

$s\bar{s}$ sea pair contribution to electromagnetic observables of the proton in the unquenched quark model

R. Bijker

Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Apartado 70-543, 04510 México, Distrito Federal, México

J. Ferretti and E. Santopinto*

INFN, Sezione di Genova, via Dodecaneso 33, I-16146 Italy

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We present an unquenched quark model calculation of the $s\bar{s}$ sea pairs' contribution to the magnetic moment of the proton (the strange magnetic moment) and its charge radius (the strange radius). In our approach the effects of the $s\bar{s}$ pairs are taken explicitly into account through a microscopic, QCD-inspired, quark-antiquark pair creation mechanism. Our results for the two "strangeness" observables are compatible with the latest experimental results and recent lattice calculations.

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I. INTRODUCTION

The strange quark contribution to the nucleon electromagnetic form factors can be expressed in terms of the strange electric and magnetic form factors G_E^s and G_M^s . In 1987 Kaplan and Manohar [1] observed that neutral current experiments could provide information on the strange matrix elements of the nucleon. The parity-violating elastic scattering (PVES) of electrons on nucleon, as suggested by Beck and McKeown in 1988 [2], has demonstrated to provide a powerful tool to measure this contribution in several experiments [3–8]. The parity-violating asymmetry $A_{LR} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$, measured in the elastic scattering cross section of left- and right-handed electrons, must be zero for the pure electromagnetic interaction; however, the presence of the weak interaction and the interference between the electromagnetic and the weak amplitudes results in a small but nonzero value. Considering the electromagnetic form factors of the proton and the neutron and also the weak form factors of the proton, determined through parity-violating electron scattering experiments [2,9], one may separate the up, down, and strange quark contributions, thus performing the so-called flavor decomposition, and extract the unknown quantities G_E^s and G_M^s [2,7,9]. In fact, the measurement of the weak form factors of the proton brings the third independent information that makes it possible to separate the contributions of the first three flavors:

$$\begin{aligned} G_{E,M}^{\gamma,p} &= \frac{2}{3} G_{E,M}^u - \frac{1}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^s, \\ G_{E,M}^{\gamma,n} &= \frac{2}{3} G_{E,M}^d - \frac{1}{3} G_{E,M}^u - \frac{1}{3} G_{E,M}^s, \\ G_{E,M}^{Z,p} &= \left(1 - \frac{8}{3} \sin^2 \theta_W\right) G_{E,M}^u + \left(-1 + \frac{4}{3} \sin^2 \theta_W\right) G_{E,M}^d \\ &\quad + \left(-1 + \frac{4}{3} \sin^2 \theta_W\right) G_{E,M}^s. \end{aligned} \quad (1)$$

The first theoretical calculation of the strange form factors of the nucleon, performed by Jaffe in 1989 [10], reported quite large results for G_E^s and G_M^s , thus triggering the interest for this kind of observable. A few years later the first measurements

of the asymmetry A_{LR} provided unusually large and positive values of G_M^s , even if with substantial errors; for example, the SAMPLE collaboration in 1999 found $G_M^s = 0.61 \pm 0.17 \pm 0.21 \pm 0.19$ at $Q^2 = 0.1$ (GeV/c)² [11].

The situation changed drastically with the latest PVES experiments, which showed smaller values for the strange form factors [5–7] (see Tables I and II). From the most recent measurements, as reported in Tables I and II, one can see that the strange quarks contribution to the magnetic form factor, G_M^s , is compatible with zero within the experimental errors and the same holds for the strange electric form factor G_E^s .

In the meantime, it has become conventional to evaluate the values of two static observables: the strange magnetic moment and the strange radius. They can be extracted from the strange form factors of the nucleon and describe their low- Q^2 behavior.

The strange magnetic moment is defined as the value of the strange magnetic form factor of the proton at $Q^2 = 0$ times the electric charge of the strange quark, that is, $\mu_s \equiv -\frac{1}{3} G_M^s(0)$. Because the nucleon has no net strange charge, one finds that $G_E^s(Q^2 = 0) = 0$. However, one can express the slope of G_E^s at $Q^2 = 0$ in terms of a "strange radius" R_s , defined as $R_s^2 \equiv 2 \frac{dG_E^s}{dQ^2} \Big|_{Q^2=0}$.

