclei having higher excitation energies of the first 2^+ levels on the order of a few MeV. On the other hand, as the mass number A increases, the 2^+ excitation energies decrease to be on the order of several tenths of a KeV. Consequently, in general the first 2^+ level energy of each nucleus can be used as an indicator of collective behavior.

A recent approach proposed by Relaño *et al*. [19] considers the spectrum of any quantum system as a set of discrete eigenvalues placed along the real energy axis. This set is ordered as any sequence of events occurring at different times. Thus, these events (energy levels) can be treated by standard time series analysis methods. An important result using the power-spectrum analysis was that quantum systems are characterized by $1/f^\alpha$ noise, where the exponent α is related to the degree of chaos. It is equal to 1 and 2 for chaotic and integrable systems, respectively, while varying between these two values for intermediate systems [20,21]. This conjecture is supported by numerical calculations [19] and theoretical studies [22]. One of the main advantages of this approach is that the spectral fluctuations can be described without reference to random matrix models. A recent review that covers this approach is given by Gómeza *et al*. [23] and references therein.

In the current study, we follow the same trend by assuming the analogy between energy levels and time series. But we use the autocorrelation function (ACF) as another technique instead of the power-spectrum analysis used in the previous references.

We describe the method of analysis in Sec. II. A brief account of the classifying parameters is given in Sec. III. Section IV is devoted to results and discussions. The conclusion of this work is outlined in Sec. V.

Method of analysis. Long-term correlations [24] have been found in the dynamics of many physical, technological, and natural systems. They are characterized by a divergent correlation time [25]. This type of persistence represents a surprising regularity since it is found in many different data, such as DNA-sequences, climatological temperatures, human heartbeat, etc. [26–28].

The autocorrelation function of a time series measures how strongly on average each data point of a time series is correlated with another, which is k time steps away in the same series. It is the ratio of the autocovariance to the variance of the data [29]. Uncorrelated data should have an ACF within $\pm 2/\sqrt{N}$ of zero (two standard deviations) for about 95% of the k values. The error bar for each point of an ACF is shown as a dotted line in Figs. 1 and 2. The ACF falls from a value of 1 at $k = 0$ to zero at large k. The value of k at which an ACF falls to $1/e \approx 37\%$ is called the correlation time τ_c [29]. In the present study, series of the 2^+ levels that are arranged in increasing order of the Z, N, A, β_2, $R_{4/2}$, and P parameters, respectively, are investigated. We expect the ACF and τ_c values to increase as the parameter under investigation is capable to order neighboring nuclei having nearly similar collective behavior.

