$\Delta^0(1232)$ production in $d + {}^{12}C$ collisions at 4.2A GeV/c

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Production of $\Delta^0(1232)$ resonances in $d + {}^{12}C$ collisions at 4.2A GeV/c was investigated with 4π acceptance, for the first time. The mass distribution of $\Delta^0(1232)$ resonances produced in $d + {}^{12}C$ collisions was reconstructed and their mass and width were extracted. The kinematical distributions of $\Delta^0(1232)$ were reconstructed in the laboratory frame and the corresponding mean kinematical characteristics were estimated. The fraction of charged π^- mesons coming from $\Delta^0(1232)$ decay as well as the relative number of nucleons excited to $\Delta^0(1232)$ in freeze-out conditions were estimated. The freeze-out temperature of $\Delta^0(1232)$ was estimated and compared to results of other experiments with different sets of colliding nuclei and various incident energies.

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I. INTRODUCTION

Resonances with a short lifetime and their coupling to a nuclear medium have unique characteristics to probe various features of the medium resulting from relativistic hadron-nucleus and nucleus-nucleus collisions. Much of the experimental and theoretical research [1–44] has been devoted to the investigation of $\Delta(1232)$ resonance production in various strong and electromagnetic processes involving pions, protons, photons, light nuclei, and heavy ions. The process responsible for meson production in central heavy-ion collisions at energies of the order of several GeV/nucleon is believed to be predominantly the excitation of baryon resonances during the early compression phase of the collision [12]. In the later expansion phase these resonances decay. The mass and width of $\Delta(1232)$ produced in a nuclear medium in relativistic hadron-nucleus and nucleus-nucleus collisions modify as compared to the mass and width ($M_{\Delta_{NN}} = 1232$ MeV/c^2, $\Gamma = 115$–120 MeV/c^2) [45] of $\Delta(1232)$ produced in nucleon-nucleon collisions. The significant decrease of the mass and width of $\Delta(1232)$ resonances produced in dense hadron matter created in central heavy-ion collisions was interpreted in terms of thermal and isobar models [12,26]. The modification of $\Delta(1232)$ properties was also related to the values of hadronic density, temperature, and various non-nucleon degrees of freedom in nuclear matter [12,26,35,46,47]. However, many experimental results on $\Delta(1232)$ production are still nontrivial and there are ambiguities in their theoretical interpretations.

Identification of structures in the invariant mass distribution of correlated proton and pion pairs provides a direct proof that nucleons are excited to high-lying resonances [12]. The major obstacle that should be overcome in reconstructing the invariant mass is the large background of noncorrelated $p\pi$ pairs [12]. In peripheral reactions with very light projectiles, e.g., p- [27,28] or ^{3}He- [29] induced reactions at ∼2 GeV bombarding energy, the $p\pi$ correlations were successfully analyzed and the mass distribution of the $\Delta(1232)$ was reconstructed. The resonance mass was found to be shifted by ∼−25 MeV/c^2 toward lower masses in reactions on various targets, compared to those on protons [27–29]. The mass reduction of the $\Delta(1232)$ resonance in $p + A$ collisions ($A =$ C, Nb, Pb) at 0.8 and 1.6 GeV incident energy [27] was traced back to the effects of Fermi motion, NN scattering, and pion reabsorption in nuclear matter. In nucleus-nucleus interactions, $\Delta(1232)$ are assumed to be mainly produced in the reaction $NN \to \Delta N$, which competes with the process of direct pion production: $NN \to NN\pi$, $NN \to NN\pi\pi$, etc. In this simple scenario $\Delta^0(1232)$ resonances are produced via the reaction $NN \to \Delta^0 N + k\pi$ ($k = 0, 1, \ldots$), decaying subsequently through the $\Delta^0 \to p\pi^-$ channel. The observed final particles, protons and π^- mesons, were used to analyze $\Delta^0(1232)$ production in the present paper.

The present paper is a continuation of a series of research papers [41–44,48,49] devoted to the experimental investigation of $\Delta(1232)$ production in relativistic hadron-nucleus and nucleus-nucleus collisions. We aim to extract the various physical properties of $\Delta^0(1232)$ resonances, produced in $d + {}^{12}C$ collisions at 4.2A GeV/c, and compare them with those of $\Delta^0(1232)$ produced with different sets of colliding nuclei at various initial energies. The paper is organized as follows. The experimental procedures are described in Sec. II. Analysis of $\Delta^0(1232)$ production is given in Sec. III. Sec. IV presents the analysis of reconstructed kinematical spectra and mean kinematical characteristics of $\Delta^0(1232)$ in $d + {}^{12}C$ collisions at 4.2A GeV/c. Finally, Sec. V summarizes the main results of the present paper.

II. THE EXPERIMENT

The experimental data were obtained on the basis of processing stereophotographs from the 2-m propane (C_3H_8) bubble chamber of the Laboratory of High Energy of the Joint Institute for Nuclear Research (JINR, Dubna, Russia) placed in a magnetic field of strength 1.5 T and irradiated with a beam of deuterons accelerated to a momentum of

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4.2 GeV/c per nucleon at the JINR synchrophasotron. To select events of inelastic $d + {}^{12}$C interactions in the total set of deuteron interactions with propane, we used criteria based on the determination of the total charge of secondary particles, the presence of protons emitted into backward hemisphere, the number of π^- mesons produced, etc., as described in detail in Refs. [50–53]. These criteria separate ∼70% of the total number of inelastic $d + {}^{12}$C events that was estimated by using the known cross sections for $d + p$ and $d + {}^{12}$C interactions and $p:{}^{12}$C ratio in the propane molecule. The remaining 30% collision events were added statistically from $d + p$ interactions on quasifree protons of C_3H_8 molecules by introducing the relevant weights. The weights were determined in such a way that the numbers of events occurring on carbon and hydrogen corresponded to the numbers expected on the basis of the known cross sections for inelastic interactions [52–55]. It should be mentioned that $d + p$ interaction events on quasifree protons of carbon nuclei constitute ∼69% of all the $d + p$ interaction events on quasifree protons of propane molecules. This follows from the fact that 18 out of a total of 26 protons in the propane molecule belong to carbon nuclei. The corrections to account for losses of particles emitted under large angles to the object plane of the camera were introduced: They amounted to ∼3% for protons with momenta $p_{\rm lab} > 300$ MeV/c and ∼15% for slow protons with $p_{\rm lab} < 300$ MeV/c [52]. The protons and π^+ mesons were separated visually based on their ionization up to a momentum $p \approx 0.75$ GeV/c. However, under conditions of the present experiment, protons and positively charged pions are identified unambiguously up to a momentum of 0.5 GeV/c. Therefore all the positively charged particles with a momentum of greater than 0.5 GeV/c were assigned weights determining the probability that a given particle is a proton or a π^+ meson. All the negatively charged particles were identified as π^- mesons. It should be mentioned that π^- mesons constitute >95% of the negatively charged particles, and the admixture of fast electrons and strange negative particles among them does not exceed 5%. Most of the pions and protons with a momentum of lower than 70 and 150 MeV/c, respectively, were not recorded because of their short range in the bubble chamber. The average error in measuring angles of the secondary particles was 0.8°, while the mean relative error in determining momenta of the particles from the curvature of a track in the magnetic field was 11%.

The momentum distribution of protons produced in $d + {}^{12}$C collisions at 4.2A GeV/c is presented in Fig. 1(a). In Ref. [52] protons with $p_{\rm lab} > 300$ MeV/c (and excluding projectile spectator protons with a momentum $p > 3$ GeV/c and an emission angle relative to the beam direction $\theta < 3^\circ$ in the laboratory frame) were classified as participant protons. The mean multiplicity per event of participant protons was determined to be 1.95 ± 0.08 [52] in $d + {}^{12}$C collisions in early work, with a statistics of 5404 inelastic events. After introducing all of the above-mentioned corrections, we obtained the mean multiplicity per event of participant protons to be 1.94 ± 0.06 in the present analysis, with a statistics of 7071 inelastic $d + {}^{12}$C events, measured under 4π acceptance, corresponding to the cross section 400 ± 20 mb. The mean multiplicity of participant protons calculated using the Dubna

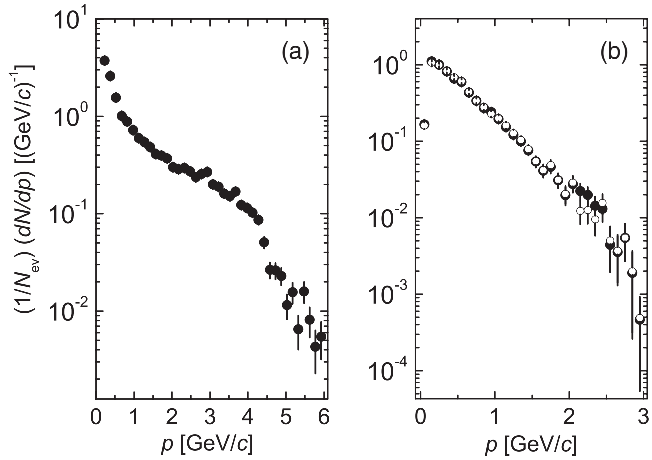

FIG. 1. Momentum distribution of (a) protons and (b) π^- (•) and π^+ (○) mesons produced in $d + {}^{12}$C collisions at 4.2 GeV/c per nucleon.

cascade model (DCM) [56,57], the FRITIOF model [58–62], and the quark-gluon-string model (QGSM) [63] were 1.97 ± 0.01 [52], 2.05 ± 0.03 [55], and 1.86 ± 0.02 [64], respectively.

The momentum distributions of π^- and π^+ mesons produced in $d + {}^{12}$C collisions at 4.2A GeV/c are presented in Fig. 1(b). Momentum distributions of π^- and π^+ mesons, as seen from Fig. 1(b), practically coincide with each other, as expected for a symmetric $d + {}^{12}$C system containing the same number of protons and neutrons. The mean multiplicities per event of π^- and π^+ mesons produced in $d + {}^{12}$C collisions at 4.2A GeV/c coincided and proved to be 0.66 ± 0.01 in the present paper. In an early work with a statistics of 2171 inelastic events, the mean multiplicity per event of π^- mesons was found to be 0.62 ± 0.03 [50] in $d + {}^{12}$C collisions at 4.2A GeV/c. The mean multiplicity of π^- mesons calculated using DCM, the FRITIOF model, and QGSM were 0.68 ± 0.02 [50], 0.70 ± 0.01 [55], and 0.63 ± 0.01 [65], respectively.