The values of μ_s and R_s^2 can be extracted from the experimental data as follows. In Ref. [3], after measuring G_M^s at $Q^2 = 0.1$ (GeV/c)², the strange magnetic moment μ_s is extrapolated by considering the momentum dependence of $G_M^s(Q^2)$ taken from Ref. [18]. The resulting value is $\mu_s(p) = (-0.003 \pm 0.097 \pm 0.103 \pm 0.023) \mu_N$. In Ref. [15] the strange magnetic moment of the proton is obtained from a fit to the complete set of parity-violating scattering experimental data [4,8,13,16,17,19,20], obtaining that $\mu_s(p) = (-0.04 \pm 0.18 \pm 0.02) \mu_N$.

The evaluation of the strange radius R_s^2 requires at least a measurement of the strange electric form factor at a small value of $Q^2 = \tilde{Q}^2$, because R_s^2 is proportional to the slope of the line that connects the points $G_E^s(Q^2 = \tilde{Q}^2)$ and $G_E^s(Q^2 = 0)$; moreover $G_E^s(Q^2 = 0) = 0$ because the nucleon has no net strange charge. We have reported the evaluations for R_s^2 , extracted by us in this way, in Table IV. Analyzing the results in Tables III and IV, one can see that the two strangeness

* corresponding author: santopinto@ge.infn.it

TABLE I. Strange magnetic form factor of the proton: Experimental data (see Refs. [3–7,11–16] for details).

$Q^2 = 0.038 \text{ (GeV}/c)^2$	$G_M^s (\mu_N)$
SAMPLE [4]	$0.23 \pm 0.36 \pm 0.40$
$Q^2 = 0.1 \text{ (GeV}/c)^2$	$G_M^s (\mu_N)$
HAPPEX [5]	0.18 ± 0.27
SAMPLE [3]	$0.14 \pm 0.29 \pm 0.31$
SAMPLE [11]	$0.61 \pm 0.17 \pm 0.21 \pm 0.19$
SAMPLE [12]	$0.23 \pm 0.37 \pm 0.15 \pm 0.19$
SAMPLE [13]	$0.37 \pm 0.20 \pm 0.26 \pm 0.07$
Global analysis [14]	0.29 ± 0.21
Global analysis [15]	-0.01 ± 0.25
$Q^2 = 0.22 \text{ (GeV}/c)^2$	$G_M^s (\mu_N)$
PVA4 [6]	$-0.14 \pm 0.11 \pm 0.11$
G0 [7]	$0.0834 \pm 0.1834 \pm 0.0855 \pm 0.0781$
$Q^2 = 0.23 \text{ (GeV}/c)^2$	$G_M^s (\mu_N)$
PVA4 [16]	-0.099 ± 0.154

observables, μ_s and R_s^2 , are compatible with zero within the errors.

The theoretical estimations of μ_s and R_s^2 , obtained through lattice QCD calculations, hadronic models and effective hadronic theory, vary widely and are not consistent in the predicted signs for $\mu_s(p)$ and $R_s^2(p)$ (see Table V and Fig. 1). The theoretical predictions for $\mu_s(p)$ are indeed distributed from negative values ($-0.12 \mu_N$ [21]) to positive ones ($0.25 \pm 0.10 \mu_N$ [22]) and the same happens for $R_s^2(p)$, which varies from $-0.08 \pm 0.01 \text{ fm}^2$ [23] to 0.12 fm^2 [24]. On the contrary, the most recent lattice QCD calculations have provided small values for both $\mu_s(p)$ and $R_s^2(p)$ [25,26].

II. UNQUENCHED QUARK MODEL

In this section, we recall the unquenched quark model of Refs. [38–40], in which the effects of quark-antiquark pairs are introduced explicitly into the constituent quark model (CQM) via a QCD-inspired 3P_0 pair-creation mechanism. The approach at issue is a generalization of the unitarized

TABLE II. Strange electric form factor of the proton: Experimental data (see Refs. [5–7,14–17] for details).

