Coriolis contribution to excited states of deformed ^{163}Dy and ^{173}Yb nuclei with multiple mass parameters

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The use of different mass parameters for the rotation and two vibrations in the Bohr Hamiltonian is important for describing properties of nuclei, especially interband $E2$ transition probabilities. For odd nuclei, the Coriolis interaction of the angular momentum of an odd nucleon with that of a nucleus can affect results noticeably. Excited-state energies and $E2$ transition probabilities have been calculated for the ^{163}Dy and ^{173}Yb nuclei, by using three different mass parameters and taking into account the Coriolis interaction, once shell-model calculations have been performed to determine the single-particle ground-state properties of ^{163}Dy and ^{173}Yb.

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I. INTRODUCTION

The Bohr Hamiltonian [1] has been used for many decades to glean an understanding of the collective properties in nuclei [1–7]. In these studies, rotational and vibrational modes are assumed to have one common mass parameter. The topic is still interesting, and a number of different methods for using this Hamiltonian have been explored lately [8–20].

Recent works show that multiple mass parameters should be used in order to well describe the properties of nuclei. In Ref. [19] it was shown that, by analyzing experimental data and comparing them with theoretical calculations, the use of different mass parameters for vibrational and rotational modes is very important for a correct description of the properties of even-even nuclei. In Ref. [11], analytical expressions for spectra and wave functions are derived for a Bohr Hamiltonian when describing the collective motion of deformed nuclei, and in these, the mass parameter is allowed to depend on the nuclear deformation. Solutions are obtained for separable potentials that contain a Davidson potential for the β variable, for cases of γ-unstable nuclei, axially symmetric prolate deformed nuclei, and triaxial nuclei, implementing the usual approximations in each case. The matrix elements of the deformation-dependent components of the mass tensor have been studied in Ref. [20], using experimental data for the energies and the $E2$ transitions of low-lying collective states.

An investigation similar to that of Ref. [19] has been carried out in Ref. [21] for odd-mass, deformed nuclei. One common mass parameter was used for the rotation and vibrations in this study and a calculation of the interband-reduced $E2$ transitions was not performed.

In Ref. [22], results for the even-mass nuclei considered in Ref. [19] were recreated, using multiple mass parameters as suggested. Projection of the angular momentum of the total spin of the nucleus to the third principal axis K was considered to be a good quantum number, such that it was possible to compare results with those of Ref. [19]. After this, the odd ^{173}Yb nucleus was considered. It was assumed that the projection of the angular momentum of nucleus K and that of external nucleon Ω are good quantum numbers, in order that the work of Ref. [22] was consistent throughout. The Hamiltonian used in Ref. [21] is simplified for use in this case. Because of these simplifications, the Coriolis interaction had not been included in the Hamiltonian.

However, the Coriolis interaction significantly affects the structure of nuclear excited states and many works are devoted to its study. It is discussed in detail in Ref. [23]. The change in the effective Coriolis matrix elements has been used to explain experimental regularities within the pseudo-L scheme in strongly deformed Bose-Fermi systems in Ref. [24], along with the relationship of this change to the Nilsson model. In recent works [25,26], quadrupole-octupole oscillations and rotations with a Coriolis coupling between an even-even core and an unpaired nucleon are used to model odd nuclei. $E2$ transition probabilities in a wide range of odd-mass nuclei have been reproduced successfully therein.

Thus, it is desirable to extend the previous work of Ref. [22] to study the effect of the Coriolis interaction when several different mass parameters are included in the model. We do this in this paper, where we study the excited states of the odd nuclei ^{163}Dy and ^{173}Yb. Because the quantum numbers K and Ω are not conserved in our present considerations, the value of angular momentum j of the external nucleon directly contributes to excited-state energies and wave functions. This is because the interaction of the total spin of the nucleus and that of the external nucleon exists in the expression of

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the rotational part of the eigenvalues, and this interaction changes from one state to another as L, K, and Ω change. The arrangement of the excited-state levels is not fixed now, as in Ref. [22], but will strongly depend on mass parameters and on the parameter which connects single-particle and collective states. Therefore, the new arrangement of the levels due to the nonconservation of K and Ω will be important in the present considerations.

The paper is organized in the following way: In Sec. II we obtain expressions for level energies, wave functions, and reduced $E2$ transition probabilities, considering K and Ω as nonconserved quantities and using three different mass parameters for rotation and vibrations.

Large-scale shell-model calculations have been performed in order to determine the main contribution from spherical shell orbitals to the ground states of both ^{163}Dy ($Z = 66$) and ^{173}Yb ($Z = 70$), using the ANTOINE [27] code. These calculations are important for ensuring that the correct value of the angular momentum of the external nucleon is used. Results are shown in Sec. III.

Having thus determined the appropriate value of j, calculated excited-state energies and reduced $E2$ transition probabilities are presented, discussed, and compared with available experimental data in Sec. IV. Finally, in Sec. V, we draw conclusions regarding the effect of the Coriolis interaction in this paper.