III. ANALYSIS OF Δ^0(1232) PRODUCTION

The measured momenta of protons and π^- mesons were used to calculate the invariant mass M of the $p\pi^-$ system, from the relation

$$M^2 = (E_p + E_\pi)^2 - (\vec{p}_p + \vec{p}_\pi)^2, \quad (1)$$

where E_p, E_π, $\vec{p}_p$, $\vec{p}_\pi$ are the energy and momentum of the proton and π^- meson, respectively. The experimental and background invariant mass distributions for $p\pi^-$ pairs in $d + {}^{12}$C collisions at 4.2A GeV/c are shown in Fig. 2(a). The experimental distribution was obtained by combining all protons and π^- mesons in each individual event. The background spectrum was obtained by an event mixing method: That is, the invariant mass of $p\pi^-$ pairs selected randomly using a proton from one event and a pion from another event was calculated. To take into account the event topology, only the events with equal particle multiplicities were combined. As seen from Fig. 2(a), the experimental invariant mass distribution for $p\pi^-$ pairs does not show a resonancelike

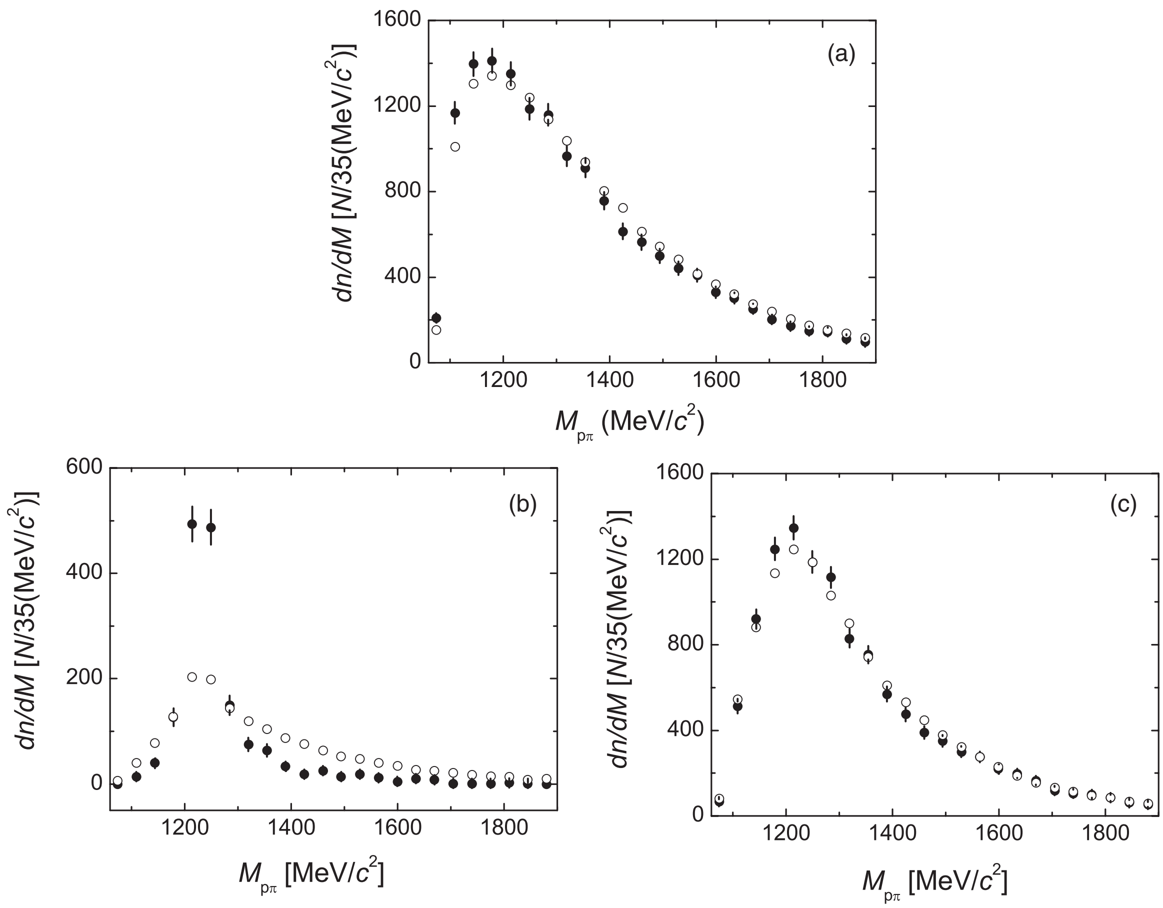

FIG. 2. (a) Experimental (•) and background (○) invariant mass distributions of $p\pi^-$ pairs in $d + {}^{12}$C collisions at 4.2 GeV/c per nucleon, and those obtained using the cutoff parameters (b) $\varepsilon = 0.05$ and (c) $\varepsilon = 0.90$.

structure near $M_\Delta = 1232$ MeV/c^2, expected for $\Delta^0(1232)$ resonance, and the maximum of experimental distribution is shifted to the values of $M < 1200$ MeV/c^2. This is due to a large combinatorial background from uncorrelated $p\pi^-$ pairs, as was also seen earlier for ^{16}O + p collisions [42] at 3.25A GeV/c, $p + {}^{12}$C [48,49], ^{12}C + ^{12}C [40], ^{4}He + ^{12}C [44], and ^{12}C + Ta [43] collisions at 4.2A GeV/c, $\pi^- + {}^{12}$C interactions at 40 GeV/c [41]. To reduce such a contribution as much as possible, the method of analyzing an angle between the proton and pion was used as done successfully in Refs. [40–44,48,49] to extract the mass distribution of $\Delta^0(1232)$.

If the Δ resonance decays in flight, the angle α between the outgoing proton and pion, in the laboratory frame, is defined by

$$\cos\alpha = \frac{1}{p_p p_\pi}\left(E_p E_\pi - \frac{M^2_{\Delta_{NN}} - M^2_\pi - M^2_p}{2}\right), \qquad (2)$$

where p_p and p_π are the proton and pion momenta, E_p and E_π are the respective energies, and $M_{\Delta_{NN}} = 1232$ MeV/c^2. This value was compared with the cosine of experimentally measured angle β,

$$\cos\beta = \frac{\vec{p}_p \cdot \vec{p}_\pi}{p_p p_\pi}. \qquad (3)$$

The experimental invariant mass distribution dn^{expt}/dM for $p\pi^-$ pairs was constructed using the sole criterion: Only the combinations satisfying the inequality

$$|\cos\beta - \cos\alpha| < \varepsilon \qquad (4)$$

were kept, where ε is an arbitrary cutoff parameter theoretically lying in the interval [0,2]. If the momenta of protons and pions are measured with high precision, the upper limit of this interval should be low.

The background spectrum was obtained by the event-mixing technique using the same criterion in relation (4), as for experimental distribution, and calculating the invariant mass of $p\pi^-$ pairs. It is necessary to mention that for each experimental distribution at certain ε the corresponding background distribution was obtained using the same value of the cutoff parameter ε. For the pairs measured in a certain event the corresponding mixed pair was chosen from events with identical proton and pion multiplicities. It is necessary to note that the condition of equal multiplicities in the experimental and mixed spectra ensures that correlations due to the reaction dynamics and pion rescattering in spectator matter are properly subtracted from the experimental spectrum [12,37,38]. The number of mixed combinations for each background spectrum was five times greater than in experimental distribution. Then the background spectrum was normalized to the total number of pairs in the experimental distribution. In this way, the set of experimental and background invariant mass distributions of $p\pi^-$ pairs was constructed for different values of the cutoff parameter ε. The distribution of differences between experimental and background invariant mass distribution, given by

$$D(M) = \frac{dn^{\text{expt}}}{dM} - a\frac{dn^b}{dM}, \quad (5)$$

was analyzed for each ε value, where a is a coefficient varying from 0 to 1. Interpreting the distribution $D(M)$ as a pure Δ^0 signal, it was fitted in region 1092–1407 MeV/c^2 by a relativistic Breit-Wigner function [39]

$$b(M) = C\frac{\Gamma M M_\Delta}{\left(M^2 - M_\Delta^2\right)^2 + \Gamma^2 M_\Delta^2}, \quad (6)$$

where M_Δ and Γ are the mass and width of the resonance, and C is the normalization coefficient. Then the data set $D(M)$ was fitted by the function $b(M)$ and the value of $\chi^2(a)$ was determined for each fit. During these fitting procedures, the parameter ε was varied from 0.01 to 1.00 with a step of 0.01, and a varied from 0.00 to 1.00 in steps of 0.01 for each ε value. The best value of the parameter a for each ε was found from the minimum of the function $\chi^2(a)$. Thus for each experimental distribution (for each ε) the best background distribution $a\,dn^b/dM$ at the best value of a was obtained. Analyzing the experimental and background spectra at different values of ε, we discovered that at $\varepsilon < 0.10$ the distributions $D(M)$ are too narrow, the width and mass being $\Gamma < 45$ MeV/c^2 and $M_\Delta \sim 1232$ MeV/c^2, respectively. This is due to the relative smallness of statistics for the experimental spectrum in the vicinity of the $\Delta^0(1232)$ peak and dominance of the effect of the chosen mass $M_{\Delta_{NN}}$ in Eq. (2) for $\varepsilon < 0.10$. As an example, the experimental and background invariant mass distributions of $p\pi^-$ pairs at $\varepsilon = 0.05$ are shown in Fig. 2(b). At the values of the cutoff parameter $\varepsilon > 0.80$ the spectra dn^{expt}/dM become similar to the experimental spectrum in Fig. 2(a)—in particular, a large combinatorial background from uncorrelated $p\pi^-$ pairs masks the resonancelike structure. As an example, experimental and background invariant mass distributions of $p\pi^-$ pairs obtained for $\varepsilon = 0.90$ are presented in Fig. 2(c). With a further increase of ε, the peak of experimental spectra shifts toward $M < 1200$ MeV/c^2, becoming wider and approaching more the shape of the experimental distribution in Fig. 2(a) obtained without using the criterion in relation (4).

The dependence of the obtained best values of parameters a and the corresponding values of M and Γ on the cutoff parameter ε is illustrated in Figs. 3(a)–3(c). One can see from Fig. 3(a) that in region $\varepsilon < 0.4$, the parameter a, which characterizes the background contribution, on the whole grows with an increase of ε. It is important to mention that in this

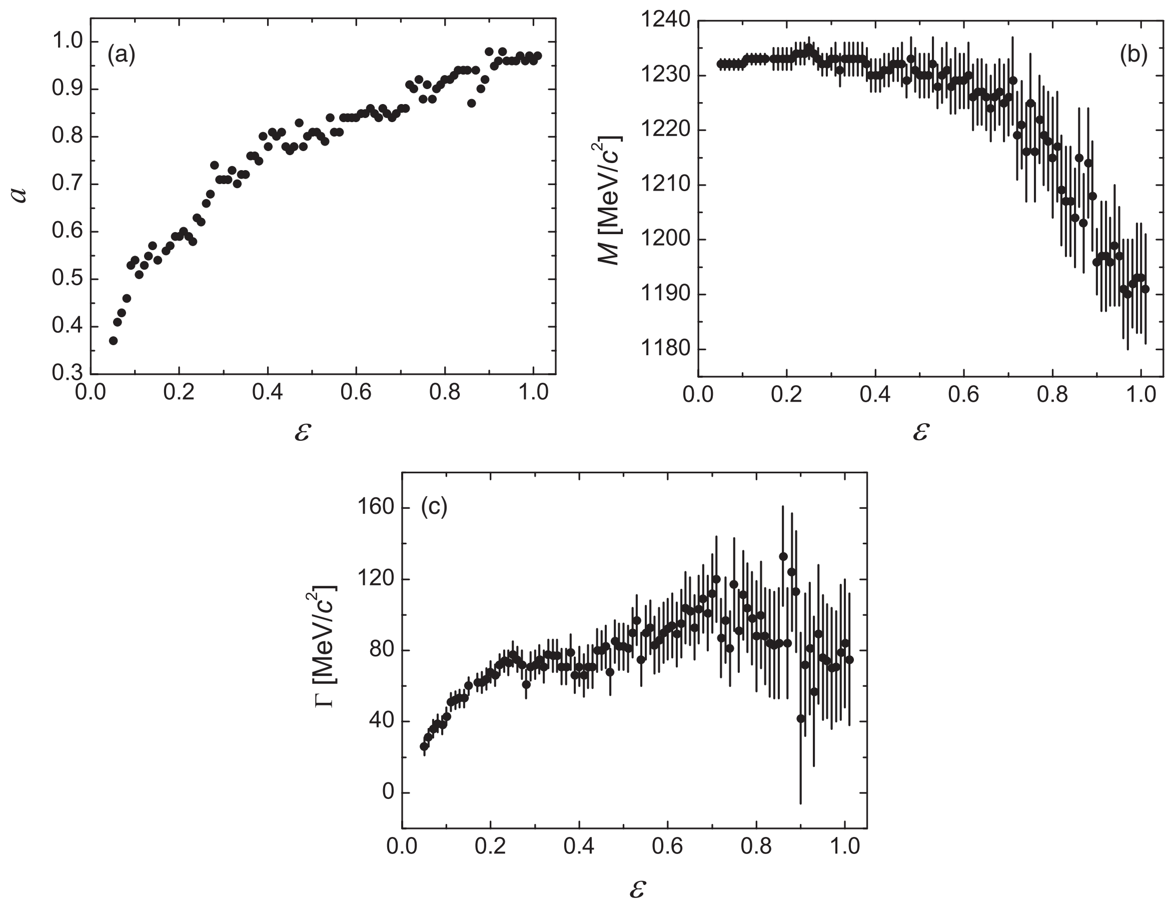

FIG. 3. Dependence of the obtained values of parameters (a) a, (b) M, and (c) Γ on cutoff parameter ε.

