α-decay systematics for superheavy elements

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In this Brief Report we extend the α-decay half-life calculation to the superheavy emitter region to verify whether these nuclei satisfy the recently observed systematics [D. N. Poenaru *et al.*, Phys. Rev. C **83**, 014601 (2011); C. Qi *et al.*, Phys. Rev. C **80**, 044326 (2009)]. To establish the systematics, we have used the α-cluster potential description, which was originally developed to study α decay in connection with nuclear energy level structure [B. Buck *et al.*, Phys. Rev. C **51**, 559 (1995)]. The quantum-mechanical tunneling calculation has been employed to obtain the half-lives, showing that with this treatment the systematics are well reproduced in the region of heavy nuclei. Finally, the half-life calculation has been extended to the superheavy emitters to verify whether the systematics can still be observed.

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The analysis of α-decay half-life is an important tool to study nuclear level structure. Almost twenty years ago a considerable effort was invested in this direction, and a sequence of papers studied nuclear potential forms to reproduce the nuclear level distribution of the nuclei involved in the process [3–5]. The α-cluster model was designed with this purpose and used to determine half-lives of a wide range of ground-state α emitters. In the last version of this model [3] a Wood-Saxon-like potential form together with the semiclassical Bohr-Sommerfeld orbit quantization rule have been successfully applied for intermediate mass and heavy emitters. In these last months, the use of this potential form to reproduce the properties of first levels in ground-state spectral bands of the ^{90}Sr and ^{98}Pd nuclei has been reported, assuming the α-cluster structure for these nuclear systems [6].

Recently α-decay studies were revisited by looking for a sort of universal decay law for the families of odd-even, even-even, and even-odd α emitters, which could be extended to heavier cluster emission processes [2,7]. A systematic behavior of half-lives was recognized by using the semiclassical approach (WKB barrier penetrability) with a nuclear potential obtained by the folding of the nucleon-nucleon interaction with the nuclear density of the nascent fragments [1,8].

A simple linear relation for experimental half-life values of α-decay in heavy-nuclei regions was observed in Ref. [9], showing that a straight line can be adjusted for the logarithm of half-lives as a function of $Z_d^{0.6}/\sqrt{Q}$. Another type of empirical analysis [10] of experimental data provided general relationships to obtain half-lives for large-mass regions of α emitters, accounting for different values of angular momentum.

In this Brief Report we use the α-cluster potential with the same parameters introduced in the original version [3]; however, we use an approach formally distinct from the Born-Sommerfeld orbit quantization treatment to determine half-lives in the original α-cluster model.

The α-cluster model potential is given by a Wood-Saxon-like form,

$$V_N(r) = V_0\left\{\beta\left[1+\exp\left(\frac{r-R}{a}\right)\right]^{-1} + (1-\beta)\left[1+\exp\left(\frac{r-R}{3a}\right)\right]^{-3}\right\}, \tag{1}$$

where the potential parameter (depth, diffuseness, and the mix parameters, V_0, a, and β,respectively) were chosen to obtain better determination of half-lives of a wide range of α emitters and simultaneously reproduce the energy level structure of some double closed shell emitters. In addition, the choice of the parameters was done in order to obtain a good agreement with the real part of the characteristic optical potential for the α-core scattering of double closed shell cores. In the present study we used the set values $V_0 = 220$ MeV, $a = 0.65$ fm, $\beta = 0.3$, extracted from Table 1 of Ref. [3]. Here no change of parameters or adjustment is employed to determine α-decay half-life.

The nuclear potential in Eq. (1) is superposed to the Coulomb potential of a homogeneous charged sphere, given by

$$V_c(r) = \begin{cases} Z_1Z_2e^2[3-(r/R)^2]/2R & \text{for} \quad r \leqq R, \\ Z_1Z_2e^2/r & \text{for} \quad r > R, \end{cases} \tag{2}$$

to naturally generate the decay barrier in the case of null angular values of the emitted α particle. When higher angular moment values are considered we have to add the contribution of the centrifugal barrier,

$$V_{\text{cent}} = \frac{\hbar^2 l(l+1)}{2\mu r^2}. \tag{3}$$

The nuclear radius appearing in Eqs. (1) and (2) is taken as $R = r_0A^{1/3}$, with $r_0 = 1.22$ fm within the range of radius used in Ref. [5]. The characteristic form of the combined nuclear and Coulomb potential presents a pocket in the inner region, separated by a barrier from the external region. Consequently,