II. LEVEL ENERGIES, WAVE FUNCTIONS, AND $E2$ TRANSITION PROBABILITIES

In this paper the Bohr Hamiltonian is written such that it includes three different mass parameters B_γ, $B_{\rm rot}$, and B_β, and it also includes a Coriolis interaction in the rotational component $2(L_1 j_1 + L_2 j_2)$. The vibrational component for this case of an odd-mass nucleus is

$$H_{\rm v} = -\frac{\hbar^2}{2}\left\{\frac{1}{B_\beta}\frac{\partial^2}{\partial\beta^2} + \frac{2}{B_\beta}\frac{1}{\beta}\frac{\partial}{\partial\beta} + \frac{2}{B_\beta}\frac{1}{\beta}\frac{\partial}{\partial\beta} + \frac{1}{B_\gamma}\frac{1}{\gamma}\frac{\partial}{\partial\gamma}\left(\gamma\frac{\partial}{\partial\gamma}\right) - \frac{1}{B_\gamma}\frac{1}{4\beta^2} \times\left(\frac{1}{\gamma^2}+\frac{1}{3}\right)(\hat{L}_3 - j_3)^2\right\} + V(\beta,\gamma), \tag{1}$$

the operator of rotational energy is

$$H_{\rm rot} = \frac{\hbar^2}{6B_{\rm rot}\beta^2}\left[L^2 + j^2 - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2)\right], \tag{2}$$

and the interaction operator is

$$H_{\rm int} = -\beta\langle T\rangle\left(3j_3^2 - j^2\right), \tag{3}$$

where L is the total angular momentum of the nucleus, L_1, L_2, and L_3 are its projections on the principal axes of the nucleus, and j, j_1, j_2, and j_3 are the operators of a single nucleon external to a core, and its projections. In Refs. [6] and [7], $T(r)$ is a function of the distance between the single nucleon and the center of the core nucleus. It appears in the Hamiltonian of Eq. (2.2) in Ref. [6] and Eq. (2) of Ref. [7]. $\langle T\rangle$, which is introduced in those papers and is used here, is the average of the $T(r)$ in the states of the extra nucleon, assuming zero nuclear surface oscillation.

If the same potential of Eq. (6) in Ref. [22] is used, eigenvalues are determined by the following expression:

$$E_{n_\beta n_\gamma L|m|\tau} = \left[2n_\beta + q^\tau_{n_\gamma}(L,|m|) + 3/2\right]\sqrt{2g_\beta}, \tag{4}$$

where

$$q^\tau_{n_\gamma}(L,|m|) = \sqrt{\Lambda - \Lambda_0 + 2g_\beta + 1/4} - 1/2, \tag{5}$$

and

$$\Lambda - \Lambda_0 = \frac{2}{\rm g}\frac{B_\beta}{B_\gamma}\left(2n_\gamma + |m| + \frac{m^2}{3}\right) + \varepsilon_{|m|L\tau} - \varepsilon_{0L_{\rm g.s}1}, \tag{6}$$

where Λ is the eigenvalue of the γ-vibrational part of the Hamiltonian plus the third term of the rotational energy, Λ_0 is that of the ground state, $L_{\rm g.s}$ is the ground-state spin of the nucleus, $g_\beta = \frac{B_\beta V_0 \beta_0^2}{\hbar^2}$, ${\rm g} = \frac{1}{\beta_0^2}\frac{\hbar^2}{\sqrt{B_\gamma C_\gamma}}$, τ distinguishes different states of the same L, and n_β and n_γ are the quantum numbers of β and γ vibrations, respectively. The values of m are connected with K and Ω through the condition $K - \Omega = 2m$ [6], where m should be an integer.

Now, the following determinant should be calculated in order to determine the eigenvalues and eigenfunctions of the rotational part of the Hamiltonian:

$$\|\langle LjKm|\hat{X}|LjK'm'\rangle - \varepsilon_{|m|L\tau}\delta_{KK'}\delta_{mm'}\| = 0, \tag{7}$$

where

$$\hat{X} = \frac{1}{3}\frac{B_\beta}{B_{\rm rot}}\big[L(L+1) + j(j+1) - L_3^2 - j_3^2 - 2(L_1 j_1 + L_2 j_2)\big] - \frac{1}{3\xi}\big[3j_3^2 - j(j+1)\big], \tag{8}$$

and $\xi = \frac{\hbar^2}{6B_\beta\beta_0^3\langle T\rangle}$. Since K and Ω are not good quantum numbers, not only do the diagonal elements of the Hamiltonian but also nondiagonal elements contribute to the energies and $E2$ transition probabilities. The diagonal elements are as follows:

$$\langle LjKm|\hat{X}|LjKm\rangle = \frac{1}{3}\frac{B_\beta}{B_{\rm rot}}[L(L+1) + j(j+1) - K^2 - (K-2m)^2 - (-1)^{L-j} \times (L+1/2)(j+1/2)\delta_{K1/2}\delta_{m0}] - \frac{1}{3\xi}[3(K-2m)^2 - j(j+1)]. \tag{9}$$

The nondiagonal elements are

$$\langle LjKm|\hat{X}|LjK\pm 1m\rangle = \frac{1}{3}\frac{B_\beta}{B_{\rm rot}}\sqrt{(L\mp K)(L\pm K+1)} \times\sqrt{(j\mp K\pm 2m)(j\pm K\mp 2m+1)}. \tag{10}$$

Indeed, only when $K = 1/2$ does the Coriolis interaction affect the diagonal elements. This is an effect of nonconserved K that is not present in our previous paper [22], where K was fixed as 5/2.

