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Revealing baryon number fluctuations from proton number fluctuations in relativistic heavy ion collisions

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Baryon number cumulants are invaluable tools to diagnose the primordial stage of heavy ion collisions if they can be measured. In experiments, however, proton number cumulants have been measured as substitutes. In fact, proton number fluctuations are further modified in the hadron phase and are different from those of the baryon number. We show that the isospin distribution of nucleons at kinetic freeze-out is binomial and factorized. This leads to formulas that express the baryon number cumulants solely in terms of proton number fluctuations, which are experimentally observable.

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The order of the phase transition of quantum chromodynamics (QCD) at nonzero temperature (T) is believed to change from crossover [1] to first order at a nonzero baryon chemical potential (μ_B). The existence of the QCD critical point is thus expected in the phase diagram on the T-μ_B plane [2]. Experiments to explore the phase structure at nonzero μ_B, especially the existence of the critical point, are now ongoing in the energy scan program at the Relativistic Heavy Ion Collider (RHIC) [3,4], and will also be performed in future facilities [5,6]. Much attention has also been paid to this problem from numerical experiments on the lattice [1,7]. The establishment of the QCD phase structure at nonzero μ_B is an important issue, not only to deepen our knowledge of the matter described by QCD, but also to gain understanding of a wide array of topics in physics which share the concepts of phase transitions and techniques to treat strongly correlated many-body systems.

Fluctuations, which are experimentally measured by event-by-event analyses in heavy ion collisions, are promising observables to probe the properties of created fireballs [8], as their behaviors are sensitive to the state of the matter. For example, because of the singularity at the critical point, fluctuations of various physical quantities, including skewness and kurtosis, behave anomalously near the critical point [9–11]. One can also argue that ratios between the cumulants of conserved charges are sensitive to the magnitudes of the charge carried by the quasiparticles composing the system, and hence they behave differently in the hadronic and quark-gluon phases [12–14]. Recently, it was also pointed out that some higher-order cumulants of conserved charges change signs around the phase boundary of QCD, which would serve as clear experimental signatures to determine the location of the matter in the phase diagram [15–17].

Among the fluctuation observables, those of conserved charges can reflect fluctuations produced in earlier stages during the time evolution of fireballs than non-conserved ones [18]. This is because the variation of a conserved charge in a volume is achieved only through diffusion, which makes the relaxation to equilibrium slower. In fact, it is argued that if the rapidity range of a detector is taken to be sufficiently large, whereas the range should be kept narrow enough so that the rest of fireballs can be regarded as the heat bath, the effects of diffusion are well suppressed and fluctuations produced in the quark-gluon phase can be detected experimentally [12,13].

The dependences of the proton number fluctuations, cumulants up to fourth order, on the beam energy $\sqrt{s}$, have been recently measured by the STAR collaboration at RHIC [3,4]. The result appears to be almost consistent with the prediction of the hadron resonance gas (HRG) model [19]; although the experimental result shows some deviation from the prediction at small $\sqrt{s}$, it is at most of the order of 20% [4]. The proton number, however, is not a conserved quantity, and in fact we will see later that its fluctuations significantly evolve in the hadronic stage, which makes the experimentally measured fluctuations close to those in the equilibrated hadronic matter. The agreement between the experiments and the HRG model in the proton number fluctuations [4] is in part due to these effects, and hence it does not immediately exclude the slow baryon number diffusion in the hadronic stage. Although the measurement of the baryon number, which is a conserved charge, is desirable to probe fluctuations generated in earlier stages, its direct experimental measurement has been considered to be impossible because of the difficulty in detecting and identifying neutrons.

In this Rapid Communication, we show that the experimentally measured proton number fluctuations are nevertheless directly related to baryon number fluctuations in earlier stages, and we present concrete formulas that relate the baryon number cumulants and these experimental observables. The key observation is that the distributions of (anti)proton and (anti)neutron numbers in the final state are well described by binomial distributions. As will be argued in detail later, this observation is well justified at least for RHIC energy, and is expected to hold for $\sqrt{s} \gtrsim 10$ GeV.

Experimentally, the electric charge can be measured directly. Electric charge fluctuations, however, contain the contribution of isospin fluctuations, which are nonsingular at the critical point, in addition to baryon number fluctuations [10]. The signals of the phase transition in this observable thus

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generally become weak owing to the nonsingular contribution (such a tendency, for example, in the third moments is seen in Ref. [15]). In this sense, the baryon number fluctuations are superior to the electric ones as probes of the QCD phase structure.

