# Uncertainties in nuclear transition matrix elements for neutrinoless $\beta\beta$ decay: The heavy Majorana neutrino mass mechanism

P. K. Rath,[1] R. Chandra,[2,3] P. K. Raina,[3,4] K. Chaturvedi,[5] and J. G. Hirsch[6]

[1]*Department of Physics, University of Lucknow, Lucknow-226007, India*

[2]*Department of Applied Physics, Babasaheb Bhimrao Ambedkar University, Lucknow-226025, India*

[3]*Department of Physics and Meteorology, Indian Institute of Technology, Kharagpur-721302, India*

[4]*Department of Physics, Indian Institute of Technology, Ropar, Rupnagar-140001, Punjab, India*

[5]*Department of Physics, Bundelkhand University, Jhansi-284128, India*

[6]*Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, 04510 México, D.F., México*

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Employing four different parametrizations of the pairing plus the multipolar type of effective two-body interaction and three different parametrizations of the Jastrow type of short-range correlations, the uncertainties in the nuclear transition matrix elements $M_N^{(0\nu)}$ due to the exchange of heavy Majorana neutrino for the $0^+ \to 0^+$ transition of neutrinoless double beta decay of $^{94}$Zr, $^{96}$Zr, $^{98}$Mo, $^{100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes in the PHFB model are estimated to be around 35%. Excluding the nuclear transition matrix elements calculated with the Miller-Spencer parametrization of Jastrow short-range correlations, the uncertainties are found to be smaller than 20%.

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## I. INTRODUCTION

In addition to establishing the Dirac or Majorana nature of neutrinos, the observation of $(\beta\beta)_{0\nu}$ decay is a convenient tool to test the lepton number conservation, possible hierarchies in the neutrino mass spectrum, the origin of neutrino mass, and *CP* violation in the leptonic sector. Furthermore, it can also ascertain the role of various gauge models associated with all possible mechanisms, namely the exchange of light neutrinos, heavy neutrinos, the right-handed currents in the left-right symmetric model (LRSM), the exchange of sleptons, neutralinos, squarks, and gluinos in the $R_p$-violating minimal supersymmetric standard model, the exchange of leptoquarks, existence of heavy sterile neutrinos, compositeness, extradimensional scenarios, and Majoron models, allowing the occurrence of $(\beta\beta)_{0\nu}$ decay. Stringent limits on the associated parameters have already been extracted from the observed experimental limits on the half-life of $(\beta^-\beta^-)_{0\nu}$ decay [1] and, presently, all the experimental attempts are directed for its observation. The experimental and theoretical studies devoted to $(\beta\beta)_{0\nu}$ decay over the past decades have been recently reviewed by Avignone *et al.* [2] and references therein.

Presently, there is an increased interest in calculating reliable NTMEs for $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of heavy Majorana neutrinos to ascertain the dominant mechanism contributing to it [3,4]. The lepton number violating $(\beta^-\beta^-)_{0\nu}$ decay was studied by Vergados by taking a Lagrangian consisting of left-handed as well as right-handed leptonic currents [5]. In the QRPA, the $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of heavy Majorana neutrinos was studied by Tomoda [6]. The decay rate of $(\beta^-\beta^-)_{0\nu}$ mode in the LRSM was derived by Doi and Kotani [7]. Hirsch *et al.* [8] have calculated all the required nuclear transition matrix elements (NTMEs) in the QRPA and limits on the effective light neutrino mass $\langle m_\nu \rangle$, heavy neutrino mass $\langle M_N \rangle$, right-handed heavy neutrino $\langle M_R \rangle$, $\langle \lambda \rangle$, $\langle \eta \rangle$, and mixing angle tan$\xi$ have been obtained. The heavy neutrino mechanism was also studied in the QRPA without [9] and with *pn* pairing [10]. In the heavy Majorana neutrino mass mechanism, Simkovic *et al.* [11] have studied the role of induced weak magnetism and pseudoscalar terms and it was found that they are quite important in $^{48}$Ca nucleus. The importance of the same induced currents in both the light and heavy Majorana neutrino exchange mechanisms was also studied using the *pn* RQRPA [12] as well as SRQRPA [3].

In spite of the remarkable success of the large-scale shell model (LSSM) calculations of the Strassbourg-Madrid group [13], there is a necessity for large configuration mixing to reproduce the structural complexity of medium and heavy mass nuclei. On the other hand, the QRPA and its extensions have emerged as successful models by including a large number of basis states and in correlating the single-$\beta$ GT strengths and half-lives of $(\beta^-\beta^-)_{2\nu}$ decay in addition to explaining the observed suppression of $M_{2\nu}$ [14,15]. In the mass region $90 \leqslant A \leqslant 150$, there is a subtle interplay of pairing and quadrupolar correlations and their effects on the NTMEs of $(\beta^-\beta^-)_{0\nu}$ decay have been studied in the interacting shell model (ISM) [16,17], deformed QRPA model [18–21], and projected-Hartree-Fock-Bogoliubov (PHFB) model [22,23].

The possibility of constraining the values of the gauge parameters using the measured lower limits on the $(\beta^-\beta^-)_{0\nu}$ decay half-lives relies heavily on the model-dependent NTMEs. Different predictions are obtained by employing different nuclear models, and within a given model, varying the model space, single-particle energies (SPEs), and effective two-body interaction. In addition, a number of issues regarding the structure of NTMEs, namely the effect of pseudoscalar and weak magnetism terms on the Fermi, Gamow-Teller, and tensorial NTMEs [24,25], the role of finite size of nucleons (FNS) as well as short-range correlations (SRC) vis-a-vis the radial evolution of NTMEs [16,26–28], and the value of the axial-vector coupling constant $g_A$ are also sources of uncertainty and remain to be investigated.

It was observed by Vogel [29] that in the case of well-studied $^{76}$Ge, the calculated decay rates $T_{1/2}^{0\nu}$ differ by a factor of 6–7 and consequently, the uncertainty in the effective neutrino mass $\langle m_\nu \rangle$ is about 2–3. Thus, the spread between the calculated NTMEs can be used as the measure of the theoretical uncertainty. In case the $(\beta\beta)_{0\nu}$ decay of different nuclei is observed, Bilenky and Grifols [30] have suggested that the results of calculations of NTMEs of the $(\beta^-\beta^-)_{0\nu}$ decay can be checked by comparing the calculated ratios of the corresponding NTMEs squared with the experimentally observed values.