We denote $E_{00L0\tau} - E_{00L_{\text{g.s.}}01} = E(L)$ as ground-state band energies, $E_{10L01} - E_{00L_{\text{g.s.}}01} = E(L_\beta)$ as β-band energies, and $E_{00L11} - E_{00L_{\text{g.s.}}01} = E(L_\gamma)$ as γ-band energies. The corresponding wave function is expanded as

$$\Psi = \beta^{-1-\frac{B_\beta}{B_\gamma}} F(\beta) \sum_{mK} A_{LK}^{m\tau} \chi_{K|m|}(\gamma) |LMjKm\rangle, \quad (11)$$

where

$$|LMjKm\rangle = \sqrt{\frac{2L+1}{16\pi^2}} \big[D_{MK}^L(\theta_i) \varphi_{K-2m}^j(x_i) + (-1)^{L-j} D_{M-K}^L(\theta_i) \varphi_{-K+2m}^j(x_i) \big], \quad i = 1, 2, \quad \text{or} \quad 3, \quad (12)$$

$$\chi_{n_\gamma |m|}(\gamma) = N_\gamma \left(\frac{\gamma^2}{\mathrm{g}} \right)^{\frac{|m|}{2}} {}_1F_1\left(-n_\gamma, |m|+1, \frac{\gamma^2}{\mathrm{g}} \right), \quad (13)$$

and

$$F_{n_\beta n_\gamma L|m|\tau}(\beta) = N_\beta \beta^{q_{n_\gamma}^\tau(L,|m|)+1} \exp\left(-\frac{\beta^2}{2b^2} \right) \times L_{n_\beta}^{q_{n_\gamma}^\tau(L,|m|)+1/2} \left(\frac{\beta^2}{b^2} \right). \quad (14)$$

Here N_β and N_γ are normalization coefficients for β and γ wave functions, respectively, ${}_1F_1(-n_\gamma, |m|+1, \frac{\gamma^2}{\mathrm{g}})$ is a confluent hypergeometric function, $b = \frac{\beta_0}{\sqrt[4]{2g_\beta}}$, $L_{n_\beta}^{q_{n_\gamma}^\tau(L,|m|)+1/2}$ are Laguerre polynomials, $D(\theta_i)$ is Wigner function, $\varphi(x_i)$ is the wave function of the single-particle states, and $A_{LK}^{m\tau}$ are the coefficients of the expansion of the wave functions.

Then, if we denote deformability with respect to the β vibration as

$$S_{n_\beta n_\gamma L|m|\tau; n'_\beta n'_\gamma L'|m'|\tau'} = \int_0^\infty F_{n_\beta n_\gamma L|m|\tau} \frac{\beta}{\beta_0} F_{n_\beta n_\gamma L|m|\tau} d\beta, \quad (15)$$

the following expressions are obtained:

$$B(E2; L_{\text{g.s.}}+2 \to L_{\text{g.s.}}) = \frac{5Q_0^2}{16\pi} \left| \sum_{KK'} A_{LK}^{01} A_{L'K'}^{01} (L2K0|L'K') \right|^2 S_{00L01;00L'01}^2 \quad (16)$$

gives the ground-state intraband transition probabilities;

$$B(E2; L'_\beta \to L) = \frac{5Q_0^2}{16\pi} \left| \sum_{KK'} A_{LK}^{01} A_{L'K'}^{01} (L2K0|L'K') \right|^2 S_{10L01;00L'01}^2 \quad (17)$$

gives the interband $E2$ transition probabilities between the β and ground-state bands; and

$$B(E2; L_\gamma \to L) = \frac{5Q_0^2}{16\pi} \mathrm{g} \left| \sum_{KK'} A_{LK}^{11} A_{L'K'}^{01} [(L2K2|L'K') + (L2K-2|L'K')] \right|^2 S_{00L01;00L'11}^2 \quad (18)$$

gives the interband $E2$ transition probabilities between the γ and ground-state bands, where

$$S_{00L01;00L'01} = \frac{2}{\sqrt[4]{2g_\beta}} \frac{1}{\sqrt{\Gamma\big(q_0^1(L,0)+3/2\big)}} \times \frac{1}{\sqrt{\Gamma\big(q_0^1(L',0)+3/2\big)}} \times \Gamma\left[\frac{q_0^1(L',0)+q_0^1(L,0)+4}{2} \right], \quad (19)$$

$$S_{10L01;00L'01} = \frac{2}{\sqrt[4]{2g_\beta}} \frac{\sqrt{q_0^1(L',0)+3/2}}{\sqrt{\Gamma\big(q_0^1(L,0)+3/2\big)}} \times \frac{1}{\sqrt{\Gamma\big(q_0^1(L',0)+3/2\big)}} \times \Gamma\left[\frac{q_0^1(L',0)+q_0^1(L,0)+4}{2} \right] \times \left\{ 1 - \frac{q_0^1(L',0)+q_0^1(L,0)+4}{2q_0^1(L',0)+1} \right\}, \quad (20)$$

and

$$S_{00L01;00L'11} = \frac{2}{\sqrt[4]{2g_\beta}} \frac{1}{\sqrt{\Gamma\big(q_0^1(L,0)+3/2\big)}} \times \frac{1}{\sqrt{\Gamma\big(q_0^1(L',1)+3/2\big)}} \times \Gamma\left[\frac{q_0^1(L',1)+q_0^1(L,0)+4}{2} \right]. \quad (21)$$

III. SHELL-MODEL PREDICTIONS OF GROUND-STATE PROPERTIES

Our Bohr Hamiltonian studies are guided by shell-model results, which we report here. In these calculations, we have taken the model space to be that of an inert ^{132}Sn core, with the remaining neutrons (ν) and protons (π) in the 82–126 and 50–82 valence shells, respectively. Nucleons in this valence space are allowed to interact via the realistic CWG Hamiltonian, which is based on the CD-Bonn force [28]. The single-neutron energies of the $1h_{9/2}$, $2f_{7/2}$, $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$, and $1i_{13/2}$ orbits are −0.894, −2.455, −0.450, −1.601, −0.799, and 0.250 MeV, and the single-proton energies of the $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ orbits are −9.663, −9.000, −7.323, −7.223, and −6.870 MeV, respectively.

