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Sensitivity of neutron to proton ratio toward the high density behavior of the symmetry energy in heavy-ion collisions

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The symmetry energy at sub- and supra-saturation densities has great importance for understanding the exact nature of asymmetric nuclear matter as well as neutron stars, but it is poorly known, especially at supra-saturation densities. We will demonstrate here whether or not the neutron-to-proton ratios from different kinds of fragments can determine the supra-saturation behavior of the symmetry energy. For this purpose, a series of Sn isotopes were simulated at different incident energies using the isospin quantum molecular dynamics (IQMD) model with either a soft or a stiff symmetry energy. It is found that the single neutron-to-proton ratio from free nucleons as well as Light Charged Particles (LCPs) is sensitive to the symmetry energy, incident energy, and isospin asymmetry of the system. However, with the double neutron-to-proton ratio, this is true only for the free nucleons. It is possible to study the high-density behavior of symmetry energy by using the neutron-to-proton ratio from free nucleons.

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I. INTRODUCTION

From the Bethe-Weizsäcker mass formula, it is well understood that the symmetry energy from bulk matter is the difference between the energy of pure neutron matter and pure symmetric matter. Mathematically, it can we written as

$$E_{\text{Sym}}(\rho,\delta) = E(\rho,\delta=1) - E(\rho,\delta=0), \tag{1}$$

where $\delta = \frac{\rho_n-\rho_p}{\rho_n+\rho_p}$ and $\rho = \rho_n + \rho_p$. ρ_n, ρ_p, and ρ are the neutron, proton, and nuclear matter densities, respectively. The symmetry energy has great importance in the dense matter existing in the neutron stars, but only indirect information can be extracted from astrophysical observations [1]. It is also important in the quark-gluon plasma (QGP) and hadron gas (HG) phases [2]. The QGP and HG phases existed in the early stage of the evolution of Universe (about 15 billion years ago) and are inaccessible nowadays. It is difficult to recreate these conditions, although numerous experiments are occurring at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) [3]. Heavy-ion reactions, during which matter goes through compression and expansion, are considered to be the true testing ground for the hot and dense matter phases. The nuclear equation of state (NEOS) and the density dependence of the symmetry energy can be probed by some observables in intermediate-energy heavy-ion collisions (HICs). The softness of the NEOS has been well described in the literature in the last couple of decades [4,5]. However, the density dependence of the symmetry energy, from the Coulomb barrier to the deconfinement of nuclear matter, is a hot topic in the present era. At sub-saturation densities, the density dependence of the symmetry energy is studied by observables such as the neutron-to-proton ratio, isotopic and isobaric scaling, isospin diffusion, isospin fractionation and/or distillation, and isospin migration [6–9]. Recently, the MSU group [7] claimed the softness of the symmetry energy at sub-saturation densities by using the double neutron-to-proton ratio and isospin diffusion from two isotopic systems, ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{124}Sn at $E = 50$ MeV/nucleon. In another study, again soft symmetry energy was claimed by using the isospin diffusion for the same set of reactions, but at $E = 35$ MeV/nucleon [8]. In a recent study, soft symmetry energy is also favored for the same set of reactions at $E = 50$ MeV/nucleon by using the neutron-to-proton ratio [9]. In all the studies, the problem of sub-saturation density dependence of the symmetry energy seems to be addressed to some extent; however, the uncertainties are still large enough to justify the large amount of work that is under way in many laboratories all over the world.

In contrast, the present status of supra-saturation density dependence of the symmetry energy is quite uncertain and interesting. The high-density behavior of the symmetry energy in the literature is studied by using two important parameters: one is the yield ratio parameter and second is the flow parameter. The yield ratio parameter has been studied in terms of single and double ratios of neutrons to protons [10–12], single and double ratios of π^-/π^+ [10,13–18], the Σ^-/Σ^+ ratio [14], the K^-/K^+ ratio [15], and isospin fractionation [12], while the flow parameter has been studied in terms of relative and differential flows (single and double ratios) of neutrons to protons or ^{3}H to ^{3}He [10,19], and in terms of the ratio [20] or difference [21] of neutron-to-proton elliptic flow. Before using the ^{3}H and ^{3}He particle yield and flow ratios for the density dependence of the symmetry energy at high incident energies, one must check the production of these particles in the supra-saturation density region, which is obtained during the highly compressed stage only. However, the production of neutrons and protons occurs in large amounts, and can explain the high density dependence of the symmetry energy with great accuracy. Favorable results with neutron and proton elliptic flow at $E = 400$ MeV/nucleon were also observed in 2011. In one of the studies, the softness of the symmetry energy with

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$\gamma_i = 0.9$ is predicted by comparing the FOPI Collaboration data with the neutron-to-proton elliptic flow ratio [20]. In the same year, Cozma *et al.* [21] predicted the softness of the symmetry energy with $x = 2$ by comparing the FOPI data with the neutron-to-proton elliptic flow difference. Even then uncertainty lies in the results, in terms of determination of the symmetry energy: in the first study, the symmetry energy is momentum independent, while in later one it is from momentum-dependent interactions. Moreover, the studies were limited to only 400 MeV/nucleon.

Let us examine some interesting features from the ratio parameters at supra-saturation densities. All the ratio parameters show sensitivity to the symmetry energy. In the literature, it is also claimed that K^0/K^+ and Σ^-/Σ^+ have more sensitivity than π^-/π^+ [14,15]. The sensitivities of all the parameters is checked in term of transverse momentum and rapidity distribution dependence [10–12,14,18,19], while pion and kaon ratio studies are extended with the isospin asymmetry of the system and the incident energy [13,15–17]. In recent years, when the pion ratio has been compared with the FOPI data by using the two well known models IBUU04 and ImIQMD, in terms of isospin asymmetry and incident energy, the predictions for the symmetry energy are found to be totally opposite. ImIQMD predicts stiff symmetry energy ($\gamma_i = 2$) [17], while IBUU04 predicts soft symmetry energy ($x = 1$) [16].