Bahcall *et al*. [31] and Avignone *et al*. [32] have calculated averages of all the available NTMEs, and their standard deviation is taken as the measure of theoretical uncertainty. On the other hand, Rodin *et al.* [33] have calculated nine NTMEs with three sets of basis states and three realistic two-body effective interactions of charge-dependent Bonn, Argonne, and Nijmen potentials in the QRPA as well as RQRPA and estimated the theoretical uncertainties by making a statistical analysis. It was noticed that the variances are substantially smaller than the average values and the results of QRPA, albeit slightly larger, are quite close to the RQRPA values. Faessler and coworkers have further studied uncertainties in NTMEs from short-range correlations using the unitary correlation operator method (UCOM) [26] and self-consistent coupled cluster method (CCM) [27].

The PHFB model has the advantage of treating the pairing and deformation degrees of freedom on equal footing and projecting out states with good angular momentum. However, the single $\beta$-decay rates and the distribution of GT strength, which require the structure of the intermediate odd $Z$-odd $N$ nuclei, cannot be studied in the present version of the PHFB model. In spite of this limitation, the PHFB model in conjunction with pairing plus quadrupole-quadrupole ($PQQ$) [34] was successfully applied to reproduce the lowest yrast states, electromagnetic properties of the parent and daughter nuclei, and the measured $(\beta^-\beta^-)_{2\nu}$-decay rates [35,36]. In the PHFB formalism, the existence of an inverse correlation between the quadrupole deformation and the size of NTMEs $M_{2\nu}$, $M^{(0\nu)}$, and $M_N^{(0\nu)}$ was observed [22,23]. Further, it was noticed that the NTMEs are usually large for a pair of spherical nuclei, almost constant for small deformation, suppressed depending on the difference in the deformation $\Delta\beta_2$ of parent and daughter nuclei, and having a well defined maximum when $\Delta\beta_2 = 0$ [22,23].

In Ref. [37], a statistical analysis was performed for extracting uncertainties in eight (twelve) NTMEs for $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of light Majorana neutrino, calculated in the PHFB model with four different parametrizations of pairing plus the multipolar type of effective two-body interaction [23] and two (three) different parametrizations of the Jastrow type of SRC [27]. In confirmation with the observation made by Simkovic *et al*. [27], it was noticed that the Miller-Spencer type of parametrization is a major source of uncertainty and its exclusion reduces the uncertainties from 10%–15% to 4%–14%. Presently, the same procedure was adopted to estimate the theoretical uncertainties associated with the NTMEs $M_N^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of heavy Majorana neutrino. In Sec. II, a brief discussion of the theoretical formalism is presented. The results for different parametrizations of the two-body interaction and SRC vis-a-vis radial evolution of NTMEs are discussed in Sec III. In the same section, the averages as well as standard deviations are calculated for estimating the theoretical uncertainties. Finally, the conclusions are given in Sec. IV.

## II. THEORETICAL FORMALISM

In the charged current weak processes, the current-current interaction under the assumption of zero mass neutrinos leads to terms which, except for vector and axial vector parts, are proportional to the lepton mass squared, and hence negligible. However, it was reported by Simkovic *et al*. [24,25] that the contribution of the pseudoscalar term is equivalent to a modification of the axial vector current because of PCAC and greater than the vector current. The contributions of pseudoscalar and weak magnetism terms in the mass mechanism can change $M^{(0\nu)}$ up to 30% and the change in $M_N^{(0\nu)}$ is considerably larger. In the shell model [16,38], IBM [39], and GCM + PNAMP [40], the contributions of these pseudoscalar and weak magnetism terms to $M^{(0\nu)}$ have also been investigated. However, it was reported by Suhonen and Civitarese [41] that these contributions are relatively small and can be safely neglected. Therefore, the investigation of this issue is of definite interest and is reported in the present work.

In the two nucleon mechanism, the half-life $T_{1/2}^{0\nu}$ for the $0^+ \to 0^+$ transition of $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of heavy Majorana neutrino between nucleons having finite size is given by [6,7]

$$\left[T_{1/2}^{0\nu}(0^+ \to 0^+)\right]^{-1} = \left(\frac{m_p}{\langle M_N \rangle}\right)^2 G_{01} \left|M_N^{(0\nu)}\right|^2, \qquad (1)$$

where $m_p$ is the proton mass and

$$\langle M_N \rangle^{-1} = \sum\nolimits_i U_{ei}^2 m_i^{-1}, \quad m_i > 1 \text{ GeV}, \qquad (2)$$

and in the closure approximation, the NTMEs $M_N^{(0\nu)}$ is of the form [12,26,27],

$$M_N^{(0\nu)} = -M_{Fh} + M_{GTh} + M_{Th}, \qquad (3)$$

where

$$M_\alpha = \sum_{n,m} \langle 0_F^+ \| O_{\alpha,nm} \tau_n^+ \tau_m^+ \| 0_I^+ \rangle, \qquad (4)$$

with

$$O_{Fh} = H_{Fh}(r_{nm}), \qquad (5)$$

$$O_{GTh} = \sigma_n \cdot \sigma_m H_{GTh}(r_{nm}), \qquad (6)$$

$$O_{Th} = [3(\sigma_n \cdot \hat{\mathbf{r}}_{nm})(\sigma_m \cdot \hat{\mathbf{r}}_{nm}) - \sigma_n \cdot \sigma_m] H_{GTh}(r_{nm}). \qquad (7)$$

The exchange of heavy Majorana neutrinos gives rise to short-ranged neutrino potentials, which with the consideration of FNS are given by

$$H_{\alpha h}(r_{nm}) = \frac{2R}{(m_p m_e)\pi} \int f_{\alpha h}(q r_{nm}) h_\alpha(q) q^2 dq, \quad (8)$$

where $f_{\alpha h}(q r_{nm}) = j_0(q r_{nm})$ for $\alpha = F$ as well as $GT$ and $f_{Th}(q r_{nm}) = j_2(q r_{nm})$.

