Residual interactions and the *K*-mixing-induced fast decay of the three-quasiparticle isomer in ^{171}Tm

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The anomalously fast decay of a $19/2^+$ three-quasiparticle isomer in ^{171}Tm was interpreted recently as an example of K mixing induced by a very small mixing matrix element but a (random) close proximity to a collective state. To understand the source of the residual interaction we have generalized the projected shell model by introducing two-body octupole and hexadecupole forces into the Hamiltonian and expanding the model space with inclusion of specific three-quasiparticle configurations. It is found that the K mixing is built up from small interactions transferred through numerous highly excited configurations that contain high-j orbitals. While the chance near-degeneracy enhances the transition strength, the octupole correlation and Coriolis coupling produce the mixing matrix element.

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I. INTRODUCTION

An interesting and important research area in nuclear physics has been the study of high-K isomers, both for understanding their structure and for evaluating their potential for applications [1,2]. High-K isomers are known specially in the mass $A = 170$–180 region of the nuclear chart because of the existence of the high-j Nilsson orbitals near the Fermi surface. A challenging problem is to understand the strength of electromagnetic transitions connecting high-K isomers to low-K states, when there is a large change in K, transitions which are, in principle, forbidden [3].

These rates are usually classified in terms of the reduced hindrance factor f_ν, where the forbiddenness $\nu = \Delta K - \lambda$ measures the shortfall between the change in K and the multipolarity λ. These reduced hindrances typically fall in the range $30 < f_\nu < 300$. Transitions with $f_\nu < 20$ need to be understood [4]. Different mechanisms have been proposed to explain the observed K-forbidden transitions, such as Coriolis mixing [5,6] based on the particle-rotor model, random mixings [6,7] due to large level densities for states significantly above the yrast line, and γ tunnelings through a barrier separating different shapes [8–12]. Specific transitions can also be enhanced in isolated cases by mixing of the initial or final states due to accidental degeneracies (or near-degeneracies) [5,13–16].

Although a unified view of the problem is difficult [17], cases of accidental degeneracies between high-K states and collective states provide a sensitive probe of K mixing and the residual interactions [5]. In a recent example, a $K^\pi = 19/2^+$, three-quasiparticle (qp) isomer was identified in ^{171}Tm [13] with a strong $E2$ decay branch to the $I^\pi = 15/2^+$ member of the $K^\pi = 7/2^+$ 1-qp band ($\pi 7/2^+[404]$), nominally a transition with $\Delta K = 6$ and $\nu = 4$. Its reduced hindrance factor f_ν is 7.6 [13], which is very low. The reason for this fast decay was attributed [13] to the chance near-degeneracy between the isomeric state and the $I^\pi = 19/2^+$ member of the $\pi 7/2^+[404]$ band. The mixing matrix element extracted using a two-level-mixing model [5] is small (12 eV), consistent with the systematics as a function of ΔK in Ref. [5].

While such a chance degeneracy with a collective state provides a way of measuring (very small) interactions, their magnitudes and sources remain to be explained. That is the focus of the present study: to find out the type(s) of interaction that induce the K mixing and the configurations involved in the mixed wave functions. The dominant structure of the isomer was suggested [13,18] to be the $\nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$ 3-qp configuration. This isomeric state decays to states that are band members based on the $\pi 7/2^+[404]$ 1-qp configuration. We note that the four quasiparticles that are involved originate from four different orbits; namely, they are from opposite parity orbits for both neutrons and protons. As is shown below, in order for the matrix elements to be nonvanishing, correlations must include the type that act between quasiparticle states of opposite parities.

To proceed in a quantitative way, we have performed projected shell model (PSM) calculations [19]. It will be shown that, with a few suitably chosen parameters in the model, we are able to achieve a precise description of the energy spectrum including the near-degeneracy of the $19/2^+$ 3-qp isomer and the $19/2^+$ member of the $\pi 7/2^+[404]$ band. We then build a microscopic "two-level model" with the two states, respectively, having the 3-qp $\nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$ and the 1-qp $\pi 7/2^+[404]$ as the major components, and mix them by a residual octupole-octupole interaction. Only by including a large number of configurations (with a qp excitation energy up to 4 MeV) into the mixing can we

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obtain a mixing matrix element with a value (∼8 eV) that is comparable to the value of 12 eV extracted from experiment [13]. Further analysis of the components in the wave functions reveals that (relatively) strong mixing occurs with those 3-qp configurations that involve the high-j orbitals $\nu i_{13/2}$ and $\pi h_{11/2}$. This suggests that the forces, which scatter the high-j particles in the mixing states up to high excitation energies, are probably of the Coriolis type.