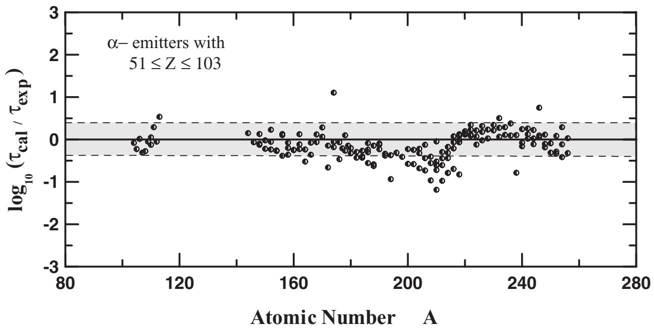

FIG. 1. Comparison between our calculated half-lives and data for 164 heavy nuclei. The points represents the logarithm of the rate between the calculated and experimental half-life values. The gray area contains 126 points having the ratio within a factor smaller than 2.5, corresponding to $\sigma = 0.20$.

we have three classical turning points R_1, R_2, and R_3 in the WKB-tunneling calculation, which are currently defined by the condition

$$V(R_1) = V(R_2) = V(R_3) = Q, \tag{4}$$

where $V(r) = V_n + V_c + V_{\rm cent}$ is the total potential and Q is the decay energy value obtained from the experimental nuclear mass values.

The α-decay half-lives were calculated as in Ref. [11], but here including explicitly the α preformation factor S,

$$\tau = \frac{\ln(2)}{\lambda_0 S P}. \tag{5}$$

From Ref. [12] we have used the values $S_{\text{odd-odd}} = 0.15$, $S_{A\text{-odd}} = 0.21$, and $S_{\text{even-even}} = 0.34$. The barrier

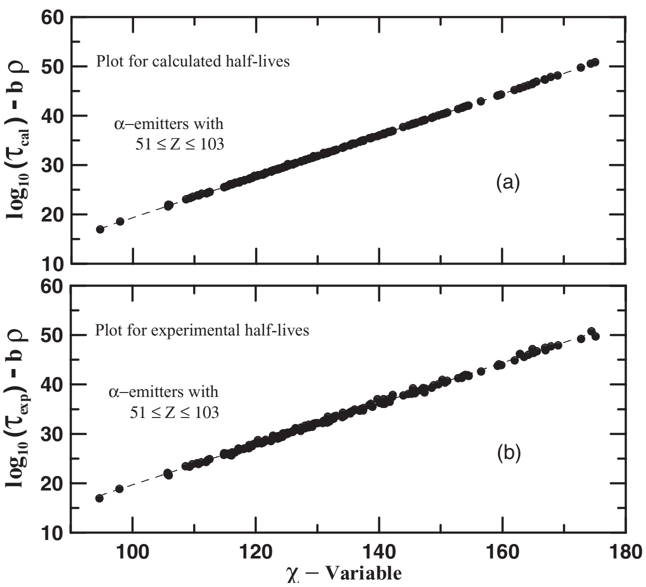

FIG. 2. The UDL systematic for half-lives, using quantities ρ and χ (see text for the definition). In part (a) the α-decay systematic is shown for our calculated decay half-lives of 164 heavy nuclei. In part (b) we show the corresponding plot by using the experimental data. A nice agreement of our results is observed with the established systematic.

TABLE I. Straight-line parameters $[\log_{10}(\tau_{\rm cal}) - b\rho = a\chi + c]$ for the UDL systematic using $b = -0.4311$ and the rms deviations with respect to data, determined by Eq. (8). Our results are compared with those from Ref. [2].

	Present Work (164 nuclei with $51 \leqslant Z \leqslant 103$)	Ref. [2] (nuclei with $78 \leqslant Z \leqslant 108$)
a	0.4171	0.4065
c	−22.3641	−20.7889
σ	0.36	0.3436

penetrability factor P is given by [13]

$$P = \exp\left(-\frac{2}{\hbar}\int_{R_2}^{R_3} \sqrt{2\mu(V-Q)}\,dr\right). \tag{6}$$

The assault rate λ_0 of the α particle to the potential barrier is obtained considering the frequency of oscillation of the α particles inside the inner pocket, resulting from the composition of the Coulomb and nuclear potential, given by

$$\lambda_0 = \sqrt{\frac{Q - V_{\rm min}}{4\mu(R_2 - R_1)^2}}. \tag{7}$$

To determine the mean kinetic energy of the α particle inside the inner potential pocket we have adjusted a harmonic oscillator, and from the virial relation we determine the kinetic energy as a function of the potential depth $V_{\rm min}$. In Eqs. (6) and (7) μ is the reduced mass of the α-cluster system.