Furthermore, the $h_F(q)$, $h_{GT}(q)$ and $h_T(q)$ are written as

$$h_F(q) = \left(\frac{g_V}{g_A}\right)^2 \left(\frac{\Lambda_V^2}{q^2+\Lambda_V^2}\right)^4, \quad (9)$$

$$\begin{aligned} h_{GT}(q) &= \frac{g_A^2(q^2)}{g_A^2}\left[1 - \frac{2}{3}\frac{g_P(q^2)q^2}{g_A(q^2)2m_p} + \frac{1}{3}\frac{g_P^2(q^2)q^4}{g_A^2(q^2)4m_p^2}\right] + \frac{2}{3}\frac{g_M^2(q^2)q^2}{g_A^2 4m_p^2} \\ &\approx \left(\frac{\Lambda_A^2}{q^2+\Lambda_A^2}\right)^4 \left[1 - \frac{2}{3}\frac{q^2}{(q^2+m_\pi^2)} + \frac{1}{3}\frac{q^4}{(q^2+m_\pi^2)^2}\right] + \left(\frac{g_V}{g_A}\right)^2 \frac{\kappa^2 q^2}{6m_p^2}\left(\frac{\Lambda_V^2}{q^2+\Lambda_V^2}\right)^4, \end{aligned} \quad (10)$$

$$\begin{aligned} h_T(q) &= \frac{g_A^2(q^2)}{g_A^2}\left[\frac{2}{3}\frac{g_P(q^2)q^2}{g_A(q^2)2m_p} - \frac{1}{3}\frac{g_P^2(q^2)q^4}{g_A^2(q^2)4m_p^2}\right] + \frac{1}{3}\frac{g_M^2(q^2)q^2}{g_A^2 4m_p^2} \\ &\approx \left(\frac{\Lambda_A^2}{q^2+\Lambda_A^2}\right)^4 \left[\frac{2}{3}\frac{q^2}{(q^2+m_\pi^2)} - \frac{1}{3}\frac{q^4}{(q^2+m_\pi^2)^2}\right] + \left(\frac{g_V}{g_A}\right)^2 \frac{\kappa^2 q^2}{12m_p^2}\left(\frac{\Lambda_V^2}{q^2+\Lambda_V^2}\right)^4, \end{aligned} \quad (11)$$

where the form factors are given by

$$\begin{aligned} g_A(q^2) &= g_A\left(\frac{\Lambda_A^2}{q^2+\Lambda_A^2}\right)^2, \\ g_M(q^2) &= \kappa g_V\left(\frac{\Lambda_V^2}{q^2+\Lambda_V^2}\right)^2, \\ g_P(q^2) &= \frac{2m_p g_A(q^2)}{(q^2+m_\pi^2)}\left(\frac{\Lambda_A^2-m_\pi^2}{\Lambda_A^2}\right), \end{aligned} \quad (12)$$

with $g_V = 1.0$, $g_A = 1.254$, $\kappa = \mu_p - \mu_n = 3.70$, $\Lambda_V = 0.850$ GeV, $\Lambda_A = 1.086$ GeV, and $m_\pi$ is the pion mass.

Substituting Eqs. (5)–(11) in Eq. (3), there is one term, associated with $h_F$ [Eq. (9)] contributing to $M_{Fh}$, while $M_{GTh}$ has four terms, denoted by $M_{GT-AA}$, $M_{GT-AP}$, $M_{GT-PP}$, and $M_{GT-MM}$, which correspond to the four terms in $h_{GT}$ [Eq. (10)]. The tensor contribution $M_{Th}$ has three terms, denoted by $M_{T-AP}$, $M_{T-PP}$, and $M_{T-MM}$, which correspond to the three terms in $h_T$ [Eq. (11)]. Their contributions to the total nuclear matrix element are discussed in Sec. III.

The short-range correlations (SRC) arise mainly from the repulsive nucleon-nucleon potential from the exchange of $\rho$ and $\omega$ mesons and have been incorporated by using the effective transition operator [42], the exchange of $\omega$ meson [43], UCOM [26,44], and the self-consistent CCM [27]. The SRC can also be incorporated phenomenologically by the Jastrow type of correlations with Miller-Spencer parametrization [45]. Furthermore, it was shown in the self-consistent CMM [27] that the SRC effects of the Argonne and CD-Bonn two-nucleon potentials are weak and it is possible to parametrize them by the Jastrow type of correlations within a few percent accuracy. Explicitly,

$$f(r) = 1 - ce^{-ar^2}(1 - br^2), \quad (13)$$

where $a = 1.1$, 1.59, and 1.52 fm$^{-2}$, $b = 0.68$, 1.45, and 1.88 fm$^{-2}$, and $c = 1.0$, 0.92, and 0.46 for Miller-Spencer parametrization, Argonne V18, and CD-Bonn $NN$ potentials, respectively. In this work the NTMEs $M_N^{(0\nu)}$ are calculated in the PHFB model for the above-mentioned three sets of parameters for the SRC, denoted as SRC1, SRC2, and SRC3, respectively.

In Fig. 1, we plot the neutrino potential $H_N(r,\Lambda) = H_{Fh}(r,\Lambda)\, f(r)$ with the three different parametrizations of SRC. It is noticed that the potentials from FNS and FNS + SRC3 are peaked at the origin whereas the peaks from FNS + SRC1 and FNS + SRC3 are at $r \approx 0.6$ fm and $r \approx 0.5$ fm, respectively. The shapes of these functions have a definite influence on the radial evolution of NTMEs $M_N^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay from the exchange of heavy Majorana neutrino as discussed in Sec. III.