II. OUTLINE OF THE MODEL

The projected shell model is a shell-model configuration-mixing method that builds its basis from (angular-momentum) projected multi-quasiparticle states

$$\hat{P}^I_{MK}|\Phi\rangle, \tag{1}$$

with $\hat{P}^I_{MK}$ being the angular-momentum projection operator and $|\Phi\rangle$ multi-qp states. The qp states are constructed from the solution of the deformed Nilsson model [20] followed by a BCS calculation. To describe qp excitations of odd-proton nuclei, the qp basis states $|\Phi\rangle$ are taken to be [19]

$$\{a^+_\pi|0\rangle, a^\dagger_\pi a^\dagger_\nu a^\dagger_\nu|0\rangle\}, \tag{2}$$

where $a^\dagger$ is the qp creation operator. In the early treatment of 3-qp states in odd-proton nuclei [21,22] and in the original PSM code [23], the two quasineutrons in a 3-qp state were restricted to those from the same major shell. Although the choice of this configuration space is suitable for a description of the rotational alignment of a pair of high-j neutrons [19], it is not general enough to allow two quasineutrons from different major shells. In the present work, the restriction has been relaxed for the 3-qp configuration part in Eq. (2) to include two quasineutrons selected either from the same major shell (with a total positive parity) or from two adjacent major shells (with a total negative parity).

The Hamiltonian is diagonalized in the projected basis (1) and the wave functions can be written as

$$|\Psi_{IM}\rangle = \sum_\kappa f_\kappa \hat{P}^I_{MK}|\Phi_\kappa\rangle. \tag{3}$$

In Eq. (3), κ labels the basis states. If the deformed Nilsson states are created with axial symmetry, as for the present case, the summation over κ may be replaced by K because each basis state $|\Phi_\kappa\rangle$ has a definite K quantum number. Then Eq. (3) implies explicitly a K-mixed wave function.

The rotational invariant Hamiltonian consists of the single-particle term h_0 and a sum of separable two-body forces,

$$\hat{H} = \hat{h}_0 - \sum_{\lambda=2}^{4}\frac{\chi_\lambda}{2}\sum_{\mu=-\lambda}^{\lambda}\hat{Q}^+_{\lambda\mu}\hat{Q}_{\lambda\mu} - G_M\hat{P}^+\hat{P} - G_Q\sum_{\mu=-2}^{2}\hat{P}^+_\mu\hat{P}_\mu, \tag{4}$$

in which h_0 is the spherical mean field. The strengths of the quadrupole and hexadecupole force ($\chi_2 \equiv \chi_Q$ and $\chi_4 \equiv \chi_H$) are determined by the self-consistent relation [24] respectively with the deformation parameters ϵ_2 and ϵ_4 that define the Nilsson states. For the present calculation for ^{171}Tm, we use $\epsilon_2 = 0.263$ and $\epsilon_4 = 0.058$, which are very close to those used in Ref. [18] and also to the values given by Möller *et al.* [25]. These deformation parameters are appropriate to reproduce the basic properties of this nucleus such as its rotational features and the bandhead energies of quasiparticle excitations. The strength of the octupole force ($\chi_3 \equiv \chi_O$) is treated as a free parameter, adjusted to reproduce the observed isomer transitions (see discussions below). For the pairing terms in the Hamiltonian, we adopt the same force strengths as for the previous PSM calculations [19]. The monopole pairing strength takes the form

$$G_M = \left(20.12 \mp 13.13\frac{N-Z}{A}\right)\Big/A(\text{MeV}),$$

with "−" for neutrons and "+" for protons. The quadrupole-pairing strength G_Q is assumed to be proportional to G_M, with the proportionality constant 0.20.