In ^{163}Dy, we allowed protons to fill in the $g_{7/2}$, $d_{5/2}$, and $s_{1/2}$ orbitals and neutrons in the $f_{7/2}$, $p_{3/2}$, $h_{9/2}$, $p_{1/2}$, and $f_{5/2}$ orbitals. Under these conditions, the ground state has a dominant configuration (∼29%) of $\pi(g_{7/2}^8 d_{5/2}^6 s_{1/2}^2) \otimes \nu(f_{7/2}^5 p_{3/2}^4 h_{9/2}^2 p_{1/2}^2 f_{5/2}^2)$. The occupancy of the $f_{7/2}$ orbital is 4.88 and that of $f_{5/2}$ is 2.07. Thus, the contribution of the neutron $f_{7/2}$ orbital is dominant in comparison to that of other orbitals.

TABLE I. The values of the parameters used in calculations.

Nucleus	ξ	g	g_β	$B_\beta/B_{\rm rot}$	B_β/B_γ
^{163}Dy	0.0347	0.0166	306	5.67	1.12
^{173}Yb	0.0347	0.0166	1187	11.1	1.54

For ^{173}Yb, we allowed protons to fill in the $g_{7/2}$, $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ orbitals, and neutrons in the $f_{7/2}$, $p_{3/2}$, $h_{9/2}$, $p_{1/2}$, and $f_{5/2}$ orbitals. In this model space, the ground state has a dominant configuration (∼59%) of $\pi(g^8_{7/2}d^6_{5/2}s^2_{1/2}d^4_{3/2}) \otimes \nu(f^8_{7/2}p^4_{3/2}h^2_{9/2}p^2_{1/2}f^5_{5/2})$. The occupancy of the $f_{7/2}$ orbital is highest. This reveals that the major contribution to the ground-state structure comes from the $f_{7/2}$ orbital. Thus, shell-model calculations predict that the orbital of greatest influence in the ground-state structure of both ^{163}Dy and ^{173}Yb is the neutron $f_{7/2}$ orbital.

It should be noted that we have to ignore the $\pi(1h_{11/2})$ and $\nu(1i_{13/2})$ orbitals in our calculation because a completely unrestricted shell-model calculation is not yet feasible for these two nuclei. However, in the pioneering work of Federman and Pittel [29], it was shown that the strong correlation between $\pi(1h_{11/2})$ and $\nu(1i_{13/2})$ orbitals plays an important role in the development of nuclear deformation in the rare-earth region. In ^{163}Dy, even with this restriction, our results show that the main contributions to the ground state are mainly from $2f_{7/2}$. This is in agreement with the ground-state Nilsson orbits contained in Table I in the recent experimental work by Cakirli *et al.* [30]. However, in ^{173}Yb, the same reasoning leads to the Nilsson orbital 5/2[512], which belongs to the $1h_{9/2}$ orbital. This is probably due to the severe truncation which we have had to perform in order to make a shell-model calculation feasible. In both cases a 5/2 ground state with negative parity is obtained, as seen experimentally.

IV. RESULTS AND DISCUSSIONS

In our earlier paper [21], we considered the Bohr Hamiltonian for odd-mass nuclei in the case when the projection of the nuclear total angular momentum onto the third axis K and that of external nucleon Ω are not conserved, and when the angular momentum of the external nucleon j is conserved. One common mass parameter was used there. Recently [22], we studied the properties of the odd-mass, deformed ^{173}Yb nucleus, using different mass parameters for rotation and vibration modes. K and Ω were considered to be conserved quantities, and thus the excited-state energies did not depend on j.

Here, where K and Ω are not conserved, the value of j affects the spectrum and to wave functions, as explained in Sec. I. As discussed in Sec. III, $j = 7/2$ is the appropriate value for the ^{163}Dy and ^{173}Yb nuclei.

As is discussed in Sec. II, both the excited-state energies in units of $E(7/2^-_{\rm g.s.})$ and reduced $E2$ transition probabilities in units of $B(E2; 9/2^- \to 5/2^-)$ depend on the mass parameter ratios $B_\beta/B_{\rm rot}$ and B_β/B_γ, and on the parameters ξ, g, and g_β. The values of these parameters used for the calculations are given in Table I.

For ^{173}Yb, the value of the mass parameter ratios, determined by analyzing experimental data for ^{172}Yb in Ref. [19], had been used in Ref. [22], and we shall use the same values here. The value of parameters ξ and g have changed here significantly as compared to those of Ref. [22]. The reason for this is that ξ connects single-particle states and β vibration states, while g connects β and γ vibration states. In Ref. [22], the values determined in Ref. [21] were used, where the difference in mass parameters for rotation and vibrations had not been taken into account. It was impossible to evaluate these parameters there, because in that case energy levels had a rigid arrangement, and the last term of Eq. (8) influenced only the values of energies, not their arrangement, due to fixing K and Ω. The arrangement of the new levels which appear because of the present approach should be taken into account. The influence of mass difference on the connection between single-particle and collective states is clearly seen in Eqs. (9) and (10) if one compares these formulas, inserting $B_\beta = B_{\rm rot}$. This causes changes to g accordingly. In the case of ^{163}Dy, the values of the mass parameter ratios obtained in Ref. [18] for ^{162}Dy are used, and g_β is thus changed accordingly.

In all that follows, two sets of calculations are presented: one with a Coriolis interaction, and one without it. Both sets use the same parameters for a given nucleus, but for those without the Coriolis interaction we omit the term $(-1)^{L-j}(L+1/2)(j+1/2)\delta_{K1/2}\delta_{m0}$ from the diagonal matrix elements of Eq. (9), and sets all off-diagonal elements to 0. This is equivalent to removing the $2(L_1 j_1 + L_2 j_2)$ term from Eq. (2). In this way, we can examine the contribution of the Coriolis interaction. A future study will be to employ, for a particular nucleus, different parameters for a Coriolis-including calculation and a Coriolis-neglecting calculation, optimized to give a best match to data in each case.