In the present era, the π^-/π^+ ratio is supposed to be a strong candidate for predicting the high-density behavior of the symmetry energy. Just as for π^- and π^+, neutrons and protons are also produced in large amounts up to 1 GeV/nucleon. Even around 400 MeV/nucleon, the production of neutrons and protons is greater than the production of pions. Unfortunately, the neutron-to-proton ratio parameter in most studies is restricted only with the transverse momentum and kinetic energy dependences [10–12]. To draw a fruitful conclusion in the near future, first it is very important to study the isospin asymmetry and incident energy dependences of the neutron-to-proton ratio, just as in the recent π^-/π^+study, and then compare the sensitivity to the symmetry energy from both ratios, as the pion ratio results were recently compared with the FOPI experimental findings. Second, one has to avoid choosing randomly any type of fragment to study the supra-saturation density dependence of the symmetry energy. For this, it is important to check whether or not a particular type of fragment is formed in the region $\rho > \rho_0$, which is simple when one addresses the sub-saturation density dependence of the symmetry energy. Finally, with increasing incident energy, the time evolution of the density has a different trend at two extremes: one at the time of maximum compression and the second at the freeze-out time ($t = 200$ fm/c). It also becomes interesting to see the different stiffnesses of the symmetry energy dependence of the neutron-to-proton ratio for incident energies at the time of maximum compression and at freeze-out time.

In the concluding remarks of this paper, we have tried to address the following goals:

(i) To check the sensitivities of different kind of fragments to the high-density behavior of the symmetry energy.

(ii) To check the behavior of the neutron-to-proton ratio at the time of maximum compression and at freeze-out time.

(iii) To check the isospin asymmetry and incident energy dependences of single and double neutron-to-proton ratios from different neutron-rich systems to the high-density behavior of the symmetry energy, and then compare the sensitivity of the symmetry energy from the neutron-to-proton ratio with that from the pion ratio. This study is similar to recent studies using the pion ratio [13,15–17].

For the present study, the isospin quantum molecular dynamics (IQMD) model is used to generate the phase space of nucleons, which is discussed in Sec. II. The results are discussed in Sec. III, and we summarize the results in Sec. IV.

II. METHODOLOGY: ISOSPIN QUANTUM MOLECULAR DYNAMICS MODEL

In the IQMD model [22,23], nucleons are represented by wave packets, just as in the QMD model of Aichelin [4]. These wave packets of the target and projectile interact via the full Skyrme potential energy, which is represented by U and is given as

$$U = U_\rho + U_{\rm Coul}. \tag{2}$$

Here $U_{\rm Coul}$ is the Coulomb energy and U_ρ originates from the density dependence of the nucleon optical potential, and is given as

$$U_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma+1}\frac{\rho^{\gamma+1}}{\rho_0^\gamma} + E_{\rm Sym}^{\rm pot}(\rho)\rho\delta^2. \tag{3}$$

The first two of the three parameters of Eq. (3) (α and β) are determined by demanding that, at normal nuclear matter densities, the binding energy should be equal to 16 MeV and the total energy should have a minimum at ρ_0. The third parameter γ is usually treated as a free parameter. Its value is given in term of the compressibility:

$$\kappa = 9\rho^2\frac{\partial^2}{\partial\rho^2}\left(\frac{E}{A}\right). \tag{4}$$

The different values of compressibility give rise to soft and hard equations of state. The soft equation of state is employed in the present study with the parameters $\alpha = -356$ MeV, $\beta = 303$ MeV, and $\gamma = 7/6$, corresponding to an isoscaler compressibility of $\kappa = 200$ MeV. In the third term $E_{\rm Sym}^{\rm pot}$ is the potential part of the symmetry energy, which is adjusted on the basis of calculations from the microscopic or phenomenological many-body theory, having the form

$$E_{\rm Sym}^{\rm pot} = \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}. \tag{5}$$

Here $C_{s,p} = 35.19$ MeV, parameterized on the basis of the experimental value of the symmetry energy, is known as the symmetry potential energy coefficient. On the basis of the γ_i value, the symmetry energy is divided into two types with $\gamma_i = 0.5$ and $\gamma_i = 1.5$, corresponding to soft and stiff symmetry energies, respectively.

The total symmetry energy per nucleon employed in the simulation is the sum of the kinetic and potential terms and is given as

$$E_{\rm Sym}(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + E_{\rm Sym}^{\rm pot}, \quad (6)$$

where $C_{s,k} = \frac{\hbar^2}{3m}(\frac{3\pi^2\rho_0}{2})^{2/3} \approx 25$ MeV is known as the symmetry kinetic energy coefficient. The kinetic symmetry energy originates from the Fermi-Dirac distribution [24].

Finally, we get a density and isospin single-particle potential in nuclear matter as follows:

$$V_\tau(\rho,\delta) = \alpha\left(\frac{\rho}{\rho_0}\right) + \beta\left(\frac{\rho}{\rho_0}\right)^{\gamma} + E_{\rm Sym}^{\rm pot}(\rho)\delta^2 + \frac{\partial E_{\rm Sym}^{\rm pot}(\rho)}{\partial\rho}\rho\delta^2 + E_{\rm Sym}^{\rm pot}(\rho)\rho\frac{\partial\delta^2}{\partial\rho_{\tau,\tau'}}. \quad (7)$$

Here $\tau \neq \tau'$, $\frac{\partial\delta^2}{\partial\rho_n} = \frac{4\delta\rho_p}{\rho^2}$, and $\frac{\partial\delta^2}{\partial\rho_p} = \frac{-4\delta\rho_n}{\rho^2}$. The potential also depends on the momentum-dependent interactions, which are optional in the IQMD model.

Note that the γ used in the determination of the equation of state and γ_i used in the determination of the symmetry energy are different parameters. The interesting feature of symmetry energy is that its value increases with decreasing γ_i at sub-saturation densities, while the opposite is true at supra-saturation densities. In other words, soft symmetry energy is more pronounced at sub-saturation densities, while stiff symmetry energy is more pronounced at supra-saturation densities.