III. RESULTS AND DISCUSSION

The calculated level scheme is shown in Fig. 1 and compared with the known experimental data [13,18]. The overall agreement is excellent. In particular, the near-degeneracy between the $19/2^+$ 3-qp isomer and the $I^\pi = 19/2^+$ member of the $\pi 7/2^+[404]$ band is reproduced. In the PSM, the quadrupole force and (monopole and quadrupole) pairing forces have been well determined and tested in many previous calculations [19]. These guarantee a correct description of the rotational feature of deformed nuclei including electromagnetic transitions within each rotational band. Here we acknowledge the role of the two additional terms in the Hamiltonian (4), which, with the chosen strengths, are needed for reproducing the fine structure in the current problem. First, we note that the hexadecupole deformation ϵ_4 helps to provide the correct deformed single-particle energies, and the self-consistent relation of the corresponding two-body force strength χ_H with ϵ_4 ensures the precise positions of the calculated bandhead energy of the side bands. The calculated

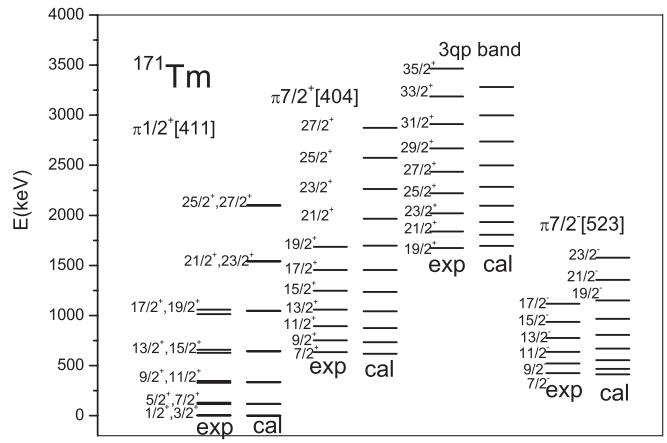

FIG. 1. Comparison of calculated ^{171}Tm levels with available data [13,18].

energies of the two $19/2^+$ states are then 11 keV apart, correctly reproducing the experimental measurement.

Next, we discuss the role of the octupole term in the Hamiltonian (4). As implied earlier, a near-degeneracy condition is not sufficient to induce mixing, the key factor is the residual interaction. One needs small (eV scale) but nonzero matrix elements to link the initial and final states of the decay. The main configuration of the initial state is $\nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$, and that of the final state is $\pi 7/2^+[404]$. In order to have an overlap between these two configurations, correlations that act between quasiparticle states of opposite parities are necessary. The quadrupole and hexadecupole interactions in the original PSM [19,23] have "basic contractions" only between orbitals of the same parity, so that they are unable to give the desired coupling matrix elements. That is the rationale for including a two-body octupole force in the Hamiltonian:

$$\hat{H}_{\rm oct} = -\frac{\chi_O}{2} \sum_{\mu=-3}^{3} \sum_{\tau\tau'} \hat{Q}^+_{3\mu}(\tau)\hat{Q}_{3\mu}(\tau'), \tag{5}$$

where τ and τ' label isospin in the summation running over protons and neutrons. The strength χ_O is assumed to be independent of τ or τ' for simplicity. The one-body operator $Q_{3\mu}$ is defined by

$$\hat{Q}_{3\mu} = \sum_{\alpha\beta} c^+_\alpha Q_{\mu\alpha\beta} c_\beta, \tag{6}$$

where c^+ and c are spherical single-particle operators, and

$$Q_{\mu\alpha\alpha'} = \sqrt{\frac{4\pi}{7}} \langle Njm| \left(\frac{r}{b}\right)^3 Y_{3\mu} |N'j'm'\rangle. \tag{7}$$

We stress that the octupole effect is introduced into the model as a residual interaction. We have not spontaneously broken the parity symmetry by introducing octupole deformations in the deformed mean field, as was done in early works [26,27]. It has indeed been found that with the strength $\chi_O = 5.46 \times 10^{-3}$ MeV, we are able to reproduce the observed isomer decay rates. The octupole strength is of the same order of magnitude as that used in other work [28–31], suggesting that our octupole strength, although allowed as a freely adjustable parameter in our model, is a reasonable one. The calculated $19/2^+$ 3-qp isomer decay rates to the states of the $\pi 7/2^+[404]$ 1-qp band are $B(E2, \frac{19}{2}^+(3\text{qp}) \rightarrow \frac{15}{2}^+(1\text{qp})) = 1.14 \times 10^{-4}$ W.u. and $B(M1, \frac{19}{2}^+(3\text{qp}) \rightarrow \frac{17}{2}^+(1\text{qp})) = 3.93 \times 10^{-8}$ W.u., which are comparable to the measured values 2.99×10^{-4} W.u. and 1.14×10^{-7} W.u., respectively.