A. Analysis of spectra

In order to to see the arrangement of bands and levels with respect to the available experimental data, we present all levels appearing in the calculation with and without a Coriolis interaction up to 26 units of $E(7/2^-_{\rm g.s.})$ for ^{163}Dy in Fig. 1 and for ^{173}Yb in Fig. 2. The values of the levels which exist both in calculation and theory are listed in Tables II–IV. The calculation with a Coriolis interaction is denoted as Calc.^{a} and that without a Coriolis interaction is denoted as Calc.^{b} in the figures and tables.

From the figures it is seen that all the levels in the ground-state band in the sequence $5/2^-, 7/2^-, \ldots, 27/2^-$ exist in both calculations and in experiment for both nuclei. The sequence is not altered for any combination of parameters. The Dependence of the values of the energy levels on g_β is much stronger than ξ. This means that the collective effects are more important for these levels. Another set of levels including $1/2^-$, $3/2^-$, etc., hereafter denoted as the $1/2^-$ band, appear above $11/2^-$ for both nuclei, whose arrangement strongly depends on the mass parameters and the parameter ξ, which connects single-particle and collective states. These levels have a single-particle character. The parameter g affects

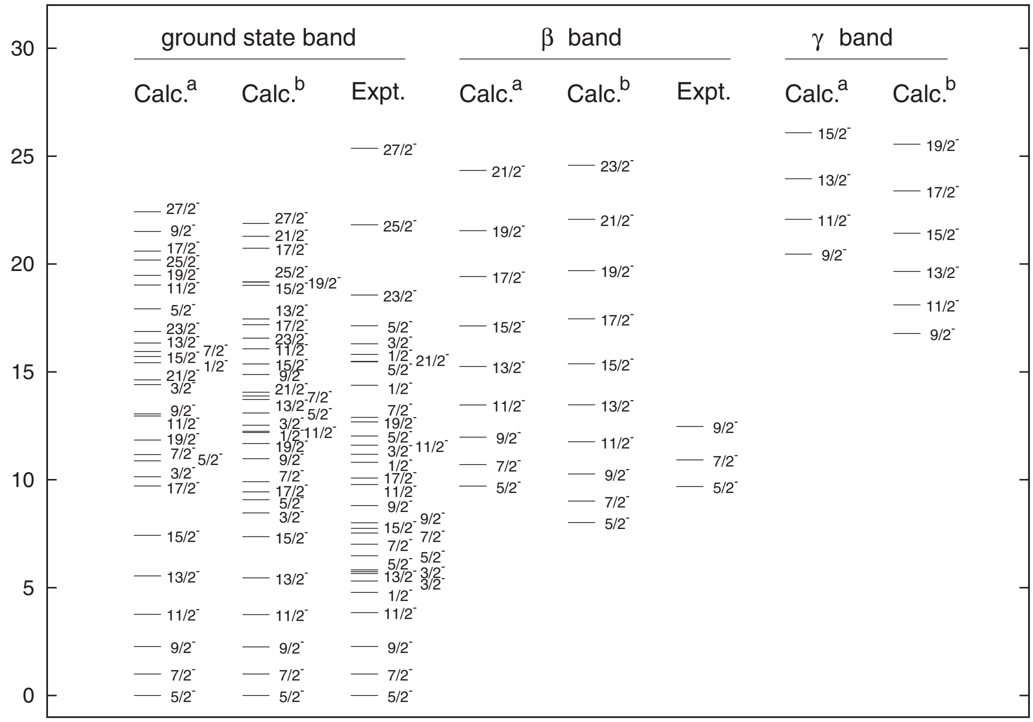

FIG. 1. Comparison of calculated values of energy levels in units of $E(7/2^-_{\rm g.s.})$ for ^{163}Dy with (Calc.[a]) and without a Coriolis interaction (Calc.[b]) with experimental data from Ref. [31].

only the γ-band energies. For these nuclei, the whole spectrum is collective single particle. In principle, the spectrum can be divided into single-particle and collective parts for very large and small values of ξ, and at given values of mass ratios. Of course, in the latter case our assumption of the conservation of j is not justified. It is deduced from Eqs. (4)–(7) that using the different mass parameters changes the connection between single-particle and collective states. Therefore, the value of ξ is