The calculation of $M_N^{(0\nu)}$ in the PHFB model was discussed in our earlier work [22,37] and one obtains the following expression for NTMEs $M_\alpha^{(0\nu)}$ of $(\beta^-\beta^-)_{0\nu}$ decay [37],

$$\begin{aligned} M_\alpha^{(0\nu)} = {} & [n^{J_i=0} n^{J_f=0}]^{-1/2} \int_0^\pi n_{(Z,N),(Z+2,N-2)}(\theta) \sum_{\alpha\beta\gamma\delta} (\alpha\beta|O_\alpha|\gamma\delta) \\ & \times \sum_{\varepsilon\eta} \frac{\left(f_{Z+2,N-2}^{(\pi)*}\right)_{\varepsilon\beta}}{\left[\left(1 + F_{Z,N}^{(\pi)}(\theta) f_{Z+2,N-2}^{(\pi)*}\right)\right]_{\varepsilon\alpha}} \\ & \times \frac{\left(F_{Z,N}^{(\nu)*}\right)_{\eta\delta}}{\left[\left(1 + F_{Z,N}^{(\nu)}(\theta) f_{Z+2,N-2}^{(\nu)*}\right)\right]_{\gamma\eta}} \sin\theta d\theta, \end{aligned} \quad (14)$$

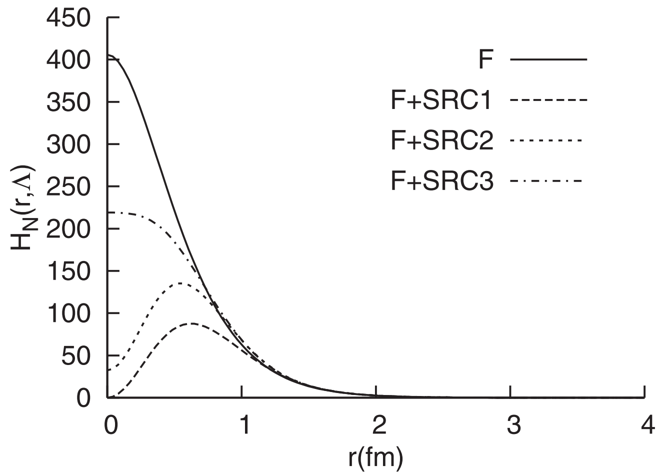


FIG. 1. Radial dependence of $H_N(r,\Lambda) = H_{Fh}(r,\Lambda) f(r)$ for the three different parametrizations of the SRC. In the case of FNS, $f(r) = 1$.

and the expressions for calculating $n^J$, $n_{(Z,N),(Z+2,N-2)}(\theta)$, $f_{Z,N}$, and $F_{Z,N}(\theta)$ are given in Refs. [22,37].

The calculation of matrices $f_{Z,N}$ and $F_{Z,N}(\theta)$ requires the amplitudes $(u_{im}, v_{im})$ and expansion coefficients $C_{ij,m}$, which specify the axially symmetric HFB intrinsic state $|\Phi_0\rangle$ with $K = 0$. Presently, they are obtained by carrying out the HFB calculations through the minimization of the expectation value of the effective Hamiltonian given by [23]

$$H = H_{sp} + V(P) + V(QQ) + V(HH), \tag{15}$$

where $H_{sp}$, $V(P)$, $V(QQ)$, and $V(HH)$ denote the single-particle Hamiltonian, the pairing, quadrupole-quadrupole, and hexadecapole-hexadecapole part of the effective two-body interaction, respectively. The $HH$ part of the effective interaction $V(HH)$ is written as [23]

$$V(HH) = -\left(\frac{\chi_4}{2}\right) \sum_{\alpha\beta\gamma\delta} \sum_{\nu} (-1)^{\nu} \langle\alpha|r^4 Y_{4,\nu}(\theta,\phi)|\gamma\rangle \times \langle\beta|r^4 Y_{4,-\nu}(\theta,\phi)|\delta\rangle\, a_\alpha^\dagger a_\beta^\dagger\, a_\delta\, a_\gamma, \tag{16}$$

with $\chi_4 = 0.2442\ \ \chi_2 A^{-2/3} b^{-4}$ for $T = 1$, and twice of this value for $T = 0$ case, following Bohr and Mottelson [46].

In Refs. [22,35,36], the strengths of the like particle components $\chi_{pp}$ and $\chi_{nn}$ of the $QQ$ interaction were kept fixed. The strength of proton-neutron $(pn)$ component $\chi_{pn}$ was varied so as to reproduce the excitation energy of the $/2^+$ state $E_{2^+}$ for the considered nuclei, namely $^{94,96}$Zr, $^{94,96,98,100}$Mo, $^{98,100,104}$Ru, $^{104,110}$Pd, $^{110}$Cd, $^{128,130}$Te, $^{128,130}$Xe, $^{150}$Nd, and $^{150}$Sm as closely as possible to the experimental values. This is denoted as $PQQ1$ parametrization. Alternatively, one can employ a different parametrization of the $\chi_{2pn}$, namely $PQQ2$ by taking $\chi_{2pp} = \chi_{2nn} = \chi_{2pn}/2$ and the excitation energy $E_{2^+}$ can be reproduced by varying the $\chi_{2pp}$. Adding the $HH$ part of the two-body interaction to $PQQ1$ and $PQQ2$ and by repeating the calculations, two more parametrizations of the effective two-body interactions, namely $PQQHH1$ and $PQQHH2$ were obtained [37].

The four different parametrizations of the effective pairing plus multipolar correlations provide us four different sets of wave functions. With three different parametrizations of the Jastrow type of SRC and four sets of wave functions, sets of 12 NTMEs $M_N^{(0\nu)}$ are calculated for estimating the associated uncertainties in the present work. The uncertainties associated with the NTMEs $M_N^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay are estimated statistically by calculating the mean and the standard deviation

TABLE I. Calculated NTMEs $M_N^{(0\nu)}$ in the PHFB model with four different parametrizations of effective two-body interaction and three different parametrizations of the Jastrow type of SRC for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes from the exchange of heavy Majorana neutrino exchange. (a), (b), (c), and (d) denote $PQQ1$, $PQQHH1$, $PQQ2$, and $PQQHH2$ parametrizations, respectively. See the footnote on p. 3 of Ref. [37] for further details.