To get insight from the theoretical results, it is important to study the effective mixing matrix element between the two near-degenerate levels, in the way introduced in Ref. [5]. In Ref. [13], the mixing matrix element V was extracted from the experimental ratio between the inter- and intraband $B(E2)$ values as well as the energy separation between the two levels. The value so-obtained of $|V|$ is 12 eV [13]. Now we use the corresponding theoretical values obtained from the calculation and follow the same procedure as in Ref. [13] to evaluate $|V|$. By using the theoretical values of interband $B(E2, \frac{19}{2}^+(3\text{qp}) \rightarrow \frac{15}{2}^+(1\text{qp})) = 1.14 \times 10^{-4}$ W.u. and the collective transition of the $\pi 7/2^+[404]$ 1-qp band $B(E2, \frac{19}{2}^+(1\text{qp}) \rightarrow \frac{15}{2}^+(1\text{qp})) = 223.52$ W.u., we obtain the mixing coefficient $\beta = 7.14 \times 10^{-4}$. Applying this β value (see Ref. [5] for calculation details), we get $|V| = 7.9$ eV. Though the theoretical matrix element remains smaller than experiment, the agreement can be considered reasonable given that very small interactions are being dealt with, that only the residual octupole force can generate the mixing, and that the 3-qp configuration space is truncated.

To understand these results and further study details of the K mixing, now we treat the octupole term as a perturbation and compute directly the mixing matrix element V. Suppose that the total Hamiltonian $\hat{H}$ in Eq. (4) can be separated into two parts:

$$\hat{H} = \hat{H}_0 + \hat{H}_{\rm oct}, \tag{8}$$

where $\hat{H}_{\rm oct}$ is the two-body octupole-octupole interaction and $\hat{H}_0$ contains all the remaining terms. We first diagonalize $\hat{H}_0$ to obtain energies and wave functions for the two near-degenerate states with $I^\pi = 19/2^+$, hereafter denoted as $|K = 7/2\rangle$ and $|K = 19/2\rangle$, respectively,

$$\begin{aligned} \hat{H}_0|K = 19/2\rangle &= E_1|K = 19/2\rangle, \\ \hat{H}_0|K = 7/2\rangle &= E_2|K = 7/2\rangle, \end{aligned} \tag{9}$$

with energies E_1 and E_2. Here, $|K = 19/2\rangle$ and $|K = 7/2\rangle$ are nonperturbed states (i.e., no octupole correlation) having the main component $\nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$ and $\pi 7/2^+[404]$, respectively. Using these two states, we can construct a two-dimensional matrix, in which the expectation values of H_0 are diagonal, and the off-diagonal elements (the mixing matrix element V) are given by

$$V = \langle K = 19/2|\hat{H}_{\rm oct}|K = 7/2\rangle. \tag{10}$$

From this perturbation treatment we get $V = 6.45$ eV, which is close to the previous result of 7.9 eV.

Thus we have found a possible source of the (tiny) interaction. This can be an intrinsic property of the nucleus, largely independent of the detailed structure variations that may produce, for example, the accidental degeneracy of particular states. To see this, we further perform calculations for $|V|$ with different hexadecapole deformations ϵ_4 (with all other calculation conditions fixed as before). The change of hexadecapole deformation in the deformed basis modifies the single-particle energies, which can cause a change in the energy separation ΔE of the two $19/2^+$ states. The results are shown in the left plot of Fig. 2. It can be seen that the change of $|V|$ is very small. In the simple two-level model as proposed in Ref. [5], $|V|$ should be independent of ΔE, as the off-diagonal elements of the Hamiltonian matrix do not have an explicit dependence on the diagonal elements. In the right plot of Fig. 2, we show the calculated values of $|V|$ as the strength of octupole correlation χ_O in Eq. (5) changes. It can be clearly seen that $|V|$ increases nearly linearly with χ_O. This can be easily understood from the perturbation treatment in which the linear dependence of the two quantities should be strict. In the limit of vanishing octupole correlation, $|V|$ naturally becomes zero. This confirms our claim that the octupole correlation is the source of the mixing.