Throughout this Rapid Communication, we use N_X to represent the number of particles X leaving the system after each collision event, where $X = p, n$, and B represent proton, neutron, and baryon, respectively, and their antiparticles, $\bar{p}$, $\bar{n}$, and $\bar{\mathrm{B}}$. The net and total numbers are denoted as $N_X^{(\mathrm{net})} = N_X - N_{\bar{X}}$ and $N_X^{(\mathrm{tot})} = N_X + N_{\bar{X}}$, respectively.

Before starting the main discussion on the cumulants of baryon and proton numbers, let us briefly consider how the proton number fluctuations evolve in the hadronic stage. The most important process responsible for the variation of the proton number is the charge exchange reactions with thermal pions mediated by $\Delta^+(1232)$ and $\Delta^0(1232)$ resonances:

$$p(n) + \pi \to \Delta^{+,0} \to n(p) + \pi. \tag{1}$$

Because of the small energy required and the large cross sections, these reactions proceed even after chemical freeze-out, as we demonstrate later. We note that these reactions do not alter the average abundances $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ if the isospin chemical potential vanishes, while they modify the fluctuations of N_p and $N_{\bar{p}}$. Because chemical freeze-out is a concept that describes ratios between particle abundances such as $\langle N_{\bar{p}} \rangle / \langle N_p \rangle$, these reactions below the chemical freeze-out temperature T_{chem} do not contradict the statistical model. The success of the model, on the other hand, indicates that the creation and annihilation of (anti)nucleons hardly occur below T_{chem}.

The importance of reactions (1) below T_{chem} is confirmed by evaluating the mean time of the nucleons for these reactions. Provided that the pions have a thermal distribution, the mean time τ that a proton at rest in the medium forms Δ^+ or Δ^0, being scattered by a thermal pion, is evaluated to be

$$\tau^{-1} = \int \frac{d^3 k_\pi}{(2\pi)^3} \sigma(E_{\mathrm{c.m.}}) v_\pi n(E_\pi), \tag{2}$$

with the Bose distribution function $n(E) = (e^{E/T} - 1)^{-1}$, pion velocity $v_\pi = k_\pi / E_\pi$, $E_\pi = \sqrt{m_\pi^2 + k_\pi^2}$, and the pion mass m_π. $\sigma(E_{\mathrm{c.m.}})$ is the sum of the cross sections for $p\pi$ reactions producing Δ^+ and Δ^0, with a center-of-mass energy $E_{\mathrm{c.m.}} = [(m_{\mathrm{N}} + E_\pi)^2 - k_\pi^2]^{1/2}$ with the nucleon mass m_{N}. To evaluate Eq. (2), we assume a cross section of Breit-Wigner type, $\sigma(E_{\mathrm{c.m.}}) = \sigma_\Delta(\Gamma^2/4)/[(E_{\mathrm{c.m.}} - E_\Delta)^2 + \Gamma^2/4]$, which is a sufficient approximation for our purpose, with hadron properties in the vacuum, $m_N = 940$ MeV, $m_\pi = 140$ MeV, $E_\Delta = 1232$ MeV, $\Gamma = 110$ MeV, and $\sigma_\Delta = 20$ fm^2 [20]. The mean time is then evaluated to be 3–4 fm for $T = 150$–170 MeV. One can also check that this mean time hardly changes even for moving protons in the range of momentum $p \lesssim 3T$. On the other hand, dynamical models for RHIC energy predict that protons stay in the hadronic gas and continue to interact for several tens of fm on average at midrapidity [21], which is significantly longer than the mean time and the lifetime of Δ, $1/\Gamma \simeq 1.8$ fm. This result shows that nucleons in the fireball indeed undergo this reaction several times on average in the hadronic stage.[1] The ratio of the probabilities that a proton in the medium produces a Δ^+ or Δ^0 and then decays into p and n is 5 : 4, which is determined by the isospin SU(2) algebra. Whereas this probability is not even, after repeating the above processes several times in the hadronic stage, the nucleons tend to completely forget their initial isospin.