$Q^2 = 0.091 \text{ (GeV}/c)^2$	G_E^s
HAPPEX [17]	$-0.038 \pm 0.042 \pm 0.010$
$Q^2 = 0.1 \text{ (GeV}/c)^2$	G_E^s
HAPPEX [5]	-0.005 ± 0.019
Global analysis [14]	-0.008 ± 0.016
Global analysis [15]	0.002 ± 0.018
$Q^2 = 0.22 \text{ (GeV}/c)^2$	G_E^s
PVA4 [6]	$0.050 \pm 0.038 \pm 0.019$
G0 [7]	$-0.0142 \pm 0.0356 \pm 0.0182 \pm 0.0176$
$Q^2 = 0.23 \text{ (GeV}/c)^2$	G_E^s
PVA4 [16]	0.061 ± 0.035

TABLE III. Strange magnetic moment of the proton. Note that $\mu_s(p)$ is reported including the strange quark charge.

Reference	$\mu_s(p) (\mu_N)$
SAMPLE [3]	$-0.003 \pm 0.097 \pm 0.103 \pm 0.023$
Global analysis [15]	$-0.04 \pm 0.18 \pm 0.02$

quark model by Törnqvist and Zenczykowski [41], and was motivated by later work by Isgur and co-workers on the flux-tube breaking model, in which they showed that the CQM emerges as the adiabatic limit of the flux-tube model to which the effects of $q\bar{q}$ pair creation can be added as a perturbation [42]. Thus, our approach is based on a CQM to which the quark-antiquark pairs with vacuum quantum numbers are added as a perturbation. The pair-creation mechanism is inserted at the quark level and the one-loop diagrams are calculated by summing over the possible intermediate states.

Under these assumptions, the baryon wave function consists of a zeroth-order three-quark configuration plus a sum over all the possible higher Fock components owing to the creation of 3P_0 quark-antiquark pairs. To leading order in pair creation, the baryon wave function can be written as

$$|\psi_A\rangle = \mathcal{N} \left[|A\rangle + \sum_{BCIJ} \int d\vec{k} |BC\vec{k}lJ\rangle \frac{\langle BC\vec{k}lJ|T^\dagger|A\rangle}{M_A - E_B - E_C} \right], \quad (2)$$

where T^\dagger is the 3P_0 quark-antiquark pair creation operator [43], A is the baryon, B and C represent the intermediate baryon and meson, respectively. The hadron wave functions are expressed in terms of three-quark (for baryons A and B) and quark-antiquark (for mesons C) wave functions of the CQM. M_A , E_B , and E_C are the energies of the three hadrons, \vec{k} and l the relative radial momentum and orbital angular momentum, respectively, of B and C , and J is the total angular momentum $\vec{J} = \vec{J}_B + \vec{J}_C + \vec{l}$.

The 3P_0 quark-antiquark pair-creation operator, T^\dagger , can be written as [43]

$$T^\dagger = -3\gamma_0 \int d\vec{p}_4 d\vec{p}_5 \delta(\vec{p}_4 + \vec{p}_5) C_{45} F_{45} e^{-r_q^2(\vec{p}_4 - \vec{p}_5)^2/6} [\chi_{45} \times \mathcal{Y}_1(\vec{p}_4 - \vec{p}_5)]_0^{(0)} b_4^\dagger(\vec{p}_4) d_5^\dagger(\vec{p}_5). \quad (3)$$

TABLE IV. Strange radius of the proton, as extracted from the experimental data listed in the first column. Note that $R_s^2(p)$ is reported including the strange quark charge.

Reference	$R_s^2(p) (\text{fm}^2)$
HAPPEX [5]	-0.004 ± 0.015
HAPPEX [17]	$-0.033 \pm 0.036 \pm 0.009$
G0 [7]	$-0.005 \pm 0.013 \pm 0.006 \pm 0.006$
PVA4 [6]	$0.018 \pm 0.013 \pm 0.007$
PVA4 [16]	0.021 ± 0.012
Global analysis [14]	-0.006 ± 0.013
Global analysis [15]	0.002 ± 0.014

TABLE V. Theoretical predictions for $\mu_s(p)$ and $R_s^2(p)$ according to various studies. Note that $\mu_s(p)$ and $R_s^2(p)$ are reported including the strange quark charge. In the first column Fit to octet baryon m.m. stands for fit to octet baryon magnetic moments, NJL stands for Nambu-Jona-Lasinio model, VMD stands for vector meson dominance model, LQCD stands for lattice QCD, PChQM stands for perturbative chiral quark model, χ QSM stands for chiral quark soliton model, and ChPT stands for chiral perturbation theory.