region the total number of combinations in the experimental spectra increases, with an increase in the number of correlated as well as uncorrelated $p\pi^-$ pairs, as ε grows. For $\varepsilon < 0.15$, it follows from Figs. 3(b) and 3(c) that the distributions $D(M)$ are narrow, the width and mass being $\Gamma < 60$ MeV/c^2 and $M_\Delta \sim 1232$ MeV/c^2, respectively. This is, as mentioned earlier, due to relatively less statistics for the experimental spectrum in the vicinity of the Δ^0 peak and to a dominance of the effect of the chosen mass $M_{\Delta_{NN}}$ in Eq. (2) for $\varepsilon < 0.15$. Further, Fig. 3(a) shows that, in the region $\varepsilon \approx 0.4$–0.7, there is almost a "plateau," the value of a remaining stable within small oscillations, the maximal spread of a in this interval being $|a_{\max} - a_{\min}| = 0.08$. With a further increase in ε, as seen from Fig. 3(a), the value of a on the whole increases, showing unstable behavior with large oscillations, and reaches the value of $a = 0.96$ at $\varepsilon = 1.0$. As seen from Fig. 3(b), the region $\varepsilon > 0.7$ is characterized by a strong decrease in M_Δ and an unstable behavior of Γ as ε increases. Figures 3(b) and 3(c) show that the fitting errors of the parameters M_Δ and Γ become large in the region $\varepsilon > 0.7$. These observations can be explained by a significant increase in the contribution of the combinatorial background from uncorrelated $p\pi^-$ pairs, whereas there are almost no further contributions from correlated $p\pi^-$ pairs to the experimental spectra with an increase in ε in region $\varepsilon > 0.7$. Therefore, at the values of the cutoff parameter $\varepsilon > 0.8$, the spectra dn^{expt}/dM become similar to the experimental spectrum in Fig. 2(a), with a large combinatorial background from uncorrelated $p\pi^-$ pairs practically masking the resonancelike structure.

The mass distribution of $\Delta^0(1232)$ resonance was obtained by choosing the best value of the parameter ε and the corresponding best value of a. These best values of the parameters were determined from an analysis of the behavior of the function $\chi^2(\varepsilon,a)$. The minimum of the $\chi^2(\varepsilon,a)$ function gave the following values: $\varepsilon(\Delta^0) = 0.52 \pm 0.16$ and $a(\Delta^0) = 0.80^{+0.06}_{-0.03}$. It is worth mentioning that the selected best value of ε lies inside the "plateau" region, $\varepsilon \sim 0.4$–0.7, with relatively stable values of a. The uncertainty in ε, $\Delta\varepsilon/\varepsilon \approx 30\%$, was estimated from an average experimental error on the momentum measurement, $\langle \Delta p/p \rangle \approx 11\%$, in $d + {}^{12}\text{C}$ collisions at 4.2A GeV/c. In what follows, all the systematic errors in the determination of the parameters and kinematical characteristics of $\Delta^0(1232)$ resonances were estimated based on the above obtained uncertainty in ε.

The uncertainty in a was determined as the maximal spread of a in the interval of ε from 0.36 ($0.52 - \Delta\varepsilon$) to 0.68 ($0.52 + \Delta\varepsilon$). The experimental invariant mass distribution dn^{expt}/dM and the best background distribution $a\, dn^b/dM$ of $p\pi^-$ pairs at the best values of ε and a are shown in Fig. 4(a). The difference distributions $D(M)$ for $p\pi^-$ pairs, using the above chosen best values of ε and a, along with the corresponding Breit-Wigner fit, are presented in Fig. 4(b), from where the mass and width of the $\Delta^0(1232)$, produced in $d + {}^{12}\text{C}$ collisions, were obtained. As seen from Fig. 4(a), the experimental invariant mass distribution of $p\pi^-$ pairs has a statistically significant resonancelike structure, expected for $\Delta^0(1232)$ resonance, at the best value of ε. The slight excess of the experimental spectrum over the background in region

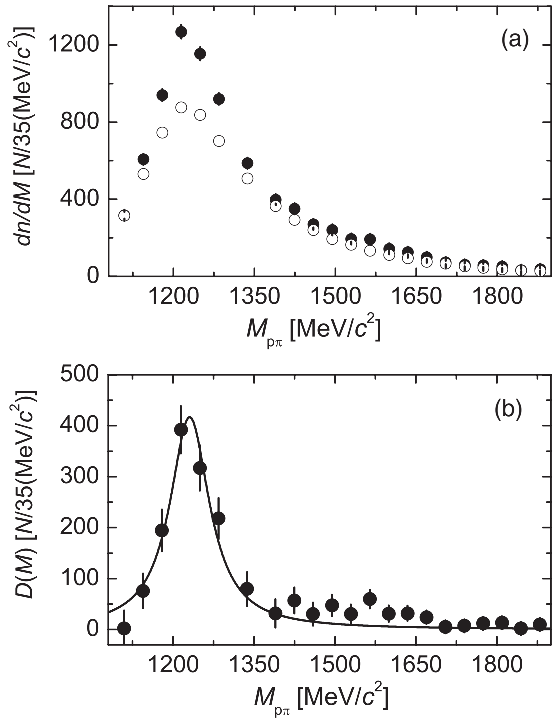

FIG. 4. (a) Experimental invariant mass distribution $\frac{dn^{\text{exp}}}{dM}$ (•) and the best background distribution $a\frac{dn^b}{dM}$ (○) for $p\pi^-$ pairs in $d + {}^{12}\text{C}$ collisions at 4.2 GeV/c per nucleon obtained using the best values of parameters ε and a. (b) The corresponding difference (•) between the experimental invariant mass distribution $\frac{dn^{\text{exp}}}{dM}$ and the best background distribution $a\frac{dn^b}{dM}$ for $p\pi^-$ pairs obtained at the best values of parameters ε and a, along with the corresponding Breit-Wigner fits (solid lines).

$M_{p\pi} > 1400$ MeV in Fig. 4(a) is due to the contribution from higher-lying resonances, the closest being the $N^*(1440)$ resonance. It should be noted that the contribution from higher-lying resonances is highly suppressed by the criterion in relation (4) and tends to vanish as $\varepsilon \to 0$ [41].

To estimate the fraction of π^- mesons coming from $\Delta^0(1232)$ decay relative to the total number of π^- mesons, produced in $d + {}^{12}\text{C}$ collisions, the following relation was applied to the above obtained best experimental and background spectra:

$$R(\Delta^0/\pi^-) = \frac{\int_{M_p+M_\pi}^{M_x} \left(\frac{dn^{\text{expt}}}{dM} - a\frac{dn^b}{dM}\right) dM}{N_{\text{in}}^{dC}\, n(\pi^-)}, \tag{7}$$

with $M_p + M_\pi$ the sum of proton and pion masses, $M_x \approx 1410$ MeV/c^2 as the lower and upper limits of integration, $n(\pi^-) = 0.66 \pm 0.01$ the mean multiplicity per event of π^- mesons in $d + {}^{12}\text{C}$ collisions, and $N_{\text{in}}^{dC} = 7071$ the total number of inelastic $d + {}^{12}\text{C}$ collision events. The value of the numerator in Eq. (7) is simply the total excess of the experimental spectrum over the background, i.e., $N_{\Delta^0 \to p\pi^-} = 1390 \pm 37(\text{stat.})^{+269}_{-336}(\text{syst.})$. The systematic error on this $N_{\Delta^0 \to p\pi^-}$ was estimated from uncertainty in a.

The parameters of the approximation of the mass distribution of $\Delta^0(1232)$ by the relativistic Breit-Wigner function are

TABLE I. Parameters of approximation of mass distributions of $\Delta^0(1232)$ resonances produced in $d + {}^{12}\mathrm{C}$ collisions at $4.2A$ GeV/c per nucleon by the relativistic Breit-Wigner function.

C	M (MeV/c^2)	Γ (MeV/c^2)	χ^2/n.d.f.
37 407 ± 3714	$1230 \pm 4^{+3}_{-6}$	$90 \pm 14^{+19}_{-24}$	0.81

presented in Table I. As seen from the value of χ^2/n.d.f. in Table I, a good approximation of mass distribution of $\Delta^0(1232)$ by the relativistic Breit-Wigner function was obtained. For the sake of comparison, the masses, widths, and fractions of π^- mesons, coming from $\Delta^0(1232)$ decay, for $\Delta^0(1232)$ resonances, produced on carbon nuclei in $p + {}^{12}\mathrm{C}$ collisions at 4.2 GeV/c [49] and $\pi^- + {}^{12}\mathrm{C}$ interactions [41] at 40 GeV/c, and produced on oxygen nuclei in ${}^{16}\mathrm{O} + p$ interactions [42] at $3.25A$ GeV/c, are presented in Table II. As seen from this table, the masses as well as the widths of $\Delta^0(1232)$ for the reactions considered coincided with each other, within error limits. Despite the relatively large uncertainties, the absolute values of the obtained widths of $\Delta^0(1232)$, presented in Table II, proved to be close to each other and agree with the average width $\sim 90 \pm 10$ MeV/c^2 of $\Delta(1232)$ obtained for the relativistic collisions involving helium, carbon, oxygen, and tantalum nuclei in bubble chamber experiments [40–44,48,49].