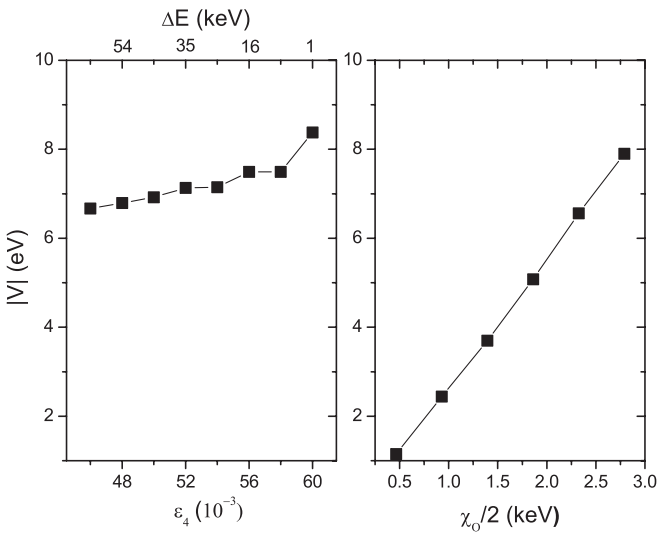

FIG. 2. (Left) The values of $|V|$ as functions of the hexadecapole deformation ϵ_4 (bottom scale). The corresponding energy separation of the two $19/2^+$ states ΔE (top scale) is also given. Note that the top scale is not linear because the relation of ΔE with ϵ_4 is not strictly linear. (Right) The values of $|V|$ for different octupole strengths.

The central point is that $|K = 19/2\rangle$ and $|K = 7/2\rangle$ in Eqs. (9) and (10) are not pure configurations, but linear superpositions of many projected qp configurations expressed as [see Eq. (3)]

$$|K = 19/2\rangle = \sum_\kappa f_\kappa^{19/2} \hat{P}^I |\Phi_\kappa\rangle,$$
$$|K = 7/2\rangle = \sum_\kappa f_\kappa^{7/2} \hat{P}^I |\Phi_\kappa\rangle. \quad (11)$$

The mixing matrix element (10) is a complex composition of octupole matrix elements between different sets of configurations, explicitly expressed as

$$V = \sum_{\kappa,\mu} f_\kappa^{19/2} f_\mu^{7/2} \langle\Phi_\kappa|\hat{P}^{I+} \hat{H}_{\rm oct} \hat{P}^I|\Phi_\mu\rangle. \quad (12)$$

The significance of the configuration mixing in $|K = 19/2\rangle$ and $|K = 7/2\rangle$ lies in the fact that, among the terms in the summation in Eq. (12), the largest contribution to V does not come from the specific term bracketed by $|\Phi_\kappa\rangle = \nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$ and $|\Phi_\mu\rangle = \pi 7/2^+[404]$. The contribution of this specific set is very small, there being many other configuration sets with much larger contributions. Those having the largest contribution to V are

$$|\Phi_\kappa\rangle = \nu 5/2^-[512]1/2^+[660] \otimes \pi 9/2^-[514],$$
$$|\Phi_\mu\rangle = \nu 5/2^+[642]7/2^+[633] \otimes \pi 7/2^+[404], \quad (13)$$

$$|\Phi_\kappa\rangle = \nu 5/2^-[512]3/2^+[651] \otimes \pi 9/2^-[514],$$
$$|\Phi_\mu\rangle = \nu 5/2^+[642]7/2^+[633] \otimes \pi 7/2^+[404], \quad (14)$$

$$|\Phi_\kappa\rangle = \nu 5/2^-[512]5/2^+[642] \otimes \pi 9/2^-[514],$$
$$|\Phi_\mu\rangle = \nu 5/2^+[642]7/2^+[633] \otimes \pi 7/2^+[404]. \quad (15)$$