The above discussion shows that the evolution of proton number fluctuations in the hadronic stage is dominantly made via the exchanges of the two isospin states of the nucleons. Now, we further assert that *isospins of all nucleons in the final state are uncorrelated*. This statement is well justified when the hadronic medium fulfills the following two conditions: (i) The medium effects on the branching ratios and formation rates of Δ are insensitive to the proton and neutron number densities n_p and n_n (and the same holds for the antiparticle sector as well), and (ii) (anti)nucleon-(anti)nucleon interactions generating correlations between two nucleons hardly occur. As we will see later, these two conditions are well satisfied below T_{chem} except for low-energy collisions. The probability distribution of finding N_p and N_n ($N_{\bar{p}}$ and $N_{\bar{n}}$) particles in the final state in each event then becomes binomial. Under the isospin symmetry [19], this fact enables to factorize the probability distribution $P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$ having N_p, N_n, $N_{\bar{p}}$, and $N_{\bar{n}}$ particles in each event as

$$P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) = F(N_{\mathrm{B}}, N_{\bar{\mathrm{B}}}) B(N_p; N_{\mathrm{B}}) B(N_{\bar{p}}; N_{\bar{\mathrm{B}}}), \tag{3}$$

where $B(k; N) = 2^{-N} N!/[k!(N-k)!]$ is the binomial distribution function with an equal probability. On the right-hand side (RHS) of Eq. (3) we have used N_{B} and $N_{\bar{\mathrm{B}}}$ defined by $N_{\mathrm{B}} = N_p + N_n$ and $N_{\bar{\mathrm{B}}} = N_{\bar{p}} + N_{\bar{n}}$. We will later elucidate this notation to use the baryon numbers N_{B} and $N_{\bar{\mathrm{B}}}$ in place of the nucleon numbers. Under the probability distribution Eq. (3), the event-by-event average of a function $f(N_p, N_{\bar{p}})$ is given by

$$\begin{aligned} \langle f(N_p, N_{\bar{p}}) \rangle &= \sum_{N_{\{p,n,\bar{p},\bar{n}\}}} P(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) f(N_p, N_{\bar{p}}) \\ &= \sum_{N_{\mathrm{B}}, N_{\bar{\mathrm{B}}}} F(N_{\mathrm{B}}, N_{\bar{\mathrm{B}}}) \sum_{N_p, N_{\bar{p}}} f(N_p, N_{\bar{p}}) \\ &\quad \times B(N_p; N_{\mathrm{B}}) B(N_{\bar{p}}; N_{\bar{\mathrm{B}}}). \end{aligned} \tag{4}$$

The factorization Eq. (3) leads to

$$\langle N_p^{(\mathrm{net})} \rangle = \tfrac{1}{2} \langle N_{\mathrm{B}}^{(\mathrm{net})} \rangle, \tag{5}$$

$$\langle (\delta N_p^{(\mathrm{net})})^2 \rangle = \tfrac{1}{4} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \rangle + \tfrac{1}{4} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle, \tag{6}$$

$$\langle (\delta N_p^{(\mathrm{net})})^3 \rangle = \tfrac{1}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^3 \rangle + \tfrac{3}{8} \langle \delta N_{\mathrm{B}}^{(\mathrm{net})} \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle, \tag{7}$$

$$\begin{aligned} \langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c &\equiv \langle (\delta N_p^{(\mathrm{net})})^4 \rangle - 3 \langle (\delta N_p^{(\mathrm{net})})^2 \rangle^2 \\ &= \tfrac{1}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^4 \rangle_c + \tfrac{3}{8} \langle (\delta N_{\mathrm{B}}^{(\mathrm{net})})^2 \delta N_{\mathrm{B}}^{(\mathrm{tot})} \rangle \\ &\quad + \tfrac{3}{16} \langle (\delta N_{\mathrm{B}}^{(\mathrm{tot})})^2 \rangle - \tfrac{1}{8} \langle N_{\mathrm{B}}^{(\mathrm{tot})} \rangle, \end{aligned} \tag{8}$$

[1] We note that some event generators employed in Ref. [3] do not take this reaction into account. As we will see later, however, it is this reaction that is responsible for our main results, Eqs. (9)–(12), expressing the baryon number fluctuations in terms of experimental observables without introducing any models.

where $\delta N_X = N_X - \langle N_X \rangle$. To derive Eqs. (5)–(8), we have used the fact that the sums over N_p and $N_{\bar{p}}$ in Eq. (4) can be taken separately with corresponding binomial functions, e.g., $\sum_{N_p} N_p B(N_p; N_B) = N_B/2$ and $\sum_{N_p} N_p^2 B(N_p; N_B) = N_B^2/4 + N_B/4$.

