

Exploration of relativistic symmetry by the similarity renormalization group

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The similarity renormalization group is used to transform Dirac Hamiltonian into a diagonal form, which the upper (lower) diagonal element becomes an operator describing Dirac (anti-)particle. The eigenvalues of the operator are verified to be in good agreement with that of the original Hamiltonian. Furthermore, the pseudospin symmetry is investigated. It is shown that the pseudospin splittings appearing in the nonrelativistic limit are reduced by the contributions from these terms relating the spin-orbit interactions, added by those relating the dynamical terms, and the quality of pseudospin symmetry origins mainly from the competition of the dynamical effects and the spin-orbit interactions. The spin symmetry of antiparticle spectrum is well reproduced in the present calculations.

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Many years ago a quasidegeneracy was observed in heavy nuclei between single-nucleon doublets with quantum numbers $(n, l, j = l + 1/2)$ and $(n - 1, l + 2, j = l + 3/2)$, where n , l , and j are the radial, the orbital, and the total angular momentum quantum numbers, respectively [1,2]. The quasidegenerate states were suggested to be pseudospin doublets $j = \tilde{l} \pm \tilde{s}$ with the pseudo-orbital angular momentum $\tilde{l} = l + 1$ and the pseudospin angular momentum $\tilde{s} = 1/2$ and have explained a number of phenomena in nuclear structure. Because of these successes, there have been comprehensive efforts to understand the origin of this symmetry. Until 1997, it was identified as a relativistic symmetry [3]. Nevertheless, a large amount of attention is still focused on this symmetry. The pseudospin symmetry (PSS) of nuclear wave functions was tested in Refs. [4,5] with the conclusion supporting the claim in Ref. [3]. The existence of broken PSS was checked in Refs. [6,7], where the quasidegenerate pseudospin doublets were confirmed to exist near the Fermi surface for spherical and deformed nuclei. The isospin dependence of PSS was investigated in Ref. [8], where it was found that PSS is better for exotic nuclei with a highly diffuse potential. PSS was shown to be approximately conserved in medium-energy nucleon scattering from even-even nuclei [9–11]. In combination with the analytic continuation method, the resonant states were exposed to hold the PSS in Refs. [12,13]. In Ref. [14], the conditions which originate the spin and pseudospin symmetries in the Dirac equation were shown to be the same that produce equivalent energy spectra of relativistic spin-1/2 and spin-0 particles in the presence of vector and scalar potentials. Furthermore, the symmetries and super-symmetries of the Dirac Hamiltonian were checked for particles moving in the spherical or axially deformed scalar and vector potentials [15]. More reviews on the PSS can be found in the literature [16] and the references therein. Recently, a perturbation method was adopted to investigate the spin and pseudospin symmetries by dividing the Dirac Hamiltonian into the part of possessing the exact (pseudo-)spin symmetry and that of breaking the symmetry [17].

Despite the large number of studies on PSS, the origin of PSS and its breaking mechanism are still not fully understood since there is no bound states in the PSS limit. Hence, much effort has been devoted to compare the contributions of different terms in the Schrödinger-like equation for the lower component of Dirac spinor to the pseudospin energy splitting. In Refs. [18–20], the PSS in real nuclei was shown in connection with the competition between the pseudocentrifugal barrier and the pseudospin-orbital potential. In Refs. [21,22], it was shown that the observed pseudospin splitting arises from a cancellation of the several energy components, and the PSS in nuclei has a dynamical character. A similar conclusion was reached in Refs. [23,24]. However, in these studies, one encounters inevitably the singularity in calculating the contribution of every component to the pseudospin splitting, and the coupling between the energy ϵ and the operator in solving the Schrödinger-like equation for the lower component of Dirac spinor [to see Eq. (4) in the following], which affects our understanding on the origin of the PSS. As seen in the following [Eq. (4)], it seems that only $\frac{\kappa}{r} \frac{\Sigma'}{4M^2}$ destroys the PSS, but the pseudospin splittings are related to every component [21–24]. In order to cure these defects, in this Rapid Communication we transform the Dirac operator into a diagonal form by use of the similarity renormalization group (SRG), in which the upper (lower) diagonal part becomes an operator describing the Dirac (anti-)particle with the singularity and the coupling disappearing. In the following, we first derive out the operator and then present its application in analyzing the PSS.