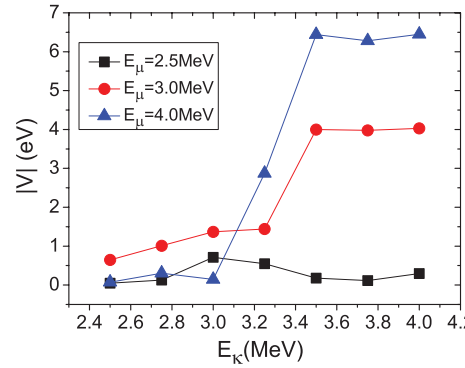

FIG. 3. (Color online) Contributions of excited qp configurations to the mixing matrix element V. E_κ and E_μ are, respectively, qp energies of bra and ket configurations in Eq. (12).

It has numerically been seen that the contribution from the above three sets exhausts nearly 75% of the total V.

Figure 3 illustrates contributions to V as a function of excitation energy of the bra and ket states in Eq. (12). The three curves represent different qp energies truncated for the ket states labeled by μ. In our calculation, the 3-qp ket state involved in Eqs. (13)–(15) has an excitation energy of 2.93 MeV. From Fig. 3, it can be clearly seen that without this special configuration, contributions to V are always negligible (curve with black squares). The three bra states labeled by κ in Eqs. (13)–(15) have excitation energies of 3.35, 3.22, and 2.86 MeV, respectively. It is noticed that these three configurations differ only in K components of the neutron $i_{13/2}$ orbit, with the $K = 5/2$ one being the closest to the Fermi level. With inclusion of contributions of more bra states, we find a jump in V after $E_\kappa = 3.22$ MeV and another one after 3.35 MeV (see curves with red dots or blue triangles). This is clearly attributed to the inclusion of the important bra states that contain the $i_{13/2}$ neutrons. Especially, the large jump after 3.35 MeV corresponds to the participation of the state with the smallest K component, $K = 1/2$.

The 3-qp bra configurations $|\Phi_\kappa\rangle$ in Eqs. (13)–(15) are mixed with the $|K = 19/2\rangle$ 3-qp isomer $\nu 5/2^-[512]7/2^+[633] \otimes \pi 7/2^-[523]$. The mixing is associated with couplings among the Nilsson states from the neutron $i_{13/2}$ or proton $h_{11/2}$ orbits. The correlations can be thought of as being of the Coriolis type, which mix Nilsson states within the same high-j set of orbitals. On the other hand, the 3-qp ket configurations $|\Phi_\mu\rangle$ in Eqs. (13)–(15) act as an "s-band" configuration of the $\pi 7/2^+[404]$ band. The mixing of the $|K = 7/2\rangle$ state with $|\Phi_\mu\rangle$ is due to the rotational alignment of a pair of quasineutrons $\nu 5/2^+[642]$ and $\nu 7/2^+[633]$, which come from the $i_{13/2}$ neutron orbit. Therefore, we may conclude that the K-mixing pattern results from propagation of the interaction through many high-lying states. [With the qp truncation energy of 4 MeV in our calculation, there are approximately 95 bra states and 120 ket states in the summation of Eq. (12).] The octupole-mixing matrix element between the two near-degenerate $I^\pi = 19/2^+$ states have large contributions from the highly excited 3-qp states which connect the two lower-lying $I^\pi = 19/2^+$ states through the

Coriolis coupling. Thus both octupole correlations and the indirect Coriolis coupling produce the residual interaction that eventually induces the anomalously fast decay of the ^{171}Tm isomer.

IV. SUMMARY

In summary, the small residual interaction that results, because of a near-degeneracy, in low reduced hindrances in the decay of the $K^{\pi} = 19/2^{+}$ 3-qp isomer in ^{171}Tm has been studied by using the projected shell model. The octupole correlation plays an important role in the mixing matrix element V, which is estimated at about 8 eV, compared to the experimental value of 12 eV. The largest theoretical contributions come indirectly from high-lying 3-qp configurations that are affected by Coriolis interactions. This theoretical development may be a step toward a quantitative understanding of interactions between states of different K, which although very small, will be important for the limits to, and the consequences of, mixing effects in nuclei [5].

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