As a first task we have to verify whether our calculation can reproduce satisfactorily the existing experimental data, and also whether the half-life systematics in Refs. [1,2,7,8] are observed. In Fig. 1 we show the logarithm of the ratio between the calculated and experimental half-lives for 164 heavy emitters with null angular decay momentum ($l = 0$). The data are taken from the collected experimental results in the literature presented in Refs. [11,12,14,15]. We can see that the maximum deviation is around one order of magnitude for the whole set of 164 α heavy nuclei emitters used. For this set of nuclei the root-mean-square deviation, given by

$$\sigma = \left\{\frac{1}{n-1}\sum_{i=1}^{n}\left[\log_{10}\left(\tau_i^{\rm cal}/\tau_i^{\rm exp}\right)\right]^2\right\}^{1/2}, \tag{8}$$

is $\sigma = 0.36$, which is on a par with others values in the literature, as for example $\sigma = 0.34$ in Ref. [2] and $\sigma = 0.32$

TABLE II. Straight-line $[\log_{10}(\tau_{\rm cal}) = -\log_{10} S - a\log_{10} P + c]$ of the NUP and the rms deviations determined by Eq. (8). Our results are compared with those from Ref. [1]. Note that in this reference the systematic parameters are determined using a data set which includes α and heavier cluster emitters.

	Present Work (164 nuclei with $51 \leqslant Z \leqslant 103$)	Ref. [1] (163 α + 27 cluster emitters)
a	0.9999	1.0000
c	−21.7615	−22.12917
σ	0.36	0.428

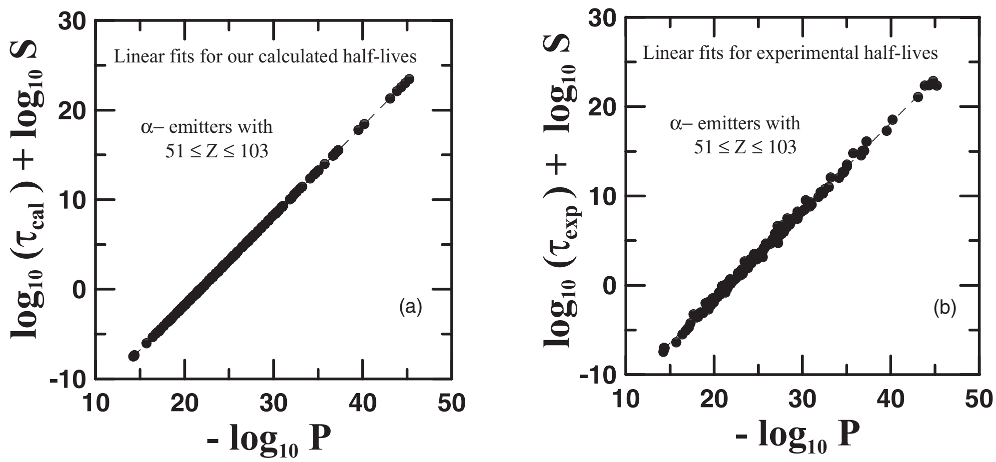

FIG. 3. The new universal plot (NUP), using the logarithm of half-life and the logarithm of the penetrability factor P. In part (a) the α-decay systematic is shown for calculated decay half-lives of 164 heavy nuclei. In part (b) we show the corresponding plot by using the experimental data. Again a rather good accordance of our results is observed with the new universal plot.

in Ref. [15]. When the rms is restricted to the set of nuclei with half-life satisfying $|\log_{10}(\tau_i^{\rm cal}/\ \tau_i^{\rm exp})| \leqslant 0.4$ (126 heavy α emitters), our result for the deviation is $\sigma = 0.20$. These three works use different approaches to determine half-lives but they have in common the use of a spherical potential to describe the α-decay process. On the other hand, calculations with an explicit deformed potential for half-life determination present quite similar rms deviations, as follows: (i) In Ref. [12], $\sigma = 0.21$, to the set of nuclei with half-life satisfying $|\log_{10}(\tau_i^{\rm cal}/\tau_i^{\rm exp})| \leqslant 0.4$, and $\sigma \lesssim 0.4$, for the whole set of 153 α emitters used; (ii) in Ref. [16], $\sigma = 0.19$, obtained for 35 very deformed nuclei; (iii) in Ref. [14], $\sigma = 0.29$, for 166 α emitters; (iv) in Ref. [17], $\sigma \lesssim 0.79$, for α emitters with $78 \leqslant Z \leqslant 102$. At this point, we note that the two systematics discussed in this Brief Report are introduced in Refs. [1,7] also using a spherical potential to determine α-decay half-lives.