Assuming the spherical symmetry, the radial Dirac equation can be cast in the form of

$$H_s \psi = \epsilon \psi, \quad (1)$$

with

$$H_s = \begin{bmatrix} M + \Sigma(r) & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & -M + \Delta(r) \end{bmatrix} \quad \text{and} \quad \psi = \begin{bmatrix} F(r) \\ G(r) \end{bmatrix}, \quad (2)$$

where $\Sigma(r) = V(r) + S(r)$ and $\Delta(r) = V(r) - S(r)$ denote the combinations of the scalar potential $S(r)$ and the vector potential $V(r)$ and κ is defined as $\kappa = (l - j)(2j + 1)$. To understand the PSS, one decouples Eq. (1) into two equations

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for the upper and lower components as follows:

$$\left\{ -\frac{1}{2M_+} \left[\frac{d^2}{dr^2} + \frac{\Delta'}{2M_+} \frac{d}{dr} - \frac{\kappa(\kappa+1)}{r^2} \right] + (M + \Sigma) - \frac{\kappa}{r} \frac{\Delta'}{4M_+^2} \right\} F(r) = \epsilon F(r), \quad (3)$$

$$\left\{ -\frac{1}{2M_-} \left[\frac{d^2}{dr^2} + \frac{\Sigma'}{2M_-} \frac{d}{dr} - \frac{\kappa(\kappa-1)}{r^2} \right] - (M - \Delta) + \frac{\kappa}{r} \frac{\Sigma'}{4M_-^2} \right\} G(r) = \epsilon G(r), \quad (4)$$

here the effective masses $2M_+ = \epsilon + M - \Delta$ and $2M_- = \epsilon - M - \Sigma$. The prime denotes derivative with respect to r . From Eq. (4), it can be seen that the system possesses exact PSS when $\Sigma' = 0$. Unfortunately, the condition cannot be realized in real nuclei, many efforts are devoted to analyze the contributions of various terms to the PSS [21–24]. However, as there exist deficiencies mentioned before, we decouple Eq. (1) by SRG.

Without loss of generality, we begin our formalism for a general Dirac Hamiltonian $H = \vec{\alpha} \cdot \vec{p} + \beta(M + S) + V$, which is fully applicable for H_s . Following Wegner's formulation of the SRG [25], the initial Hamiltonian H is transformed by the unitary operator $U(l)$ according to

$$H(l) = U(l) H U^\dagger(l), \quad H(0) = H, \quad (5)$$

where l is a flow parameter. Differentiation Eq. (5) gives the flow equation as

$$\frac{d}{dl} H(l) = [\eta(l), H(l)], \quad (6)$$

with the generator $\eta(l) = \frac{dU(l)}{dl} U^\dagger(l)$. There are several possibilities to choose the $\eta(l)$ so $H(l)$ becomes diagonal in the limit $l \rightarrow \infty$. In Wegner's original formulation [25], $\eta(l)$ was chosen as the commutator of the diagonal part of $H(l)$ with $H(l)$ itself, i.e., $\eta(l) = [H_{\text{diag}}(l), H(l)]$. An alternative to Wegner's formulation is $\eta(l) = [G, H(l)]$, where G is a fixed (l -independent) Hermitian operator. It is straightforward to show that $H(l)$ converges to a final Hamiltonian which commutes with G . Here, we hope to transform Dirac Hamiltonian into a diagonal form, which must commute with the β matrix. Thus, it is appropriate to choose $\eta(l)$ in the form

$$\eta(l) = [\beta M, H(l)]. \quad (7)$$

In order to solve Eq. (6), the technique in Ref. [26] is adopted. The Hamiltonian $H(l)$ is presented as a sum of an even operator $\epsilon(l)$ and odd operator $o(l)$:

$$H(l) = \epsilon(l) + o(l), \quad (8)$$

where the even or oddness is defined by the commutation relations of the respective operators, i.e., $\epsilon(l)\beta = \beta\epsilon(l)$ and $o(l)\beta = -\beta o(l)$. To put Eqs. (7) and (8) into Eq. (6) gives