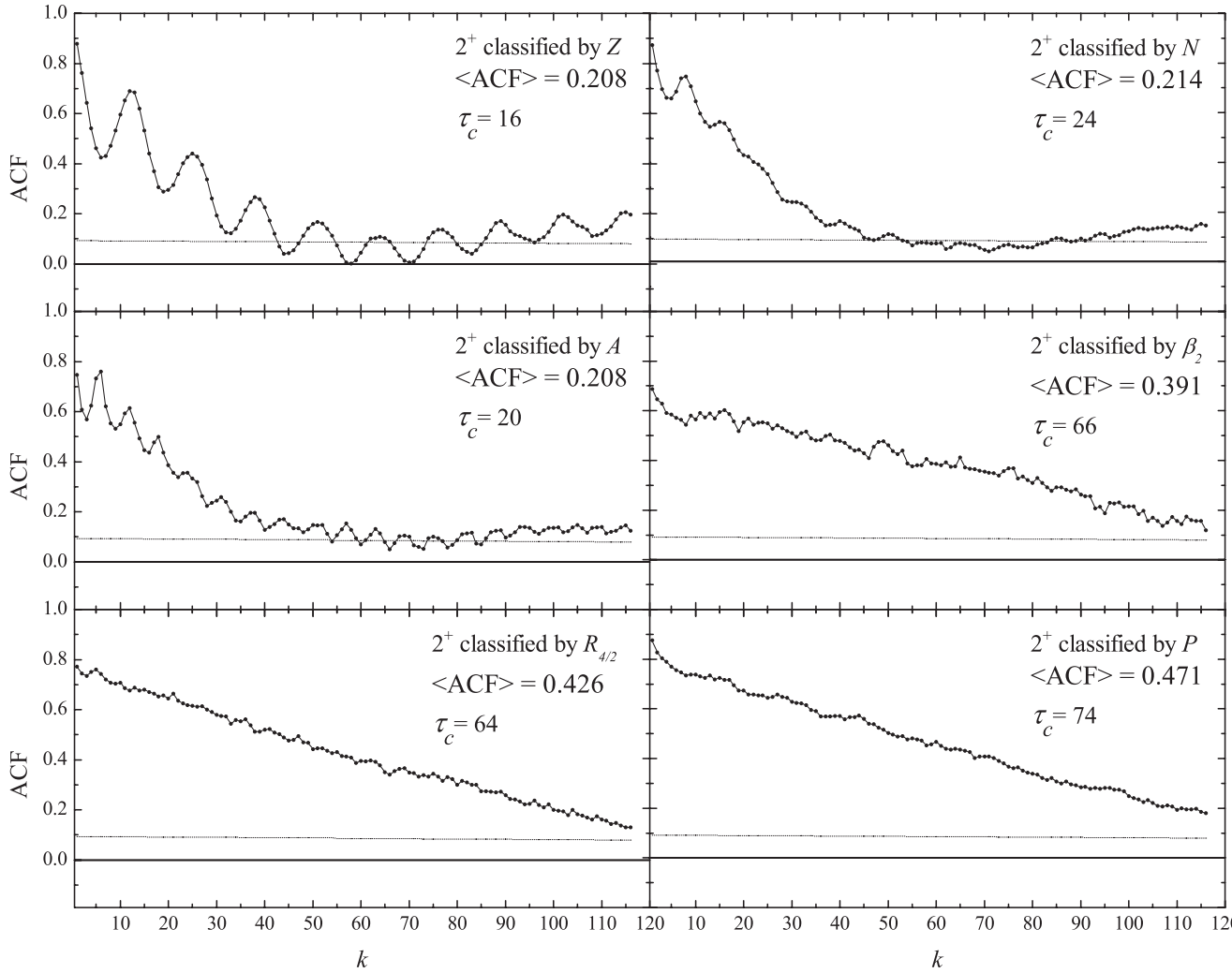

FIG. 1. Shows the ACF and τ_c versus the lag number k for the 2^+ states of all 466 nuclei.

The ACF could be represented as

$$\mathrm{ACF} = \frac{\sum_{i=1}^{N-k}(E_i - \overline{E})(E_{i+k} - \overline{E})}{\sum_{i=1}^{N}(E_i - \overline{E})^2}, \tag{1}$$

where N is the series length, which is equal to the number of nuclei, i.e., 466. k is a lag number which should not exceed $N/4$ in order to get significant results (see [29] and references within). Here E is the first 2^+ energy state of ith nuclei, and $\overline{E}$ the average of 2^+ energy levels of nuclei contributed in the calculation of the ACF. A weak point of the autocorrelation analysis is its sensitivity to outliers within each series under investigation with respect to the average. In the current study, the energy level values are ranging from 0.0415 to 7.012 MeV for ^{248}Cf and ^{14}C, respectively. To avoid such a situation, a common method to deal with time series cases where all values are positive, such as energy levels, is to compose a new series by taking the natural logarithm of the original series and performing the analysis on the new series.

Classifying parameters. (a) Z, N, and A parameters. The shell model [30,31] was suggested to resolve the extraordinary stability of nuclei having certain values for either the number of protons Z or the number of neutrons N or both. These so called magic numbers are 2, 8, 20, 28, 50, 82, and 126.