In Fig. 2 we exhibit the systematic of Ref. [2], the universal decay law (UDL), using the quantities $Y = \log_{10}\tau - b\rho$ (with $\rho = [A_r Z_\alpha Z_d(A_e^{1/3} + A_d^{1/3})]^{1/2}$, and τ in seconds) versus the variable χ, defined as

$$\chi = Z_\alpha Z_d/\sqrt{A_r} \quad \text{with} \quad A_r = A_\alpha A_d/A. \tag{9}$$

Here Z_α and A_α are respectively the atomic and mass number of the α particle, and Z_d and A_d the corresponding values for the daughter nucleus. We can see in part (a) of the figure that our calculated half-lives for the 164 heavy nuclei with $l = 0$ angular momentum nicely obey the systematic. For comparison in part (b) we display the same systematic by using the experimental half-life values. The straight lines in the plot are represented by the equations

$$\log_{10}(\tau_{\rm cal}) - b\rho = 0.4171\chi - 22.3641 \tag{10}$$

for the straight line in part (a) and

$$\log_{10}(\tau_{\rm exp}) - b\rho = 0.4123\chi - 21.5659 \tag{11}$$

in part (b). For the parameter b, in order to have a direct comparison of our systematic with the original one from Ref. [2], we have taken the same value obtained there, namely $b = -0.4311$.

Another type of systematic for half-lives has been observed and presented in Refs. [1,8], which was called recently the new universal plot (NUP) [1], pointing out the linear relation between the logarithm of the half-lives to the logarithm of the barrier penetrability factor. In Fig. 3 we show that our

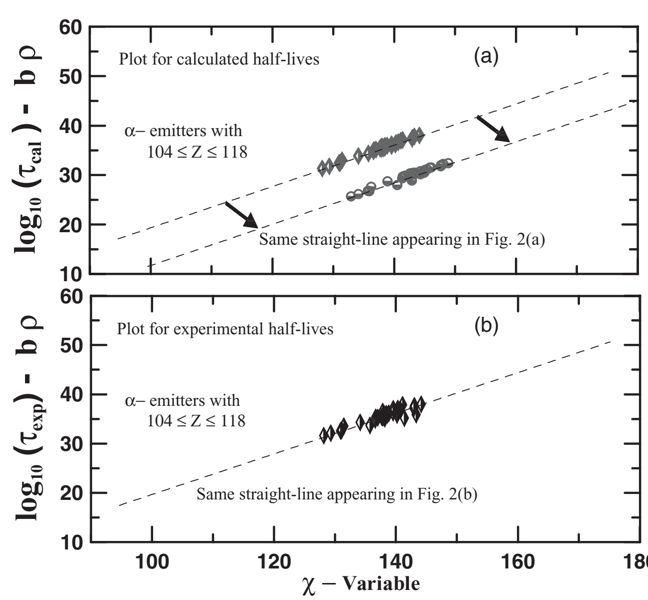

FIG. 4. The UDL systematic for superheavy nuclei. In part (a) the α-decay systematic is shown for our calculated decay half-lives of 35 superheavy nuclei (represented by full diamonds). The systematic using calculated half-lives from Ref. [12] is also shown (represented by circles). Intentionally this last result is drawn shifted from the straight line of our results to make easier the comparison. In part (b) we show the corresponding plot by using the experimental data. We are showing that superheavy elements follow satisfactorily the same observed systematic using ρ and χ, which is obeyed for heavy nuclei.