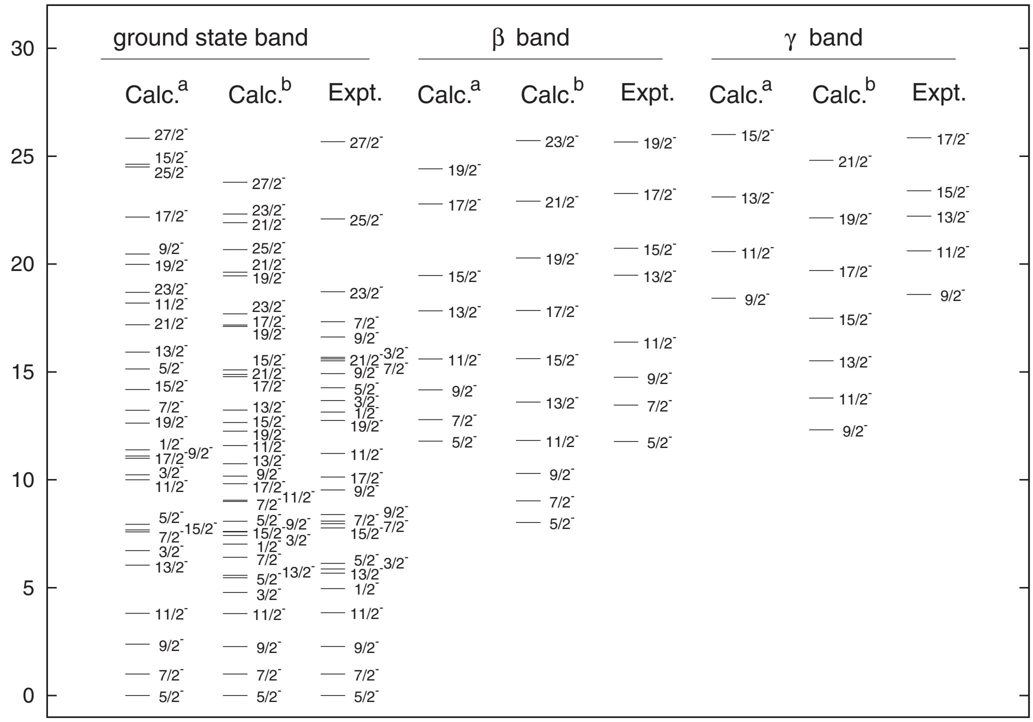

FIG. 2. Comparison of calculated values of energy levels in units of $E(7/2^-_{\rm g.s.})$ for ^{173}Yb with (Calc.[a]) and without a Coriolis interaction (Calc.[b]) with experimental data from Ref. [31].

TABLE II. The calculated and experimental values of the $E(L_{\rm g.s.})/E(7/2^-_{\rm g.s.})$ in ^{163}Dy and ^{173}Yb. The experimental values are taken from Ref. [31].

$L_{\rm g.s.}$	^{163}Dy			$L_{\rm g.s.}$	^{173}Yb		
	Calc.[a]	Calc.[b]	Expt.		Calc.[a]	Calc.[b]	Expt.
$9/2^-$	2.27	2.26	2.28	$9/2^-$	2.38	2.27	2.28
$11/2^-$	3.77	3.75	3.84	$11/2^-$	3.82	3.80	3.84
$13/2^-$	5.55	5.46	5.66	$13/2^-$	6.04	5.58	5.67
$15/2^-$	7.43	7.36	7.75	$15/2^-$	7.68	7.59	7.77
$17/2^-$	9.71	9.44	10.13	$17/2^-$	10.99	9.82	10.13
$19/2^-$	11.84	11.68	12.68	$19/2^-$	12.62	12.26	12.75
$21/2^-$	14.63	14.06	15.49	$21/2^-$	17.18	14.88	15.61
$23/2^-$	16.87	16.56	18.56	$23/2^-$	18.69	17.69	18.72
$25/2^-$	20.18	19.17	21.82	$25/2^-$	24.49	20.66	22.09
$27/2^-$	22.42	21.88	25.36	$27/2^-$	25.83	23.79	25.67
$1/2^-$	15.43	15.17	15.78	$1/2^-$	11.39	7.02	13.14
$3/2^-$	14.41	15.49	16.29	$3/2^-$	10.23	5.42	13.66
$5/2^-$	17.92	16.03	17.14	$5/2^-$	15.24	6.08	14.26
				$7/2^-$	13.23	9.00	15.51
				$9/2^-$	20.46	10.17	16.62

smaller than in Refs. [6,7,21], where the same mass parameter for rotation and vibrations is used.

The calculated values of the excited-state energies in the ground-state band, which exist in both calculation and in experiment, are given in Table II in units of $E(7/2^-_{\rm g.s.})$. We omit the first and second excited-state energies, since they are always 0 and 1, respectively, in the units used. Separate bands built on $1/2^-$ with the energies 15.78 and 13.14 in units of $E(7/2^-_{\rm g.s.})$ exist in the experiment [31] for ^{163}Dy and ^{173}Yb, respectively. These states appear as part of the ground-state band in the calculation. Therefore, we have presented them in the figures as a part of the ground-state band both in the calculation and experiment. We have presented them separately in the table, under a horizontal rule, in order to compare numerical values with data. The experimental values shown are taken from Ref. [31].

TABLE III. The calculated and experimental values of the $E(L_\beta)/E(7/2^-_{\rm g.s.})$ in ^{163}Dy and ^{173}Yb. The experimental values are taken from Ref. [31].

L_β	^{163}Dy			L_β	^{173}Yb		
	Calc.[a]	Calc.[b]	Expt.		Calc.[a]	Calc.[b]	Expt.
$5/2^-$	9.70	8.02	9.70	$5/2^-$	11.78	8.03	11.78
$7/2^-$	10.70	9.02	10.91	$7/2^-$	12.78	9.03	
$9/2^-$	11.98	10.27	12.47	$9/2^-$	14.16	10.30	13.46
$11/2^-$	13.47	11.76		$11/2^-$	15.60	11.83	14.75
$13/2^-$	15.25	13.47		$13/2^-$	17.83	13.60	16.38
$15/2^-$	17.13	15.38		$15/2^-$	19.46	15.61	19.48
$17/2^-$	19.42	17.46		$17/2^-$	22.78	17.84	20.73
$19/2^-$	21.55	19.70		$19/2^-$	24.40	20.28	23.27
$21/2^-$	24.33	22.07		$21/2^-$	28.96	22.91	27.99
$23/2^-$	26.58	24.58		$23/2^-$	30.47	25.72	30.63
$25/2^-$	29.89	27.19		$25/2^-$	36.27	28.69	33.15

TABLE IV. The calculated and experimental values of the $E(L_\gamma)/E(7/2^-_{\rm g.s.})$ in ^{163}Dy and ^{173}Yb. The experimental values are taken from Ref. [31].