Equation (3) also enables to represent the baryon number cumulants by those of the net and total proton numbers as

$$\langle N_B^{(\mathrm{net})} \rangle = 2\langle N_p^{(\mathrm{net})} \rangle, \tag{9}$$

$$\langle (\delta N_B^{(\mathrm{net})})^2 \rangle = 4\langle (\delta N_p^{(\mathrm{net})})^2 \rangle - 2\langle N_p^{(\mathrm{tot})} \rangle, \tag{10}$$

$$\langle (\delta N_B^{(\mathrm{net})})^3 \rangle = 8\langle (\delta N_p^{(\mathrm{net})})^3 \rangle - 12\langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle + 6\langle N_p^{(\mathrm{net})} \rangle, \tag{11}$$

$$\langle (\delta N_B^{(\mathrm{net})})^4 \rangle_c = 16\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c - 48\langle (\delta N_p^{(\mathrm{net})})^2 \delta N_p^{(\mathrm{tot})} \rangle + 48\langle (\delta N_p^{(\mathrm{net})})^2 \rangle + 12\langle (\delta N_p^{(\mathrm{tot})})^2 \rangle - 26\langle N_p^{(\mathrm{tot})} \rangle, \tag{12}$$

where we have used relations for mixed cumulants such as $\langle \delta N_B^{(\mathrm{net})} \delta N_B^{(\mathrm{tot})} \rangle = 4\langle \delta N_p^{(\mathrm{net})} \delta N_p^{(\mathrm{tot})} \rangle - 2\langle N_p^{(\mathrm{tot})} \rangle$, which are obtained with Eq. (3). Since the RHSs of Eqs. (9)–(12) consist of only $N_p^{(\mathrm{net})}$ and $N_p^{(\mathrm{tot})}$, which are experimentally observable, these are formulas that express baryon number cumulants solely in terms of the experimental observables. We remind that no specific form of $F(N_B, N_{\bar{B}})$ is assumed in deriving these results.

We remark that $N_B^{(\mathrm{net})}$ ($N_B^{(\mathrm{tot})}$) in Eqs. (5)–(12) are interpreted to be the sum of all net (total) baryon numbers entering a region in the phase space in the final state of each event. If the diffusion of the baryon number in the hadronic stage is slow [12,13], the information on the primordial fluctuations remains in $F(N_B, N_{\bar{B}})$ in Eq. (3) and, as a result, in baryon number cumulants.

Next, let us inspect the validity of Eq. (3) in more detail. First, we consider the conditions (i) and (ii) introduced above Eq. (3). In the medium, the decay rate of Δ acquires the statistical factor

$$[1 - f(E_N)][1 + n(E_\pi)], \tag{13}$$

where $f(E) = (e^{(E-\mu_B)/T} + 1)^{-1}$ is the Fermi distribution function and E_N and E_π are the energies of the nucleon and pion produced by the decay, respectively. The first term in Eq. (13) represents the Pauli blocking effect. At RHIC energy, the Boltzmann approximation is well applied to nucleons below T_{chem} since $T \ll m_N$ and $|\mu_B| \ll m_N$. Thus, the Pauli blocking effect can be almost ignored. The Bose factor $[1 + n(E_\pi)]$ in Eq. (13), on the other hand, has a non-negligible contribution since $m_\pi \simeq T_{\mathrm{chem}}$. The density of the pions, however, is more than one order larger than that of the nucleons below T_{chem}. The Bose factor thus must be insensitive to n_p and n_n, while it leads to the enhancement of the decay of Δ in the medium, which acts in favor of the isospin randomization. The large pion density also means that the mean time for a nucleon to form Δ is insensitive to n_p and n_n. Condition (i) is thus well satisfied below T_{chem} at RHIC energy. The validity of condition (ii) is conjectured from the success of the statistical model as follows. The statistical model indicates that the pair annihilation of an N and an $\bar{\mathrm{N}}$ terminates at T_{chem}. NN and N$\bar{\mathrm{N}}$ reactions are then also expected to terminate there, because the elastic cross section of N$\bar{\mathrm{N}}$ is significantly smaller than the inelastic one, and the total cross section of NN behaves similarly to that of N$\bar{\mathrm{N}}$ for $E_{\mathrm{c.m.}} < 1$ GeV [20]. Condition (ii) thus should also be satisfied for $T < T_{\mathrm{chem}}$. Intuitively speaking, in a hot medium the nucleons are so dilutely distributed that they do not feel one another's existence, while there are so many pions which can be regarded as the heat bath when the nucleon sector is concerned. The large pion density also enables to use the binomial distribution independently of the initial nucleon isospin density.