| Nuclei | | $F$ | $F+S$ | | |
|---|---|---|---|---|---|
| | | | SRC1 | SRC2 | SRC3 |
| $^{94}$Zr | (a) | 236.9498 | 77.5817 | 138.2606 | 191.3897 |
| | (b) | 220.3794 | 72.4285 | 128.7496 | 178.0783 |
| | (c) | 205.8370 | 72.9303 | 124.3248 | 168.5705 |
| | (d) | 211.0437 | 68.9323 | 122.9710 | 170.3572 |
| $^{96}$Zr | (a) | 177.7479 | 56.4909 | 102.4434 | 142.8831 |
| | (b) | 185.5251 | 59.5338 | 107.2877 | 149.3117 |
| | (c) | 170.8199 | 54.2382 | 98.4051 | 137.2870 |
| | (d) | 175.4730 | 56.0746 | 101.2963 | 141.1240 |
| $^{98}$Mo | (a) | 355.1915 | 117.0804 | 208.2494 | 287.5615 |
| | (b) | 346.1118 | 116.4967 | 204.5667 | 281.0515 |
| | (c) | 358.5109 | 118.0563 | 210.1150 | 290.2080 |
| | (d) | 343.4160 | 115.2077 | 202.6977 | 278.7158 |
| $^{100}$Mo | (a) | 365.8004 | 122.2000 | 215.8882 | 296.9869 |
| | (b) | 361.9877 | 122.6611 | 214.7455 | 294.4297 |
| | (c) | 368.4056 | 123.2364 | 217.5391 | 299.1598 |
| | (d) | 328.9795 | 111.4464 | 195.1601 | 267.5869 |
| $^{104}$Ru | (a) | 274.0700 | 89.7666 | 160.7925 | 222.1151 |
| | (b) | 264.9015 | 88.1515 | 156.2893 | 215.1076 |
| | (c) | 258.2796 | 84.6746 | 151.6002 | 209.3600 |
| | (d) | 247.0603 | 82.3208 | 145.8435 | 200.6645 |
| $^{110}$Pd | (a) | 424.6601 | 140.3359 | 249.6835 | 344.3187 |
| | (b) | 379.9404 | 127.4915 | 224.6563 | 308.6907 |
| | (c) | 407.2163 | 134.6824 | 239.4733 | 330.1888 |
| | (d) | 390.3539 | 130.6314 | 230.5392 | 316.9996 |
| $^{128}$Te | (a) | 190.5325 | 62.4373 | 111.5143 | 154.1796 |
| | (b) | 231.8024 | 77.4559 | 136.7936 | 188.1893 |
| | (c) | 220.7156 | 73.5158 | 130.0810 | 179.0960 |
| | (d) | 235.4814 | 78.6367 | 138.9052 | 191.1366 |
| $^{130}$Te | (a) | 236.0701 | 81.5493 | 141.3447 | 192.7610 |
| | (b) | 231.5921 | 79.3844 | 138.1901 | 188.8492 |
| | (c) | 233.0024 | 80.4020 | 139.4400 | 190.2194 |
| | (d) | 230.5282 | 78.9888 | 137.5307 | 187.9675 |
| $^{150}$Nd | (a) | 163.8037 | 55.8968 | 97.8169 | 133.6912 |
| | (b) | 130.1364 | 43.8840 | 77.3178 | 105.9993 |
| | (c) | 160.2720 | 54.6713 | 95.6942 | 130.8005 |
| | (d) | 131.9781 | 44.6741 | 78.5433 | 107.5715 |

defined by

$$\overline{M}_N^{(0\nu)} = \frac{\sum_{i=1}^{k} M_N^{(0\nu)}(i)}{N}, \tag{17}$$

and

$$\Delta\overline{M}_N^{(0\nu)} = \frac{1}{\sqrt{N-1}}\left[\sum_{i=1}^{N}\left(\overline{M}_N^{(0\nu)} - M_N^{(0\nu)}(i)\right)^2\right]^{1/2}. \tag{18}$$

## III. RESULTS AND DISCUSSIONS

The model space, SPEs, parameters of the $PQQ$ type of effective two-body interactions, and the method to fix them have already been given in Refs. [22,35,36]. It turns out that with $PQQ1$ and $PQQ2$ parametrizations, the experimental excitation energies of the $2^+$ state $E_{2^+}$ [47] can be reproduced within about 2% accuracy. The electromagnetic properties, namely reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, deformation parameters $\beta_2$, static quadrupole moments $Q(2^+)$, and gyromagnetic factors $g(2^+)$ are in overall agreement with the experimental data [48,49].

### A. Short-range correlations and radial evolutions of NTMEs

In the approximation of the finite size of nucleons with the dipole form factor ($F$) and finite size plus SRC ($F + S$), the theoretically calculated 12 NTMEs $M_N^{(0\nu)}$ using the four sets of HFB wave functions generated with $PQQ1$, $PQQHH1$, $PQQ2$, and $PQQHH2$ parametrizations of the effective two-body interaction and three different parametrizations of the Jastrow type of SRC for $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes are presented in Table I.

To analyze the role of different components of NTME $M_N^{(0\nu)}$, the decomposition of the latter into Fermi, different terms of Gamow-Teller, and tensor matrix elements of $^{100}$Mo are presented in Table II for $PQQ1$ parametrization. From the inspection of Table II, the following observations emerge.

(i) The contribution of conventional Fermi matrix elements $M_{Fh} = M_{F-VV}$ is about 20% to the total matrix element.

(ii) The Gamow-Teller matrix element is noticeably modified by the inclusion of the pseudoscalar and weak magnetism terms in the hadronic currents. While $M_{GT-PP}$ increases the absolute value of $M_{GT-AA}$, $M_{GT-AP}$ has a significant contribution with the opposite sign in all cases. The term $M_{GT-MM}$ is smaller than others, and the introduction of short-range correlations changes its sign.

(iii) The tensor matrix elements have a very small contribution, smaller than 2%, to the total transition matrix elements.

(iv) The inclusion of short-range correlations changes the nuclear matrix elements significantly, whose effects are large for the Gamow-Teller and Fermi matrix elements but small in the case of the tensor ones.

(v) The Miller-Spencer parametrization of the short-range correlations, SRC1, cancels out a large part of the radial function $H_N$, as shown in Fig. 1. The same cancellation reduces the calculated matrix element to about one-third of its original value. The other two parametrizations of the short-range correlations, namely SRC2 and SRC3, have a sizable effect, which is in all cases much smaller than SRC1.