As more valence nucleons of both types are added beyond a major shell, more complex excitations—including collective modes—develop. Absolutely critical in this process is the key role of the valence, namely, the residual proton-neutron interaction, which is essential to the development of collectivity and deformation in nuclei. Its competition with the like nucleon pairing force largely determines the evolution of nuclear structure. [32].

(b) β_2 parameter. Bohr and Mottelson [33] proposed the geometric collective model in which the collective motion can be interpreted as vibrations and rotations of the nuclear surface. The moving nuclear surface may be described by an expansion in spherical harmonics with time-dependent shape parameters as coefficients. The nuclear radius for axially symmetric nuclei can be written as

$$R(\theta, \phi) = R_{av}[1 + \beta_2 Y_{20}(\theta, \phi)], \tag{2}$$

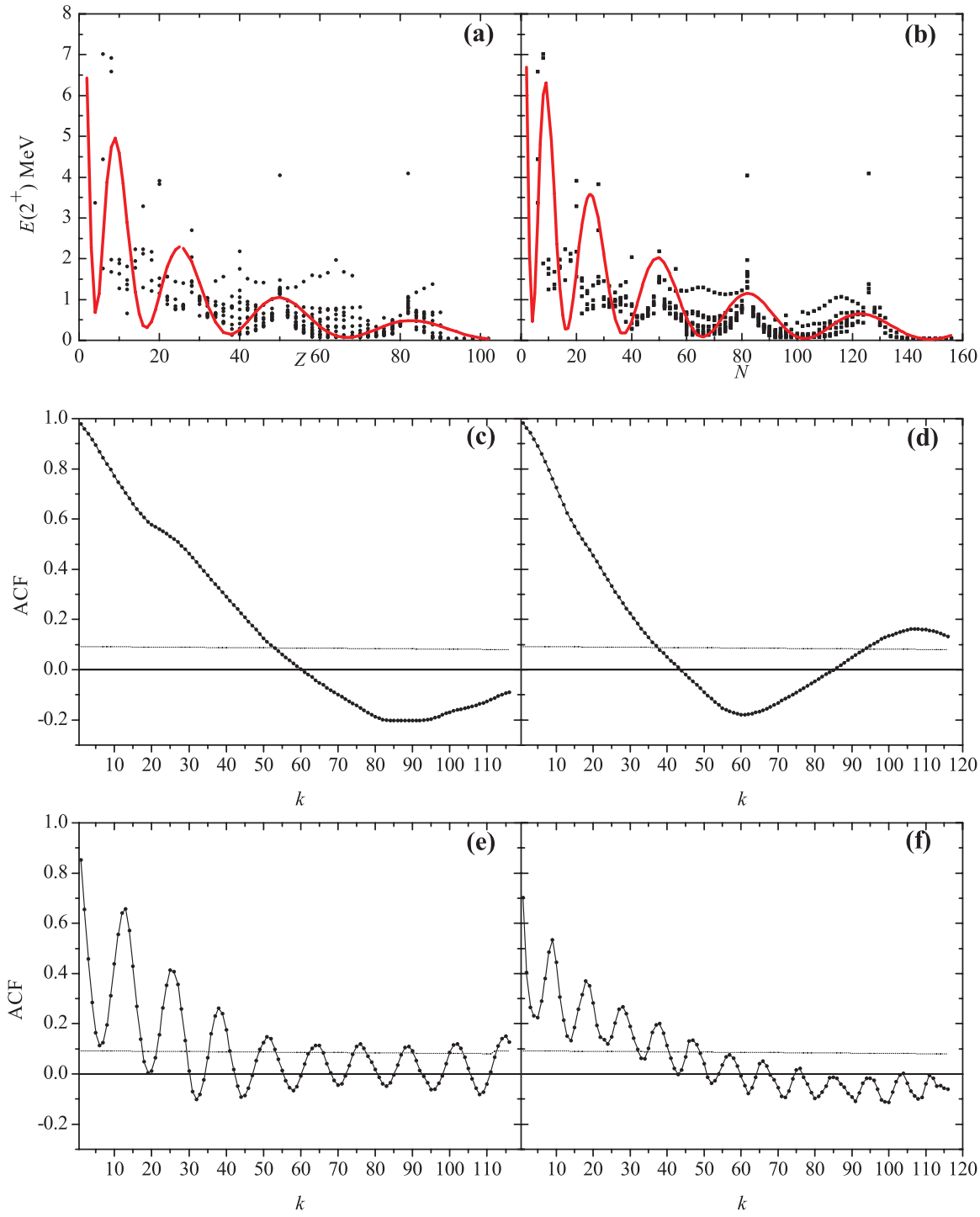

FIG. 2. (Color online) (a) Shows the 2^+ states of all 466 nuclei classified by atomic number Z and the 2^+ states calculated by Eq. (4). (b) The same as (a) but for neutron number N. (c), (d) The ACFs of Eq. (4) for Z and N, respectively. (e), (f) The ACFs for Eqs. (5) and (6), respectively.

where β_2 is the quadrupole deformation parameter. Positive and negative β_2 values correspond to prolate and oblate shapes, respectively.

(c) $R_{4/2}$ ratio. The interacting boson model (IBM) [34] presents three dynamical symmetries where there are analytical solutions. These dynamical states are U(5), SO(6), and SU(3) which are assigned to vibrational, gamma unstable, and rotational nuclei, respectively. We here follow the suggestion of several authors to differentiate between these configurations using the $R_{4/2}$ ratio [35,36]. The $R_{4/2}$ ratio is defined as the ratio of the first 4^+ to the 2^+ excitation energy. When the $R_{4/2}$ ratio increases, the collective behavior is favored.