Second, while so far we have limited our attention to the nucleon reactions mediated by Δ, other interactions can also take place in the medium. It is also possible that Δ interacts with a thermal pion to form another resonance before the decay [22]. All these reactions with thermal pions, however, proceed with a certain probability determined by the isospin SU(2) symmetry as long as they are caused by the strong interaction, and the reactions of a baryon make its isospin random. Strange baryons, on the other hand, decay via the weak or electromagnetic interaction outside the fireball. In particular, Λ and Σ are important among them. Λ decays into p and n with a branching ratio of 16 : 9. Provided that the three isospin states of Σ are produced with an equal probability in the medium, the ratio of probabilities that a Σ decays into p and n is about 1 : 1.6 [20]. Although these ratios are not even, because the abundances of Λ and Σ are small compared to the nucleons, to a first approximation it is suitable for our purpose to regard these probabilities to be equal and to incorporate nucleons produced by the decays of Λ and Σ in N_p and N_n in Eq. (3). This promotes the nucleon numbers to those of the baryons in Eq. (3). The treatment of strange baryons, however, may require more detailed arguments, especially on their quantitative effects on higher-order cumulants, which will be addressed elsewhere. Inclusion of higher baryonic resonances and light nuclei such as deuterons will not affect our conclusions owing to their negligible abundances.

While the factorization Eq. (3) is fully established for RHIC energy, the binomiality will eventually break down as the beam energy is decreased. At very low beam energy, pions are not produced enough and nucleons will not undergo charge exchange reactions sufficiently below T_{chem}. We deduce that this happens when $T_{\mathrm{chem}} \lesssim m_\pi$. When the reactions hardly occur, the isospin correlations generated at the hadronization will remain until the final state. At low beam energy, also the nucleon density becomes comparable with that of the pions, and the latter can no longer be regarded as the heat bath to absorb the isospin fluctuations of the former. From the $\sqrt{s}$ dependence of the chemical freeze-out line on the T-μ_B plane [23], and considering the validity of these two conditions, we deduce that Eq. (3) is well applicable to the range of beam energy $\sqrt{s} \gtrsim 10$ GeV.

In the argument to derive Eqs. (5)–(12), we have implicitly assumed that the hadronic medium is isospin symmetric. While the effect of nonzero isospin density should be well suppressed for large $\sqrt{s}$ where a large number of particles having nonzero isospin charges are produced, at lower energies this effect gives rise to a non-negligible modification of Eqs. (5)–(12). When the system has nonzero isospin density, the probability that a nucleon at the early stage of the hadron phase becomes a proton or a neutron in the final state is no longer even. This effect is, as long as conditions (i) and (ii) introduced above Eq. (3) hold, incorporated into our results by simply replacing the binomial function $B(N_p; N_{\rm B})$ in Eq. (3) with that having a probability $k = \langle N_p\rangle/\langle N_p + N_n\rangle$, and a similar replacement to $B(N_{\bar p}; N_{\bar{\rm B}})$. Our explicit analysis indicates that the effect of nonzero isospin density on Eqs. (5)–(12) is relatively small and well suppressed when $T_{\rm chem} > m_\pi$ and a sufficient number of pions having isospin charges are produced at chemical freeze-out. Since this modification requires a straightforward but lengthy calculation, we will elucidate the analysis in a forthcoming paper.

Now, let us apply our results to the latest experimental data from STAR [3,4]. To estimate how the binomial nature of nucleon isospins affects the proton number fluctuations, we first consider Eqs. (5)–(8). In order to estimate the contributions of terms including $N_{\rm B}^{\rm (tot)}$ in these equations, we temporarily postulate that $N_{\rm B}$ and $N_{\bar{\rm B}}$ have thermal distributions fixed at chemical freeze-out as the statistical model suggests, while the distribution of their combination, $N_{\rm B}^{\rm (net)}$, deviates from the thermal one, reflecting the baryon number conservation. Under this assumption, the distributions of $N_{\rm B}$ and N_p are Poissonian, and hence the cumulants of the baryon and proton numbers satisfy

$$\begin{aligned}\langle N_{\rm B}\rangle &= \langle(\delta N_{\rm B})^2\rangle = \langle(\delta N_{\rm B})^3\rangle = 2\langle N_p\rangle_{\rm HG}\\ &= 2\langle(\delta N_p)^2\rangle_{\rm HG} = 2\langle(\delta N_p)^3\rangle_{\rm HG},\end{aligned} \tag{14}$$