The situation with respect to relativistic nucleus-nucleus collisions, especially for heavy-ion collisions, is different and more complex. This is because of the dense nuclear matter created in nucleus-nucleus collisions in which the properties of $\Delta(1232)$ can modify significantly [12]. As was mentioned in Ref. [12], the nuclear environment causes, in general, a mass shift which can be either positive or negative and depends on the hadronic density. For example, in near-central Ni + Ni and Au + Au collisions [12,25] at energies between 1 and $2A$ GeV the average mass shift of $\Delta(1232)$ was -60 and -80 MeV/c^2, respectively, with widths of the order of 50 MeV/c^2. The mass shift of $\Delta(1232)$ in nucleus-nucleus collisions was found to be roughly proportional to the number of participants that became smaller with increasing impact parameter b [12,24,26]. In our case [40–42,44,48,49], we deal with the relatively light nuclei, and the results are averaged over all impact parameter b values, with most of the collision events being peripheral. If we relate the significant decrease in the mass and width of $\Delta(1232)$ as due to the high values of the hadronic densities created in near-central relativistic heavy-ion collisions, then the smaller modification of the mass and width of $\Delta(1232)$ resonance in the present paper can be explained by the predominantly peripheral character of the collisions, and hence by the smaller hadronic densities roughly equaling the normal nuclear densities [42].

TABLE II. The masses, widths, and fractions of π^- mesons, coming from $\Delta^0(1232)$ decay, for $\Delta^0(1232)$ resonances, produced in $d + {}^{12}\mathrm{C}$ and $p + {}^{12}\mathrm{C}$ [48,49] collisions at $4.2A$ GeV/c and $\pi^- + {}^{12}\mathrm{C}$ interactions [41] at 40 GeV/c, and produced on oxygen nuclei in ${}^{16}\mathrm{O} + p$ interactions [42] at $3.25A$ GeV/c.

Reaction, p_0	M (MeV/c^2)	Γ (MeV/c^2)	$R(\Delta^0/\pi^-)$ (%)
$d + {}^{12}\mathrm{C}$, $4.2A$ GeV/c	$1230 \pm 4^{+3}_{-6}$	$90 \pm 14^{+19}_{-24}$	$30 \pm 2^{+6}_{-7}$
$p + {}^{12}\mathrm{C}$, 4.2 GeV/c	$1222 \pm 5^{+10}_{-14}$	$89 \pm 14^{+32}_{-43}$	$39 \pm 3^{+10}_{-7}$
$\pi^- + {}^{12}\mathrm{C}$, 40 GeV/$c$	1226 ± 3	87 ± 7	6 ± 1
${}^{16}\mathrm{O} + p$, $3.25A$ GeV/c	1224 ± 4	96 ± 10	41 ± 4

Table II shows that $(30 \pm 2^{+6}_{-7})$% of π^- mesons, produced in $d + {}^{12}\mathrm{C}$ collisions at $4.2A$ GeV/c, are estimated to come from the decay of $\Delta^0(1232)$. This value is compatible with the corresponding fraction, $(39 \pm 3^{+10}_{-7})$%, estimated in our recent paper [49] on $\Delta^0(1232)$ production in $p + {}^{12}\mathrm{C}$ collisions at the same initial momentum. A significant fraction of π^- mesons, (41 ± 4)%, as seen from this table, was estimated to come from the decay of $\Delta^0(1232)$ resonances produced on oxygen nuclei in ${}^{16}\mathrm{O} + p$ collisions [42] at $3.25A$ GeV/c. On the other hand, as seen from Table II, only (6 ± 1)% of π^- mesons come from $\Delta(1232)$ decay in $\pi^- + {}^{12}\mathrm{C}$ interactions at 40 GeV/c [41]. This result was expected [41], since at projectile energies as high as 40 GeV, as compared to energies of the order of a few GeV/nucleon, more channels open up for the favorable production of other higher mass resonances as well as ρ^0, ω^0, and f^0 mesons [66,67]. A comparison of the above-mentioned fractions of π^- mesons coming from $\Delta^0(1232)$ decays with the other published data, including central heavy-ion collisions, is illustrated in Fig. 5. As seen from this figure, the fraction of π^- coming from $\Delta^0(1232)$ decays generally decreases with an increase in incident energy. Our result for $R(\Delta^0/\pi^-)$ in $d + {}^{12}\mathrm{C}$ collisions at 4.2 GeV/c, as seen in Fig. 5, is in agreement within uncertainties with the corresponding results for $p + {}^{12}\mathrm{C}$, ${}^4\mathrm{He} + {}^{12}\mathrm{C}$, and ${}^{12}\mathrm{C} + {}^{12}\mathrm{C}$ collisions at the same incident energy per nucleon and is in line with the results of

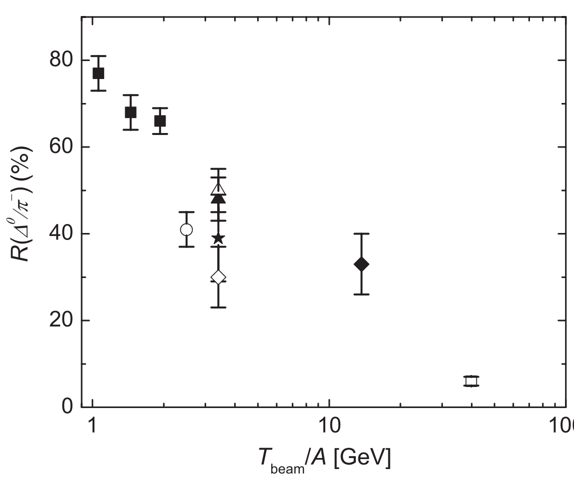

FIG. 5. Dependence of fraction of π^- mesons coming from Δ^0 decay on beam kinetic energy per nucleon obtained as follows: (◇) in the present work for $d + {}^{12}\mathrm{C}$ collisions at $4.2A$ GeV/c ($T_{\mathrm{beam}} \approx 3.4A$ GeV); (★) for $p + {}^{12}\mathrm{C}$ collisions at 4.2 GeV/c ($T_{\mathrm{beam}} \approx 3.4A$ GeV); (○) for ${}^{16}\mathrm{O} + p$ collisions at $3.25A$ GeV/c ($T_p \approx 2.5$ GeV in an oxygen nucleus rest frame); (■) in central Ni + Ni collisions [25] at 1.06, 1.45, and $1.93A$ GeV; (♦) in central ${}^{28}\mathrm{Si}$ + Pb collisions [23] at $p_{\mathrm{lab}} = 14.6A$ GeV ($T_{\mathrm{beam}} \approx 13.7A$ GeV); (▲) and (△) in ${}^4\mathrm{He} + {}^{12}\mathrm{C}$ and ${}^{12}\mathrm{C} + {}^{12}\mathrm{C}$ collisions, respectively, at $4.2A$ GeV/c ($T_{\mathrm{beam}} \approx 3.4A$ GeV); and (□) in π^- + C collisions at 40 GeV/c ($T_{\mathrm{beam}} \approx 39.9$ GeV).

other experiments with different colliding nuclei at various incident energies.

From the results presented above we can estimate the relative number of nucleons excited to Δ^0 at freeze-out, $n(\Delta)/n(\text{nucleon} + \Delta)$. The abundance of Δ^0 can be evaluated using the relation [25]

$$n(\Delta) = n(\pi^-) f_{\text{isobar}} \frac{\pi^-_{\Delta}}{\pi^-_{\text{all}}}, \tag{8}$$

where $n(\pi^-)$ is the average number of π^- per event, $\pi^-_{\Delta}/\pi^-_{\text{all}}$ is the fraction of π^- coming from $\Delta^0(1232)$ decays, and f_{isobar} is the prediction of the isobar model [68]:

$$f_{\text{isobar}} = \frac{n(\pi^- + \pi^0 + \pi^+)}{n(\pi^-)} = \frac{6(Z^2 + N^2 + NZ)}{5N^2 + NZ}, \tag{9}$$

with N and Z being the number of neutrons and protons, respectively. The mean multiplicity of π^- in $d + {}^{12}$C collisions at 4.2A GeV/c is $n(\pi^-) = 0.66 \pm 0.01$. For the ($d + {}^{12}$C) system, $f_{\text{isobar}} \approx 3.0$.

Assuming $\langle n_p^{\text{part}} \rangle / \langle n_n^{\text{part}} \rangle = 1$ because of an equal number of protons and neutrons in the colliding system, where $\langle n_n^{\text{part}} \rangle$ is the mean multiplicity per event of participant neutrons in $d + {}^{12}$C collisions at 4.2A GeV/c, we can evaluate

$$n(\text{nucleon} + \Delta) = 2\langle n_p^{\text{part}} \rangle + \langle n_{\Delta} \rangle, \tag{10}$$

with $\langle n_p^{\text{part}} \rangle = 1.94 \pm 0.06$ and $\langle n_{\Delta} \rangle = 0.20 \pm 0.02^{+0.04}_{-0.07}$ being the mean multiplicities of participant protons and Δ^0, respectively, in $d + {}^{12}$C collisions at 4.2A GeV/c. Using the above results, the relative number of nucleons excited to Δ^0 at freeze-out was found to be $n(\Delta)/n(\text{nucleon} + \Delta) = (15 \pm 2^{+3}_{-4})\%$. This result is in good agreement with the fraction of nucleons, $(16 \pm 3^{+4}_{-3})\%$, excited to Δ^0 in freeze-out conditions estimated in Ref. [48] for $p + {}^{12}$C collisions at 4.2 GeV/c from the fraction of pions coming from Δ^0 decay [49]. A comparison of this result with those obtained in earlier papers for different sets of colliding nuclei at various incident energies is shown in Fig. 6. As seen from this figure, the estimated relative number of nucleons excited to Δ^0 at freeze-out in $d + {}^{12}$C collisions at 4.2A GeV/c is compatible with those obtained in Refs. [23,25,40,44,48].

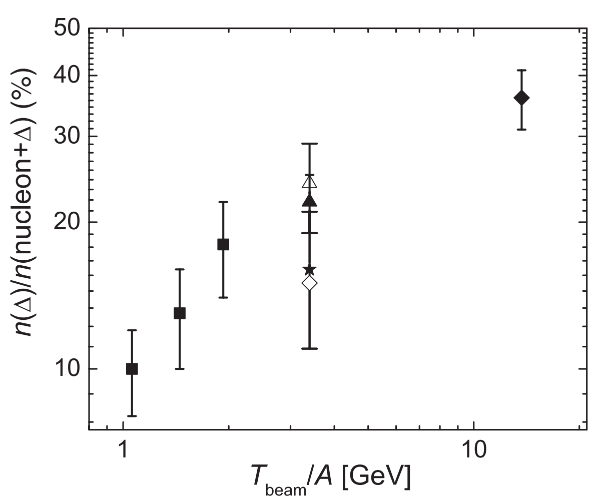

FIG. 6. The relative number of nucleons excited to Δ^0 at freeze-out as a function of beam energy obtained as follows: (◇) in the present work for $d + {}^{12}$C collisions at 4.2A GeV/c ($T_{\text{beam}} \approx 3.4A$ GeV); (★) for $p + {}^{12}$C collisions at 4.2 GeV/c ($T_{\text{beam}} \approx 3.4A$ GeV); (■) by the FOPI collaboration for central Ni + Ni collisions; (♦) in the E814 experiment for central ^{28}Si + Pb collisions at $p_{\text{lab}} =$ 14.6A GeV/c; and (▲) and (Δ) for ^{4}He + ^{12}C and ^{12}C + ^{12}C collisions, respectively, at 4.2A GeV/c.