In the calculations, we use the isospin-dependent in-medium cross section in the collision term and the Pauli blocking effects as in the QMD model [4]. The cluster yields are calculated by means of the coalescence model, in which particles with relative momentum smaller than $P_{\rm Fermi}$ and relative distance smaller than R_0 are coalesced into a one cluster. The values of R_0 and $P_{\rm Fermi}$ for the present work are 3.5 fm and 268 MeV/c, respectively.

III. RESULTS AND DISCUSSIONS

The neutron-to-proton ratio is among the first observables that was proposed as a possible sensitive probe for symmetry energy prediction at sub-saturation densities [6,7]; however, some studies are also performed using rapidity distribution and transverse momentum dependences at supra-saturation densities. In this article, the sensitivities of free nucleons, light charged particles (LCPs, having charge number between 1 and 2), and intermediate mass fragments (IMFs, having charge number between 3 and $Z_{\rm Total}/6$) to the high-density behavior of the symmetry energy are checked, providing the results of incident energy and isospin asymmetry dependences of single and double neutron-to-proton ratios with the high-density sensitive fragments.

To perform the study, thousand of events are simulated for the isotopes of Sn, namely ^{112}Sn + ^{112}Sn, ^{124}Sn + ^{124}Sn, and ^{132}Sn + ^{132}Sn between incident energies of 50 and 600 MeV/nucleon at semicentral geometry by using the soft and stiff symmetry energies of $\gamma_i = 0.5$ and 1.5, respectively. As discussed earlier, a soft equation of state with an isospin- dependent nucleon-nucleon (NN) cross section of $\sigma_{\rm med} = (1 - 0.2\frac{\rho}{\rho_0})\sigma_{\rm free}$ is employed. The incident energy and isospin asymmetry dependences of single and double neutron-to-proton ratios, just as for the π^-/π^+ ratio [16,17], are considered as a point of importance in the present study. The single ratio is just the ratio of neutrons to protons and is represented in the study by $R(N/Z)$, while double ratio is the ratio of the single ratios of any two isotopes of Sn. In order to study the systematics of the isospin effects, the single ratio of the isotope with a greater number of neutrons is always mentioned in the numerator when the double ratio is calculated. Mathematically, the double ratio is represented by $DR(N/Z)$ and is given as

$$DR(N/Z) = \frac{R(N/Z)^{\rm neutron\ rich}}{R(N/Z)^{\rm neutron\ weak}}. \quad (8)$$

To predict the high-density behavior of the symmetry energy, the very first point is to understand the time evolution of the average density at different incident energies. With increasing incident energy, the density will be expected to be greater than the normal nuclear matter density in the most compressed region. As we know, the density of the environment surrounding the nucleons of a fragment plays a crucial role in determining the physical process behind its formation. In Fig. 1, we display the average density $\langle\rho/\rho_0\rangle$ reached in a typical reaction as a function of time at different incident energies for ^{132}Sn + ^{132}Sn by using the soft symmetry energy $\gamma_i = 0.5$. The average nucleon density is calculated

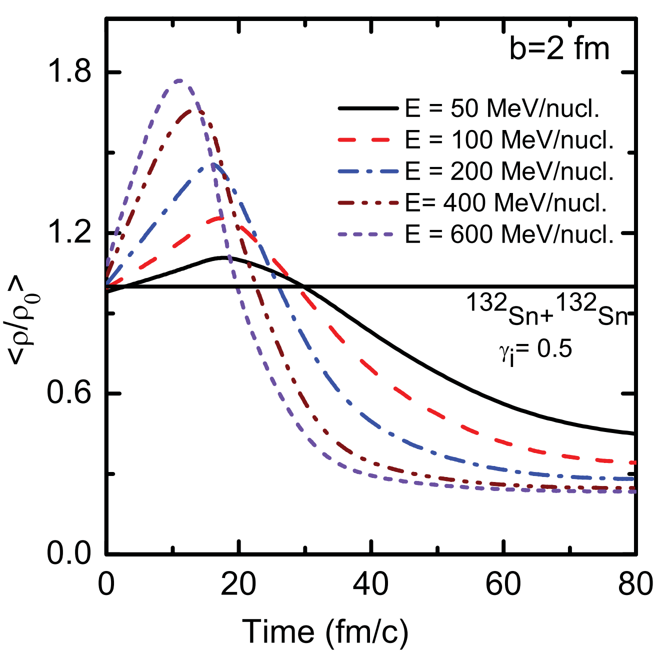

FIG. 1. (Color online) Time evolution of the average density for the ^{132}Sn + ^{132}Sn reaction with the soft symmetry energy ($\gamma_i = 0.5$) at semicentral geometry. The different lines represent different incident energies ranging from 50 to 600 MeV/nucleon.

as [4]

$$\langle \rho \rangle = \left\langle \frac{1}{A_T + A_P} \sum_{i=1}^{A_T+A_P} \sum_{j=1}^{A_T+A_P} \frac{1}{(2\pi L)^{3/2}} \times \exp[-(\vec{r}_i - \vec{r}_j)^2/2L] \right\rangle, \quad (9)$$

with $\vec{r}_i$ and $\vec{r}_j$ being the position coordinates of the ith and jth nucleons, respectively.

As we expected, with increasing incident energy, the density is found to increase in the compression zone. Interestingly, at lower beam energies, the maximum density reached is lower and the reaction time is longer. With increasing incident energy, the lifetime of the high-density nuclear matter gets shorter due to instability. For example, at $b = 2$ fm the average density reaches a maximum, and is close to normal nuclear matter density at $t = 18$ and 33 fm/c, respectively, for $E =$ 50 MeV/nucleon; but for the case of $E = 600$ MeV/nucleon, the respective times are 10 and 20 fm/c. This means that the difference between the two times is almost 15 fm/c at $E = 50$ MeV/nucleon, while it is only 10 fm/c at $E =$ 600 MeV/nucleon. This clearly indicates that the matter shows high-density behavior only for a small time interval, which decreases with increasing incident energy. Since we are interested in the sensitivities of different kinds of fragments and their neutron-to-proton ratios, only those fragments that lie in the high-density region ($\rho > \rho_0$) will be sensitive to the high-density behavior of the symmetry energy.