Theoretical model + Ref.	$\mu_s(p)$ (μ_N)	$R_s^2(p)$ (fm^2)
Fit to octet Baryon m.m. [21]	-0.12	-
QCD equalities [22]	0.25 ± 0.10	-
NJL soliton m. [24]	0.15	0.12
VMD [10]	0.10 ± 0.03	-0.05 ± 0.02
VMD [23]	0.08 ± 0.01	-0.08 ± 0.01
VMD [27]	0.087	-0.067
VMD [28]	0.047	-0.018
VMD [29]	-0.105	-0.002
LQCD [25]	0.015 ± 0.006	$0.001 \pm 0.004 \pm 0.004$
LQCD [26]	0.006 ± 0.007	0.0008 ± 0.0005
Kaon loops +VMD [30]	0.09 ± 0.01	0.014 ± 0.001
Skyrme m. [31]	0.043	0.037
PCQM [32]	0.016 ± 0.004	0.004 ± 0.001
χ QSM [33]	-0.038	0.032
Sum rules [34]	-0.11	-
Kaon cloud m. [35]	0.118 ± 0.015	0.010 ± 0.001
Kaon loops [36]	≈ -0.05	-
ChPT [37]	-0.035 ± 0.025	-0.02 ± 0.03

Here $b_4^\dagger(\vec{p}_4)$ and $d_5^\dagger(\vec{p}_5)$ are the creation operators for a quark and an antiquark with momenta \vec{p}_4 and \vec{p}_5 , respectively. The quark and antiquark pair is characterized by a color singlet wave function C_{45} , a flavor singlet wave function F_{45} , a spin triplet wave function χ_{45} with spin $S = 1$, and a solid spherical harmonic $\mathcal{Y}_1(\vec{p}_4 - \vec{p}_5)$ that indicates that the quark and antiquark are in a relative P wave. The operator T^\dagger creates a pair of constituent quarks with an effective size, thus the pair creation point is smeared out by a Gaussian factor whose width

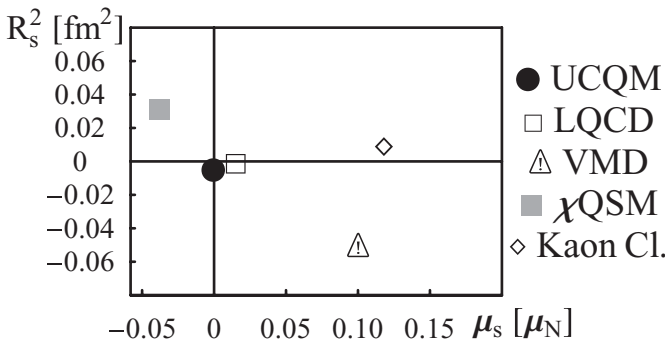


FIG. 1. The unquenched CQM (UCQM) results for the strange magnetic moment and radius of the proton, plotted in the $R_s^2(p)$ - $\mu_s(p)$ plane, are compared with other theoretical results: [25] (lattice QCD), [10] (vector meson dominance model), [33] (chiral quark soliton model), and [35] (kaon cloud model).

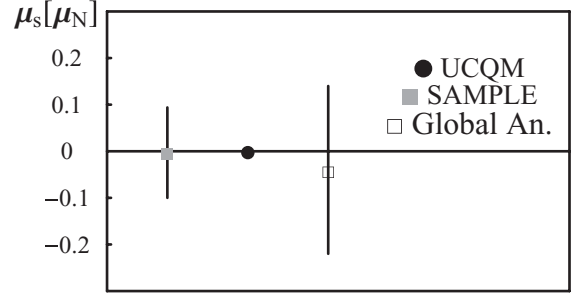


FIG. 2. Comparison between our resulting value for the strange magnetic moment of the proton in the UCQM and the results from Table III: [3] (SAMPLE) and [15] (global analysis).

r_q was determined from meson decays to be approximately 0.25–0.35 fm [42,44,45]. In our calculations we took an average value, $r_q = 0.30$ fm. The dimensionless constant γ_0 , that is, the pair-creation strength, was fitted to the strong decays of baryons. The matrix elements of the pair-creation operator T^\dagger were derived in explicit form in the harmonic oscillator basis as in Ref. [43], using standard Jacobi coordinates. We use algebraic models of hadron structure [46,47] to calculate the baryon and meson energies appearing in the denominator of Eq. (2). The baryon and meson wave functions have good flavor symmetry and depend on a single oscillator parameter which, following [42], is taken to be $\hbar\omega_{\text{baryon}} = 0.32$ GeV for the baryons and $\hbar\omega_{\text{meson}} = 0.40$ GeV for the mesons.