IV. THE KINEMATICAL SPECTRA OF $\Delta^0(1232)$

The $\Delta(1232)$ decay kinematics was found to be responsible for the low transverse momentum (p_t) enhancement of pions in hadron-nucleus and nucleus-nucleus collisions at incident beam energies from 1 to 15A GeV [23,33,69]. It was noted that the pions coming from $\Delta(1232)$ decay populated mainly the low transverse momentum part of the spectrum [23,33,69]. Therefore we extracted the transverse momentum spectra of π^- mesons from correlated (p,π^-) pairs contributing to mass distributions of $\Delta^0(1232)$ resonances in $d + {}^{12}$C collisions at 4.2A GeV/c shown in Fig. 4. The reconstructed transverse momentum distribution of π^- mesons coming from $\Delta^0(1232)$ decay along with the spectrum for all π^- produced in $d + {}^{12}$C collisions is presented in Fig. 7. As seen from Fig. 7, π^- mesons from $\Delta^0(1232)$ decay occupy predominantly the low transverse momentum region ($p_t < 0.6$ GeV/c) and are indeed responsible for the low p_t enhancement of π^- spectra in $d + {}^{12}$C collisions at 4.2A GeV/c.

To extract the mean kinematical characteristics of $\Delta^0(1232)$ produced in $d + {}^{12}$C collisions, we reconstructed the momentum, transverse momentum, kinetic energy, rapidity, and emission angle distributions in the carbon nucleus rest frame. To do this, we used all $p\pi^-$ pairs contributing to the experimental invariant mass distribution in Fig. 4(a), obtained using the best value of cutoff parameter ε. The momentum, transverse momentum, kinetic energy, rapidity, and emission angles of $\Delta^0(1232)$ were calculated using the

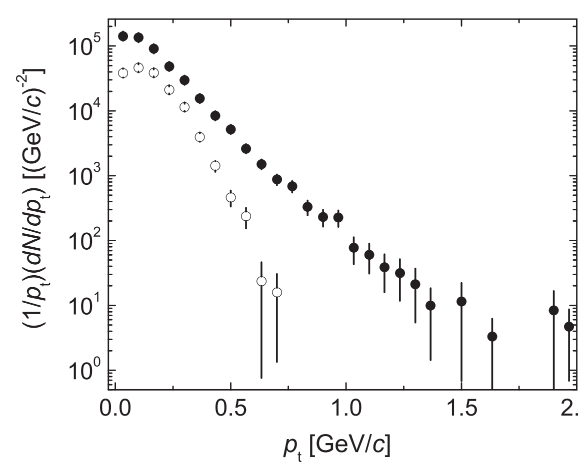

FIG. 7. Transverse momentum distribution of all π^- mesons (●) and π^- mesons coming from $\Delta^0(1232)$ decay (○) in $d + {}^{12}$C collisions at 4.2A GeV/c.

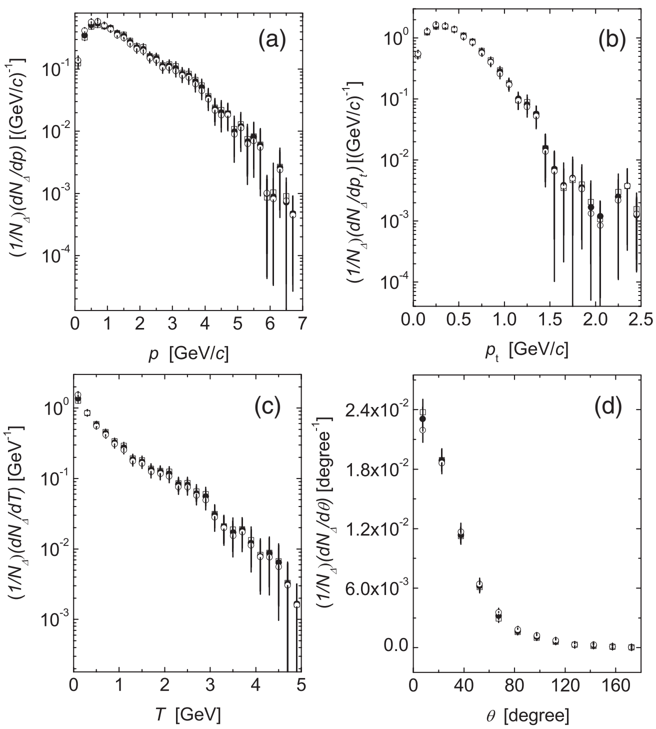

FIG. 8. (a) Reconstructed momentum, (b) transverse momentum, (c) kinetic energy, and (d) emission angle distributions of $\Delta^0(1232)$ resonances produced in $d + {}^{12}\mathrm{C}$ collisions at 4.2A GeV/c in the laboratory frame [normalized to the total number of $\Delta^0(1232)$] obtained for values of $\varepsilon = 0.41$ (□), $\varepsilon = 0.52$ (•), and $\varepsilon = 0.68$ (○).

relations

$$p_\Delta = |\vec{p}_p + \vec{p}_\pi|, \tag{11}$$

$$p_{\Delta t} = |\vec{p}_{pt} + \vec{p}_{\pi t}|, \tag{12}$$

$$T_\Delta = \left(p_\Delta^2 + M_\Delta^2\right)^{1/2} - M_\Delta, \tag{13}$$

$$Y_\Delta = \frac{1}{2}\ln\left(\frac{E_\Delta + p_\Delta^L}{E_\Delta - p_\Delta^L}\right), \tag{14}$$

$$\theta_\Delta = \arccos\left(\frac{p_\Delta^L}{p_\Delta}\right), \tag{15}$$

respectively, where $\vec{p}_p$ and $\vec{p}_\pi$ are the proton and pion momenta, $\vec{p}_{pt}$ and $\vec{p}_{\pi t}$ are the proton and pion transverse momenta, M_Δ represents the invariant mass of the $p\pi^-$ pair, and E_Δ and p_Δ^L is the total energy and longitudinal momentum of $\Delta^0(1232)$, respectively. To account for the background contribution into the experimental invariant mass distribution of $p\pi^-$ pairs in Fig. 4(a), the calculated kinematical characteristics of $\Delta^0(1232)$ for each $p\pi^-$ pair were taken with a weight

$$w = \left(\frac{\frac{dn^{\rm expt}}{dM} - a\frac{dn^b}{dM}}{\frac{dn^{\rm expt}}{dM}}\right)_{M=M_{p\pi}}, \tag{16}$$

determined at $M = M_{p\pi}$ using the experimental and background invariant mass distributions of $p\pi^-$ pairs obtained at the best values of ε and a. We estimated the systematic uncertainties in the obtained kinematical characteristics of

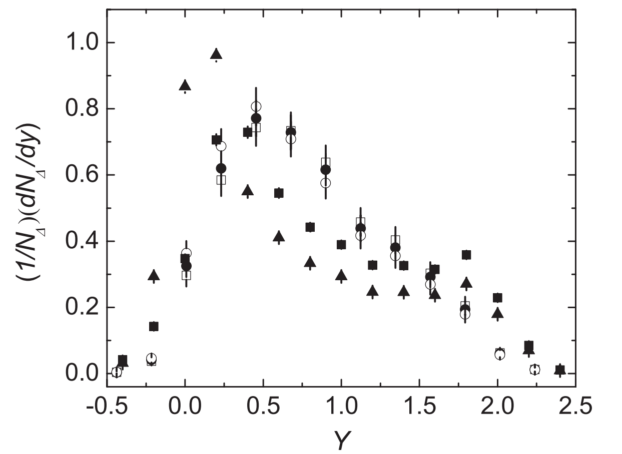

FIG. 9. Reconstructed rapidity distributions of $\Delta^0(1232)$ resonances produced in $d + {}^{12}\mathrm{C}$ collisions at 4.2A GeV/c in the laboratory frame [normalized to the total number of $\Delta^0(1232)$] obtained for values of $\varepsilon = 0.41$ (□), $\varepsilon = 0.52$ (•), and $\varepsilon = 0.68$ (○). Rapidity distribution of all protons (▲) and participant protons (■) in $d + {}^{12}\mathrm{C}$ collisions at 4.2A GeV/c (normalized to the total number of respective protons).

$\Delta^0(1232)$ at the best value of the cutoff parameter ε by comparison with those obtained at $\varepsilon = 0.41$ and 0.68, for which we obtained the minimal and maximal width of $\Delta^0(1232)$, respectively, within the ε uncertainty interval 0.36–0.68. The so-obtained (for $\varepsilon = 0.52$, 0.41, and 0.68) momentum, transverse momentum, kinetic energy, and emission angle distributions of $\Delta^0(1232)$, normalized to the total number of $\Delta^0(1232)$, are presented in Figs. 8(a)–8(d). As seen from Fig. 8, all the reconstructed kinematical distributions coincided within the uncertainties with each other for three different cutoff parameter values.

The reconstructed rapidity distributions of $\Delta^0(1232)$ are presented in Fig. 9. As seen from Fig. 9, the peak of the

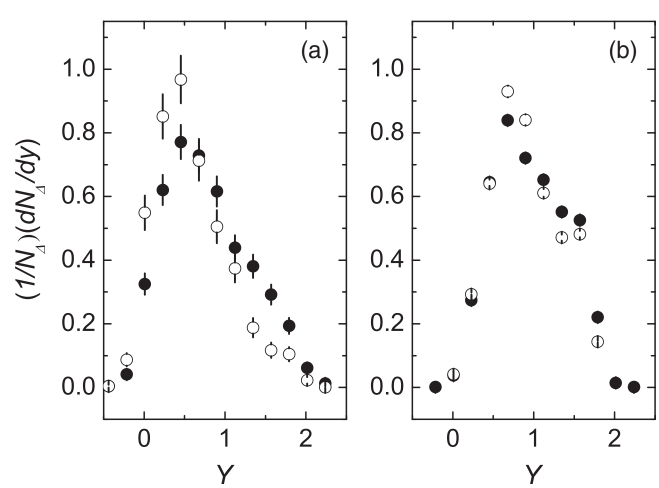

FIG. 10. Reconstructed rapidity distributions of $\Delta^0(1232)$ resonances produced in (•) $d + {}^{12}\mathrm{C}$ and (○) $p + {}^{12}\mathrm{C}$ collisions at 4.2A GeV/c in the laboratory frame [normalized to the total number of $\Delta^0(1232)$] in experiment (a) and calculated using the modified FRITIOF model (b).

TABLE III. The mean values of momentum, kinetic energy, transverse momentum, emission angle, and rapidity of Δ^0(1232) resonances, produced in $d + {}^{12}$C collisions at 4.2AGeV/c.

ε	0.52	0.41	0.68
$\langle p \rangle$ (MeV/c)	$1483 \pm 29^{+43}_{-63}$	1526 ± 30	1420 ± 29
$\langle T \rangle$ (MeV)	$797 \pm 22^{+30}_{-45}$	827 ± 24	752 ± 23
$\langle p_t \rangle$ (MeV/c)	$455 \pm 8^{+5}_{-4}$	460 ± 8	446 ± 8
$\langle \theta \rangle$ (deg)	$29 \pm 1^{+2}_{-1}$	28 ± 1	31 ± 1
$\langle Y \rangle$	$0.78 \pm 0.01^{+0.02}_{-0.03}$	0.80 ± 0.01	0.75 ± 0.01

reconstructed rapidity distributions of Δ^0(1232) produced in $d + {}^{12}$C collisions at 4.2A GeV/c appears almost at a position of the center-of-mass rapidity (y~0.45) of the collision ($d + {}^{12}$C) system, which is significantly lower than the nucleon-nucleon center-of-mass rapidity ($y_{\text{c.m.}} = 1.1$ for 4.2 GeV/c). In a simplistic picture the rapidity distribution of Δ^0(1232) resonances would have a peak at the nucleon-nucleon center-of-mass rapidity $y_{\text{c.m.}}$ if Δ^0(1232) had been produced in first chance nucleon-nucleon collisions. However, the location of the peak of the rapidity distribution of Δ^0(1232) close to the target fragmentation region indicates that the major fraction of Δ^0(1232) is produced in secondary collisions on carbon nuclei as well as in peripheral $d + {}^{12}$C interactions. A shoulderlike enhancement is also noticeable in the rapidity distribution of Δ^0(1232) in the region $y \sim 1.2$–1.9, which is likely to be due to Δ^0(1232) produced on nucleons of impinging deuterons.