To check the sensitivities of different kind of fragments, in Fig. 2 we display time evolution of free nucleons (top), LCPs (middle), and IMFs (bottom) at semicentral geometry for incident energies ranging from 50 to 600 MeV/nucleon. The behavior for all kinds of fragments is consistent with the results in the literature [25]. The production of free nucleons increases with incident energy, and LCP production decreases after 400 MeV/nucleon. In Ref. [23], LCP production is correlated with the nuclear stopping and is also found to have a maximum at 400 MeV/nucleon. IMF production is found to decrease after 100 MeV/nucleon. This is due to the different origins of the production of IMFs as compared to free nucleons and LCPs. For more details about the incident energy dependence of IMFs, please see Ref. [25].

Our main task is to check the sensitivities of the fragments in the high-density region. For this, we apply the limit that at least one particle must be produced before the time 20 fm/c, because, in an average, after that time the density becomes lower than normal nuclear matter density for all the incident energies under consideration. The free nucleons are highly sensitive at all the energies. This is not true for LCPs and IMFs. LCPs are produced in this region only after the incident energy reaches 200 MeV/nucleon. In contrast, no IMFs are produced in the supra-saturation density region. This means that IMFs are not so sensitive to the high-density dependence of the symmetry energy; however, they can be used at subsaturation and saturation densities [7]. Interestingly, the single neutron-to-proton ratio from IMFs is found to change with the incident energy (not shown here), but this is mainly due to Coulomb interactions. Here we conclude that the neutron-to-proton ratio

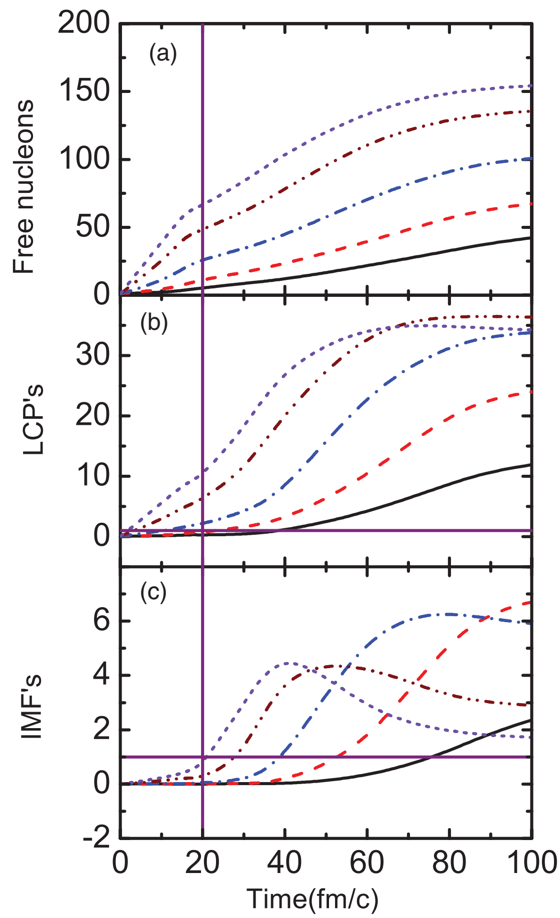

FIG. 2. (Color online) Time evolution of free nucleons (top), LCPs (middle), and IMFs (bottom) at semicentral geometry for ^{132}Sn + ^{132}Sn using the soft symmetry energy ($\gamma_i = 0.5$). The different lines have the same meaning as in Fig. 1. The vertical line in each panel represents our time limit before which the system can be in the supra-saturation region (refer to Fig. 1).

from free nucleons as well as LCPs can act as a probe of the high-density behavior of the symmetry energy.

One more interesting observation is obtained from Fig. 1. With increasing incident energy, the time evolution of the density is exactly opposite during the compression and expansion stages. That is, in the expansion stage the average density is found to decrease with increasing incident energy, which was earlier increasing in the compressed zone. Now, we have two aspects of the basis of the time evolution of density: one is the compressed-zone time and second is the freeze-out time. Interestingly, if the density behavior is opposite at the two times, then it would supposedly affect the magnitude of the symmetry energy as well as its effect on the nuclear matter during the whole time evolution.

To see the virtual change in the symmetry energy due to the change in the density, we display in Fig. 3 the incident energy dependence of $(\rho/\rho_0)^{\gamma_i}$, which is proportional to the symmetry energy, at the time of the maximum compression (left panel) and at the freeze-out time (right panel). At the time of maximum compression, the symmetry energy rises with the increasing incident energy (increase in density). As

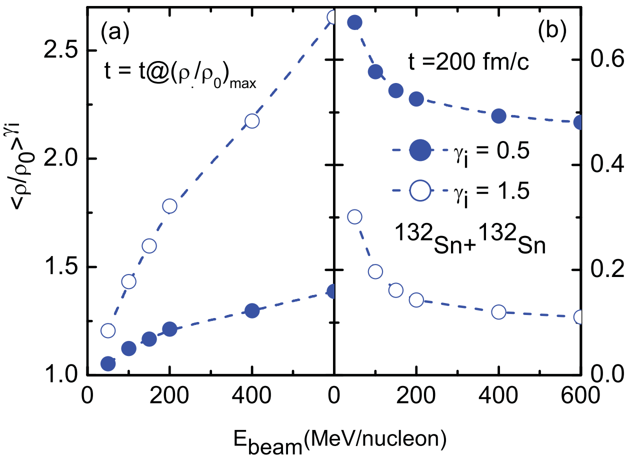

FIG. 3. (Color online) Excitation function of $(\rho/\rho_0)^{\gamma_i}$, which is proportional to E_{Sym}, at semicentral geometry. The left panel is at the time of maximum compression, while the right one is at the freeze-out time. Solid and open circles with a dashed line represent the contributions of the soft ($\gamma_i = 0.5$) and stiff ($\gamma_i = 1.5$) symmetry energies, respectively, for the system ^{132}Sn + ^{132}Sn.