L_γ	^{163}Dy		L_γ	^{173}Yb		
	Calc.[a]	Calc.[b]		Calc.[a]	Calc.[b]	Expt.
$9/2^-$	20.46	16.62	$9/2^-$	18.40	12.32	18.59
$11/2^-$	22.07	17.94	$11/2^-$	20.56	13.79	20.61
$13/2^-$	23.94	19.49	$13/2^-$	23.09	15.52	22.22
$15/2^-$	26.08	21.26	$15/2^-$	25.98	17.49	23.40
$17/2^-$	28.46	23.23	$17/2^-$	29.23	19.70	25.85
$19/2^-$	31.08	25.39	$19/2^-$	32.82	22.14	28.56
$21/2^-$	33.92	27.73	$21/2^-$	36.73	24.81	30.44
$23/2^-$	36.96	30.23	$23/2^-$	40.96	27.69	32.31

The comparison of the calculated values with and without a Coriolis interaction in the first band shows that for ^{163}Dy there are minor increases in all levels when the Coriolis interaction is applied. This brings the results slightly closer to the experimental values. For ^{173}Yb, a greater increase is observed. For both nuclei, the increases in energy becomes larger as the spin increases.

When the Coriolis interaction is neglected, the arrangement of the levels corresponds to the experiment in the $1/2^-$ band for both nuclei. However, when the Coriolis interaction is included, the following changes occur: In ^{163}Dy, levels of spin parity $3/2^-$ and $5/2$ are interchanged, and for ^{173}Yb two pairs of levels, $3/2^-$ and $5/2^-$, and $7/2^-$ and $9/2^-$, are interchanged.

The results of the calculations for β and γ bands are given in Tables III and IV, respectively, in units of $E(7/2^-_{\rm g.s.})$, for both ^{163}Dy and ^{173}Yb. For both nuclei the experimental data are available for excited-state energies in the ground-state band in Ref. [31]. For ^{163}Dy, only three states in the β band have been measured, but no measurements have been made in the γ band.

For both nuclei, the Coriolis interaction pushes the energies upward in all bands, though this effect is stronger in the β and γ bands than in the ground-state band. As a consequence, the positions of the β and γ bands change relative to the

TABLE V. The calculated and experimental values of the $B(E2;L_{\rm g.s.}+2\rightarrow L_{\rm g.s.})$ in units of $B(E2;9/2^-_{\rm g.s.}\rightarrow 5/2^-_{\rm g.s.})$ for ^{163}Dy. The experimental values Expt.[a] and Expt.[b] are taken from Refs. [31] and [32], respectively.

$\frac{B(E2;L_{\rm g.s.}+2\rightarrow L_{\rm g.s.})}{B(E2;9/2^-_{\rm g.s.}\rightarrow 5/2^-_{\rm g.s.})}$	^{163}Dy			
	Calc.[a]	Calc.[b]	Expt.[a]	Expt.[b]
$11/2^-\rightarrow 7/2^-$	1.54	1.54	–	1.63(11)
$13/2^-\rightarrow 9/2^-$	1.85	1.86	1.89(35)	2.04(44)
$15/2^-\rightarrow 11/2^-$	2.04	2.06	2.61(48)	2.80(31)
$17/2^-\rightarrow 13/2^-$	2.18	2.20	2.25(41)	2.44(18)
$19/2^-\rightarrow 15/2^-$	2.28	2.31	2.34(43)	2.6(4)
$21/2^-\rightarrow 17/2^-$	2.38	2.41	2.25(41)	2.44(24)
$23/2^-\rightarrow 19/2^-$	2.45	2.50	2.07(38)	2.28(35)

TABLE VI. The calculated and experimental values of the $B(E2; L_{g.s.}+2 \rightarrow L_{g.s.})$ in units of $B(E2; 9/2^-_{g.s.} \rightarrow 5/2^-_{g.s.})$ for ^{173}Yb. The experimental values Expt.[a] and Expt.[b] are taken from Refs. [31] and [33], respectively.

$\frac{B(E2;L_{g.s.}+2\rightarrow L_{g.s.})}{B(E2;9/2^-_{g.s.}\rightarrow 5/2^-_{g.s.})}$	^{173}Yb			
	Calc.[a]	Calc.[b]	Expt.[a]	Expt.[b]
$11/2^- \rightarrow 7/2^-$	1.52	1.53	2.03(33)	1.75(12)
$13/2^- \rightarrow 9/2^-$	1.83	1.84	2.06(33)	1.79(12)
$15/2^- \rightarrow 11/2^-$	2.00	2.01	2.31(36)	2.00(13)
$17/2^- \rightarrow 13/2^-$	2.11	2.14	2.93(50)	2.52(15)
$19/2^- \rightarrow 15/2^-$	2.21	2.23	3.21(50)	2.82(19)
$21/2^- \rightarrow 17/2^-$	2.25	2.30	3.26(54)	2.77(16)
$23/2^- \rightarrow 19/2^-$	2.33	2.36	3.37(70)	2.72(6)
$25/2^- \rightarrow 21/2^-$	2.35	2.42		2.77(7)

ground-state band, which is seen in Figs. 1 and 2 and Tables III and IV.

B. $E2$ transition probability analysis

In Tables V–VIII, the calculated values of reduced $E2$ transition probabilities are given for the intraband ground-state band transitions, interband transitions from the β to the ground-state band, and interband transitions from the γ to the ground-state band, respectively, all in units of $B(E2; 9/2^-_{g.s.} \rightarrow 5/2^-_{g.s.})$. Two sources of experimental data are available only for the intraband transition probabilities in Refs. [31] and [32]. Thus, in Tables VII and VIII we tabulated only the calculated values, with and without a Coriolis interaction.