(d) P factor. Coupling of pairs of identical nucleons which have spherically symmetric wave functions drives the nucleus towards a spherical shape since it forms the $J = 0^+$ ground state. Deformation and collectivity, on the other hand, arise from configuration mixing. Configuration mixing itself is largely driven by the valence p-n interaction. Hence it is a pairing p-n competition that tends to drive the structural evolution of nuclei [37–39]. This idea was used to estimate the locus of collectivity in nuclei. The valence p-n interactions can be represented by the P factor defined in Eq. (3):

$$P = \frac{N_n N_p}{N_n + N_p}, \tag{3}$$

where N_p and N_n are the numbers of valence protons and neutrons, respectively. Thus, one expects significant collectivity and the onset of deformation when the P factor [40–42] increases.

Results and discussions. In Fig. 1, the ACF of the logarithm of the 2^+ states of all 466 even-even nuclei is calculated for each classifying parameter. The results in Fig. 1 show that the ACF and the correlation times τ_c are nearly compatible for the Z, N, and A parameters and are characterized by varying periodic behavior. This fast decaying ACF may be explained in terms of a time series analysis as a short-range correlation, which may be referred mainly to the shell closure effect buildup on the variation of valence protons and neutrons N_p and N_n or simply on the P-factor values. On the other hand, the ACF plot of 2^+ energy states classified by the β_2, $R_{4/2}$, and P parameters show a certain kind of what is known as long-range correlation which is characterized by a higher correlation time.

To interpret the trend in Fig. 1, we try to model the 2^+ states of even-even nuclei in the case of Z and N classifying parameters as a single variable. We propose that the data set of the 2^+ states can be modeled by a simple empirical formula (4). This formula is a combination of a decaying exponential function multiplied by a varying period cosine function in the following way:

$$E(2_i^+) = \alpha e^{-\beta\sqrt{Z_i}}[\gamma - \cos(\delta\sqrt{Z_i})], \tag{4}$$

where α, β, γ, and δ are constant parameters and could be calculated by a least-squares fit. We replace the atomic number Z by the neutron number N in Eq. (4) to calculate the 2^+ states according to the neutron number classification. Figures 2(a) and 2(b) show the experimental 2^+ energy states (scattered plot) and the obtained 2^+ states from Eq. (4) against Z and N, respectively.