the density is more than the normal nuclear matter density in this region, the stiff symmetry energy is stronger than the soft one. With increasing incident energy (increase in density), the stiff symmetry energy is changing drastically, while, the soft symmetry energy shows little change. This exactly coincides with the ideal picture of density dependence of the symmetry energy. On the other hand, if we look at the energy dependence of $(\rho/\rho_0)^{\gamma_i}$ at $t = 200$ fm/c, the situation is totally different. The symmetry energy is found to decrease with increasing incident energy (decrease in density). Now the density is lower than the normal nuclear matter density, so the magnitude of the soft symmetry energy is greater than that of the stiff symmetry energy. In other words, the supra-saturation (sub-saturation) density region is more neutron rich with $\gamma_i = 1.5$ ($\gamma_i = 0.5$). The effect from the sub- and supra-saturation density behaviors of the symmetry energy will compete and contribute in the final observables. Due to the different behavior of density at different times, it is important to observe the isospin effects at the time of maximum compression and at the freeze-out time to understand the high-density behavior of the symmetry energy. For this purpose, in the coming sections, the incident energy and isospin asymmetry dependences of the single and double neutron-to-proton ratios from free nucleons and LCPs are analyzed.

A. Incident energy and isospin asymmetry dependences of the single neutron-to-proton ratio

In order to address the sensitivity of the symmetry energy at the time of maximum compression and at freeze-out time, we display the incident energy and isospin asymmetry dependences of the single neutron-to-proton ratio at different times in Figs. 4–6. The left and right panels are at the time of maximum compression and freeze-out time, respectively. In Fig. 4, the ratios from free nucleons and LCPs are displayed

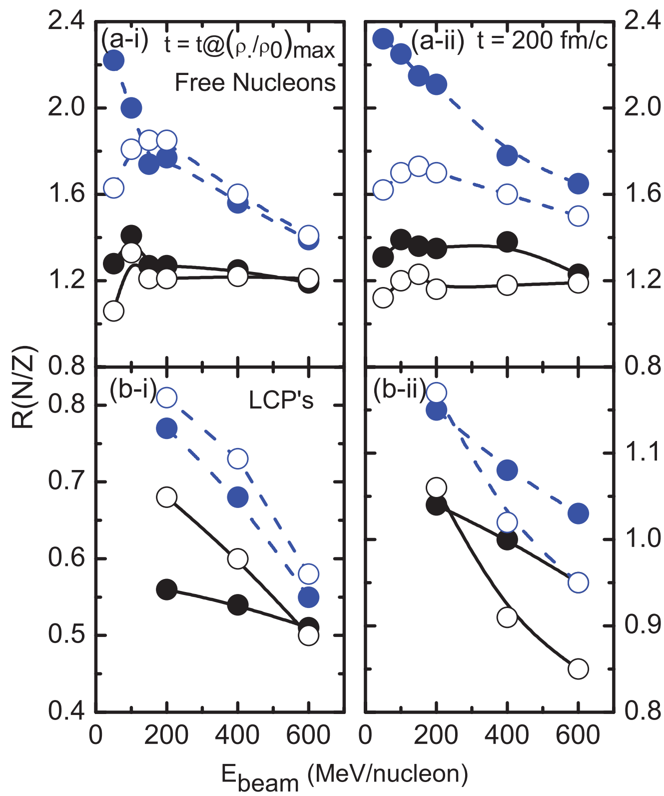

FIG. 4. (Color online) Excitation function of the single neutron-to-proton ratio for free nucleons (top panel) and LCPs (bottom panel) at semicentral geometry. The left and right panels are the same except they are at the time of maximum compression and at the freeze-out time, respectively. Solid and open circles represent the soft and stiff symmetry energies, respectively. Solid and dashed lines corresponds to the systems of ^{112}Sn + ^{112}Sn and ^{124}Sn + ^{124}Sn, respectively.

in the top and bottom panels. Many interesting facts are revealed in the figure. The incident energy dependences of the ratios are found to be highly sensitive to the symmetry energy for the two different times. As we know, the relative strength of the symmetry energy is opposite at sub- and supra-saturation densities with $\gamma_i = 0.5$ and $\gamma_i = 1.5$. In the range of 50–150 MeV/nucleon, only the low-density part up to about $1.1\rho_0$ contributes. Therefore, in the low-energy region, for free nucleons, we can see the high ratio with the soft symmetry energy at both the times under consideration. At and above 200 MeV/nucleon, a broad range of densities up to $1.8\rho_0$ is involved. Of course, at about 200 MeV/nucleon and above, for the behavior of the high-density symmetry energy, there is a combined effect for particles going through both low- and high-density regions. At higher energies, a higher N/Z ratio is observed with the stiff symmetry energy for the neutron-rich system ^{132}Sn + ^{132}Sn at the time of maximum compression, which is true with the soft symmetry energy at the freeze-out time. The result is similar for free nucleons and LCPs. However, LCPs are not as sensitive and the ratio is even less than the ratio of the system. This is due to the excess number of protons involved in the production of LCPs compared to free nucleons. These protons will lower the ratio for LCPs.

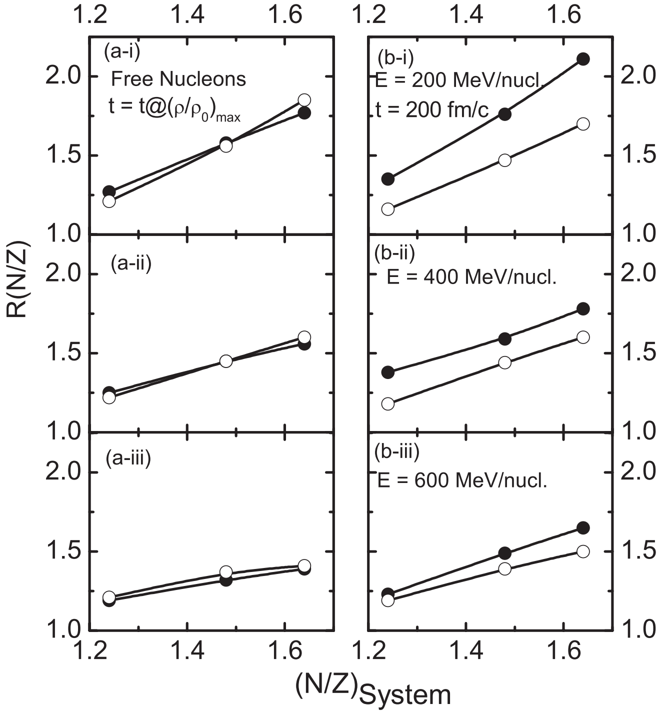

FIG. 5. Isospin asymmetry dependence of the single neutron-to-proton ratio for free nucleons at different incident energies. The left panel is at the time of maximum compression, while the right panel is at the freeze-out time. Solid and open circles represent the soft and stiff symmetry energies, respectively.