and the same for antibaryon numbers, where $\langle\cdot\rangle_{\rm HG}$ is the expectation value for free hadron gas (HG) composed of mesons and nucleons at $T_{\rm chem}$, i.e., a simplified version of the HRG model [19]. Equations (6) and (7) are then expressed as

$$\left\langle\left(\delta N_p^{\rm (net)}\right)^2\right\rangle = \tfrac{1}{4}\left\langle\left(\delta N_{\rm B}^{\rm (net)}\right)^2\right\rangle + \tfrac{1}{2}\left\langle\left(\delta N_p^{\rm (net)}\right)^2\right\rangle_{\rm HG}, \tag{15}$$

$$\left\langle\left(\delta N_p^{\rm (net)}\right)^3\right\rangle = \tfrac{1}{8}\left\langle\left(\delta N_{\rm B}^{\rm (net)}\right)^3\right\rangle + \tfrac{3}{4}\left\langle\left(\delta N_p^{\rm (net)}\right)^3\right\rangle_{\rm HG}. \tag{16}$$

To derive these results, we decomposed, for example, the second term in Eq. (7) as

$$\begin{aligned}\left\langle\delta N_{\rm B}^{\rm (net)}\delta N_{\rm B}^{\rm (tot)}\right\rangle &= \langle(\delta N_{\rm B})^2\rangle - \langle(\delta N_{\bar{\rm B}})^2\rangle\\ &= 2\langle(\delta N_p)^3\rangle_{\rm HG} - 2\langle(\delta N_{\bar p})^3\rangle_{\rm HG}\\ &= 2\left\langle\left(\delta N_p^{\rm (net)}\right)^3\right\rangle_{\rm HG},\end{aligned} \tag{17}$$

The results in Eqs. (15) and (16) show that the second terms on the RHSs, which come from the binomial distributions of nucleon isospin, make a large contribution to the cumulants of the proton number, and they become more significant as the order increases. Although one cannot derive a similar result for the fourth-order relation, from the factor $1/16$ in the first term of Eq. (8) it is clear that the effect of the fourth-order baryon number cumulant on the proton number one is more suppressed in this order. The suppression of the first term in Eqs. (6)–(8) may be one of the reasons why the results of the STAR experiment and the HRG model appear to be consistent with each other. In this sense, it is interesting that the experimental results for skewness and kurtosis have small but significant deviations from the HRG predictions at $\sqrt{s} \lesssim 50$ GeV [4]. The deviation, for example, in skewness, can be a consequence of $\langle(\delta N_{\rm B}^{\rm (net)})^3\rangle$ in Eq. (16), which possibly reflects the properties of matter in the early stage. Baryon number cumulants are, of course, directly determined with experimental observables using Eqs. (9)–(12). It is worth emphasizing that the RHSs of Eqs. (11) and (12) have terms which would lead to the negative cumulants discussed in Refs. [15,17], or the suppression of the ratio $\langle(\delta N_{\rm B}^{\rm (net)})^4\rangle/\langle(\delta N_{\rm B}^{\rm (net)})^2\rangle$ [14].

We note that when the distribution of $N_{\rm B}^{\rm (net)}$ also follows that in the HG in addition to the above postulation, the RHSs of Eqs. (16) and (17) reduce to $\langle(\delta N_p^{\rm (net)})^2\rangle_{\rm HG}$ and $\langle(\delta N_p^{\rm (net)})^3\rangle_{\rm HG}$. A way to check this is to use the fact that the (anti)nucleon numbers in the HG are well described by the Poisson distribution owing to the Boltzmann approximation, and that the Poisson distribution with an average λ, $P_\lambda(N)$, satisfies $P_\lambda(N_1)P_\lambda(N_2) = P_{2\lambda}(N_1+N_2)B(N_1; N_1+N_2)$. The HG thus corresponds to a special case of Eq. (3), where

$$F(N_{\rm B}, N_{\bar{\rm B}}) = P_{\langle N_{\rm B}\rangle}(N_{\rm B})P_{\langle N_{\bar{\rm B}}\rangle}(N_{\bar{\rm B}}). \tag{18}$$

In this Rapid Communication, we derived relations between the baryon and proton number cumulants, Eqs. (5)–(8) and (9)–(12), respectively, on the basis of the binomial nature of (anti)nucleon isospin numbers in the final state. These results enable to immediately determine the baryon number cumulants with experimental results in heavy ion collisions, which will provide significant information about the QCD phase diagram. Though these results are obtained for the isosymmetric case, incorporation of nonzero isospin density is straightforward and will be discussed elsewhere.

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