With respect to the point nucleon case. the change in $M_N^{(0\nu)}$ is about 30%–34% because of the FNS. With the inclusion of effects from FNS and SRC, the NTMEs change by about 75%–79%, 58%–62%, and 43%–47% for $F$ + SRC1, $F$ + SRC2, and $F$ + SRC3, respectively. It is noteworthy that the SRC3 has practically negligible effect on the finite-size case. Furthermore, the maximum variation in $M_N^{(0\nu)}$ from $PQQHH1$, $PQQ2$, and $PQQHH2$ parametrization with respect to PQQ1 interaction is about 24%, 18%, and 26%, respectively.

In the QRPA [26,27], ISM [16], and PHFB [28,37], the radial evolution of $M^{(0\nu)}$ from the exchange of light Majorana neutrino was already studied. In both QRPA and

TABLE II. Decomposition of NTMEs for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{100}$Mo including finite size effect ($F$) and SRC ($F + S$) for the $PQQ1$ parametrization.

| NTMEs | $F$ | $F+S$ | | |
|---|---|---|---|---|
| | | SRC1 | SRC2 | SRC3 |
| $M_F$ | 68.6223 | 35.8191 | 54.0101 | 64.2516 |
| $M_{GT-AA}$ | −370.5960 | −144.5650 | −242.3340 | −316.3250 |
| $M_{GT-AP}$ | 174.4640 | 43.3631 | 93.9737 | 137.5100 |
| $M_{GT-PP}$ | −66.3082 | −8.3767 | −28.5936 | −48.0727 |
| $M_{GT-MM}$ | −41.7693 | 16.3949 | 7.6213 | −13.3421 |
| $M_{GT}$ | −304.2095 | −93.1837 | −169.3326 | −240.2298 |
| $M_{T-AP}$ | 9.4369 | 9.2393 | 10.0332 | 10.0610 |
| $M_{T-PP}$ | −3.6622 | −3.5528 | −3.9226 | −3.9386 |
| $M_{T-MM}$ | 1.2567 | 1.1163 | 1.3438 | 1.3722 |
| $M_T$ | 7.0314 | 6.8028 | 7.4544 | 7.4945 |
| $\left\|M_N^{(0\nu)}\right\|$ | 365.8004 | 122.2000 | 215.8882 | 296.9869 |

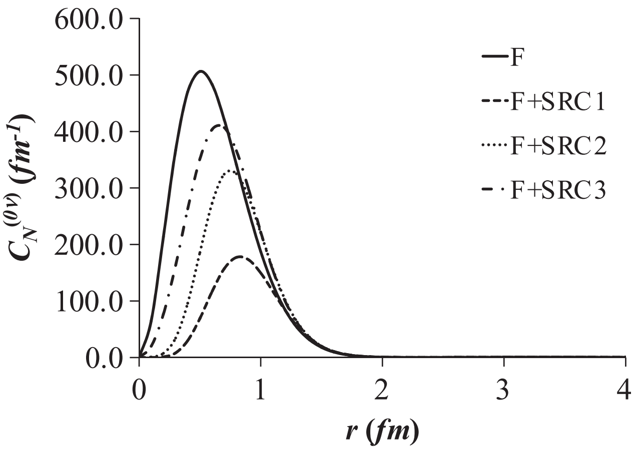


FIG. 2. Radial dependence of $C_N^{(0\nu)}(r)$ for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{100}$Mo isotope.

ISM calculations, it was established that the contributions of decaying pairs coupled to $J = 0$ and $J > 0$ almost cancel beyond $r \approx 3$ fm and the magnitude of $C^{(0\nu)}$ for all nuclei undergoing $(\beta^-\beta^-)_{0\nu}$ decay have their maximum at about the internucleon distance $r \approx 1$ fm. These observations were also made in the PHFB model [28,37]. Similarly, the radial evolution of $M_N^{(0\nu)}$ can be studied by defining

$$M_N^{(0\nu)} = \int C_N^{(0\nu)}(r)\, dr. \tag{19}$$

The radial evolution of $M_N^{(0\nu)}$ was studied for four cases, namely $F$, $F$ + SRC1, $F$ + SRC2, and $F$ + SRC3. To make the effects of finite size and SRC more transparent, we plot them for $^{100}$Mo in Fig. 2. In the case of finite-sized nucleons, the $C_N^{(0\nu)}$ are peaked at $r \approx 0.5$ fm and with the addition of SRC1 and SRC2, the peak shifts to about 0.8 fm. However, the position of peak is shifted to 0.7 fm for SRC3. In Fig. 3, we plot the radial dependence of $C_N^{(0\nu)}$ for six nuclei, namely $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd and the same observations remain valid. In addition, the same features in the radial distribution of $C_N^{(0\nu)}$ are noticed in the cases of $PQQ2$, $PQQHH1$, and $PQQHH2$ parametrizations.

### B. Uncertainties in NTMEs

The uncertainties associated with the NTMEs $M_N^{(0\nu)}$ for $(\beta^-\beta^-)_{0\nu}$ decay are estimated by preforming a statistical analysis by using Eqs. (17) and (18). In Table I, sets of 12 NTMEs $M_N^{(0\nu)}$ of $^{94,96}$Zr, $^{98,100}$Mo, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes are displayed, which are employed to calculate the average values $\overline{M}_N^{(0\nu)}$ as well as uncertainties $\Delta\overline{M}_N^{(0\nu)}$ tabulated in Table III for the bare axial vector coupling constant $g_A = 1.254$ and quenched value of $g_A = 1.0$.