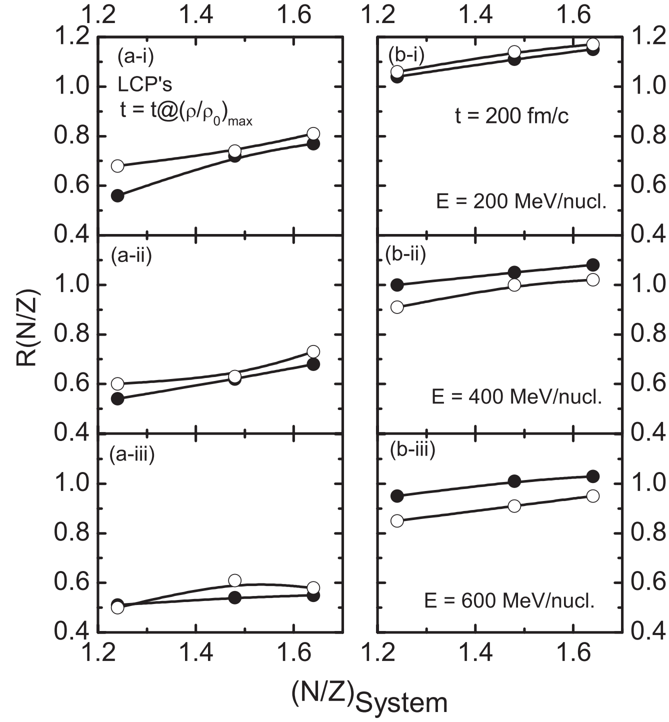

FIG. 6. Same as in Fig. 5 but for the LCPs.

It is clearly visible that the ratio at both times is almost the same with the stiff symmetry energy, but changes drastically with the soft symmetry energy. This is due to the fact that, at the time of maximum compression, the density is in the supra-saturation region and the stiff symmetry energy is much higher (see Fig. 3) than the soft symmetry energy. Therefore, the stiff symmetry energy is able to separate most of the neutrons near the time of maximum compression and then accelerate the neutrons toward higher kinetic energy at later times. However, the soft symmetry energy is not so high, and the separation of the neutrons takes place for a longer time. After 50–60 fm/c (see Fig. 1), the density drops to the sub-saturation density region and now the soft symmetry energy has a quite high magnitude (see Fig. 3) compared to the stiff one. The soft symmetry energy in this region is still separating the neutrons as well as accelerating them toward high kinetic energy. That is why the ratio with the soft symmetry energy drastically changes when one goes from compression to freeze-out time, but remains almost constant with the stiff symmetry energy.

Mainly, the neutron-to-proton ratio is found to decrease with the incident energy for free nucleons as well as for LCPs, just like the π^-/π^+ ratio. The decrease in the ratio may be due to two reasons:

(i) One reason may be the role of Coulomb interactions with incident energy. With increasing incident energy, chances of break-up of initial correlations among the nucleons become stronger, and the production of free nucleons including neutrons and protons will increase. However, at very low incident energy, the production of neutrons is more due to the symmetry energy because of its repulsive (attractive) nature for neutrons (protons). In short, due to Coulomb interactions, a shift of protons takes place from low to high incident energies. The effect of the Coulomb interactions can be checked by taking the double ratio, which is discussed in Fig. 7.

(ii) The contribution of pions from secondary-chance nucleon-nucleon collisions might increase with the beam energy. If a first-chance nucleon-nucleon collision converts a neutron to a proton by producing a π^-, then subsequent collisions of the energetic protons can convert them back to neutrons by producing a π^+. Therefore, at sufficiently high energy, the neutrons, which are produced due to the symmetry energy, are changing into the protons and further producing π's, which will lead to a decease in the neutron-to-proton ratio. This can be confirmed by using the double ratio concept. If the double ratio is still deceasing with incident energy, then it means that, in addition to the Coulomb interactions, the phenomenon of secondary nucleon-nucleon collisions is also very important.

One more point of interest is that the difference between the soft and stiff symmetry energies at freeze-out time is found to decrease with incident energy for free nucleons, while it increases for LCPs. Of the above two reasons, the first one is applicable for free nucleons as well as for LCPs. The second one is applicable only for free nucleons, as the energy in this study is up to 600 MeV/nucleon, which is quite sufficient to produce pions.

To see the effect of the high-density behavior of the symmetry energy on the isospin asymmetry dependence, we display the ratio from free nucleons and LCPs in Figs. 5 and 6 at