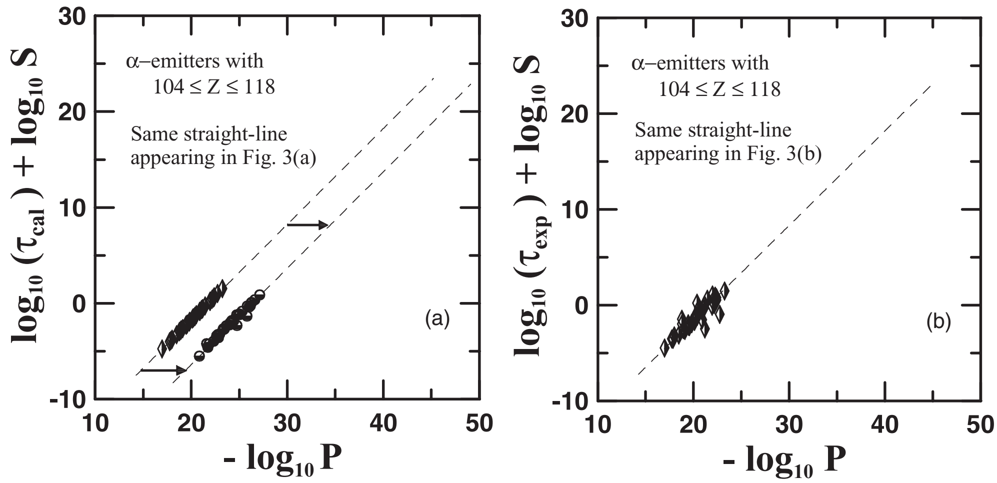

FIG. 5. The NUP systematic for half-lives of superheavy nuclei. In part (a) the α-decay systematic is shown for our calculated half-lives of 35 superheavy nuclei (represented by full diamonds). The systematic using calculated half-lives from Ref. [12] is also shown (represented by circles). Intentionally the last result is drawn shifted from the straight line of our results to make easier the comparison. In part (b) we show the corresponding plot by using the experimental data. We are showing that superheavy elements follow satisfactorily the same universal systematic observed for heavy nuclei.

calculation using the α-cluster potential model without any free parameter reproduces this systematic in rather good agreement. A single linear fit is able to represent both the theoretical prediction and experimental data, since no significant difference can be noted for the fits of points in part (a) and part (b). These fits are given by the equations

$$\log_{10}(\tau_{\rm cal}) + \log_{10} S = -0.9999 \log_{10} P - 21.7615 \qquad (12)$$

for the straight-line in part (a) and

$$\log_{10}(\tau_{\rm exp}) + \log_{10} S = -0.9861 \log_{10} P - 21.2613 \qquad (13)$$

for part (b).

A satisfactory agreement between our results and the data are shown in Fig. 1, and the rather good accordance with the systematic can be observed in Fig. 2 and Fig. 3. We have also verified that our calculated half-lives can reproduce quite well the linear behavior attained in Ref. [9]. In addition, the empirical relations presented in Ref. [10] where verified. In Tables I and II the straight-line parameters of both systematics (UDL and NUP) and the half-life rms deviation with respect to data of the heavy nuclei are compared with results from others works.

In the recent study of superheavy α decay, half-lives were determined also using the WKB approximation with the barrier generated by a Wood-Saxon form of the nuclear potential [12]. However, there the two-potential approach (TPA) was employed to determine the superheavy half-lives. Here, our aim is to go further, not only determining superheavy half-lives, but also verifying whether the decay of these system satisfies the observed universal systematic for heavy nuclei, recently revisited in Ref. [1,8], and the other one pointed out by previous works [2,7]. So we extended our calculation to the whole set of known superheavy emitters nowadays, a collection of 35 nuclei. In Fig. 4 we show that these nuclei follow properly the first type of systematic observed in Fig. 2 for the heavy-nuclei emitters. Similar accordance is observed for the case of the new universal plot systematic, as we can see from Fig. 5. In part (a) of these figures we show the systematic using our half-life results. The result using superheavy half-lives determined in Ref. [12] is also shown, with similar agreement.

Furthermore, we verified that for the used set of superheavy nuclei, the rms deviation of our calculated half-lives with respect to data ($\sigma = 0.72$)is still compatible with the value in Ref. [12] ($\sigma = 0.66$), including nuclear deformation effect. We also verified that our half-life calculation offers deviation quite similar to that obtained for few superheavy elements in Ref. [18], which incorporated explicitly the deformation effect. Indeed, we can see in Fig. 4(a) and Fig. 5(a) that the use of the spherical potential to determine the superheavy half-life still permits the establishment of both systematics in reasonable agreement with results using half-lives calculated by including deformation effect (shifted plots).

In summary, we have shown that superheavy α emitters satisfactorily fit both systematics recently pointed out, with half-lives determined by the tunneling approach in a potential barrier generated by a nuclear potential form previously designed and adjusted to reproduce the nuclear level structure of heavy emitters. We remark that no adjustable parameters are introduced in the calculation, nor were the original values of the nuclear potential parameters modified.

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