$$\frac{d\epsilon(l)}{dl} = 4M\beta o^2(l), \quad (9)$$

$$\frac{do(l)}{dl} = 2M\beta [o(l), \epsilon(l)]. \quad (10)$$

The system of Eqs. (9) and (10) can be solved perturbatively in $1/M$ [26]. It is convenient to introduce a dimensionless flow parameter $\lambda = lM^2$. Since $\epsilon(0) = \beta(M + S) + V$, and $o(0) = \vec{\alpha} \cdot \vec{p}$ and the expansion of $\epsilon(\lambda)/M$ in a series in $1/M$

contains terms starting with the zeroth-order term

$$\frac{1}{M} \epsilon(\lambda) = \sum_{i=0}^{\infty} \frac{1}{M^i} \epsilon_i(\lambda), \quad (11)$$

whereas the expansion of $o(\lambda)/M$ starts with the first order,

$$\frac{1}{M} o(\lambda) = \sum_{j=1}^{\infty} \frac{1}{M^j} o_j(\lambda). \quad (12)$$

Differentiation Eqs. (11) and (12) yield the following equations:

$$\frac{d\epsilon_n(\lambda)}{d\lambda} = 4\beta \sum_{k=1}^{n-1} o_k(\lambda) o_{n-k}(\lambda), \quad (13)$$

$$\frac{do_n(\lambda)}{d\lambda} = -4o_n(\lambda) + 2\beta \sum_{k=1}^{n-1} [o_k(\lambda), \epsilon_{n-k}(\lambda)]. \quad (14)$$

The solutions of Eqs. (13) and (14) are obtained as

$$\epsilon_n(\lambda) = \epsilon_n(0) + 4\beta \int_0^\lambda d\lambda' \sum_{k=1}^{n-1} o_k(\lambda') o_{n-k}(\lambda'), \quad (15)$$

$$o_n(\lambda) = o_n(0) e^{-4\lambda} + 2\beta e^{-4\lambda} \times \int_0^\lambda d\lambda' \sum_{k=1}^{n-1} [e^{4\lambda'} o_k(\lambda'), \epsilon_{n-k}(\lambda')], \quad (16)$$

with the initial conditions

$$\begin{aligned} \epsilon_0(0) &= \beta, & \epsilon_1(0) &= \beta S + V, & \epsilon_n(0) &= 0 \quad \text{if } n \geq 2, \\ o_1(0) &= \vec{\alpha} \cdot \vec{p}, & o_n(0) &= 0 \quad \text{if } n \geq 2. \end{aligned} \quad (17)$$

From Eqs. (15) and (16) with the initial condition in Eq. (17), we obtain $\epsilon_0(\lambda) = \beta$, $\epsilon_1(\lambda) = \beta S + V$, and $o_1(\lambda) = o_1(0) e^{-4\lambda}$. Hence, it is easy to verify that $o_n(\lambda)$ exponentially goes to zero when $\lambda \rightarrow \infty$. So the diagonalized Dirac operator is obtained as

$$\begin{aligned} \epsilon(\infty) &= M\epsilon_0(\infty) + \epsilon_1(\infty) + \frac{1}{M}\epsilon_2(\infty) + \frac{1}{M^2}\epsilon_3(\infty) \\ &\quad + \frac{1}{M^3}\epsilon_4(\infty) + \dots \\ &= M\epsilon_0(0) + \epsilon_1(0) + \frac{1}{2M}\beta o_1^2(0) \\ &\quad + \frac{1}{8M^2}[[o_1(0), \epsilon_1(0)], o_1(0)] \\ &\quad + \frac{1}{32M^3}\beta(-4o_1^4(0) + o_1(0)[[o_1(0), \epsilon_1(0)], \epsilon_1(0)] \\ &\quad + [[o_1(0), \epsilon_1(0)], \epsilon_1(0)]o_1(0) \\ &\quad - 2[o_1(0), \epsilon_1(0)][o_1(0), \epsilon_1(0)]) + \dots \end{aligned}$$