For the sake of comparison, we also plotted in Fig. 9 the rapidity distribution of protons in $d + {}^{12}$C collisions at 4.2A GeV/c, which is characterized by a prominent peak at $y \approx 0.2$ due to slow target protons with $p_{\text{lab}} < 300$ MeV/c, as was shown in Ref. [52]. A broad shoulder, but of a significantly lesser magnitude, can be seen in the region $y \sim 1.2$–2.1, which is mostly due to protons coming from projectile deuterons [52]. It should be mentioned that the rapidity distribution of protons in $d + {}^{12}$C collisions at 4.2A GeV/c was described quite satisfactorily by DCM, the FRITIOF model, and QGSM in Refs. [52,55,64]. The shape of the rapidity distribution of protons in Fig. 9 confirms that a major fraction of protons in $d + {}^{12}$C collisions is produced in secondary collisions in carbon nuclei as well as in peripheral interactions, whereas a noticeable part of the protons originates from impinging deuterons, as in the case of Δ^0(1232) resonances. A comparison of the rapidity distribution of Δ^0(1232) produced

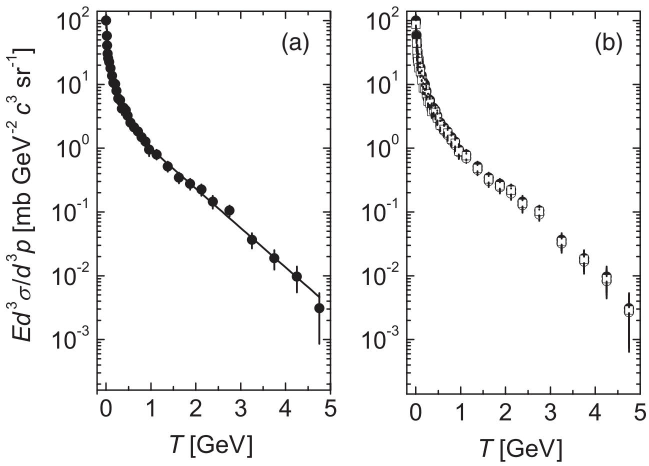

FIG. 11. (a) Reconstructed invariant cross sections of Δ^0(1232) resonances produced in $d + {}^{12}$C collisions at 4.2A GeV/c vs their kinetic energy T in the laboratory frame for $\varepsilon = 0.52$: Solid line, fit by the function in Eq. (17). (b) Reconstructed invariant cross sections of Δ^0(1232) resonances for values of $\varepsilon = 0.41$ (□), $\varepsilon = 0.52$ (•), and $\varepsilon = 0.68$ (○).

in $p + {}^{12}$C [49] and $d + {}^{12}$C collisions at 4.2A GeV/c is shown in Fig. 10(a). A rapidity distribution of Δ^0(1232) in $d + {}^{12}$C collisions is characterized by a significant enhancement in the region $y \sim 1.2$–1.9 as compared to that in $p + {}^{12}$C collisions. This enhancement is also reproduced qualitatively in Fig. 10(b), which represents a corresponding comparison of rapidity distributions of Δ^0(1232) calculated by using the modified FRITIOF model with allowance for the production of Δ(1232) isobars [70]. As seen from Figs. 10(a) and 10(b), this enhancement is pronounced more in experiment as compared to calculated spectra. In Ref. [49] it was deduced that practically all Δ^0(1232) in $p + {}^{12}$C collisions at 4.2 GeV/c are produced on (nucleons of) carbon nuclei. On the other hand, as seen from Fig. 10(a), the main peak of the rapidity distribution of Δ^0 in $p + {}^{12}$C collisions at y~0.45 is noticeably higher compared to that in $d + {}^{12}$C collisions. The mean value of Δ^0 rapidity was found to be $0.78 \pm 0.01^{+0.02}_{-0.03}$ in $d + {}^{12}$C collisions, which is significantly larger than the corresponding value, $0.59 \pm 0.02^{+0.02}_{-0.01}$, obtained in Ref. [49] for Δ^0 in $p + {}^{12}$C collisions. The colliding ($d + {}^{12}$C) system differs from the ($p + {}^{12}$C) system by one additional neutron inside the projectile deuteron. Hence, based on the above observations, in the case of $d + {}^{12}$C collisions, a major fraction

TABLE IV. The mean values of momentum, transverse momentum, rapidity, and emission angle of participant protons in $d + {}^{12}$C collisions at 4.2A GeV/c in the present experiment compared to those calculated in Refs. [52,55,64] using DCM, the FRITIOF model, and QGSM.

Kinematical characteristics	Experiment	DCM	FRITIOF	QGSM
$\langle p \rangle$ (MeV/c)	1397 ± 9	1370 ± 5	1250 ± 2	1376 ± 8
$\langle p_t \rangle$ (MeV/c)	436 ± 2	468 ± 2	440 ± 1	520 ± 2
$\langle Y \rangle$	0.83 ± 0.01	0.77 ± 0.01	0.75 ± 0.01	0.75 ± 0.01
$\langle \theta \rangle$ (deg)	36 ± 1	37.8 ± 0.2	38.4 ± 0.2	40.6 ± 0.2

TABLE V. Parameters of approximation of reconstructed invariant cross sections of $\Delta^0(1232)$ produced in $d+^{12}$C collisions at 4.2A GeV/c by the function in Eq. (17) for values of $\varepsilon = 0.52$, 0.41, and 0.68.

ε	0.52	0.41	0.68
A_1 (mb GeV^{-2} c^3 sr^{-1})	101.18 ± 42.52	88.08 ± 41.53	78.79 ± 32.72
T_1 (GeV)	$0.013 \pm 0.005^{+0.004}_{-0.001}$	0.012 ± 0.005	0.017 ± 0.007
A_2 (mb GeV^{-2} c^3 sr^{-1})	25.96 ± 4.84	21.09 ± 4.07	28.80 ± 5.95
$T_2 = T_0$ (GeV)	$0.147 \pm 0.021^{+0.003}_{-0.007}$	0.150 ± 0.023	0.140 ± 0.020
A_3 (mb GeV^{-2} c^3 sr^{-1})	4.00 ± 0.56	3.76 ± 0.55	3.58 ± 0.52
T_3 (GeV)	$0.703 \pm 0.034^{+0.001}_{-0.004}$	0.704 ± 0.035	0.699 ± 0.035
χ^2/n.d.f.	0.23	0.25	0.20

of Δ^0 is produced on carbon nuclei, while still a noticeable number of Δ^0 originate from neutrons of impinging deuterons, as is reflected by the shoulder in rapidity distribution in the region $y \sim 1.2 - 1.9$ in Figs. 9 and 10.

Table III presents the mean values of momentum, kinetic energy, transverse momentum, emission angle, and rapidity of $\Delta^0(1232)$, produced in $d + ^{12}$C collisions at 4.2A GeV/c, in the laboratory frame obtained for the values of $\varepsilon = 0.52$, 0.41, and 0.68.

For a comparison, the mean values of momentum, transverse momentum, rapidity, and emission angle of participant protons in $d + ^{12}$C collisions are presented in Table IV, along with calculations done using DCM, the FRITIOF model, and QGSM. As seen from Tables III and IV, $\Delta^0(1232)$ are characterized by the higher values of mean momentum as compared to those for participant protons, while their mean transverse momenta and rapidities are close to each other. On the other hand, as seen from Tables III and IV, the mean emission angle of $\Delta^0(1232)$ is smaller compared to that of participant protons in $d + ^{12}$C collisions.

We reconstructed in a similar way the spectrum of invariant cross sections $E\, d^3\sigma/d^3p$ of $\Delta^0(1232)$ resonances produced in $d + ^{12}$C collisions as a function of their kinetic energy in the laboratory frame for values of $\varepsilon = 0.52$, 0.41, and 0.68. The reconstructed spectrum for the best value, $\varepsilon = 0.52$, is presented in Fig. 11(a). In this figure we accounted only for $\Delta^0(1232)$ decaying via the $\Delta^0 \to p + \pi^-$ channel. To account for the $\Delta^0 \to n + \pi^0$ decay channel, one can simply multiply the spectrum in Fig. 11(a) by a factor of 3, since we have, using the respective Clebsh-Gordan coefficients [45], the following probabilities of $\Delta^0(1232)$ decay into different isospin channels: $\Delta^0 \to \frac{2}{3}(n + \pi^0) + \frac{1}{3}(p + \pi^-)$. As seen from Fig. 11(a), the spectrum of invariant cross sections of $\Delta^0(1232)$ can in general be characterized by three regions with different slopes, as was also observed recently in Ref. [49] for $p + ^{12}$C collisions: $T < 0{-}50$, $T \sim 50{-}400$, and $T > 400$ MeV. As seen from Fig. 11(b), the spectra of invariant cross sections obtained for $\varepsilon = 0.41$ and 0.68 show a behavior similar to the spectrum obtained for $\varepsilon = 0.52$. It is interesting to mention that a separate steep slope appeared in the region $T \sim 0{-}50$ MeV. Such a slope was also observed in the region $T \sim 0{-}50$ MeV of reconstructed invariant cross sections of $\Delta^0(1232)$ produced in $p + ^{12}$C collisions at 4.2 GeV/c [49]. We fitted the spectrum of invariant cross sections of $\Delta^0(1232)$ versus their kinetic energy in the region $T = 0$–5 GeV by the function with three slopes T_1, $T_2 = T_0$, and T_3:

$$f(T) = A_1 \exp\left(-\frac{T}{T_1}\right) + A_2 \exp\left(-\frac{T}{T_2}\right) + A_3 \exp\left(-\frac{T}{T_3}\right). \tag{17}$$

As seen from Fig. 11(a) and the value of χ^2/n.d.f. in Table V, the spectrum of invariant cross sections of $\Delta^0(1232)$ resonances, produced in $d + ^{12}$C collisions, is fitted well by the function in Eq. (17) with values of $T_1 = (13 \pm 5^{+4}_{-1})$ MeV, $T_2 = T_0 = (147 \pm 21^{+3}_{-7})$ MeV, and $T_3 = (703 \pm 34^{+1}_{-4})$ MeV. Here T_1 corresponds obviously to the region $T \sim 0{-}50$ MeV, $T_2 = T_0$ corresponds to $T \sim 50{-}400$ MeV, and T_3 corresponds to the region $T > 400$ MeV, where $\Delta 0(1232)$ are most likely produced in processes of hard $d + ^{12}$C scattering and/or on nucleons of impinging deuterons. It is interesting to note that the spectrum in the region $T \sim 0{-}50$ MeV has a slope $T_1 = (13 \pm 5^{+4}_{-1})$ MeV, which is of the order of temperatures $\sim$5–8 MeV, typical for nucleons coming from evaporation. This result is in very good agreement with the value $T_1 = (13 \pm 5^{+1}_{-4})$, obtained for a slope of invariant cross sections of $\Delta^0(1232)$, produced in $p + ^{12}$C collisions at 4.2 GeV/c, in the region

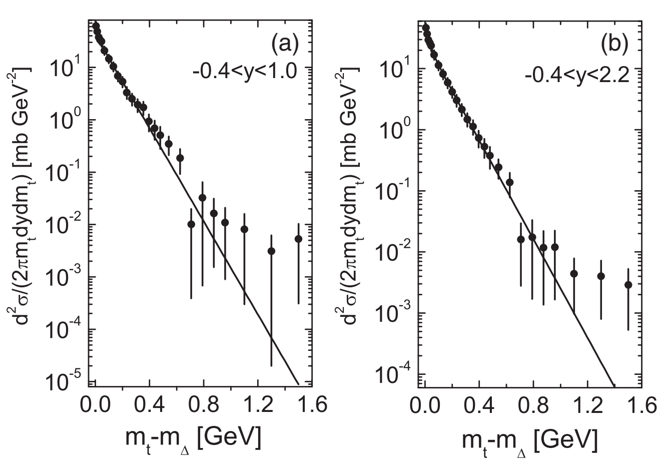

FIG. 12. Reconstructed invariant cross sections $f(m_t)$ of $\Delta^0(1232)$ vs their reduced transverse mass obtained for the best value of $\varepsilon = 0.52$ and fits by the sum of two exponential functions with two slopes.