It turns out that in all cases, the uncertainties $\Delta\overline{M}^{(0\nu)}$ are about 35% for $g_A = 1.254$ and $g_A = 1.0$. Furthermore, we estimate the uncertainties for eight NTMEs $\overline{M}_N^{(0\nu)}$ calculated using the SRC2, and SRC3 parametrizations and the uncertainties in NTMEs reduce to about 16%–20% with

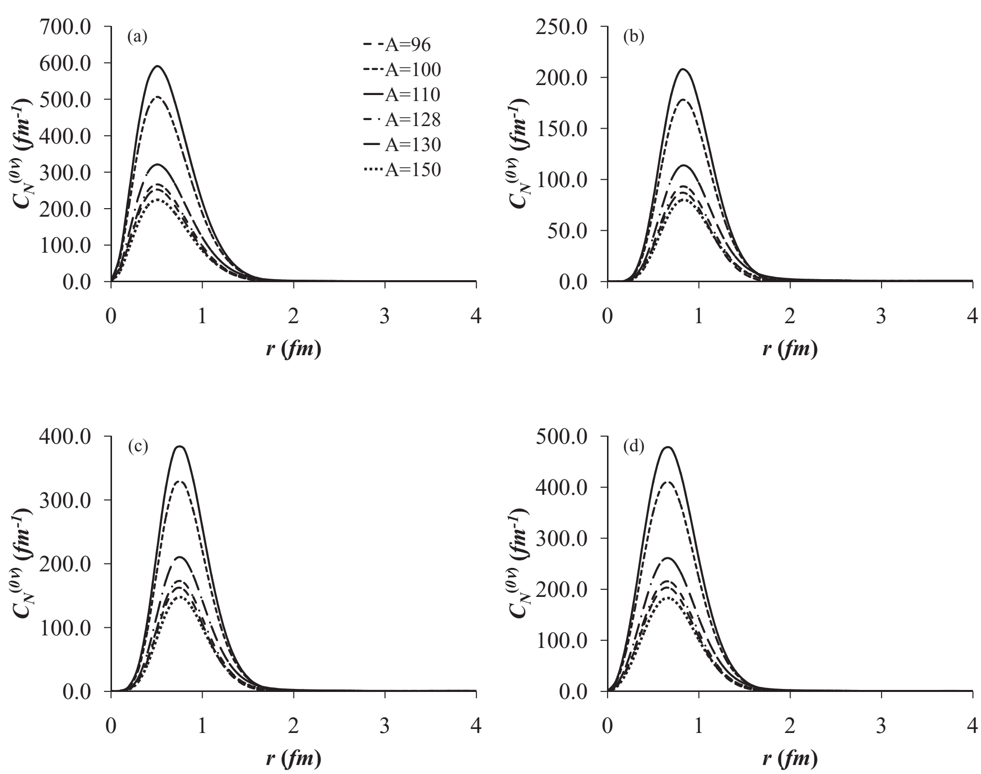


FIG. 3. Radial dependence of $C_N^{(0\nu)}(r)$ for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{96}$Zr, $^{100}$Mo, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes. In this figure, (a), (b), (c), and (d) correspond to $F$, $F$ + SRC1, $F$ + SRC2, and $F$ + SRC3, respectively.

TABLE III. Average NTMEs $\overline{M}_N^{(0\nu)}$ and uncertainties $\Delta\overline{M}_N^{(0\nu)}$ for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes. Both bare and quenched values of $g_A$ are considered. Case I and Case II denote calculations with and without SRC1, respectively.

| $\beta^-\beta^-$ emitters | $g_A$ | Case I | | Case II | |
|---|---|---|---|---|---|
| | | $\overline{M}_N^{(0\nu)}$ | $\Delta\overline{M}_N^{(0\nu)}$ | $\overline{M}_N^{(0\nu)}$ | $\Delta\overline{M}_N^{(0\nu)}$ |
| $^{94}$Zr | 1.254 | 126.2146 | 44.9489 | 152.8378 | 27.1912 |
| | 1.0 | 142.9381 | 49.1752 | 172.1620 | 29.3965 |
| $^{96}$Zr | 1.254 | 100.5313 | 36.8858 | 122.5048 | 21.9209 |
| | 1.0 | 114.4851 | 40.3246 | 138.6328 | 23.5263 |
| $^{98}$Mo | 1.254 | 202.5006 | 71.6345 | 245.3957 | 41.8882 |
| | 1.0 | 230.1520 | 78.3244 | 277.2795 | 44.9878 |
| $^{100}$Mo | 1.254 | 206.7533 | 73.0792 | 250.1870 | 43.7119 |
| | 1.0 | 235.0606 | 79.9885 | 282.7964 | 47.1334 |
| $^{104}$Ru | 1.254 | 150.5572 | 53.9389 | 182.7216 | 31.9382 |
| | 1.0 | 171.8075 | 59.0467 | 207.1750 | 34.3939 |
| $^{110}$Pd | 1.254 | 231.4743 | 82.4924 | 280.5688 | 49.1588 |
| | 1.0 | 263.4339 | 90.3033 | 317.3947 | 53.0150 |
| $^{128}$Te | 1.254 | 126.8285 | 46.3381 | 153.7370 | 29.4676 |
| | 1.0 | 143.9772 | 50.6942 | 173.5263 | 31.8554 |
| $^{130}$Te | 1.254 | 136.3856 | 46.9164 | 164.5378 | 27.2226 |
| | 1.0 | 154.3797 | 51.2511 | 185.2849 | 29.1907 |
| $^{150}$Nd | 1.254 | 85.5467 | 31.4473 | 103.4294 | 20.9802 |
| | 1.0 | 97.3640 | 34.5024 | 117.0160 | 22.8729 |

the exclusion of the Miller-Spencer type of parametrization. In Table IV, average NTMEs for case II along with NTMEs calculated in other models have been presented. It is noteworthy that in the models employed in Refs. [6,8,9], effects from higher order currents have not been included. We also extract lower limits on the effective mass of heavy Majorana neutrino $\langle M_N\rangle$ from the largest observed limits on half-lives $T_{1/2}^{0\nu}$ of $(\beta^-\beta^-)_{0\nu}$ decay. The extracted limits are $\langle M_N\rangle > 5.67^{+0.94}_{-0.94} \times 10^7$ GeV and $> 4.06^{+0.64}_{-0.64}\times 10^7$ GeV, from the limit on half-life $T_{1/2}^{0\nu} > 3.0 \times 10^{24}$ yr of $^{130}$Te [56] for $g_A = 1.254$ and $g_A = 1.0$, respectively.