Here, only a spherical system is considered,

$$\varepsilon_1(0) = \begin{pmatrix} \Sigma(r) & 0 \\ 0 & \Delta(r) \end{pmatrix}, \quad o_1(0) = \begin{pmatrix} 0 & -\frac{d}{dr} + \frac{\kappa}{r} \\ \frac{d}{dr} + \frac{\kappa}{r} & 0 \end{pmatrix},$$

the diagonalized Dirac operator becomes

$$\varepsilon(\infty) = \begin{pmatrix} H_1 + M & 0 \\ 0 & H_2 - M \end{pmatrix}, \quad (18)$$

where

$$\begin{aligned} H_1 = & \Sigma(r) + \frac{p^2}{2M} - \frac{1}{2M^2} \left(Sp^2 - S' \frac{d}{dr} \right) - \frac{\kappa}{r} \frac{\Delta'}{4M^2} \\ & + \frac{\Sigma''}{8M^2} + \frac{S}{2M^3} \left(Sp^2 - 2S' \frac{d}{dr} \right) + \frac{\kappa}{r} \frac{S\Delta'}{2M^3} \\ & - \frac{\Sigma'^2 - 2\Sigma'\Delta' + 4S\Sigma''}{16M^3} - \frac{p^4}{8M^3} \end{aligned} \quad (19)$$

is an operator describing Dirac particle with $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa+1)}{r^2}$, and

$$\begin{aligned} H_2 = & \Delta(r) - \frac{p^2}{2M} + \frac{1}{2M^2} \left(Sp^2 - S' \frac{d}{dr} \right) + \frac{\kappa}{r} \frac{\Sigma'}{4M^2} \\ & + \frac{\Delta''}{8M^2} - \frac{S}{2M^3} \left(Sp^2 - 2S' \frac{d}{dr} \right) - \frac{\kappa}{r} \frac{S\Sigma'}{2M^3} \\ & + \frac{\Delta'^2 - 2\Sigma'\Delta' - 4S\Delta''}{16M^3} + \frac{p^4}{8M^3} \end{aligned} \quad (20)$$

is an operator describing Dirac antiparticle with $p^2 = -\frac{d^2}{dr^2} + \frac{\kappa(\kappa-1)}{r^2}$. The Hamiltonian for Dirac antiparticle in this case is, in fact, $-H_2$ with eigenvalue $-\epsilon$. This is consistent with the transformations of potentials $S \rightarrow S$, $\Sigma \rightarrow -\Delta$, and $\Delta \rightarrow -\Sigma$ under charge conjugation from H_1 [22]. Here, the primes have the same meaning as that in Eq. (4) and the double primes denote second-order derivatives with respect to r .

The first two terms of $H_1(H_2)$ correspond to the operator describing Dirac (anti-)particle in the nonrelativistic limit. The relativistic effect begins to show up from the order of $1/M^2$ in the perturbation expansion of $\varepsilon(\infty)$, which are presented in $H_1(H_2)$ from the third to fifth terms. In order to obtain better result, the perturbation expansion up to order $1/M^3$ is also included in Eqs. (19) and (20).

In Eq. (19), it can be seen the spin symmetry is exact for the Dirac particle when $\Delta' = 0$, which agrees with Eq. (3). The same result is obtained for the Dirac antiparticle in comparing with Refs. [22,27], which can be observed from Eq. (20) with $\Sigma' = 0$. Particularly, the singularity disappears in every component of Eqs. (19) and (20), the operators H_1 and H_2 are Hermitian. In addition, there is no the coupling between the energy ϵ and the operator $H_1(H_2)$. Thus, the energy spectra of H_1 and H_2 can be calculated conveniently.

The energy spectra of $H_1(H_2)$ agree the results of Eq. (1) very well, and the energy splittings of pseudospin partners are in agreement with the exact relativistic case. The contribution of every component to the pseudospin splittings can be calculated, which is helpful to analyze the origin of PSS. In order to convince the conclusion, a Woods-Saxon type potential is adopted for $\Sigma(r)$ and $\Delta(r)$, i.e., $\Sigma(r) = \Sigma_0 f(a_\Sigma, r_\Sigma, r)$ and