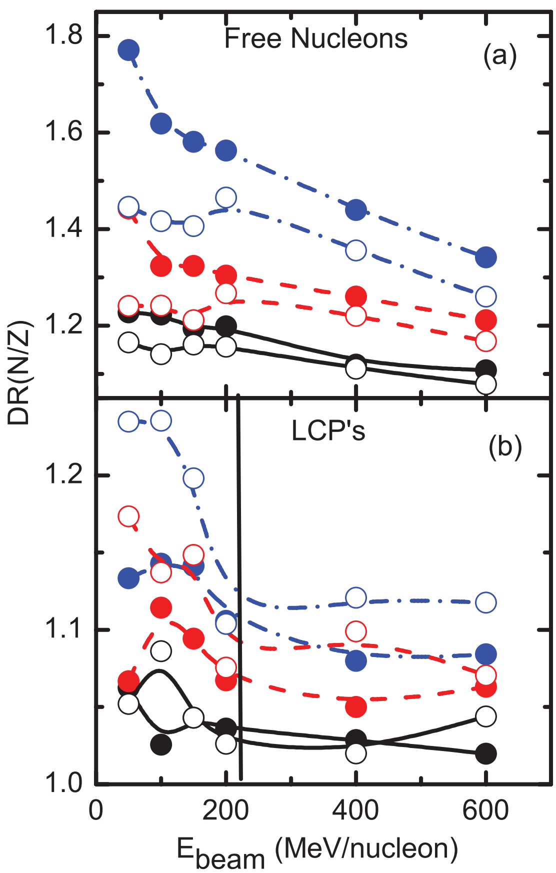

FIG. 7. (Color online) Excitation function of the double neutron-to-proton ratio from different isotopes of Sn for free nucleons (top) and LCPs (bottom). The vertical lines in the bottom panels represent the energy limit above which $DR(N/Z)$ of LCPs becomes more or less insensitive. Solid and open circles represent the soft and stiff symmetry energies, respectively. The solid, dashed, and dot-dashed lines corresponds to double ratios from ^{132}Sn + ^{132}Sn to ^{124}Sn + ^{124}Sn, ^{124}Sn + ^{124}Sn to ^{112}Sn + ^{112}Sn, and ^{132}Sn + ^{132}Sn to ^{112}Sn + ^{112}Sn, respectively.

only high energies (200, 400, and 600 MeV/nucleon). Due to the instability of the highly compressed zone, we are not able to differentiate between the results of the symmetry energy obtained at the time of maximum compression; however, we had earlier obtained some important conclusions from Fig. 4, where incident energy dependence was discussed.

The results from Figs. 5 and 6 at the freeze-out time reveal many important points. The isospin asymmetry dependence of the ratio from free nucleons is highly sensitive to the symmetry energy compared to LCPs, i.e., the ratio from free nucleons is found to be sharply increasing with the isospin asymmetry of the system compared to LCPs. This is due to the fact that isospin effects on the ratio from free nucleons are strongly affected by the symmetry energy and weakly affected by Coulomb interactions, while the opposite is true for the ratio from LCPs. As discussed earlier, the ratio is found to decease with the incident energy, which is also true here for the isospin asymmetry dependence.

The difference between the soft and stiff symmetry energy results comes from the behavior of free nucleon emission with the isospin asymmetry of the system, i.e., the greater the isospin asymmetry of the system, the greater the contribution of neutrons in the ratio due to the symmetry energy. The soft symmetry energy is stronger at the freeze-out time, which will lead to an increase in the ratio more sharply than the stiff symmetry energy. This effect is again weakly observed in the ratio from LCPs.

B. Incident energy and isospin asymmetry dependences of the double neutron-to-proton ratio

In order to cancel the Coulomb effects and to see the effect of the symmetry energy, we show in Fig. 7 the incident energy dependence of the double ratio from different isotopes of Sn with different combinations, namely, ^{132}Sn + ^{132}Sn and ^{124}Sn + ^{124}Sn, ^{124}Sn + ^{124}Sn and ^{112}Sn + ^{112}Sn, ^{132}Sn + ^{132}Sn and ^{112}Sn + ^{112}Sn, having differences of 8, 12, and 20 neutrons and the same number of protons. The upper and lower panels are for free nucleons and LCPs. The double ratio is found to increase with the difference of the number of neutrons in the different combinations. It is similar to the results obtained at sub-saturation densities by different models [6,7]. The main point to be discussed here is that the double ratio is found to decrease with increasing incident energy. As we have discussed in Fig. 4, there may be two reasons for the decrease of the single ratio with increasing incident energy. One was the Coulomb effect, which is canceled out here. The second was the pion effect, which is still active in the double ratio and becomes more and more dominant with increasing incident energy. Due to that effect, the double ratio is found to decrease with the incident energy. It indicates that the pion production effect is very important at high incident energy and is equally useful for understanding the high-density behavior of the incident energy [16,17].

In contrast, this effect is valid only for the double ratio from the free nucleons and not from the LCPs. The double ratio from LCPs is found to be constant above 200 MeV/nucleon. This indicates that the effect of the symmetry energy for the ratio from LCPs can be analyzed only near sub-saturation densities close to $1.1\rho_0$. The decrease in the single ratio for the LCPs was only due to the Coulomb interactions at higher incident energies, which is canceled out by taking the double ratio; the double ratio from the symmetry energy becomes independent of the incident energy after 200 MeV/nucleon. This type of dependence for the single π^-/π^+ ratio can be observed above 1 GeV/nucleon [13]. The behavior of the symmetry energy for the double ratio is exactly the same as that for the single ratio. This indicates that LCP production is also not a sensitive probe for investigating the high-density behavior of the symmetry energy. The only possible probe from the fragments is the double ratio of neutrons to protons from free nucleons. Another possible probe is the π^-/π^+ ratio, which recently was compared with the experimental data of the FOPI by the IBUU04 and ImIQMD calculations [16,17].

In order to strengthen our conclusion, in Fig. 8 we display the isospin asymmetry dependence of the double ratio from free nucleons at different incident energies. All the curves are fitted with a power law of the form $y = ax^{\tau}$, where y is the double ratio from free nucleons and x is the double