Table V shows that for ^{163}Dy, similarly to the case of energy levels, the inclusion of the Coriolis interaction has only a small impact on the $E2$ transition probabilities, decreasing them. However, this behavior becomes more pronounced as spins increase. The data from Ref. [31] (Expt.[a]) is in better agreement with the calculated results than that of Ref. [32] (Expt.[b]).

In the same manner Table VI shows that the inclusion of the Coriolis interaction decreases the $E2$ transition probabilities of ^{173}Yb as well. Again, this behavior becomes more pronounced

TABLE VII. The calculated values of the $B(E2; L_\beta \rightarrow L_{g.s.})$ and $B(E2; L_\gamma \rightarrow L_{g.s.})$ in units of $B(E2; 9/2^-_{g.s.} \rightarrow 5/2^-_{g.s.})$ for ^{163}Dy.

^{163}Dy					
$\frac{B(E2;L_\beta\rightarrow L_{g.s.})}{B(E2;9/2^-_{g.s.}\rightarrow 5/2^-_{g.s.})}$ $\times 10^3$	Calc.[a]	Calc.[b]	$\frac{B(E2;L_\gamma\rightarrow L_{g.s.})}{B(E2;9/2^-_{g.s.}\rightarrow 5/2^-_{g.s.})}$ $\times 10^3$	Calc.[a]	Calc.[b]
$9/2^- \rightarrow 5/2^-$	2.36	1.49	$9/2^- \rightarrow 5/2^-$	0.79	0.72
$13/2^- \rightarrow 9/2^-$	1.16	0.25	$9/2^- \rightarrow 9/2^-$	30.35	30.49
$23/2^- \rightarrow 19/2^-$	0.96	4.12	$9/2^- \rightarrow 13/2^-$	11.87	12.91
$9/2^- \rightarrow 9/2^-$	9.64	9.63	$11/2^- \rightarrow 9/2^-$	12.55	12.29
$13/2^- \rightarrow 13/2^-$	17.49	17.46	$11/2^- \rightarrow 13/2^-$	32.12	34.02
$17/2^- \rightarrow 17/2^-$	20.07	20.05	$13/2^- \rightarrow 9/2^-$	2.03	1.93
$9/2^- \rightarrow 13/2^-$	29.45	34.37	$13/2^- \rightarrow 17/2^-$	10.37	11.64
$13/2^- \rightarrow 17/2^-$	68.17	79.92	$15/2^- \rightarrow 13/2^-$	16.59	15.88

TABLE VIII. The calculated values of the $B(E2; L_\beta \rightarrow L_{g.s.})$ and $B(E2; L_\gamma \rightarrow L_{g.s.})$ in units of $B(E2; 9/2^-_{g.s.} \rightarrow 5/2^-_{g.s.})$ for ^{173}Yb.

^{173}Yb					
$\frac{B(E2;L_\beta\rightarrow L_{g.s.})}{B(E2;9/2^-_{g.s.}\rightarrow 5/2^-_{g.s.})}$ $\times 10^3$	Calc.[a]	Calc.[b]	$\frac{B(E2;L_\gamma\rightarrow L_{g.s.})}{B(E2;9/2^-_{g.s.}\rightarrow 5/2^-_{g.s.})}$ $\times 10^3$	Calc.[a]	Calc.[b]
$9/2^- \rightarrow 5/2^-$	1.61	0.80	$9/2^- \rightarrow 5/2^-$	1.00	0.77
$13/2^- \rightarrow 9/2^-$	0.98	0.48	$9/2^- \rightarrow 9/2^-$	31.09	31.77
$17/2^- \rightarrow 13/2^-$	0.07	0.28	$9/2^- \rightarrow 13/2^-$	10.46	12.90
$9/2^- \rightarrow 9/2^-$	5.01	5.01	$11/2^- \rightarrow 9/2^-$	14.30	12.95
$13/2^- \rightarrow 13/2^-$	9.08	9.08	$11/2^- \rightarrow 13/2^-$	29.86	34.31
$17/2^- \rightarrow 17/2^-$	10.37	10.41	$13/2^- \rightarrow 9/2^-$	2.67	2.06
$9/2^- \rightarrow 13/2^-$	14.02	18.01	$13/2^- \rightarrow 17/2^-$	8.86	11.32
$13/2^- \rightarrow 17/2^-$	33.56	42.75	$15/2^- \rightarrow 13/2^-$	17.77	16.43

as spin parities increase. In contrast to the case of ^{163}Dy, here the data of Ref. [33] better fits the calculated results than that of Ref. [31].

From Tables VII and VIII it is seen that the Coriolis contributions are essential for all the transitions, except for the transitions from a state of a particular spin to a state of the same spin, where the difference between both calculations is approximately zero.

V. SUMMARY

We have studied the effect of the Coriolis contribution toward the excited states of deformed nuclei, using the Hamiltonian with different mass parameters for the rotation and vibrations. Analytical expressions have been obtained for the excited-state energies and $E2$ transition probabilities. Also large-scale shell-model calculations have been carried out in order to determine ground-state properties of the considered nuclei.

The spectra and $E2$ transition probabilities are discussed with and without a Coriolis interaction. We have found good agreement with experimental data if we include a Coriolis interaction, while for all bands without a Coriolis interaction the energy levels are more compressed. Further, without a Coriolis interaction the β and γ vibrational bands are shifted down in comparison to the experimental data.

For reduced $E2$ transition probabilities of odd-mass deformed nuclei the Coriolis interaction contributes significantly. However, the effect of the Coriolis interaction on the intraband $E2$ transition probabilities between bands of the same spin is insignificant.

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