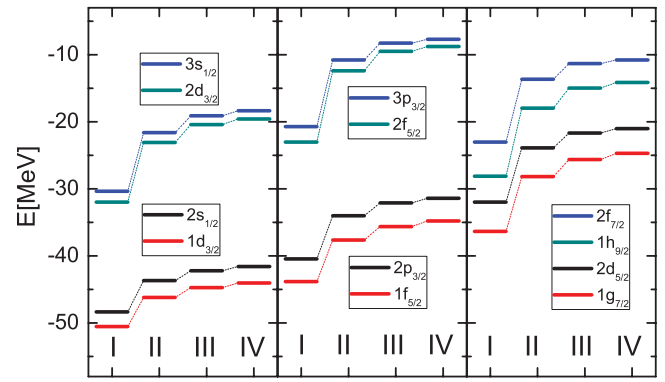


FIG. 1. (Color online) The energy spectrum of H_1 for the six pseudospin partners. The first column in each subfigure indicates that H_1 is approximated to the nonrelativistic limit. The second and third columns in each subfigure indicate that H_1 is approximated to the orders $1/M^2$ and $1/M^3$, respectively. The fourth column in each subfigure indicates the exact relativistic spectra.

$\Delta(r) = \Delta_0 f(a_\Delta, r_\Delta, r)$ with

$$f(a_0, r_0, r) = \frac{1}{1 + \exp\left(\frac{r-r_0}{a_0}\right)}. \quad (21)$$

The corresponding parameters are determined by fitting the energy spectrum from the RMF calculations for ^{208}Pb (to see Ref. [12]). The energy spectra of $H_1(H_2)$ are calculated by expansion in harmonic oscillator basis. The energy spectra of the six pseudospin partners are shown in Fig. 1, where the first column in each subfigure corresponds to that H_1 is approximated to the nonrelativistic limit. The second and third columns in each subfigure correspond to that H_1 is approximated to the order $1/M^2$ and $1/M^3$, respectively. The exact relativistic spectra [the eigenvalues of Eq. (1)] are displayed in the fourth column. From Fig. 1, it can be seen that the deviations between the nonrelativistic limit (the first column) and the exact relativistic case (the fourth column) are very large, i.e., the relativistic effect is apparent in the present system. With the increasing perturbation order, the calculated result is closer to the exact relativistic one. When H_1 is approximated to the order $1/M^3$, the calculated spectra are considerably agreeable with those from the exact relativistic calculations. The pseudospin energy splitting is in especially good agreement with the exact relativistic result. These show the operator H_1 presents a good description for Dirac particle and can be used to analyze the PSS.

In order to disclose the origin of PSS, we check the contribution of every component in H_1 to the pseudospin energy splitting. Based on the considerations of attribute and Hermitian, we decompose H_1 into the eight components as follows: $\Sigma(r) + \frac{p^2}{2M}$, $-\frac{1}{2M^2}(Sp^2 - S' \frac{d}{dr})$, $-\frac{\kappa}{r} \frac{\Delta'}{4M^2}$, $\frac{\Sigma''}{8M^2}$, $\frac{S}{2M^3}(Sp^2 - 2S' \frac{d}{dr})$, $\frac{\kappa}{r} \frac{S\Delta'}{2M^3}$, $-\frac{\Sigma'^2 - 2\Sigma'\Delta' + 4S\Sigma''}{16M^3}$, $-\frac{p^4}{8M^3}$, which are, respectively, labeled as O_1, O_2, \dots, O_8 . For the k state with eigenvector ψ_k , the contribution of O_i to the level E_k is calculated by the formula $\langle k | O_i | k \rangle = \int_0^\infty \psi_k^* O_i \psi_k d^3\vec{r}$, which is denoted as $\epsilon_i(k)$. To reduce the length of the article, only the data for two pseudospin doublets are listed in Table I.