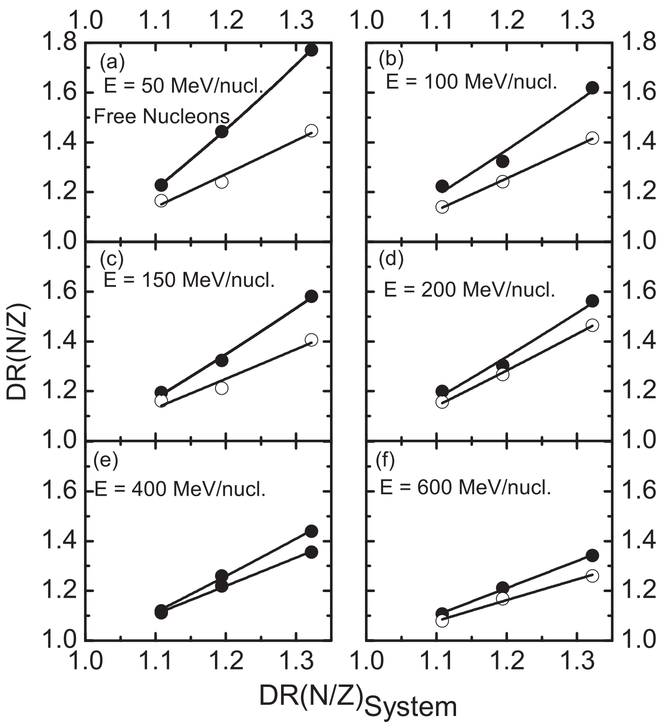

FIG. 8. Isospin asymmetry dependence of the double neutron-to-proton ratio from free nucleons at different incident energies. The different symbols have the same meanings as in Fig. 5.

ratio of the systems. The power-law exponent τ is found to vary drastically with the symmetry energy, which is to be discussed later in Fig. 9. After canceling the Coulomb effects, the trend for the double ratio is the same as that of the single ratio in Fig. 5. It reflects the fact that the isospin effect for free nucleons is stronger for more neutron-rich systems and is mainly due to the symmetry energy. However, the decrease in the isospin effect with the increase of incident energy is due to the production of pions at sufficiently high energy. The difference in the double ratio obtained with the soft and stiff symmetry energies here is also found to increase from the neutron-poor to the neutron-rich system, just like the single neutron-to-proton ratio in Fig. 5 as well as the single-pion ratio in the literature [16,17].

C. Incident energy dependence of the power-law exponent τ

To see the clear systematics of the incident energy toward the symmetry energy, we plot the incident energy dependence of the power-law exponent τ in Fig. 9, which is extracted from the curves of Fig. 8. With increasing incident energy, the sensitivity of the symmetry energy goes on decreasing toward the double ratio; however, the soft symmetry energy is more sensitive in comparison with the stiff one. In brief, when one goes from the sub-saturation to the supra-saturation density region, the soft symmetry still has a crucial role to play compared to the stiff one. This is due to the density (Fig. 1), which undergoes a sudden change between the supra- and sub-saturation density regions with time at higher incident energies.

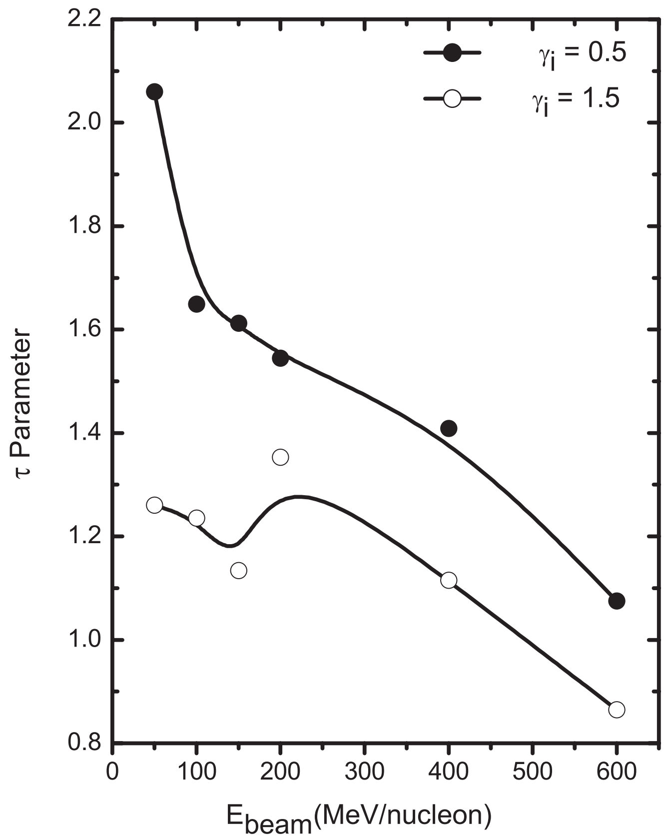

FIG. 9. Incident energy dependence of the power-law exponent τ from Fig. 8. The symbols and lines are the same as in Figs. 5 and 8.

Finally, from this study, we confirm that the high-density behavior of the symmetry energy can be studied by using the single and double ratios of neutrons to protons from free nucleons. In comparison, the double ratio is more accurate for this purpose, due to its greater sensitivity to the soft symmetry energy. Meanwhile, the lighter and heavier fragment ratios can be considered good candidates at sub-saturation densities, and also have been used in the literature many times by different groups [6,7].

IV. CONCLUSION

In order to investigate the high-density behavior of the symmetry energy, isospin asymmetry and beam energy dependences of neutron-to-proton ratios (single and double) from different kinds of fragments are studied by using the IQMD model. The single neutron-to-proton ratio from free nucleons and LCPs is found to decrease (increase) with incident energy (with the isospin asymmetry of the system). Stronger isospin effects are observed with the soft symmetry energy. Similar results with the π^-/π^+ ratio are also observed by Li *et al.* and Feng *et al.*, but with opposite behavior for the symmetry energy. The double neutron-to-proton ratio from free nucleons is highly sensitive to the symmetry energy, incident energy, and isospin asymmetry of the system. However, the sensitivity of the neutron-to-proton double ratio from LCPs to the nuclear symmetry energy is almost beam-energy independent above 200 MeV/nucleon. The same trend is observed for the single

π^-/π^+ ratio above 1 GeV/nucleon. The sensitivity of the soft symmetry energy to the ratio parameter is strongly affected by the choice of times, which is not true for the stiff symmetry energy. In simple words, just like the π^-/π^+ ratio, the neutron-to-proton double ratio from free nucleons can act as a useful probe to constrain the high-density behavior of the symmetry energy. Experiments are planned at MSU, GSI, RIKEN, and FRIB to determine the high-density behavior of the symmetry energy by using the neutron-to-proton ratio.

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