The matrix elements of an observable $\hat{\mathcal{O}}$ can be calculated as

$$\mathcal{O} = \langle \psi_A | \hat{\mathcal{O}} | \psi_A \rangle. \quad (4)$$

To calculate the effects of quark-antiquark pairs on an observable, one has to evaluate the sum over all possible intermediate states in Eq. (2). The sum over intermediate meson-baryon states includes for baryons all radial and orbital excitations up to a given oscillator shell combined with all possible SU(6) spin-flavor multiplets, and for mesons all radial and orbital excitations up to a given oscillator shell and all possible nonets. This problem was solved by means of group theoretical techniques to construct an algorithm to

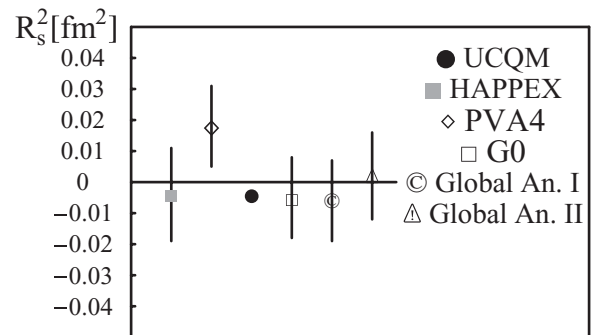


FIG. 3. Comparison between our resulting value for the strange radius of the proton in the UCQM and some of the results from Table IV: [5] (HAPPEX), [6] (PVA4), [7] (G0), [14] (global analysis I), and [15] (global analysis II).

generate a complete set of intermediate meson-baryon states in spin-flavor space for an arbitrary oscillator shell.

III. STRANGE MAGNETIC MOMENT AND RADIUS OF THE PROTON IN THE UCQM

In this section we discuss our calculation of the strange magnetic moment, $\mu_s(p)$, and radius, $R_s^2(p)$, of the proton. The results for the two strangeness observables are obtained in a calculation involving a sum over intermediate states up to four oscillator shells for both baryons and mesons. In the UCQM formalism, the strange magnetic moment of the proton is defined as the expectation value of the operator

$$\vec{\mu}_s = \sum_i \mu_{i,s} [2\vec{s}(q_i) + \vec{\ell}(q_i) - 2\vec{s}(\bar{q}_i) - \vec{\ell}(\bar{q}_i)] \quad (5)$$

on the proton state of Eq. (2); $\mu_{i,s}$ is the magnetic moment of the quark i times a projector on strangeness and the strange quark magnetic moment is set as in Ref. [38].

In the UCQM the strange magnetic moment of the proton arises from the sea quarks. There is a contribution from the quark spins of the $s\bar{s}$ pair $-0.0004 \mu_N$, as well as from the orbital motion of the $s\bar{s}$ pair $-0.0002 \mu_N$. Both contributions

are small and give a total strange magnetic moment $\mu_s(p) = -0.0006 \mu_N$ (see Fig. 2).

Similarly, the strange radius of the proton is defined as the expectation value of the operator

$$R_s^2 = \sum_{i=1}^5 e_{i,s} (\vec{r}_i - \vec{R}_{c.m.})^2 \quad (6)$$

on the proton state of Eq. (2), where $e_{i,s}$ is the electric charge of the quark i times a projector on strangeness, \vec{r}_i and $\vec{R}_{c.m.}$ are the coordinates of the quark i and of the intermediate state center of mass, respectively. The expectation value of R_s^2 on the proton is equal to -0.004 fm^2 . In Fig. 3 our result is compared with the experimental data.

In conclusion, the effects of the higher Fock components on the “strangeness” observables of the proton are found to be negligible. Our results are compatible with the latest experimental data and recent lattice calculations.

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