TABLE VI. Comparison of parameters of approximation of invariant cross sections $f(m_t)$ vs $m_t - m_\Delta$ for the best value of $\varepsilon = 0.52$ by the sum of two exponentials with two slopes for rapidity intervals $-0.4 < y < 1.0$ and $-0.4 < y < 2.2$.

Parameter	Rapidity interval	
	$-0.4 < y < 1.0$	$-0.4 < y < 2.2$
A_1 (mb GeV^{-2})	29.03 ± 16.85	23.36 ± 9.62
T_1 (GeV)	0.019 ± 0.015	0.027 ± 0.016
A_2 (mb GeV^{-2})	41.32 ± 6.13	27.64 ± 4.77
T_2 (GeV)	0.098 ± 0.005	0.108 ± 0.006
χ^2/n.d.f.	0.747	0.427

$T \sim 0$–50 MeV. This similarity for $d + {}^{12}$C and $p + {}^{12}$C collisions can be due to the reason that $\Delta^0(1232)$ with relatively small kinetic energies 0 − 50 MeV are produced at momentum transfers comparable to Fermi momenta of nucleons in carbon. Hence these $\Delta^0(1232)$ are produced on (nucleons of) carbon nuclei under similar conditions in peripheral collisions, irrespective of the type of projectile (d or p), bringing about the coincidence of the slopes of their spectra in the region $T \sim 0$–50 MeV.

The freeze-out temperature T_0 of $\Delta^0(1232)$ produced on oxygen nuclei in ^{16}O + p interactions at 3.25 A was estimated roughly [42] by fitting their spectrum of invariant cross sections by an exponential with a single slope T_0 in the region of relatively small kinetic energies $T = 30$–400 MeV in the oxygen nucleus rest frame. Naturally only the particles with relatively small kinetic energies in the fragmenting nucleus rest frame can originate from some equilibrium state of nuclear matter, and hence contain valuable information about the excited nuclear matter. The particles with relatively high kinetic energies (momenta) are most likely produced in processes of hard scattering, and hence do not carry information about the freeze-out temperature of nuclear matter. However, we showed in our recent paper [49] that T_0 estimated from kinetic energy spectra of $\Delta^0(1232)$ in $p + {}^{12}$C collisions at 4.2 GeV/c was an overestimation due to the influence of the region of relatively high kinetic energies with a quite large value of parameter T and longitudinal momentum transferred to the system by impinging protons. Indeed, as seen from Table V, the value of T_0 is quite large and cannot be a reasonable estimate of freeze-out temperature for such a small system as $d + {}^{12}$C. Therefore, in Ref. [49] we obtained a better estimate of freeze-out temperature, $T_0 = (79 \pm 8)$ MeV, from a detailed analysis of reduced transverse mass spectra of $\Delta^0(1232)$ in $p + {}^{12}$C collisions at 4.2 GeV/c. So we reconstructed the invariant cross sections $f(m_t) = d^2\sigma/(2\pi m_t\, dy\, dm_t)$ of $\Delta^0(1232)$, produced in $d + {}^{12}$C at 4.2A GeV/c, as a function of their reduced transverse mass $m_t - m_\Delta$, where $m_t = \sqrt{m_\Delta^2 + p_t^2}$ is the transverse mass of $\Delta^0(1232)$, following the same procedure as in Ref. [49]. In Figs. 12(a) and 12(b) invariant cross sections $f(m_t)$ of $\Delta^0(1232)$ versus their reduced transverse mass obtained for the best value of $\varepsilon = 0.52$ and the corresponding fits by the function

$$A_1 \exp[-(m_t - m_\Delta)/T_1] + A_2 \exp[-(m_t - m_\Delta)/T_2] \quad (18)$$

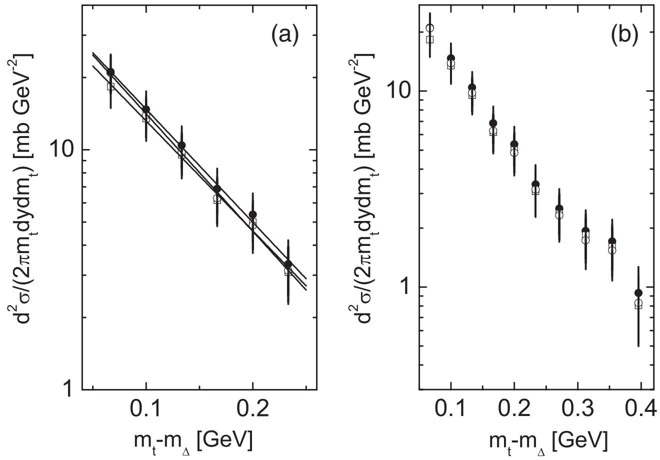

FIG. 13. Reconstructed invariant cross sections $f(m_t)$ of $\Delta^0(1232)$ vs their reduced transverse mass obtained for values of $\varepsilon = 0.41$ (□), $\varepsilon = 0.52$ (•), and $\varepsilon = 0.68$ (○) for ranges (a) $m_t - m_\Delta = 50 - 250$ MeV and (b) $m_t - m_\Delta = 50 - 400$ MeV for the rapidity interval $-0.4 < y < 1.0$, and the corresponding fits (solid lines) with a single slope Boltzmann distribution for the fitting range $m_t - m_\Delta = 50 - 250$ MeV.

TABLE VII. Comparison of parameters of approximation of invariant cross sections $f(m_t)$ vs $m_t - m_\Delta$ for values of $\varepsilon = 0.52$, 0.68, and 0.41 by an exponential with a single slope Boltzmann distribution for different ranges of $m_t - m_\Delta$ for the rapidity interval $-0.4 < y < 1.0$.

ε	Parameter	$m_t - m_\Delta$ range (MeV)			
		50 − 250	50–300	100–300	50–400
0.52	A_0 (mb GeV^{-2})	43.59 ± 9.94	42.53 ± 8.70	42.02 ± 12.15	36.78 ± 6.26
	T_0 (GeV)	0.092 ± 0.013 ± 0.003	0.094 ± 0.011	0.095 ± 0.014	0.105 ± 0.009
	χ^2/n.d.f.	0.041	0.045	0.055	0.258
0.68	A_0 (mb GeV^{-2})	43.54 ± 10.15	42.04 ± 8.80	39.91 ± 11.79	36.12 ± 6.29
	T_0 (GeV)	0.089 ± 0.012	0.092 ± 0.011	0.093 ± 0.014	0.102 ± 0.009
	χ^2/n.d.f.	0.036	0.051	0.048	0.262
0.41	A_0 (mb GeV^{-2})	37.80 ± 8.72	36.89 ± 7.64	37.58 ± 11.11	32.62 ± 5.58
	T_0 (GeV)	0.095 ± 0.014	0.097 ± 0.012	0.096 ± 0.015	0.106 ± 0.009
	χ^2/n.d.f.	0.067	0.069	0.079	0.230

with two slopes are shown for rapidity intervals $-0.4 < y < 1.0$ and $-0.4 < y < 2.2$. The corresponding parameters of approximation of $f(m_t)$ by a function in expression (18) are presented in Table VI. As can be seen from Table VI, the values of T_2 are significantly lower than the corresponding value in Table V. It should be noted that the fitting of spectra $f(m_t)$ of $\Delta^0(1232)$ in Fig. 12 by a sum of three exponentials with three slopes gives almost the same values of parameters T_1 and T_2, as in fitting by the function in (18), with a parameter $T_3 \approx T_2$ but with an unacceptably large value of χ^2/n.d.f. $\gg 1$.

For an even better estimate of T_0, we searched for region of a single slope in different intervals within the range $m_t - m_\Delta = 50{-}400$ MeV and rapidity interval $-0.4 < y < 1.0$. This rapidity interval characterizes $\Delta^0(1232)$ produced in the region of target carbon fragmentation, and has almost no contribution of $\Delta^0(1232)$ coming from neutrons of projectile deuterons. The spectra of $f(m_t)$ of $\Delta^0(1232)$ versus their reduced transverse mass obtained for the values of $\varepsilon = 0.52$, 0.68, and 0.41 and the corresponding fits with a single slope Boltzmann distribution $A_0 \exp[-(m_t - m_\Delta)/T_0]$ in the region $m_t - m_\Delta = 50{-}250$ MeV are presented in Fig. 13(a). The corresponding parameters of the approximation by an exponential with a single slope T_0 for fitting ranges $m_t - m_\Delta =$ 50−250, 50−300, 100−300, and 50−400 MeV and rapidity interval $-0.4 < y < 1.0$ are presented in Table VII. As seen from this table, we obtained the minimal values of χ^2/n.d.f. for the fitting range $m_t - m_\Delta = 50{-}250$. The corresponding value of T_0 proved to be $(92 \pm 13 \pm 3)$ MeV, which was stable within + 3 MeV for all three ε values. As observed from Fig. 13(b), the spectra at $m_t - m_\Delta > 300$ MeV start to deviate from a single slope behavior of $f(m_t)$, which is due to the onset of contribution from hard scattering, as was also the case for $p + {}^{12}$C collisions [49]. This is also supported by χ^2/n.d.f. values given in Table VII, which decrease several fold as the fitting range is changed from $m_t - m_\Delta = 50 - 400$ to 50−250 MeV. The so estimated freeze-out temperature $T_0 = (92 \pm 13 \pm 3)$ MeV, as seen from Table VII, has χ^2/n.d.f. values which are at least five times smaller than that obtained for $T_2 = T_0$ in Table V. We also fitted the spectra of $f(m_t)$ in the region $m_t - m_\Delta = 0 - 250$ MeV by the sum of two exponentials with two slopes for all three ε values for the rapidity interval $-0.4 < y < 1.0$. The corresponding spectra and fits are shown in Fig. 14 and the parameters of approximation of invariant cross sections $f(m_t)$ are presented in Table VIII. As seen from Table VIII, the values of $T_2 = T_0$ are in very good agreement with the above obtained estimate of a freeze-out temperature of $\Delta^0(1232)$. The data of Table VIII confirm the existence of a steep slope in the region of $m_t - m_\Delta <$ 30–40 MeV. However, in this case, this region of small $m_t - m_\Delta$ could be partly influenced by a contribution from $\Delta^0(1232)$ moving in the longitudinal direction with kinetic energies $T > 50$ MeV and a very small transverse momentum. Therefore, the values of T_1 in Table VIII have large uncertainties, agreeing nevertheless within uncertainties with the value $T_1 = (13 \pm 5^{+4}_{-1})$ MeV in Table V obtained for a region of kinetic energies $T < 50$ MeV.