TABLE I. The contribution of the operator O_i to the level E_k : $\epsilon_i(k) = \langle k | O_i | k \rangle$, where O_i can be seen in text, k represents the single-particle states $2d_{5/2}$, $1g_{7/2}$, $2f_{7/2}$, and $1h_{9/2}$, and $\Delta\epsilon = \epsilon_i(a) - \epsilon_i(b)$. The data listed in the last line is a sum from the first line to the eighth line.

i	$2d_{5/2}(a)$ $1g_{7/2}(b)$			$2f_{7/2}(a)$ $1h_{9/2}(b)$		
	$\epsilon_i(a)$	$\epsilon_i(b)$	$\Delta\epsilon$	$\epsilon_i(a)$	$\epsilon_i(b)$	$\Delta\epsilon$
1	-30.872	-35.649	4.777	-21.303	-27.035	5.732
2	7.511	6.746	0.765	8.448	8.003	0.445
3	-0.509	0.721	-1.230	-0.732	1.083	-1.815
4	0.018	0.042	-0.023	0.004	0.042	-0.038
5	2.732	2.349	0.383	3.021	2.717	0.303
6	-0.240	0.410	-0.650	-0.303	0.581	-0.884
7	0.001	0.018	-0.017	-0.006	0.015	-0.022
8	-0.316	-0.257	-0.059	-0.440	-0.389	-0.051
Total	-21.675	-25.621	3.946	-11.312	-14.982	3.671

The contribution of O_1 to $\Delta\epsilon$ is very large, which means that the spectra of H_1 in the nonrelativistic limit do not have the PSS; namely PSS is not a nonrelativistic symmetry. It has a relativistic origin, which agrees with the claim in Ref. [3]. The contributions of O_3 and O_6 to $\Delta\epsilon$ are negative, which implies the pseudospin splittings in the nonrelativistic limit are reduced by the contributions from these terms relating the spin-orbit interactions and agrees with the relativistic interpretation of PSS. The contributions of O_2 and O_5 to $\Delta\epsilon$ are positive, which means the pseudospin splittings are added by the contributions relating the dynamical terms and supports a relativistic origin for this symmetry. Compared with O_2 , O_3 , O_5 , and O_6 , the contributions of O_4 , O_7 , and O_8 to $\Delta\epsilon$ are relatively minor. These show the quality of PSS origins mainly from the competition of the spin-orbit interactions and the dynamical effects. Although the contributions of O_4 , O_7 , and O_8 to $\Delta\epsilon$ are minor, their influences on PSS cannot be ignored, which supports, in part, the claim in Refs. [21–23], i.e., the observed pseudospin splitting arises from a cancellation of the several energy components, and the PSS in nuclei has a dynamical character.

In order to check further the applicability and validity for the present formalism, we have calculated the energy spectrum of H_2 for Dirac antiparticle, which is shown in Fig. 2. The good spin symmetry is displayed clearly, which is in agreement with Refs. [22,27].

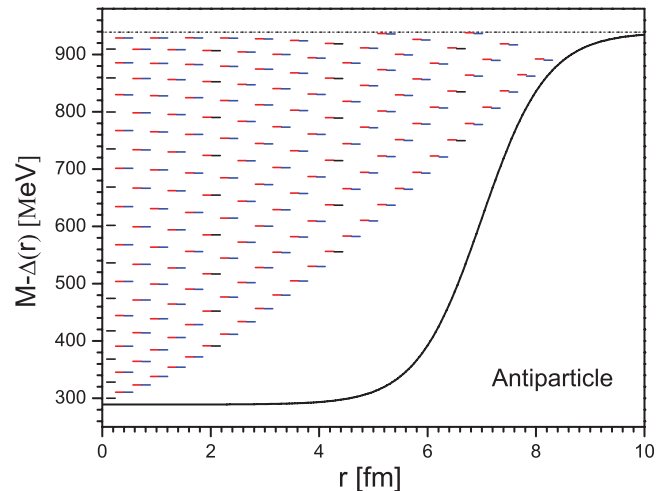


FIG. 2. (Color online) The energy spectrum of H_2 for the Dirac antiparticle.

In summary, the similarity renormalization group is used to transform the spherical Dirac operator into a diagonal form. The upper (lower) diagonal element becomes an operator describing Dirac (anti-)particle, which holds the form of the Schrödinger-like operator with the singularity disappearing in every component. The energy spectra of the operator are calculated in good agreement with the exact relativistic ones. By comparing the contributions of the various components to the energy splittings, PSS is shown to be a relativistic symmetry. The quality of PSS is correlated with the contribution of every component of H_1 to the pseudospin splitting, especially the competition of the spin-orbit interactions and the dynamical effects, which supports the claim of a dynamical character. The spin symmetry of antiparticle spectrum is also well reproduced in the present calculations.

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