The fit parameters are equal to 7.10, 0.38, 1.20, and 3.08 for Z, and equal to 6.96, 0.28, 1.11, and 3.10 for N, respectively. We observe that Eq. (4) could model the shell effect of magic numbers as seen from the peaks at nearly 2, 8, 50, 82, and 126, while it is around 26 for the two closed successive shells 20 and 28. The ACFs of the 2^+ energy states calculated by Eq. (4) for Z and N are performed in Figs. 2(c) and 2(d), respectively. It is observed that such a step cannot explain the periodic variation of the ACF given in Fig. 1. This drawback comes from neglecting the isotopes and isotones of each element that take place in our study.

We turn then to add a term that represents the number of valence protons and neutrons N_p and N_n to Eq. (4) to satisfy the variety of isotopes and isotones. In order to simulate the positive values of the 2^+ energy states, we add the following two terms for Eq. (4):

$$E(2_i^+) = \alpha e^{-\beta\sqrt{Z_i}}[\gamma - \cos(\delta\sqrt{Z_i})] + \left(30 - N_{n_i}\right), \tag{5}$$

$$E(2_i^+) = \alpha e^{-\beta\sqrt{N_i}}[\gamma - \cos(\delta\sqrt{N_i})] + \left(18 - N_{p_i}\right). \tag{6}$$

The constant numbers 30 and 18 are the maximum N_n and N_p values in our study, respectively. The energy of the 2^+ states calculated by Eqs. (5) and (6) are shifted in their values by the two terms added to Eq. (4), but this procedure will not affect the ACF calculation since it is a normalized function according to its definition in Eq. (1).

The ACFs of the 2^+ levels calculated by Eqs. (5), and (6) are performed and given in Figs. 2(e) and 2(f), respectively. It is apparent that the ACFs are similar in pattern and have the same periods of variation as Fig. 1 for the Z and N classifications. Another point could be extracted from the observed fluctuations of the ACF with respect to Z classification compared to N classification. This may be referred to the larger available number of isotopes for each element of fixed Z compared to the case of a lesser number of isotones for each element of fixed N. We expect a more complex combination of residual valance interaction of protons and neutrons to build up the pattern of the ACF with respect to the mass number A.

The importance of the residual valence interaction as shown in Eqs. (5) and (6) is essential to describe the variation of the ACF. As a result, we expect that the classification of energy states according to ascending P-factor values will decrease sharply the effect of shell closure, so nuclei of similar structure or collective mode will be arranged nearby each other and the ACF of these similar 2^+ energy states will increase remarkably. Figure 1 shows that the ACF increases from 0.391 and 0.426 for β_2 and $R_{4/2}$, respectively, to reach 0.471 in the case of the P-factor classification as it is expected. This could be used as an indicator of the ability of the P factor to represent the collective behavior within nuclear structure. This finding is supported by the work of the authors of references [43–45]. They suggested an empirical formula which describes the essential trends of the excitation energies of the lowest natural parity even multipole states, such as the 2^+ states, in even-even nuclei throughout the periodic table. This formula depends on the valence nucleon numbers N_p and N_n and the mass number A.

Conclusion. In the current study we try to probe the variety of nuclear structure evolution with one of the standard time series techniques—the autocorrelation analysis. We assume that a series of the 2^+ states of even-even nuclei classified by different parameters related to nuclear structure, such as Z, N, A, β_2, $R_{4/2}$, and P parameters, could be treated as a time series, where the time step is replaced by the parameter step. We take the log transformation to stabilize the variability of each series. Then the 2^+ states are analyzed using the autocorrelation function defined in Eq. (1). We terminate the lag number k at $N/4$ for each series. The 95% confidence level—dotted line—indicates that autocorrelations that lie outside this range are statistically significant. The ACF and

the correlation time τ_c have reached enhanced values for the P factor. This indicates that the P factor could cumulate nuclei having nearly the same properties nearby each other. This finding is in need of more studies to find a theoretical basis. We may use this result in future work to forecast some of the missed or unknown nuclear properties according to the autocorrelation plot.

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