In central heavy-ion collisions at energies between 1 and $2A$ GeV [12], it was deduced that the apparent temperature of

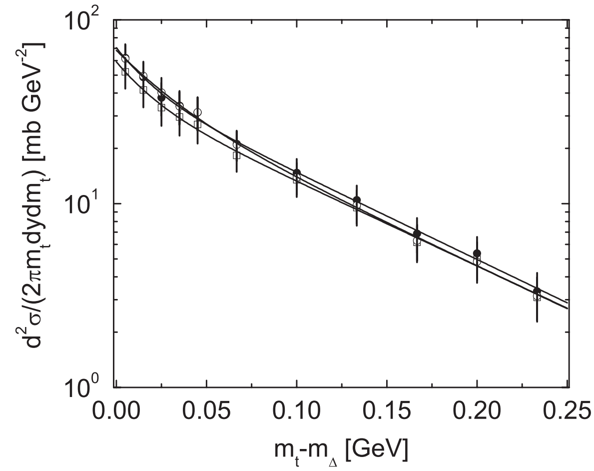

FIG. 14. Reconstructed invariant cross sections $f(m_t)$ of $\Delta^0(1232)$ vs their reduced transverse mass obtained for values of $\varepsilon = 0.43$ (□), $\varepsilon = 0.61$ (•), and $\varepsilon = 0.68$ (○), and the corresponding fits by the sum of two exponentials with two slopes for the fitting range $m_t - m_\Delta = 0{-}250$ MeV and the rapidity interval $-0.4 < y < 1.0$.

the reconstructed $\Delta(1232)$ resonances, $T_\Delta \approx$ 40–60 MeV, was smaller than that of the protons. This was attributed to the fact that the reconstructed $\Delta(1232)$ are stemming from rather late "colder" phase of the collisions. The $\Delta(1232)$ produced earlier are more difficult to reconstruct, because the decay pions may scatter before leaving the collision zone. However, in the present paper the estimated apparent temperature of $\Delta^0(1232)$ resonances, $T_0 = (92 \pm 13 \pm 3)$ MeV, proved to be larger than that of protons, $T_p = (67 + 2)$ MeV, estimated in Ref. [71], in $d + {}^{12}$C collisions at $4.2A$ GeV/c. This fact and the fact that the minimal kinetic energy of $\Delta^0(1232)$ extracted in the present analysis proved to be sufficiently small, $(T_\Delta)_{\min} \approx 1$ MeV, suggest that $\Delta^0(1232) \to p\pi^-$ could be reconstructed quite efficiently in the present paper.

In Ref. [25] the freeze-out temperatures of $\Delta(1232)$, produced in Ni + Ni collisions at beam kinetic energies between 1 and $2A$ GeV, were estimated within the context of a hadrochemical equilibrium model [72,73] using the obtained fractions $n(\Delta)/n(\text{nucleon} + \Delta)$ of nucleons excited to $\Delta(1232)$. The freeze-out temperatures of $\Delta(1232)$, produced in Ni + Ni collisions at 1–$2A$ GeV and Au + Au collisions at $1.06A$ GeV, were extracted in Refs. [33,34] from the radial

TABLE VIII. Parameters of approximation of invariant cross sections $f(m_t)$ of $\Delta^0(1232)$ resonances versus $m_t - m_\Delta$ by the sum of two exponentials with two slopes for fitting range $m_t - m_\Delta =$ 0−250 MeV and rapidity interval $-0.4 < y < 1.0$.

Parameters	$\varepsilon = 0.52$	$\varepsilon = 0.68$	$\varepsilon = 0.41$
A_1 (mb GeV^{-2})	26.78 ± 19.08	29.38 ± 23.22	21.14 ± 16.51
T_1 (GeV)	0.018 ± 0.022	0.026 ± 0.032	0.018 ± 0.025
A_2 (mb GeV^{-2})	43.67 ± 14.55	38.90 ± 26.02	37.96 ± 13.00
T_2 (GeV)	0.092 ± 0.017	0.093 ± 0.030	0.095 ± 0.019
χ^2/n.d.f.	0.061	0.037	0.062

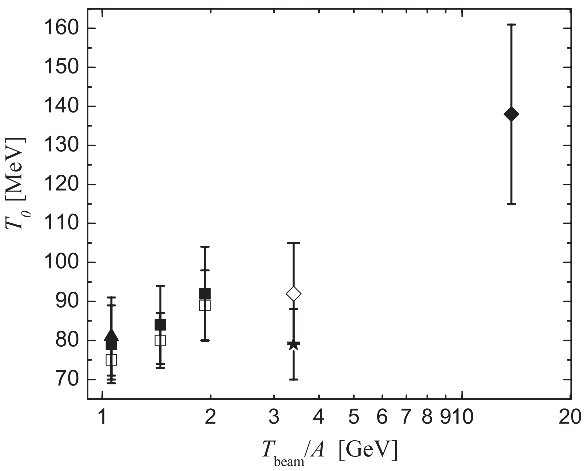

FIG. 15. Dependence of T_0 on beam kinetic energy per nucleon: (◇) obtained in the present paper for $d + {}^{12}$C collisions at 4.2A GeV/c ($T_{beam} \approx 3.4A$ GeV); (★) obtained for $p + {}^{12}$C collisions at 4.2 GeV/c ($T_{beam} \approx 3.4A$ GeV); T_0 obtained from the radial flow analysis (■) and using the hadrochemical equilibrium model (□) for Δ resonances produced in Ni + Ni collisions; (▲) T_0 obtained from the radial flow analysis for Δ^0 resonances produced in Au + Au collisions at 1.06A GeV; (◆) T_0 obtained for Δ resonances produced in central ^{28}Si + Pb collisions at $p_{lab} = 14.6A$ GeV/c.

flow analysis. Further, in Ref. [23], the pion enhancement at low transverse momentum was used to determine Δ(1232) abundance in central ^{28}Si + Al,Pb collisions at p_{lab} = 14.6 GeV/c per nucleon ($T_{beam} \approx 13.7A$ GeV), and the freeze-out temperature of the system was extracted using the measured Δ(1232) abundance. It was concluded [23] that in central ^{28}Si + Pb collisions at 14.6A GeV/c a fireball is formed with substantial excitation of Δ baryons which freezes out at $T_0 = 138^{+23}_{-18}$ MeV. All of the above-mentioned freeze-out temperatures, along with the values of T_0 for Δ^0(1232) produced in $d + {}^{12}$C and $p+{}^{12}$C collisions at 4.2A GeV/c ($T_{beam} \approx 3.4$ A GeV), are presented in Fig. 15. As seen from Fig. 15, the freeze-out temperatures for Δ(1232) produced in central heavy-ion collisions increase with an increase in beam energy. The freeze-out temperature $T_0 = (92 \pm 13 \pm 3)$ MeV estimated in the present paper agrees within uncertainties with the corresponding result for $p+{}^{12}$C collisions [49] at the same initial momentum. However, the freeze-out temperatures estimated for $d + {}^{12}$C and $p+{}^{12}$C collisions do not follow an increasing behavior with an increase in beam energy observed for central heavy-ion collisions. This is likely due to the mainly peripheral character of such collisions with carbon nuclei and the relative smallness of the interacting system.

V. CONCLUSIONS

The mass distribution of Δ^0(1232) produced in $d + {}^{12}$C collisions at 4.2A GeV/c was reconstructed using the angular criterion, for the first time. The mass and width of Δ^0(1232) were extracted to be $(1230 \pm 4^{+3}_{-6})$ and $(90 \pm 14^{+19}_{-24})$ MeV/c^2, respectively. The relative fraction of π^- mesons coming from Δ^0(1232) decay in $d + {}^{12}$C collisions was estimated to be $(30 \pm 2^{+6}_{-7})$% and is compatible with the corresponding fractions estimated for Δ^0(1232) produced in $p+{}^{12}$C, ^{4}He+^{12}C, and ^{12}C+^{12}C collisions at the same incident momentum per nucleon. Comparing this fraction with the corresponding results for different sets of colliding nuclei and incident energy range $T_{beam}/A \approx$ 1–40 GeV, we observed an overall decrease in the fraction of π^- from Δ^0(1232) decay as the incident energy increased. The relative number of nucleons excited to Δ^0 at freeze-out in $d + {}^{12}$C collisions was estimated to be $(15 \pm 2^{+3}_{-4})$%, in a good agreement with the corresponding result $(16 \pm 3^{+4}_{-3})$% obtained for $p + {}^{12}$C collisions at 4.2 GeV/c.

The transverse momentum distribution of π^- mesons coming from Δ^0(1232) decay was reconstructed and compared with the transverse momentum spectrum of all π^- produced in $d + {}^{12}$C collisions. This comparison revealed that Δ^0(1232) decay kinematics was responsible for the low transverse momentum enhancement of π^- spectra in $d + {}^{12}$C collisions at 4.2A GeV/c, as was also shown in earlier papers with various colliding nuclei and incident energies ranging from 1 to 15A GeV.

The momentum, kinetic energy, transverse momentum, emission angle, and rapidity distributions of Δ^0(1232) produced in $d + {}^{12}$C collisions at 4.2A GeV/c were reconstructed and their mean values estimated. Analyzing the rapidity distribution of Δ^0(1232), we observed that a large fraction of Δ^0 is produced on (nucleons of) carbon nuclei, while still a noticeable number of Δ^0 originate from neutrons of impinging deuterons, as is expressed by the shoulder in Δ^0 rapidity distribution in the region $y \sim 1.2-1.9$. We observed that the spectrum of invariant cross sections of Δ^0(1232) in region $T \sim 0-50$ MeV has a slope, $T_1 = (13 \pm 5^{+1}_{-4})$ MeV, of the order of temperatures 5−8 MeV, typical for evaporated nucleons, as was also found in $p + {}^{12}$C collisions [49]. The freeze-out temperature of Δ^0(1232) estimated in the present analysis, $T_0 = (92 \pm 13 \pm 3)$ MeV, was larger than that of protons, $T_p = (67 \pm 2)$ MeV, obtained in Ref. [71] for $d + {}^{12}$C collisions at 4.2A GeV/c, and close to the value, $T_0 = (79 \pm 8)$ MeV, deduced for Δ^0(1232) produced in $p + {}^{12}$C collisions at the same incident energy. This temperature was less than expected from the increasing behavior of T_0 with an increase in beam energy observed for Δ^0(1232) produced in central heavy-ion collisions. This can be due to the mainly peripheral character of $d+{}^{12}$C collisions and the relative smallness of the colliding system.

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