## IV. CONCLUSIONS

We have employed the PHFB model, with four different parametrizations of pairing plus multipole effective two-body interaction, to generate sets of four HFB intrinsic wave functions, which reasonably reproduced the observed spectroscopic properties, namely the yrast spectra, reduced $B(E2{:}0^+ \to 2^+)$ transition probabilities, static quadrupole moments $Q(2^+)$, and $g$ factors $g(2^+)$ of participating nuclei in $(\beta^-\beta^-)_{2\nu}$ decay, as well as their $M_{2\nu}$ [35,36]. Considering three different parametrizations of the Jastrow type of SRC, sets of 12 NTMEs $M_N^{(0\nu)}$ for the study $(\beta^-\beta^-)_{0\nu}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes in the heavy Majorana neutrino mass mechanism have been calculated.

The study of effects because of finite size of nucleons and SRC reveal that in the case of heavy Majorana neutrino exchange, the NTMEs change by about 30%–34% from finite size of nucleons and the SRC1, SRC2, and SRC3 change them by 75%–79%, 58%–62%, and 43%–47%, respectively. Furthermore, it was noticed through the study of radial evolution of NTMEs that the FNS and SRC play a more crucial role in the heavy than in the light Majorana neutrino exchange mechanism.

TABLE IV. Average NTMEs $\overline{M}_N^{\prime(0\nu)} (= (g_A/1.254)^2\overline{M}_N^{(0\nu)})$ for the $(\beta^-\beta^-)_{0\nu}$ decay of $^{94,96}$Zr, $^{98,100}$Mo, $^{110}$Pd, $^{128,130}$Te, and $^{150}$Nd isotopes. Both bare and quenched values of $g_A$ are considered. The superscripts $a$ and $b$ denote the Argonne and CD-Bonn potentials.

| $\beta^-\beta^-$ emitters | $g_A$ | $\overline{M}_N^{\prime(0\nu)}$ | QRPA [6] | QRPA [8] | QRPA [9] | QRPA [10] | SRQRPA$^a$ [3] | SRQRPA$^b$ [3] | $T_{1/2}^{0\nu}$ (yr) | Ref. | $\langle m_N\rangle$ (GeV) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $^{94}$Zr | 1.254 | $152.84 \pm 27.19$ | | | | | | | $1.9 \times 10^{19}$ | [50] | $2.57^{+0.46}_{-0.46} \times 10^4$ |
| | 1.0 | $109.48 \pm 18.69$ | | | | | | | | | $1.84^{+0.31}_{-0.31} \times 10^4$ |
| $^{96}$Zr | 1.254 | $122.50 \pm 21.92$ | | | | 99.062 | | | $9.2 \times 10^{21}$ | [51] | $2.68^{+0.48}_{-0.48} \times 10^6$ |
| | 1.0 | $88.16 \pm 14.96$ | | | | | | | | | $1.93^{+0.33}_{-0.33} \times 10^6$ |
| $^{98}$Mo | 1.254 | $245.40 \pm 41.89$ | | | | | | | $1.0 \times 10^{14}$ | [52] | $9.70^{+1.70}_{-1.70}$ |
| | 1.0 | $176.33 \pm 28.61$ | | | | | | | | | $6.97^{+1.13}_{-1.13}$ |
| $^{100}$Mo | 1.254 | $250.19 \pm 43.71$ | 155.960 | 333.0 | 56.914 | 76.752 | 259.8 | 404.3 | $4.6 \times 10^{23}$ | [53] | $3.43^{+0.60}_{-0.60} \times 10^7$ |
| | 1.0 | $179.84 \pm 29.97$ | | | | | 191.8 | 310.5 | | | $2.47^{+0.41}_{-0.41} \times 10^7$ |
| $^{110}$Pd | 1.254 | $280.57 \pm 49.16$ | | | | | | | $6.0 \times 10^{17}$ | [54] | $2.43^{+0.43}_{-0.43} \times 10^4$ |
| | 1.0 | $201.84 \pm 33.71$ | | | | | | | | | $1.75^{+0.29}_{-0.29} \times 10^4$ |
| $^{128}$Te | 1.254 | $153.74 \pm 29.47$ | 122.669 | 303.0 | | 101.233 | | | $1.1 \times 10^{23}$ | [55] | $2.06^{+0.39}_{-0.39} \times 10^6$ |
| | 1.0 | $110.35 \pm 20.26$ | | | | | | | | | $1.48^{+0.27}_{-0.27} \times 10^6$ |
| $^{130}$Te | 1.254 | $164.54 \pm 27.22$ | 108.158 | 267.0 | | 92.661 | 239.7 | 384.5 | $3.0 \times 10^{24}$ | [56] | $5.67^{+0.94}_{-0.94} \times 10^7$ |
| | 1.0 | $117.83 \pm 18.56$ | | | | | 176.5 | 293.8 | | | $4.06^{+0.64}_{-0.64} \times 10^7$ |
| $^{150}$Nd | 1.254 | $103.43 \pm 20.98$ | 153.085 | 422.0 | | | | | $1.8 \times 10^{22}$ | [57] | $5.99^{+1.21}_{-1.21} \times 10^6$ |
| | 1.0 | $74.41 \pm 14.55$ | | | | | | | | | $4.31^{+0.84}_{-0.84} \times 10^6$ |

Finally, a statistical analysis was performed by employing the sets of 12 NTMEs $M_N^{(0\nu)}$ to estimate the uncertainties for $g_A = 1.254$ and $g_A = 1.0$. It turns out that the uncertainties are about 35% for all the considered nuclei. Exclusion of the Miller-Spencer parametrization of the Jastrow type of SRC, reduces the maximum uncertainties to a value smaller than 20%. The best extracted limit on the effective heavy Majorana neutrino mass $\langle M_N \rangle$ from the available limits on experimental half-lives $T_{1/2}^{0\nu}$ using average NTMEs $\overline{M}_N^{(0\nu)}$ calculated in the PHFB model is $>5.67^{+0.94}_{-0.94} \times 10^7$ GeV and $>4.06^{+0.64}_{-0.64} \times 10^7$ GeV for the $^{130}$Te isotope.


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